Tensor Networks, resonances, and bulk geometry in Many Body Localizations

Dynamics of Quantum Information (KITP)

Bryan Clark (bkclark@illinois.edu)
University of Illinois at Urbana Champaign
with Xiongjie Yu, Benjamin Villalonga, and David Pekker

Title on Schedule:

Tensor networks, resonances, bulk geometry and MBL

Title I sent:

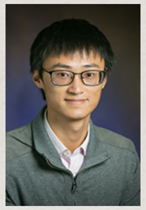
Tensor networks, resonances, and bulk geometry in MBL

What I'll actually talk about

Using RG tensor networks for bulk geometry in MBL

+

Resonances in MBL



Xiongjie Yu

David Pekker

Benjamin Correa

Not this talk (*but happy to chat about them later*)

Beyond MBL: Finding mixed log and area law states in a spin-disordered Hubbard model.

Automatically finding parent Hamiltonians from wave-functions.

Combining deep neural networks with back-flow to generate better wave-functions.

Real-space tensor network RG.

First half of talk:

Take a MBL Hamiltonian

MBL Hamiltonian

Generate the Wegner-Wilson Flow

Turn the unitary into a tensor network

Unitary

MBL Hamiltonian

Unitary†

Call it a geometric bulk.

In the bulk,

- can you see the l-bits?
- how fast do operators spread?
- see the level spacing?

Compare/contrast MBL Bulk with ergodic bulk and critical bulk.

Second half of talk:

As we adiabatically tune from deep in the MBL phase to the ergodic phase, are there resonances?

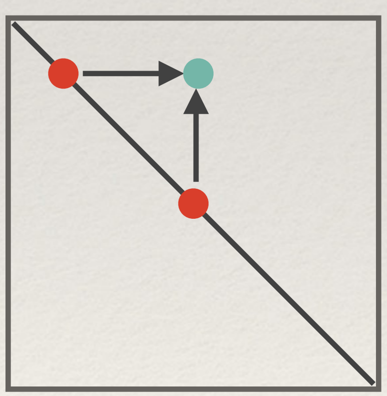
Can we understand how we build up entanglement from these resonances?

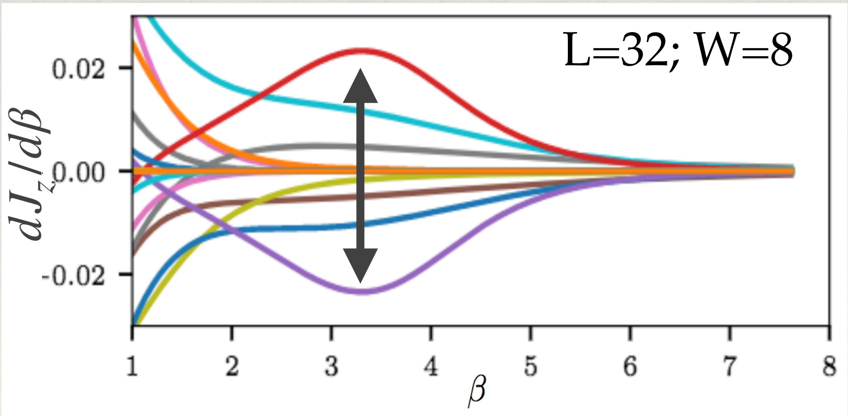
Wegner-Wilson Flow

The Wegner-Wilson Flow is a unitary RG process which diagonalizes a Hamiltonian.

$$H(t) = H_D(t) + H_{OD}(t)$$
$$\eta(t) = [H_D(t), H_{OD}(t)]$$
$$U(\Delta t) = \exp\left(i\Delta t \eta(t)\right)$$
$$H(t+\Delta t) = U(\Delta t) H(t) U^\dagger(\Delta t)$$
$$U(t+\Delta t) = U(\Delta t) U(t)$$

Why RG: It disentangles the largest diagonal energy scales connected by a `non-zero' off-diagonal element first.

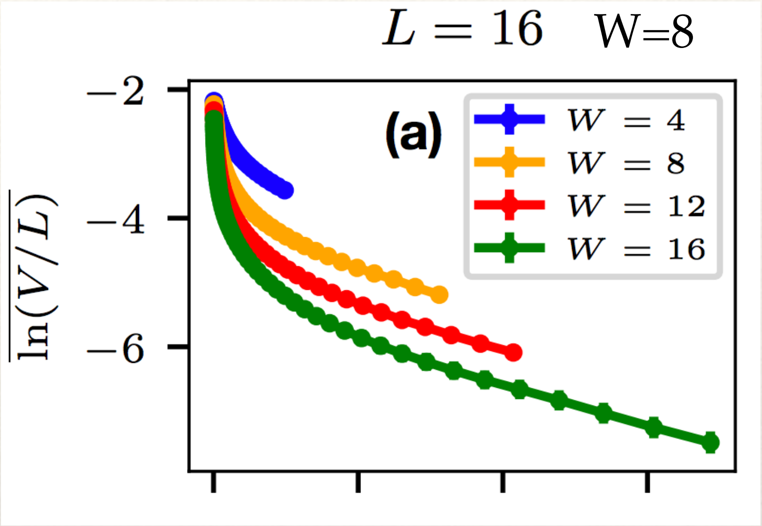




$$H = \sum_i J_i \sigma_z^i + [\ldots]$$

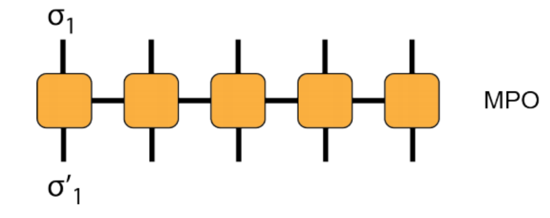
The variance $V(\beta) \equiv (1/N) \sum_{i \neq j} H_{ij}(\beta)$ decreases monotonically with β

as $V(\beta) = \exp[-\beta(E_i - E_j)^2]$

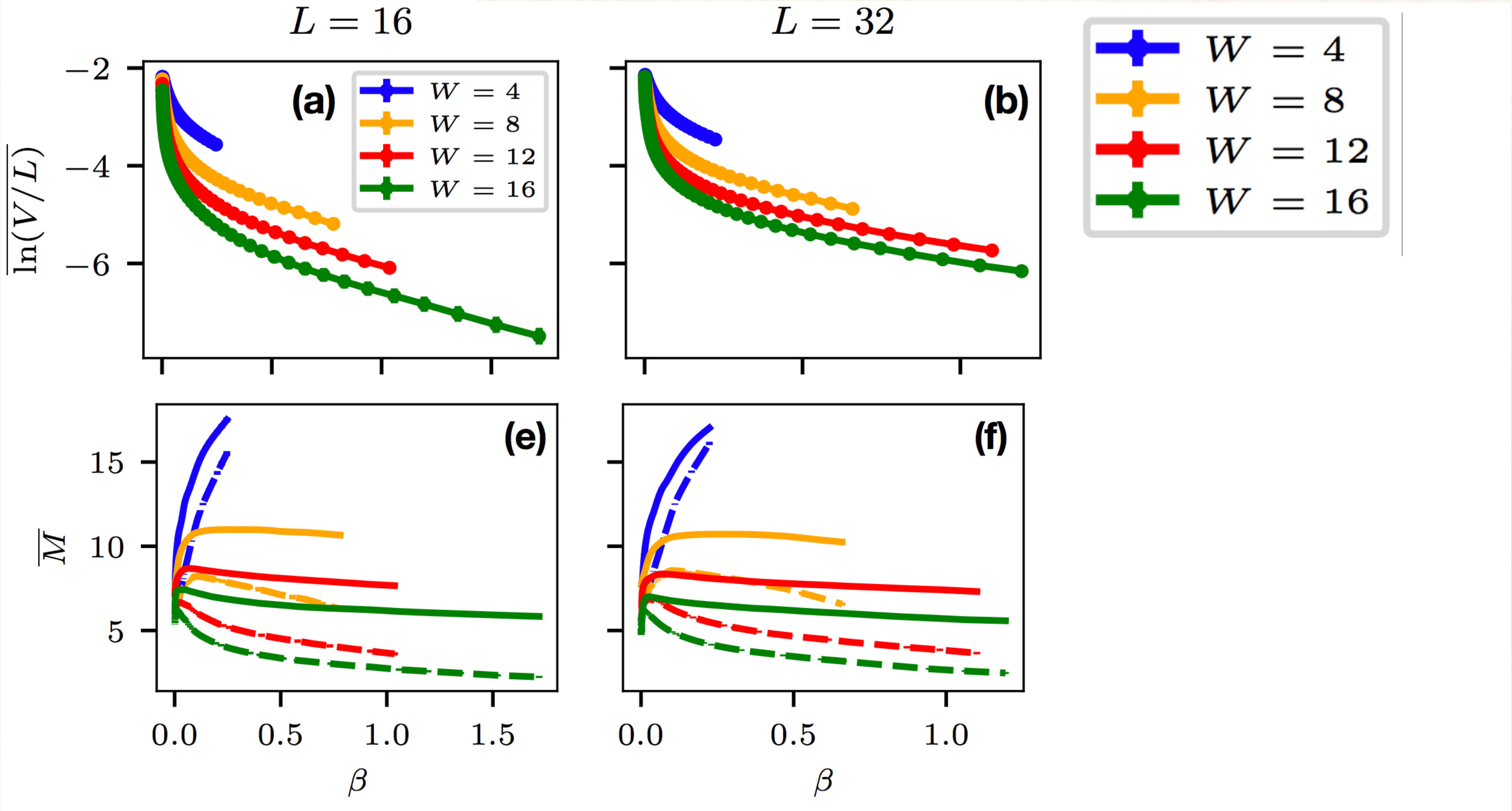


Turn the unitary into a tensor network

Take Hamiltonian as MPO

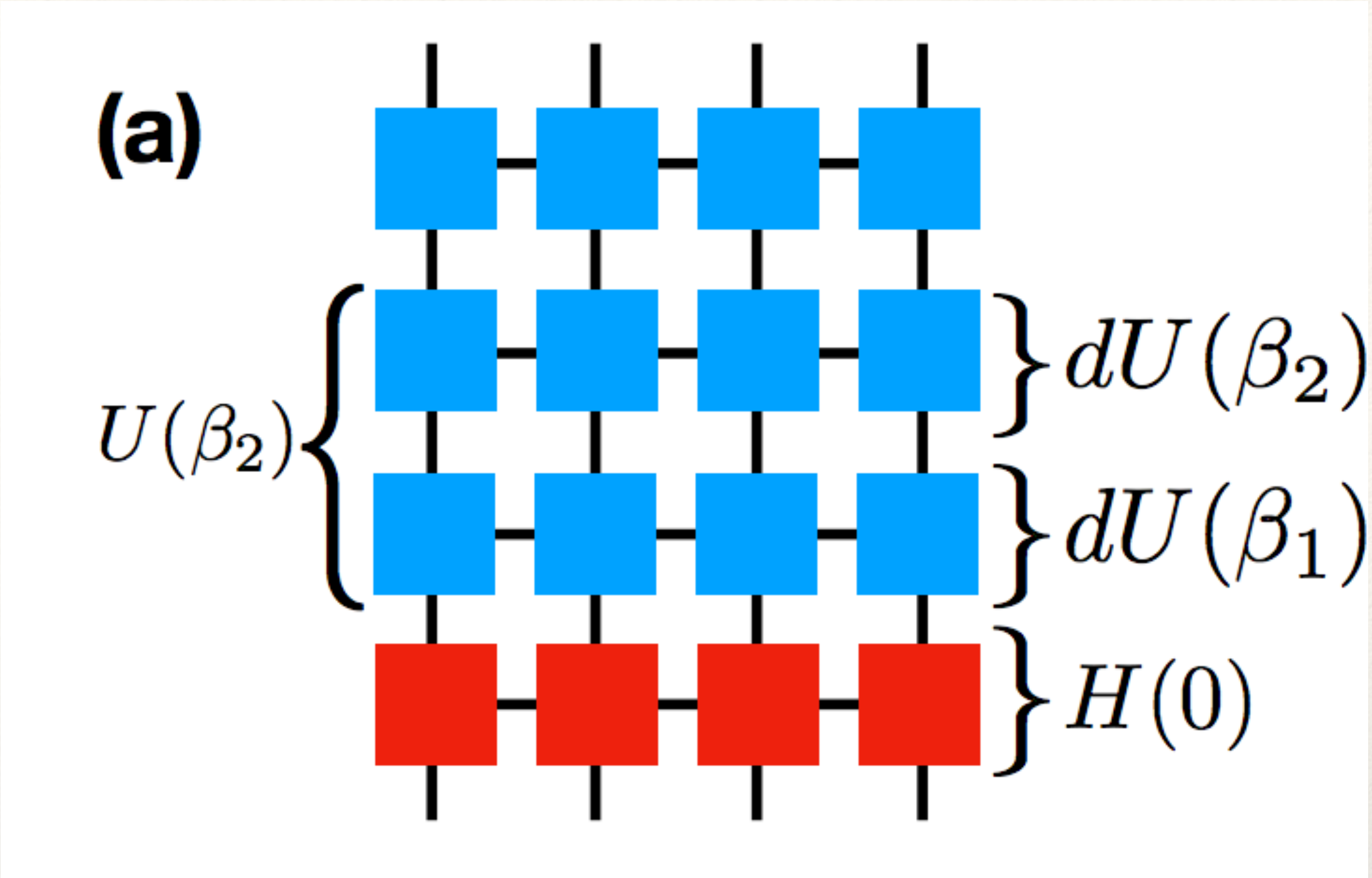


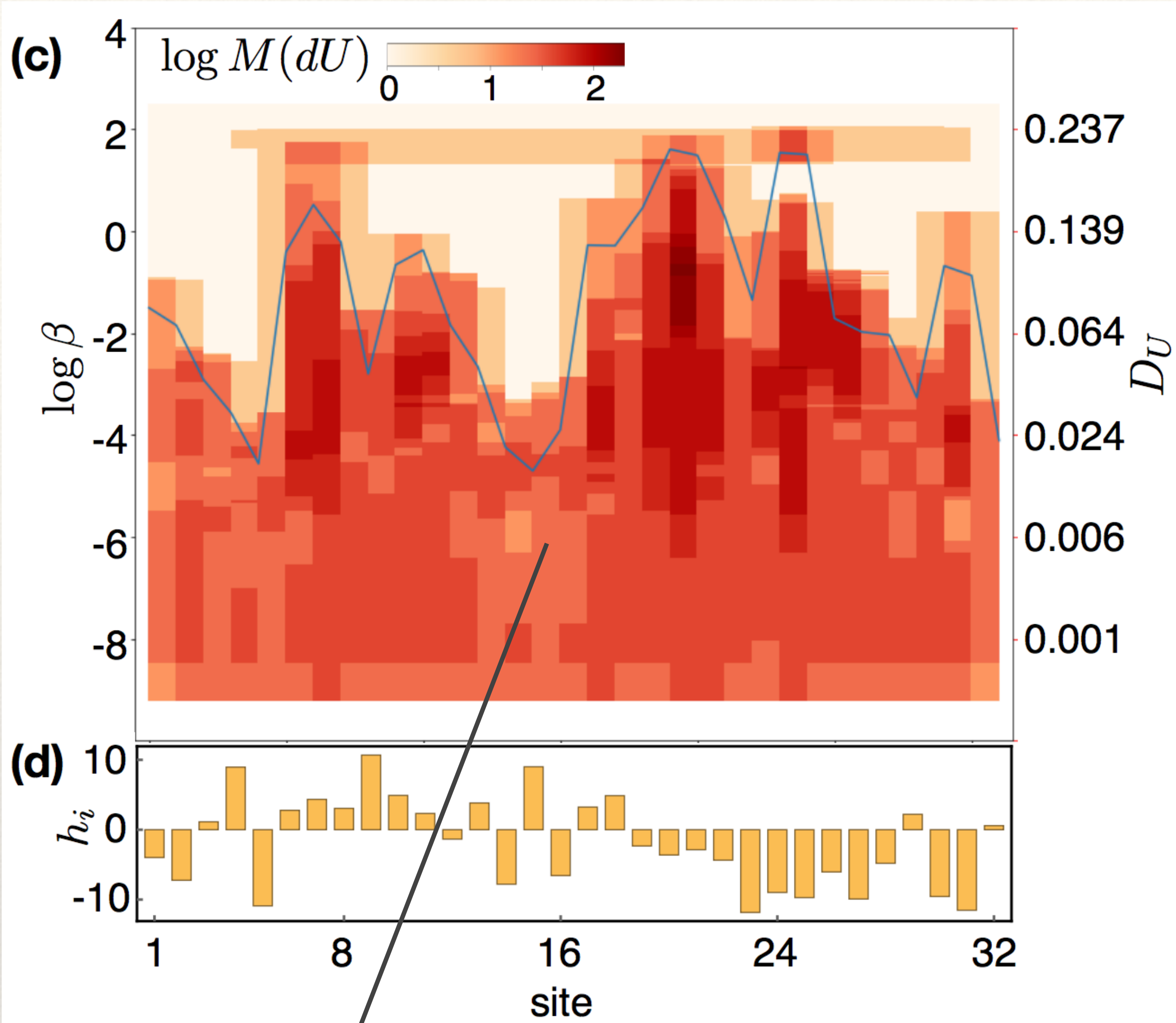
$$
\begin{aligned}
&H(\beta+\Delta\beta)\\
&= H + \Delta\beta[\eta, H] + \frac{\Delta\beta^2}{2!}[\eta,[\eta,H]] + \frac{\Delta\beta^3}{3!}[\eta,[\eta,[\eta,H]]] + \cdots\\
&= H + \Delta\beta\left[\eta,\ H + \frac{\Delta\beta}{2}\left[\eta,\ H + \frac{\Delta\beta}{3}\left[\eta,\ H + \cdots\right]\right]\right]
\end{aligned}
$$



and it works in the MBL phase

Gives l-bits, eigenstates, etc.


(a)
$U(\beta_2)$
$dU(\beta_2)$
$dU(\beta_1)$
$H(0)$



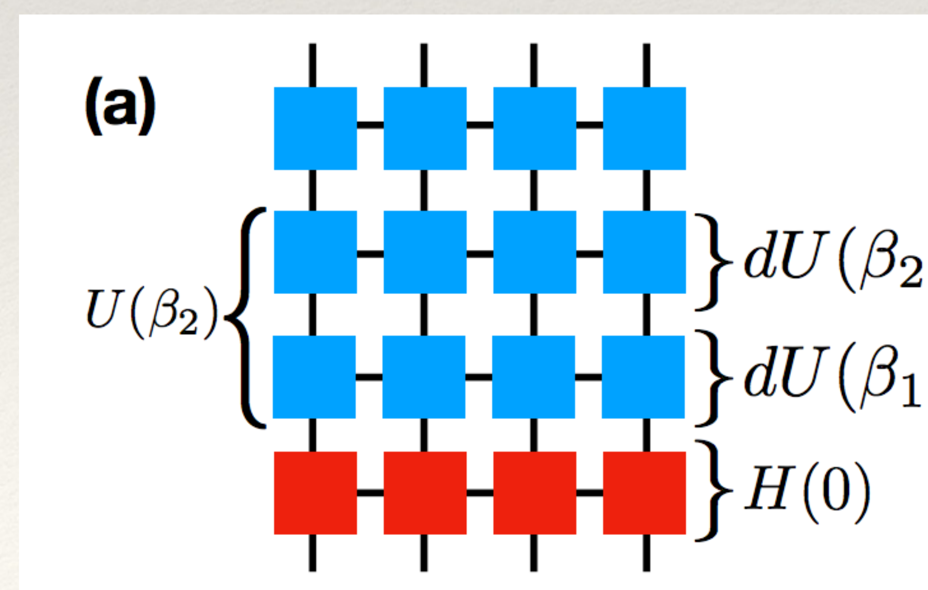
$$D_U(\beta) = \int_0^\beta \sqrt{\frac{Tr(\eta(\tau)\eta^\dagger(\tau))}{\dim(H)L}}d\tau = \int_0^\beta \sqrt{-\frac{1}{2L}\frac{dV(\tau)}{d\tau}}d\tau$$

(Generalized) version of distance used in cMERA [Ryu, et. al]

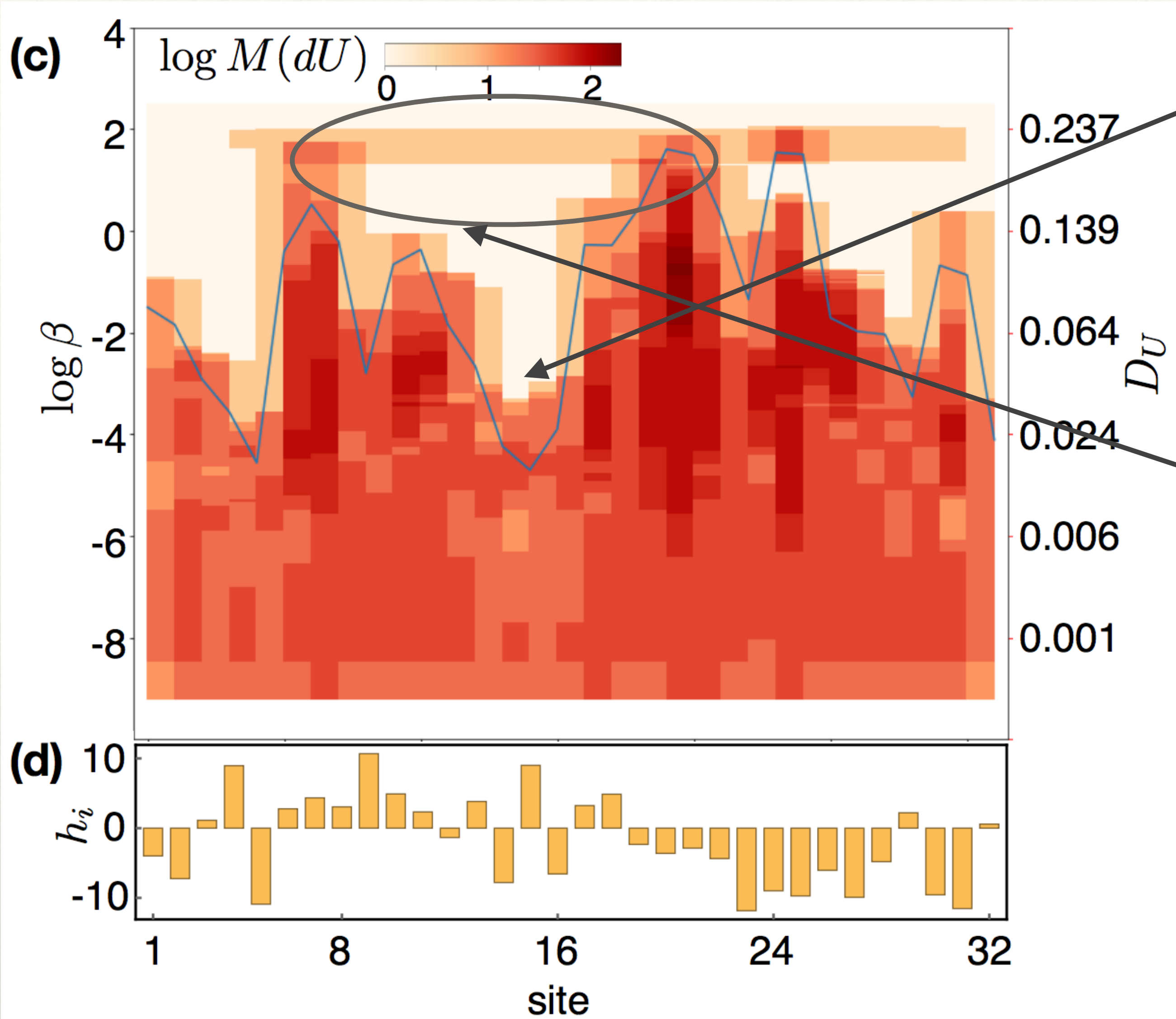
Here it is connected to variance.

RT analogue - geodesics related to entanglement.

Log(Bond Dimension)



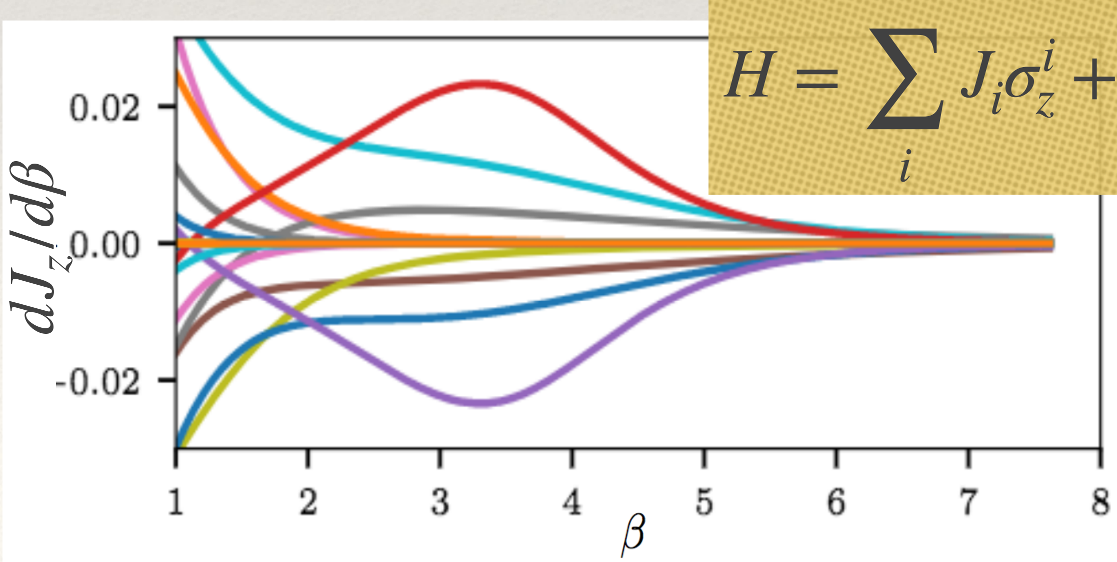
In the bulk, can you see the l-bits?



Here these l-bit's are diagonal (i.e. popped out of the system)

They have stopped rotating.

This bar is essentially (up to control-bits) connecting distance l-bits which disentangle at the lowest energy scales.

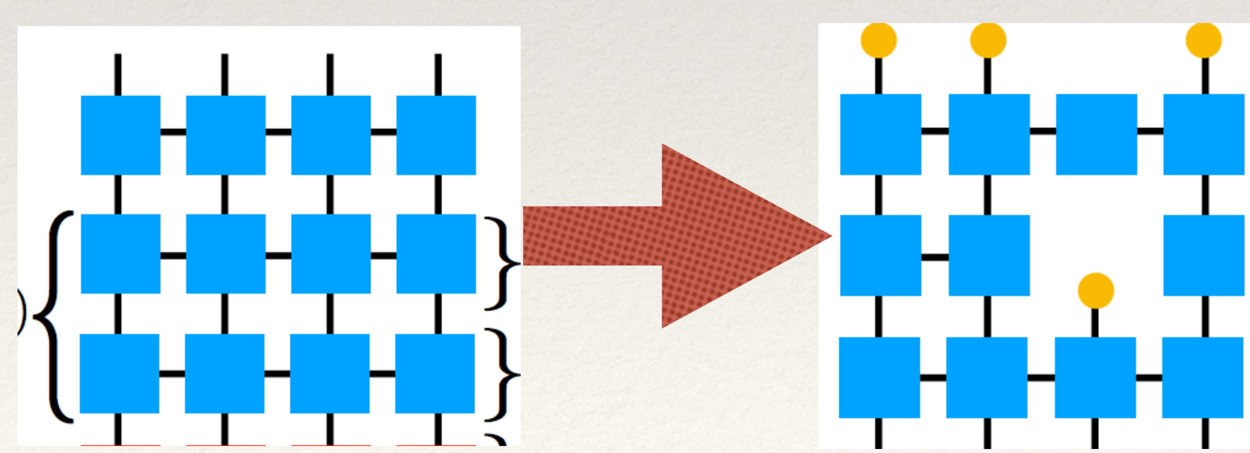


$$H = \sum_i J_i \sigma_z^i + [\ldots]$$

As $E \sim 1/\beta^2$ gives an energy scale for the l-bits

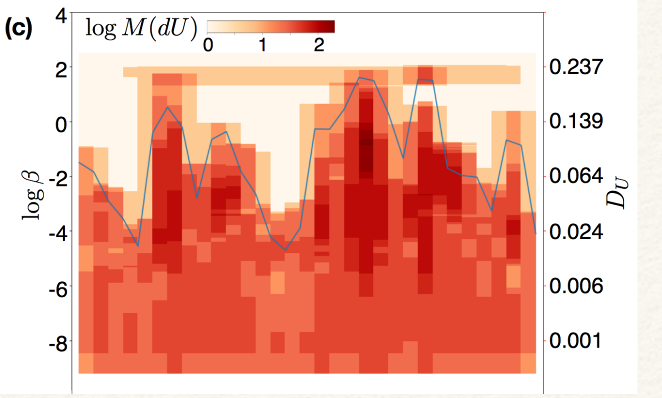
If I fix the l-bit eigenvalues, I get a state and something MERA-like.

Unitary **State**



A real space picture…

Unitary distance

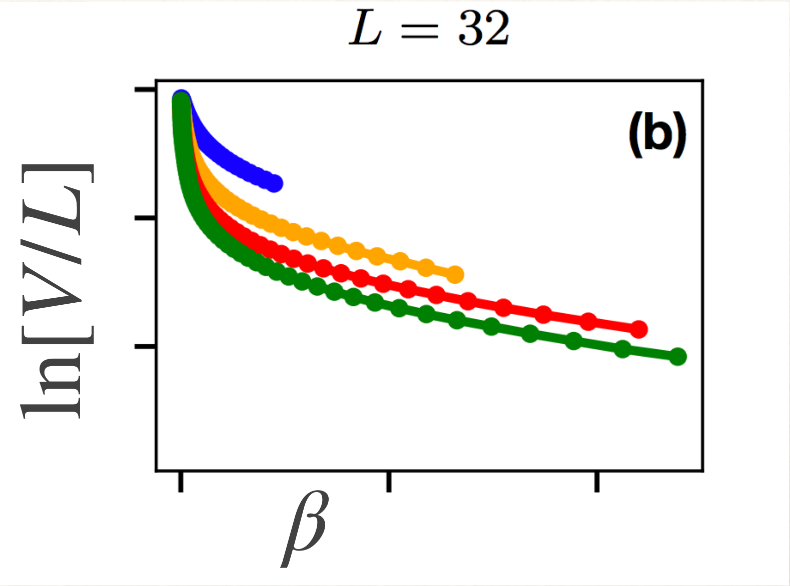


To explore rare regions, we define a distance measure in the RG time…

$$D_U(\beta) = \int_0^\beta \sqrt{\frac{Tr(\eta(\tau)\eta^\dagger(\tau))}{\dim(H)L}}d\tau = \int_0^\beta \sqrt{-\frac{1}{2L}\frac{dV(\tau)}{d\tau}}d\tau$$

where $U(\Delta t) = \exp[i\eta(t)\Delta t]$

(Generalized) version of distance used in cMERA.
Here it is connected to variance.

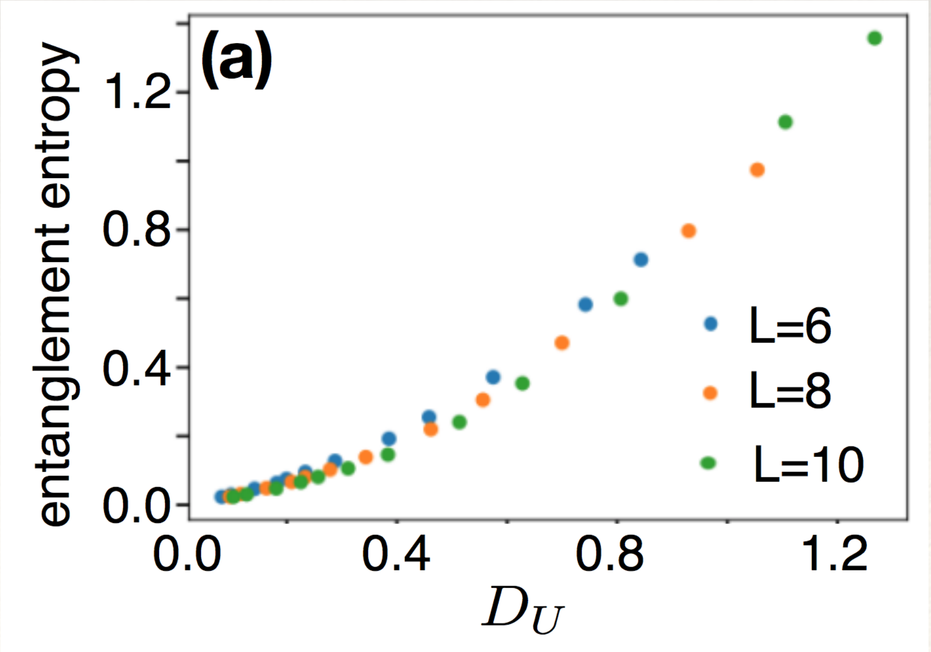


There is a RT theorem analogue.

Most the geodesics are through the ceiling

$$\langle S\rangle \propto D_u^2$$

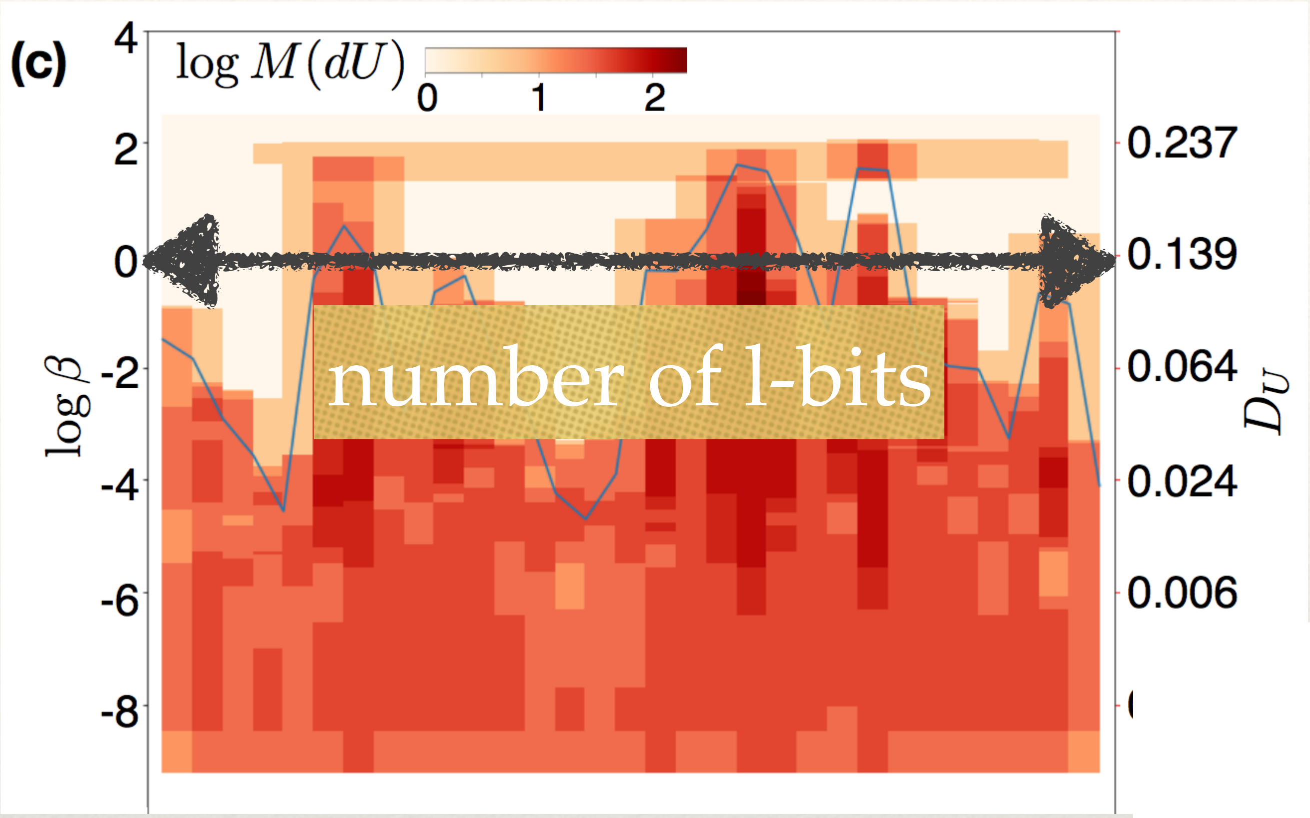
Average over all eigenstates.



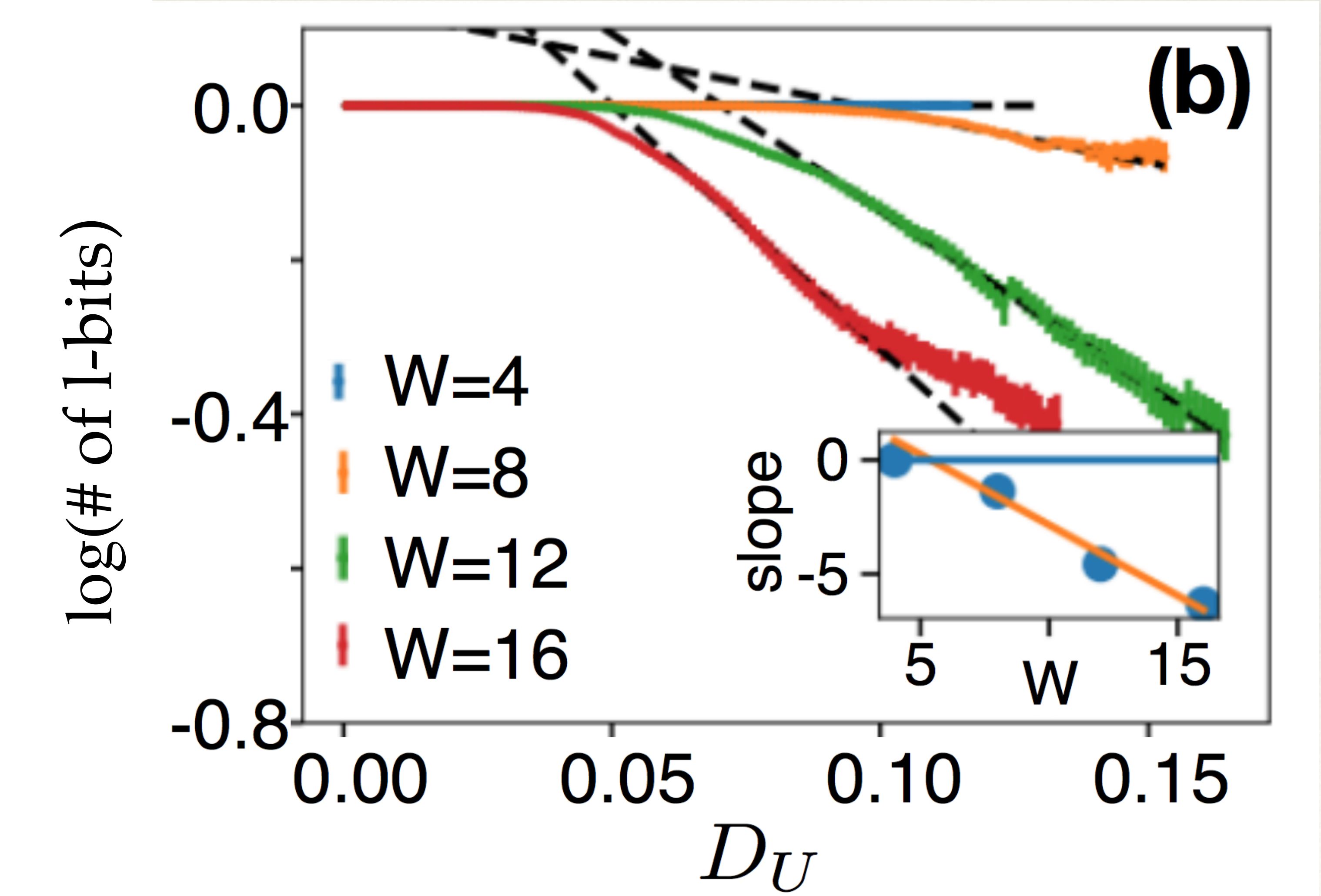
In the bulk, can you see the l-bits?

At what rate do the l-bits pop out of MBL bulk?

Number of l-bits remaining decay exponentially



Evidence of rare regions at all scale!



In the bulk, can you see the l-bits?

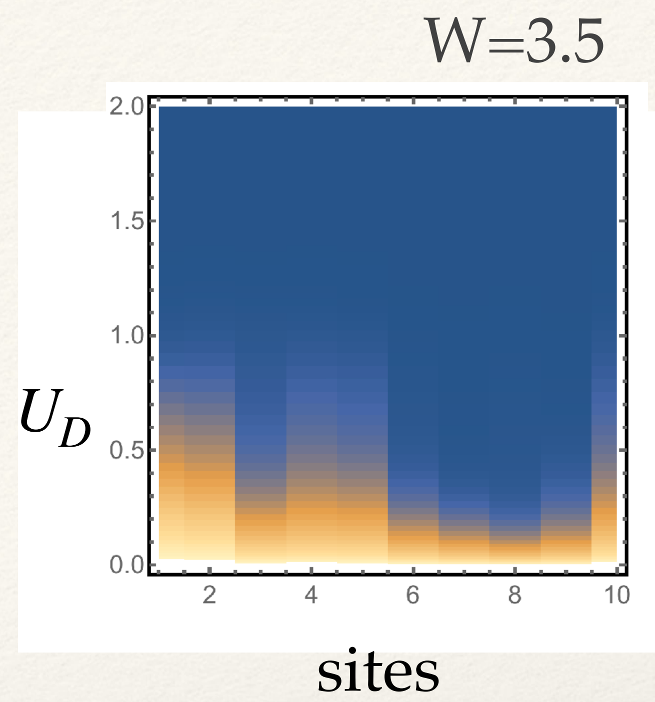
At what rate do the l-bits pop out of …. ergodic bulk?

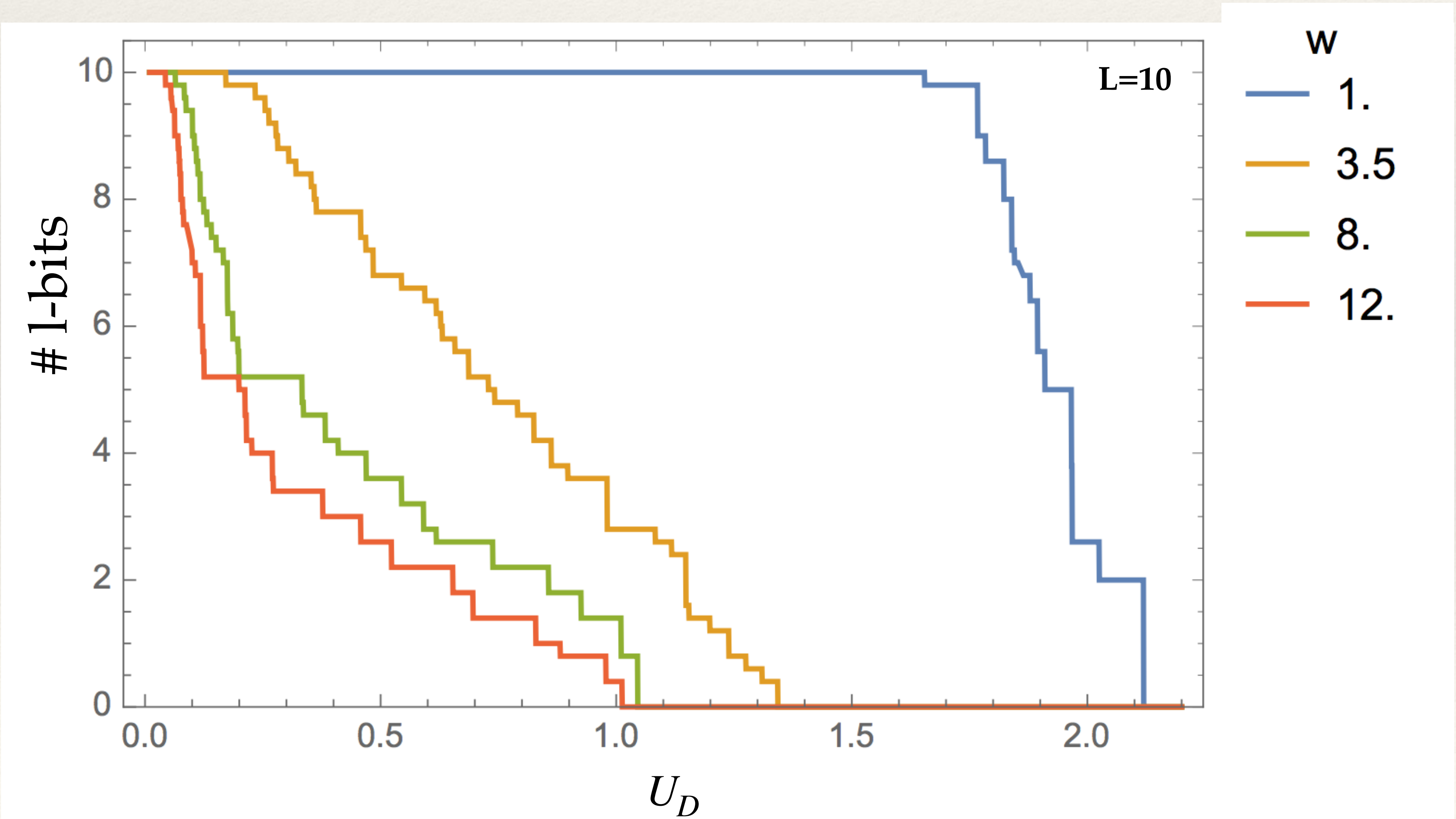
At what rate do the l-bits pop out of …. critical bulk?

Ergodic: All the `l-bits' pop out at the end.

Transition: `l-bits' pop uniformly.

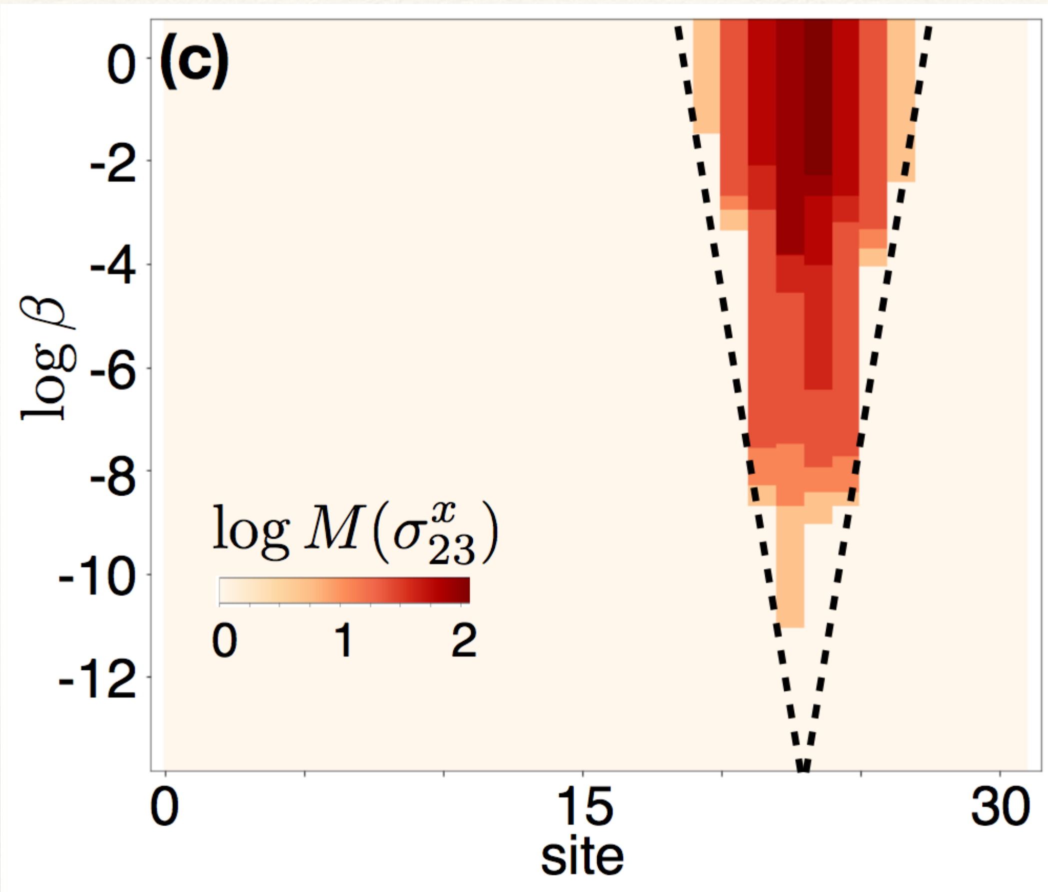
MBL: `l-bits' pop exponentially quickly.



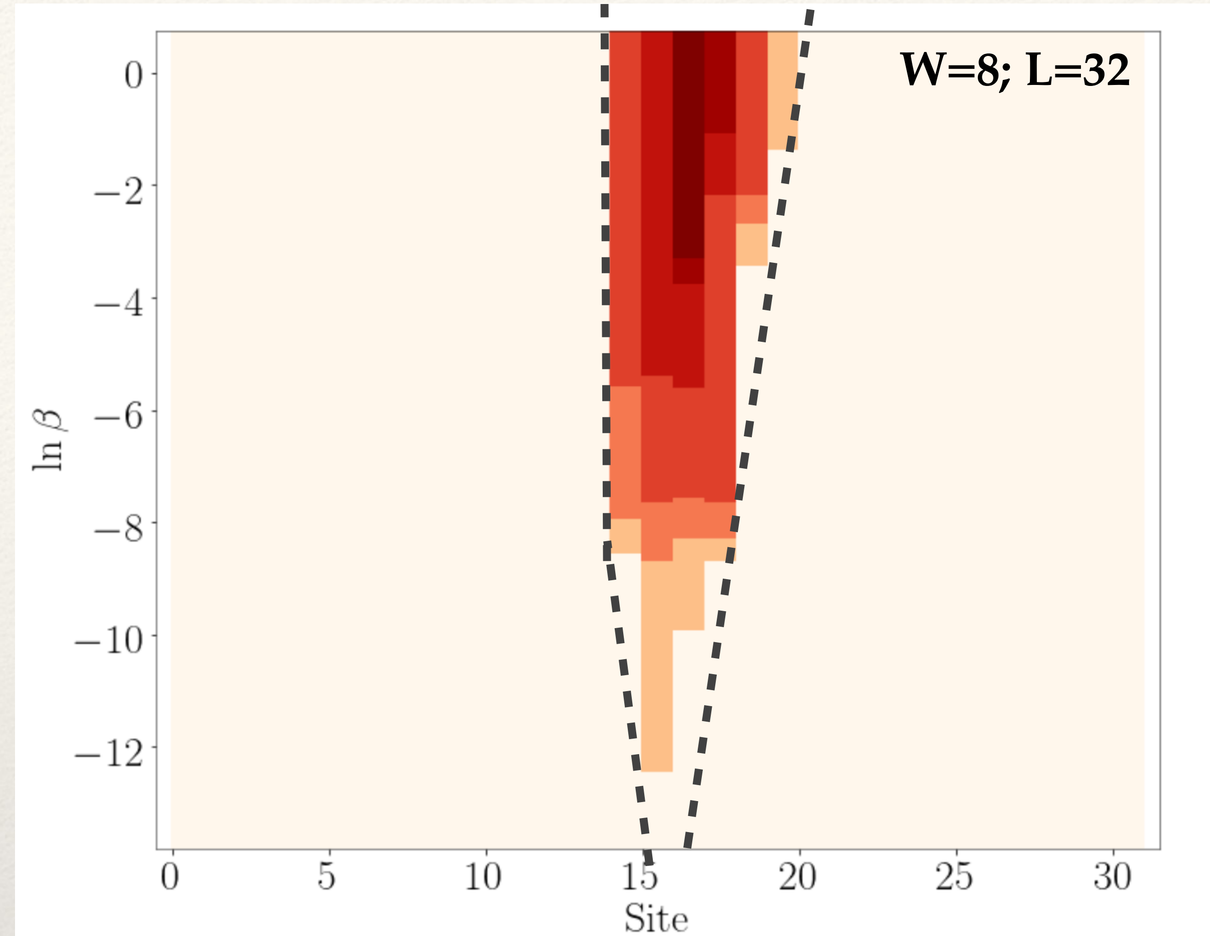


In the MBL bulk, how fast do operators spread?

In the MBL phase



Operator light cone spread as $\log(\beta)$

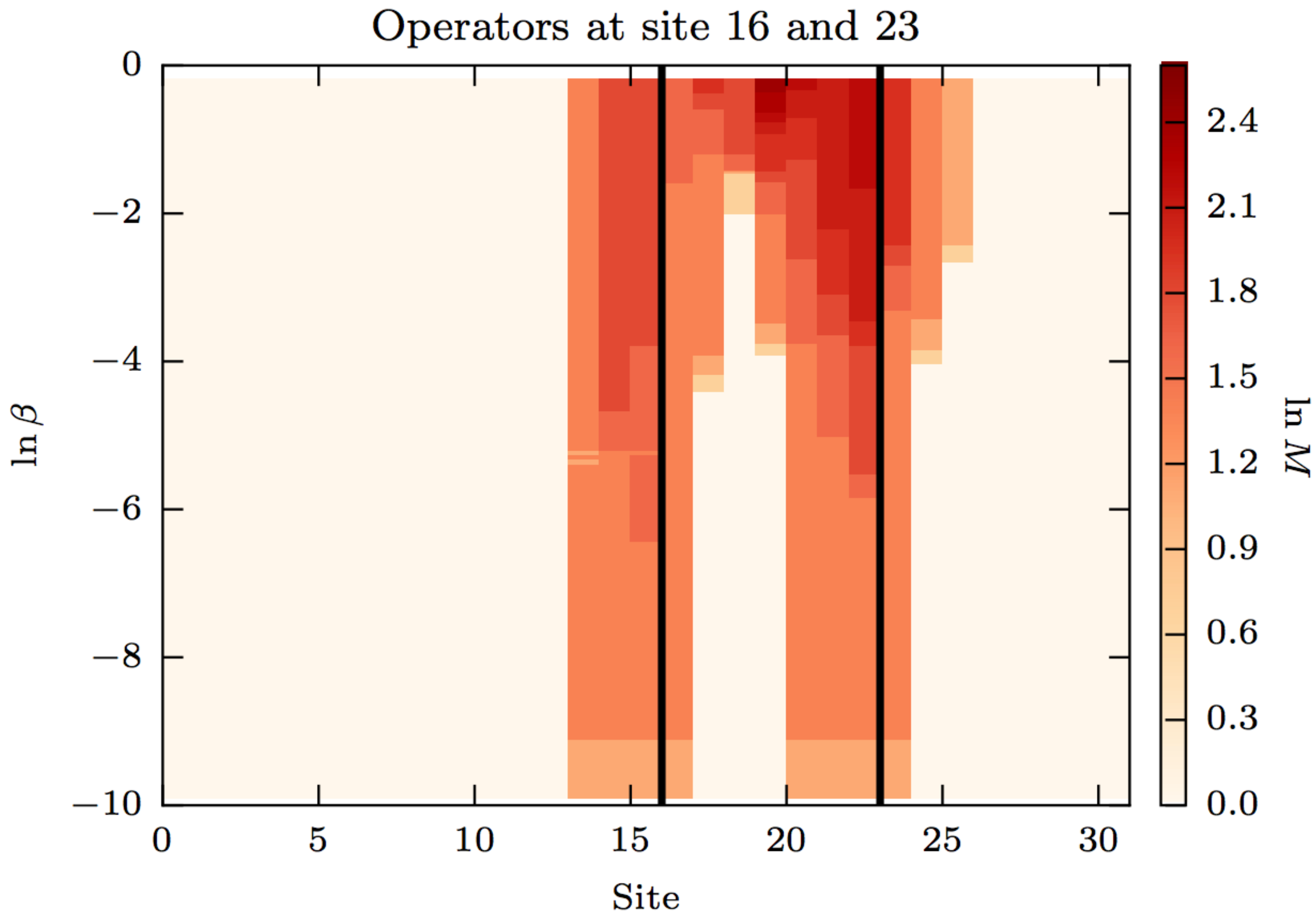


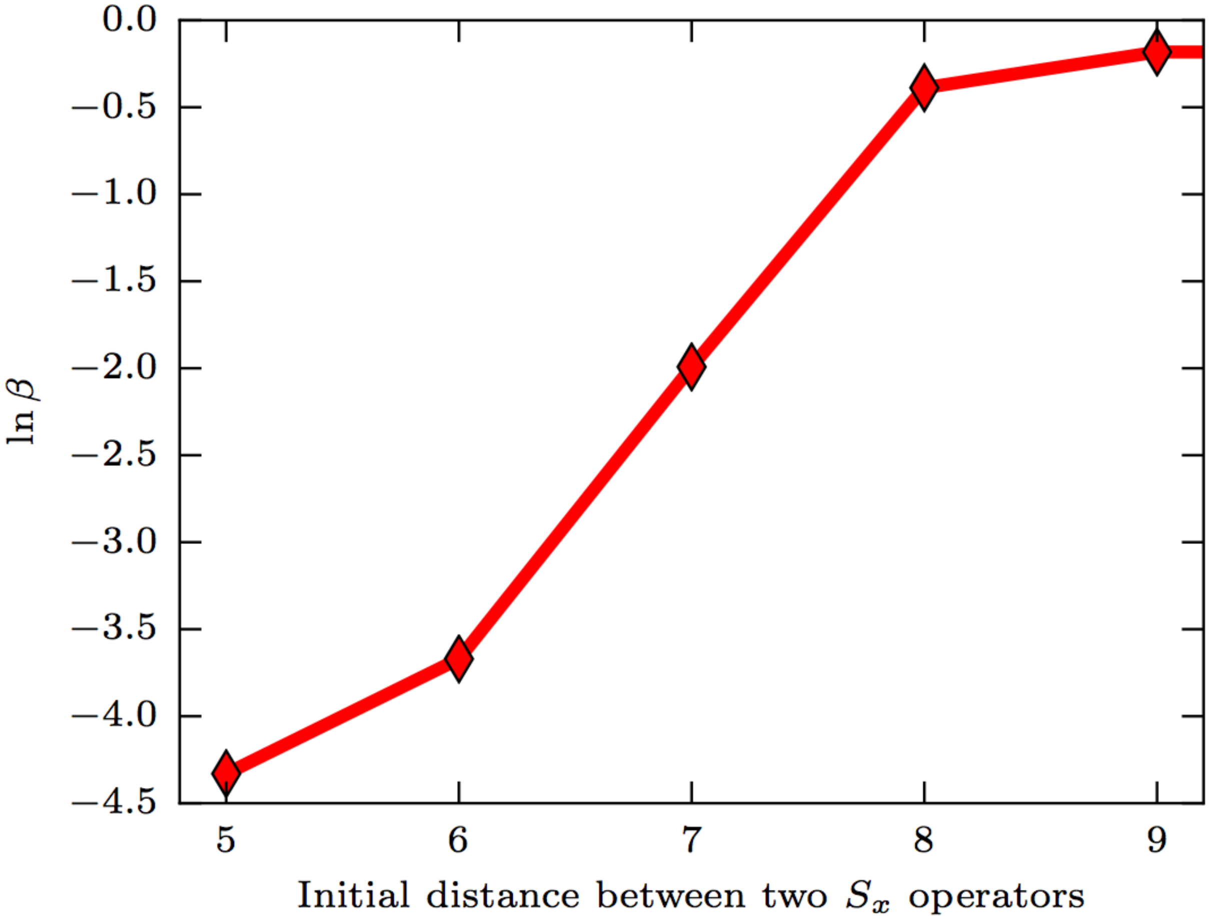
until they hit a wall.

Energy scale of coupled l-bits: e^{-L_0} where L_0 is cutoff

In the MBL bulk, how fast do operators spread?

You can also see it by looking at how long it takes light cones from two operators to collide.



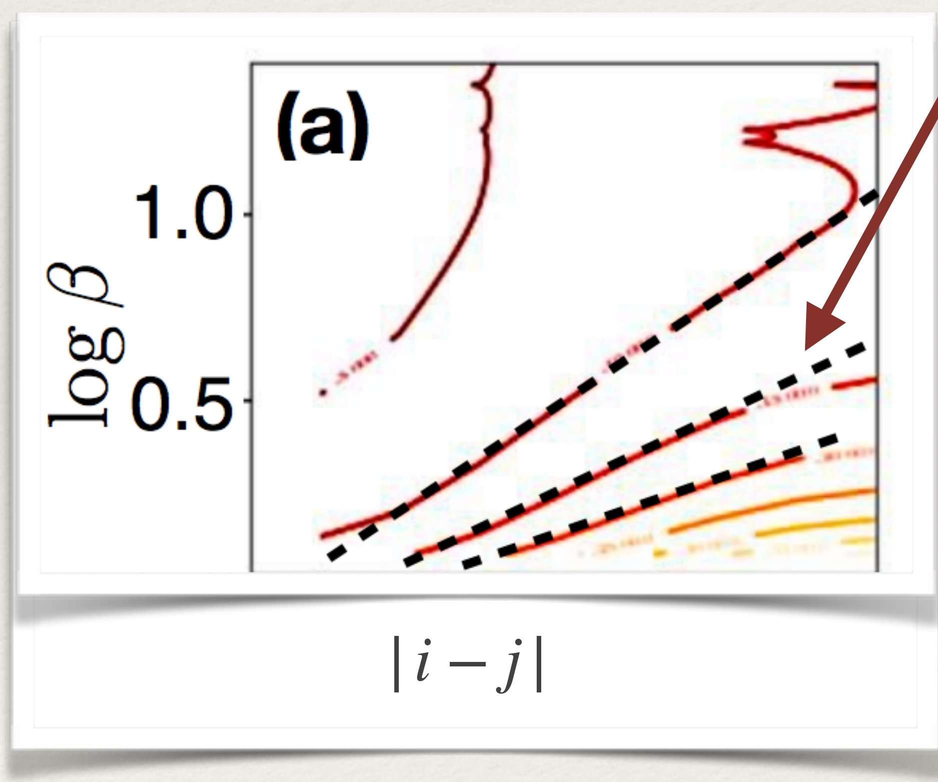


In the MBL bulk, how fast do operators spread?

You can also see it by looking at contours of the spread of the operator in its Pauli expansion.

$$U(\beta)\hat{O}U^{\dagger}(\beta) = [\ldots] + \sum_{ij} V_{ij}(\beta)\sigma_i\sigma_j$$

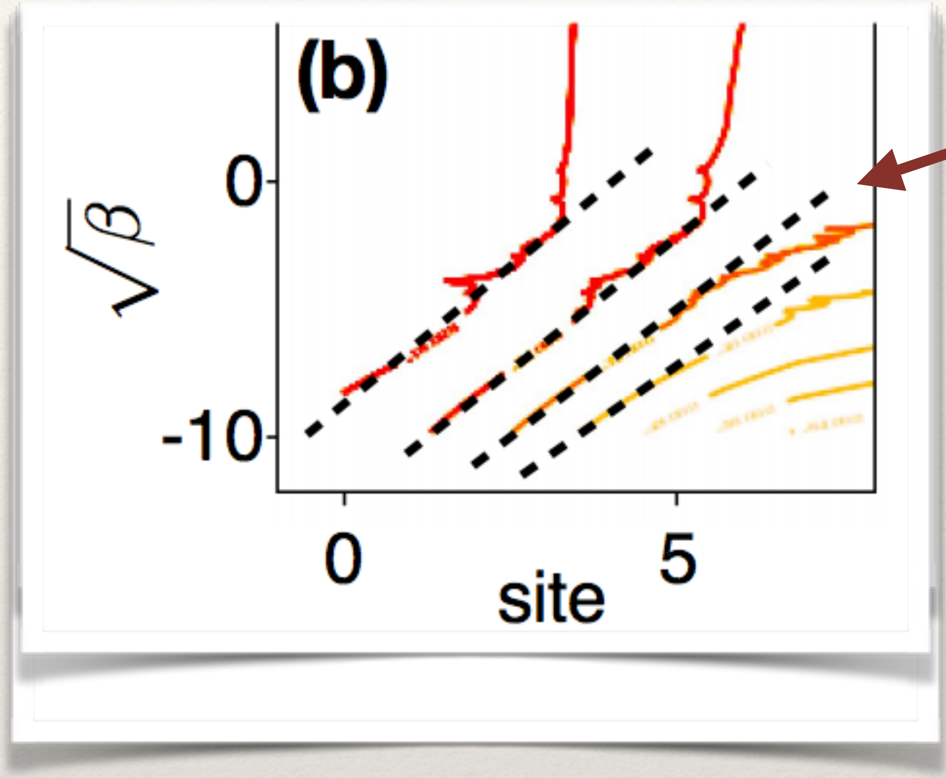
Contours of V



In the ergodic bulk, how fast do operators spread?

operator light cone spread as $\sqrt{\beta}$

$$U(\beta)\hat{O}U^{\dagger}(\beta) = [\ldots] + \sum_{ij} V_{ij}(\beta)\sigma_i\sigma_j$$



Contours of V

l-bit energy scales as 1/E

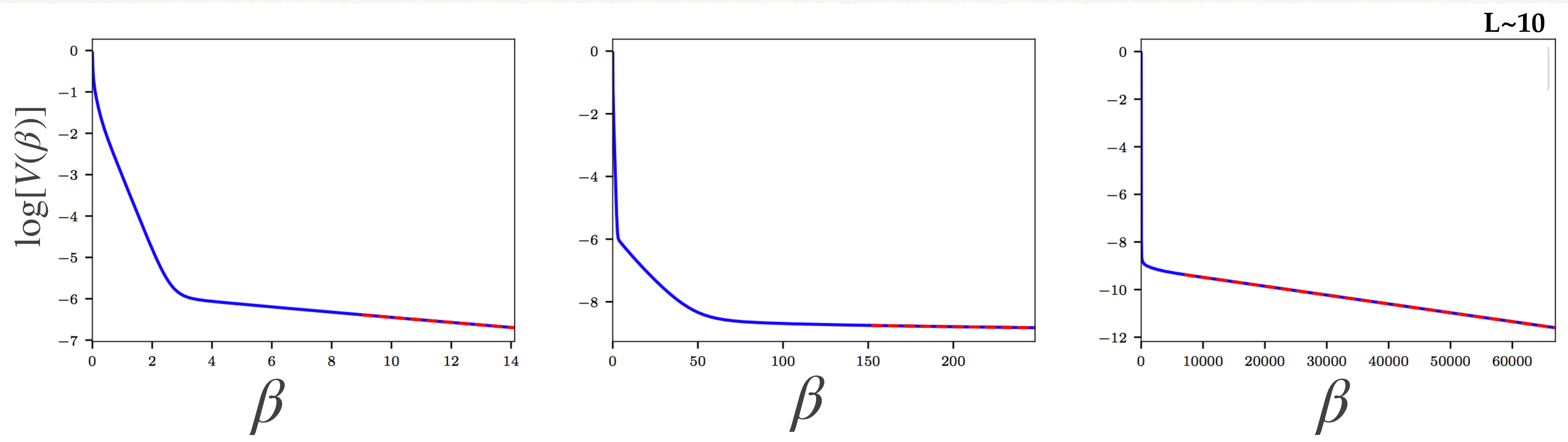
In the critical bulk, how fast do operators spread? *(speculation)*

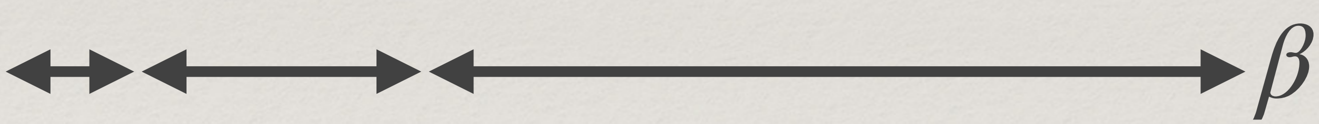
Log without any ceiling.

In the bulk, can you see the level spacing?

Variance shrinks as $V(\beta) = \exp[-\beta(\Delta E)^2]$

Unitary Distance shrinks as $\Delta E \exp[-\beta/2(\Delta E)^2]$



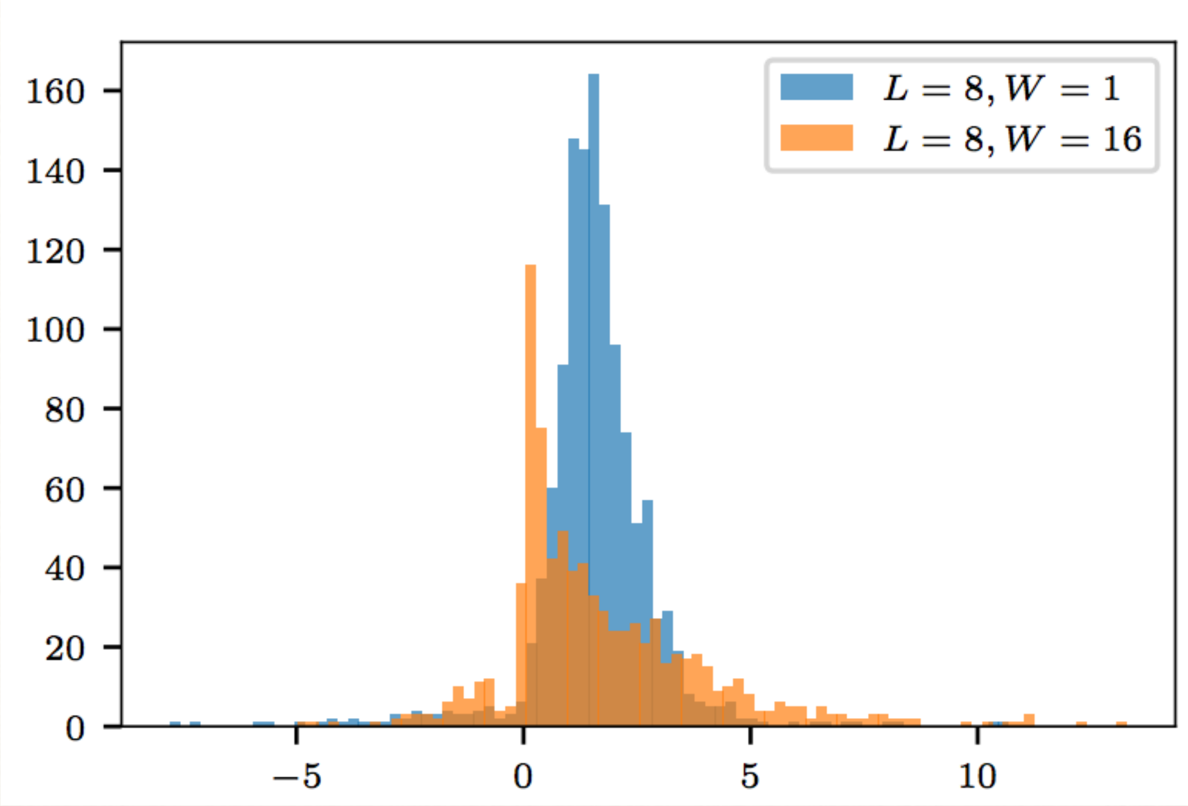


Different Epochs in the bulk

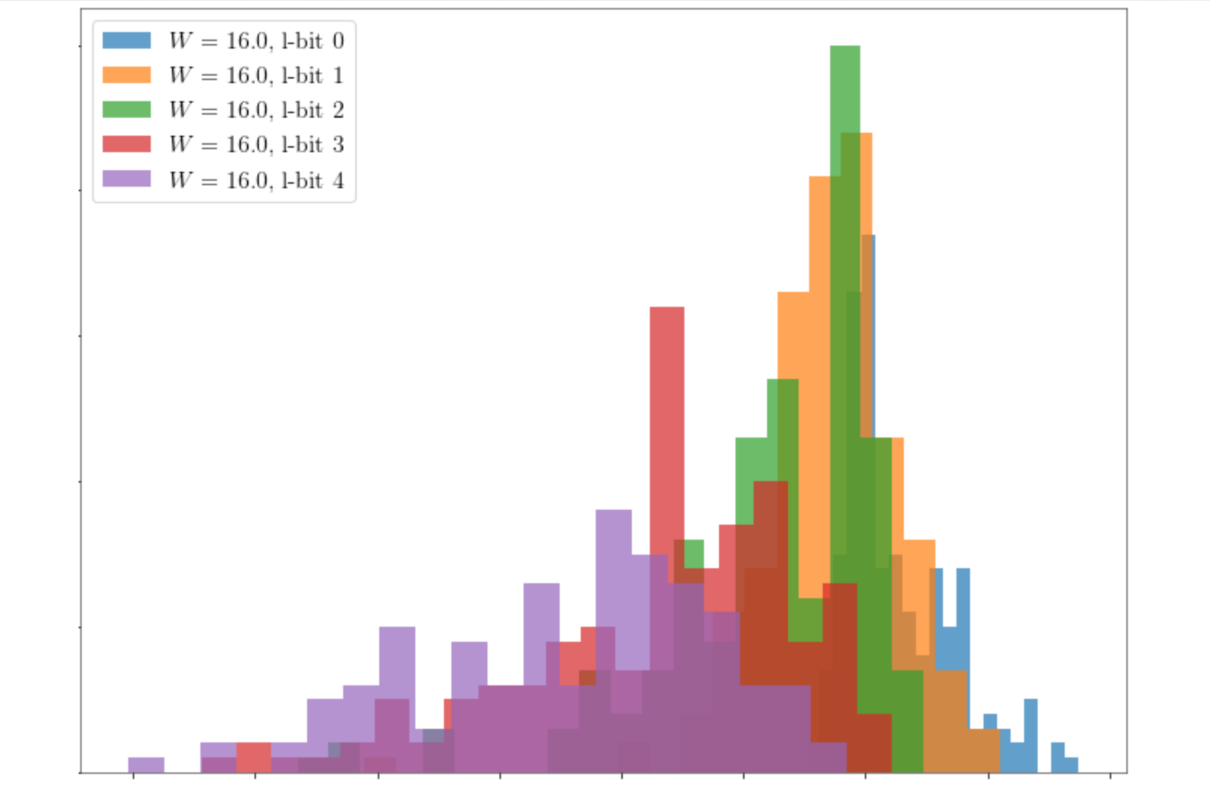
In the bulk, can you see the level spacing?

Energy must get down to `inter-level spacing'.

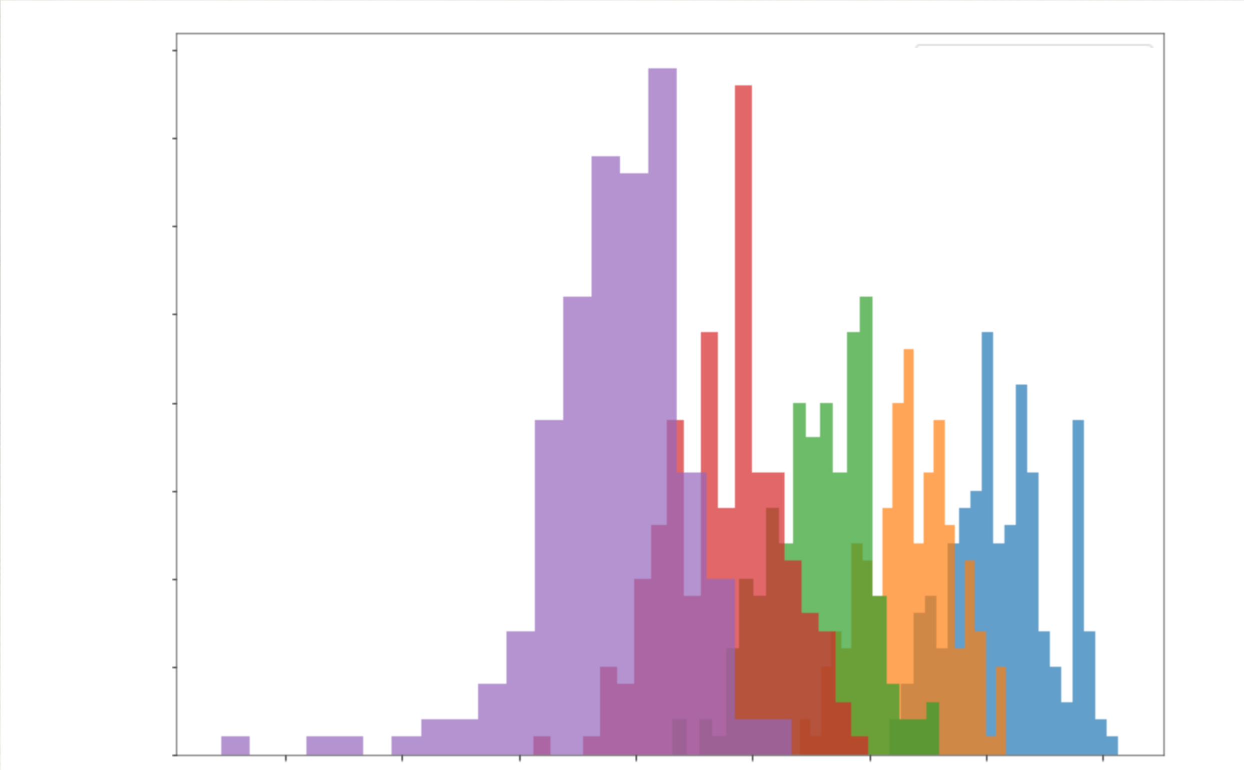
- Ergodic: RG energy scale drops exponentially
- Critical: RG energy scale drops as 1/e
- MBL: RG energy scale drops as poisson?



$\log(\Delta E_{i+1}/\Delta E_{i+2})$



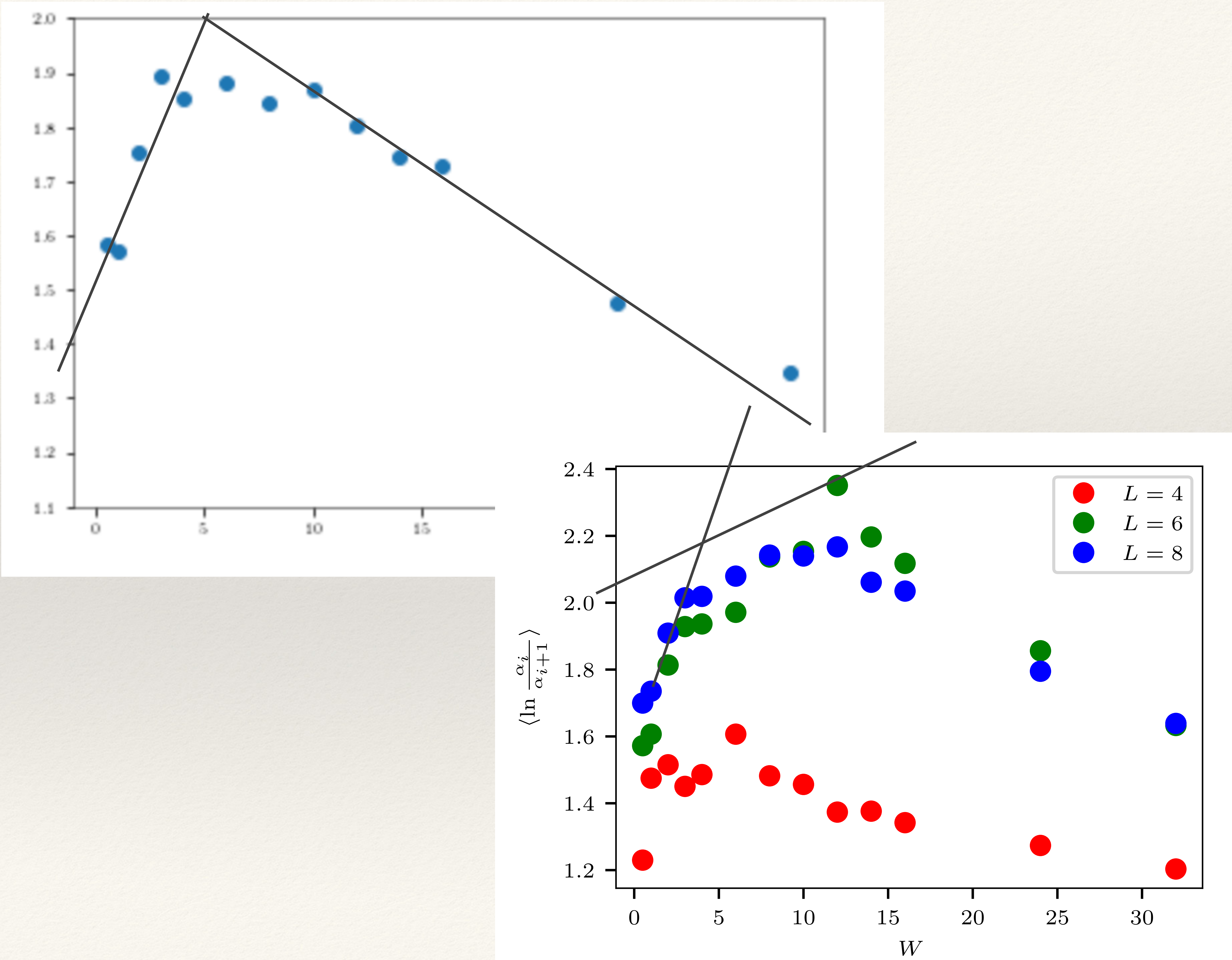
MBL $\ln \Delta E$



$\ln \Delta E$

Ergodic

A energy space picture…

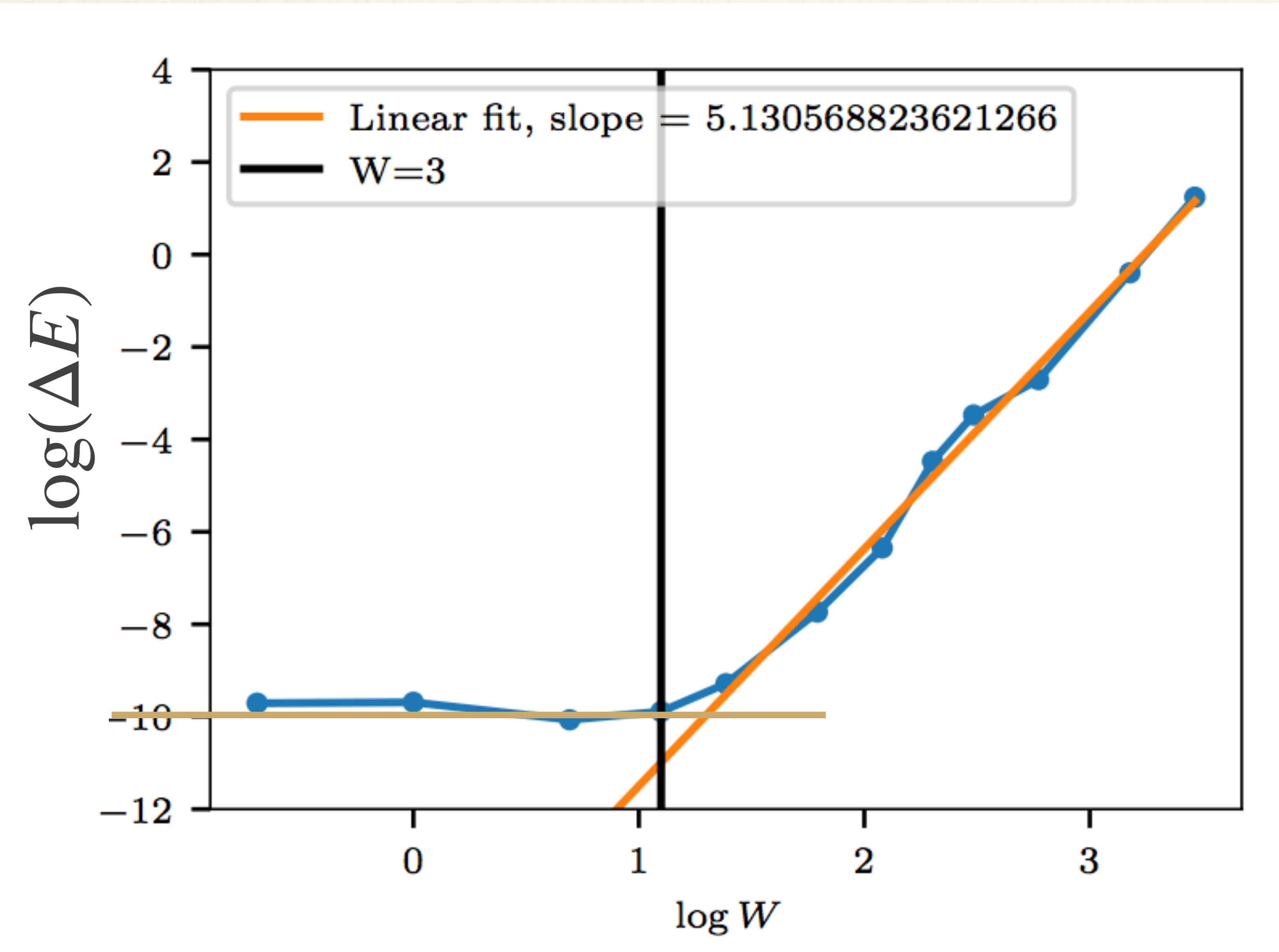


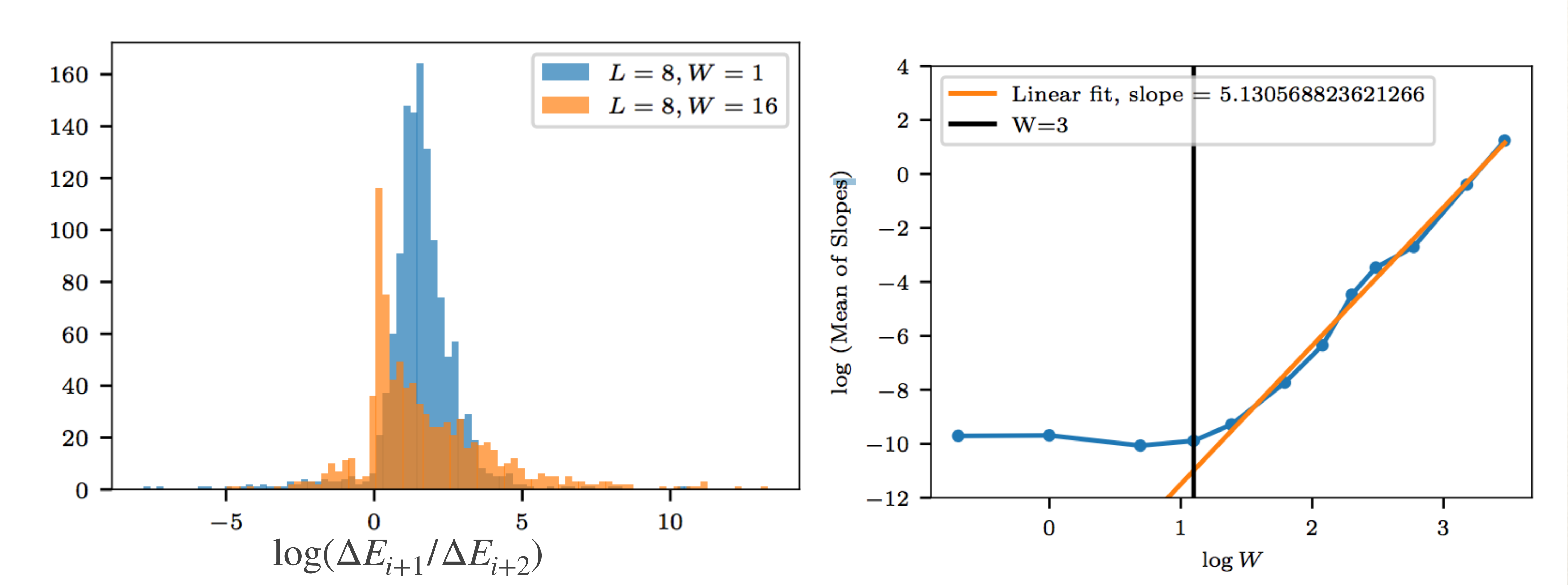
In the bulk, can you see the level spacing?

Lowest energy scale of RG tells you about MBL vs. ergodic.

Different Epochs in the bulk

MBL vs. ergodic distinguished by the `infinite’ distance in the bulk.

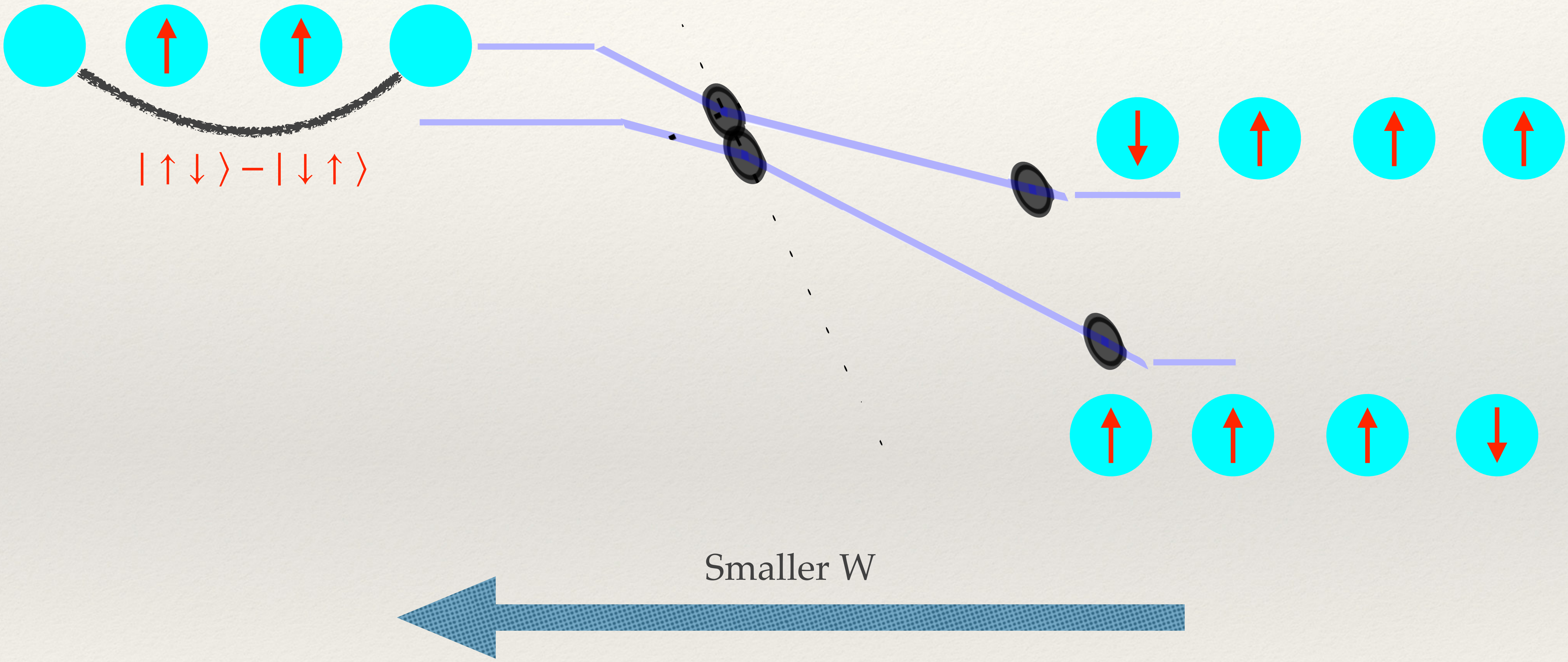




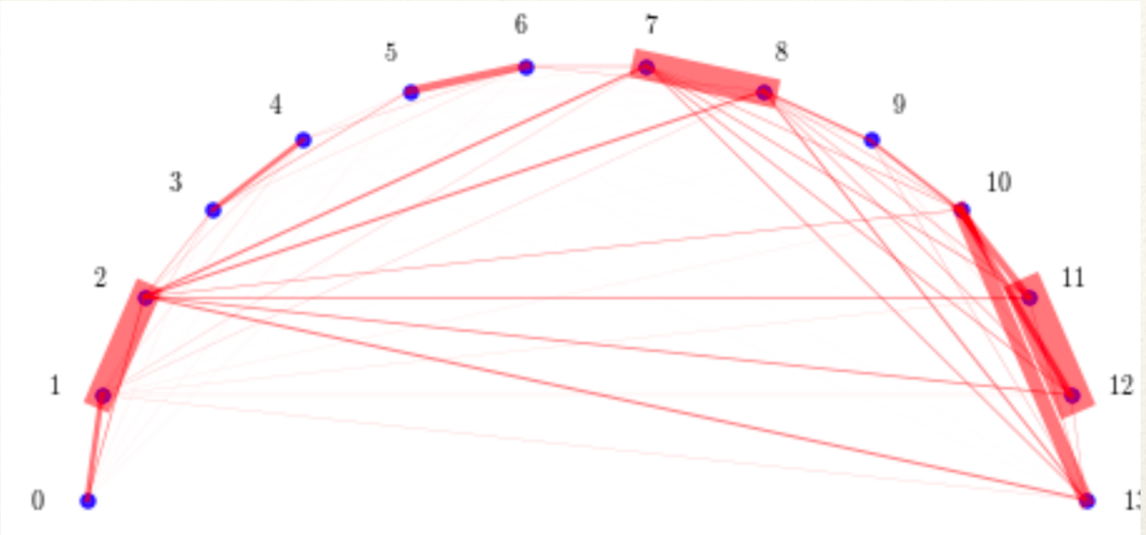
L = 8, W = 1
L = 8, W = 16
log(ΔE_{i+1}/ΔE_{i+2})
Linear fit, slope = 5.130568823621266
W=3
log (Mean of Slopes)
log W

Part 2

Q: Is the transition being driven by resonances?



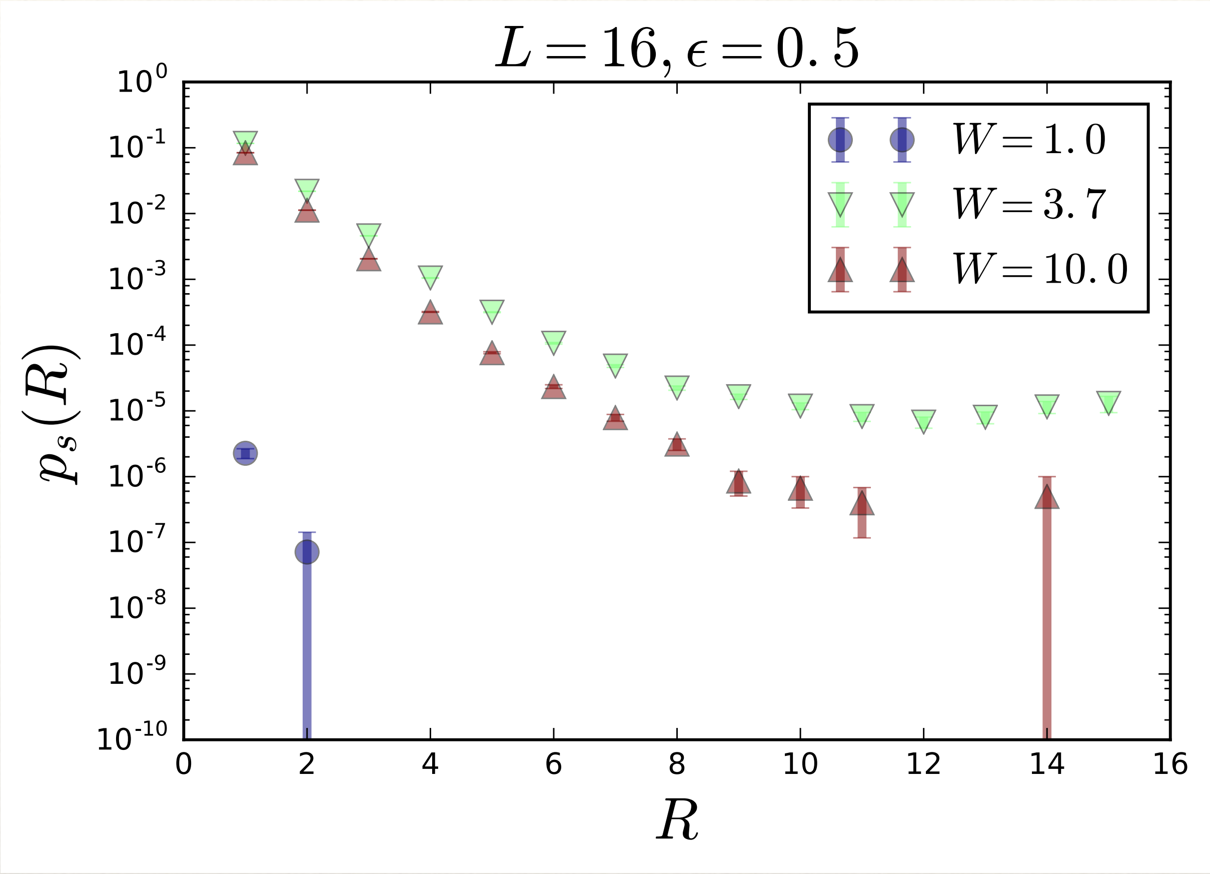
Find the resonances



Use mutual information…

De Tomasi, G., Bera, S., Bardarson, J. H., Pollmann, F. (2017). Quantum mutual information as a probe for many-body localization. *Physical review letters*, *118*(1), 016804.

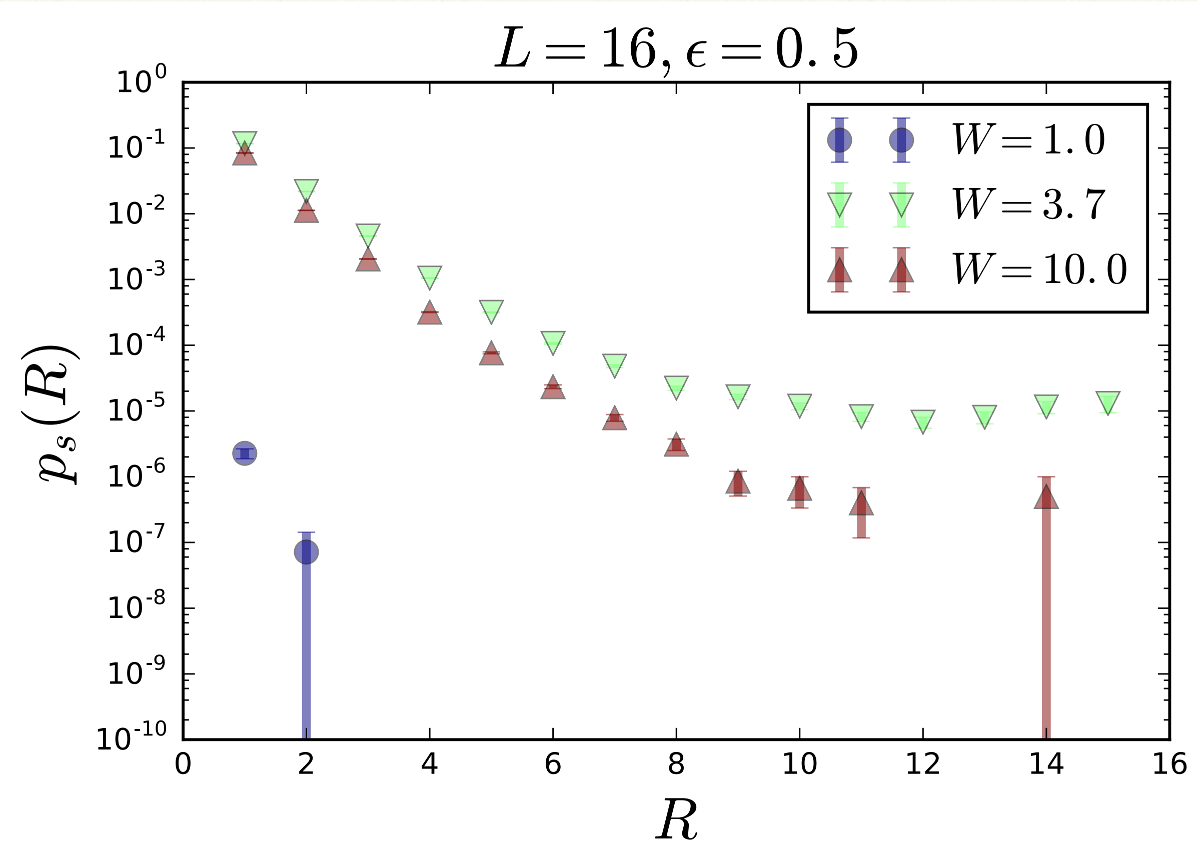
1. Find the resonances



For a given threshold, long range singlets are improbable

But around the transition, they become scale invariant.

1. Find the resonances



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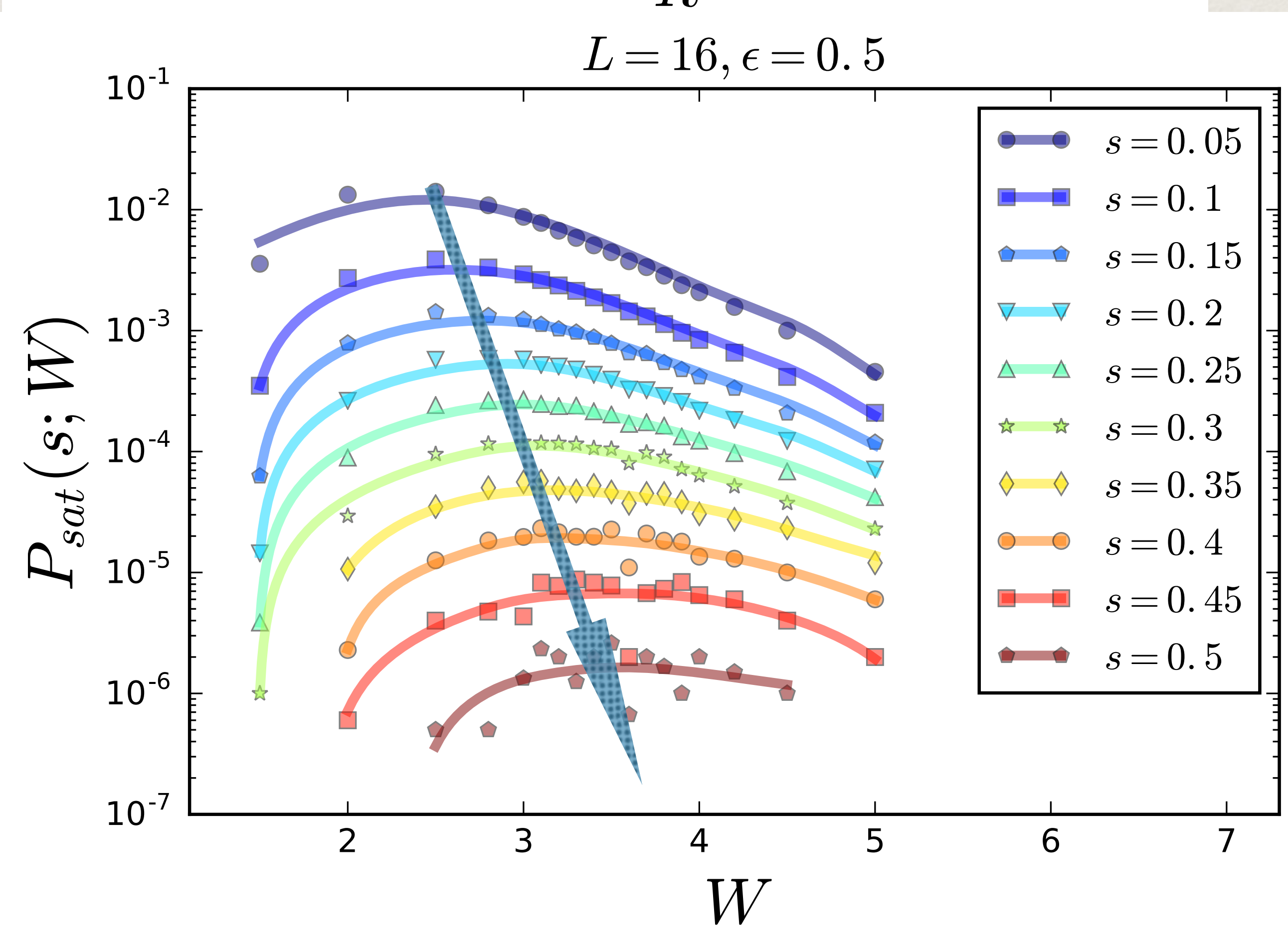
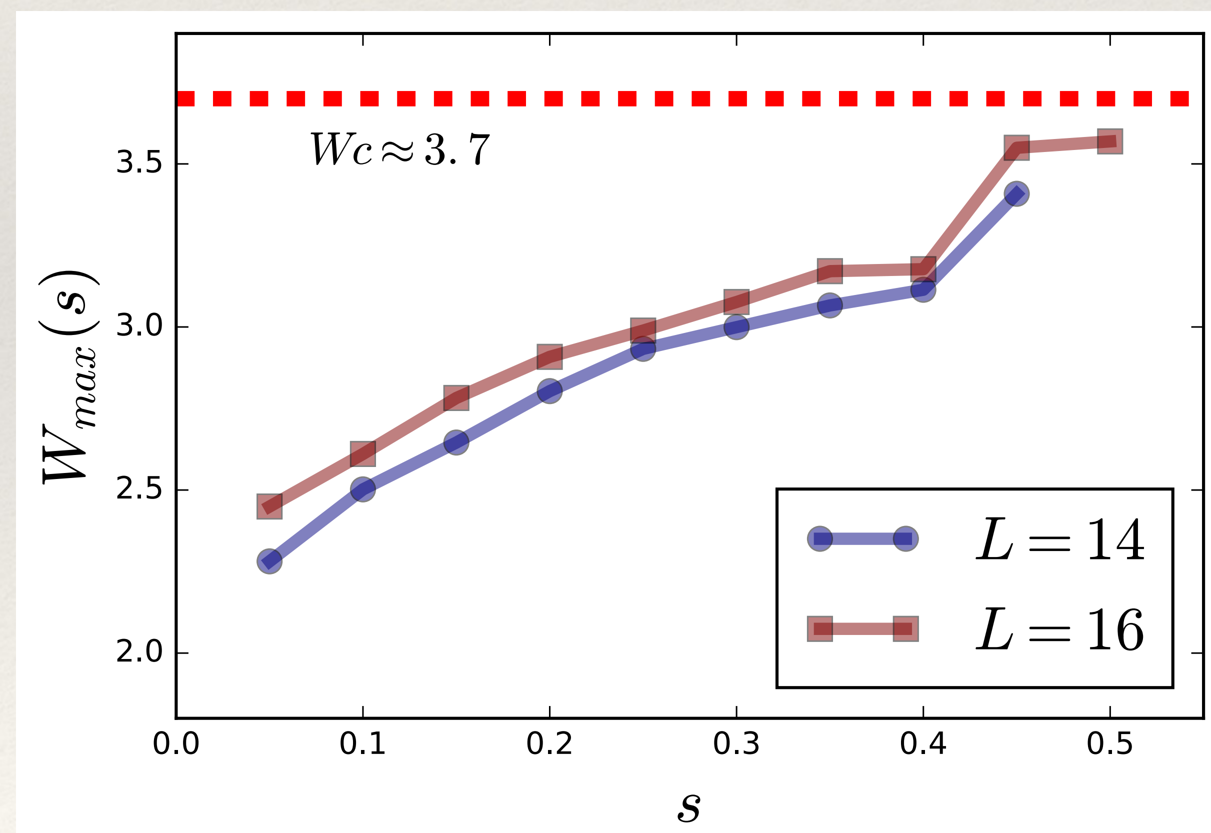
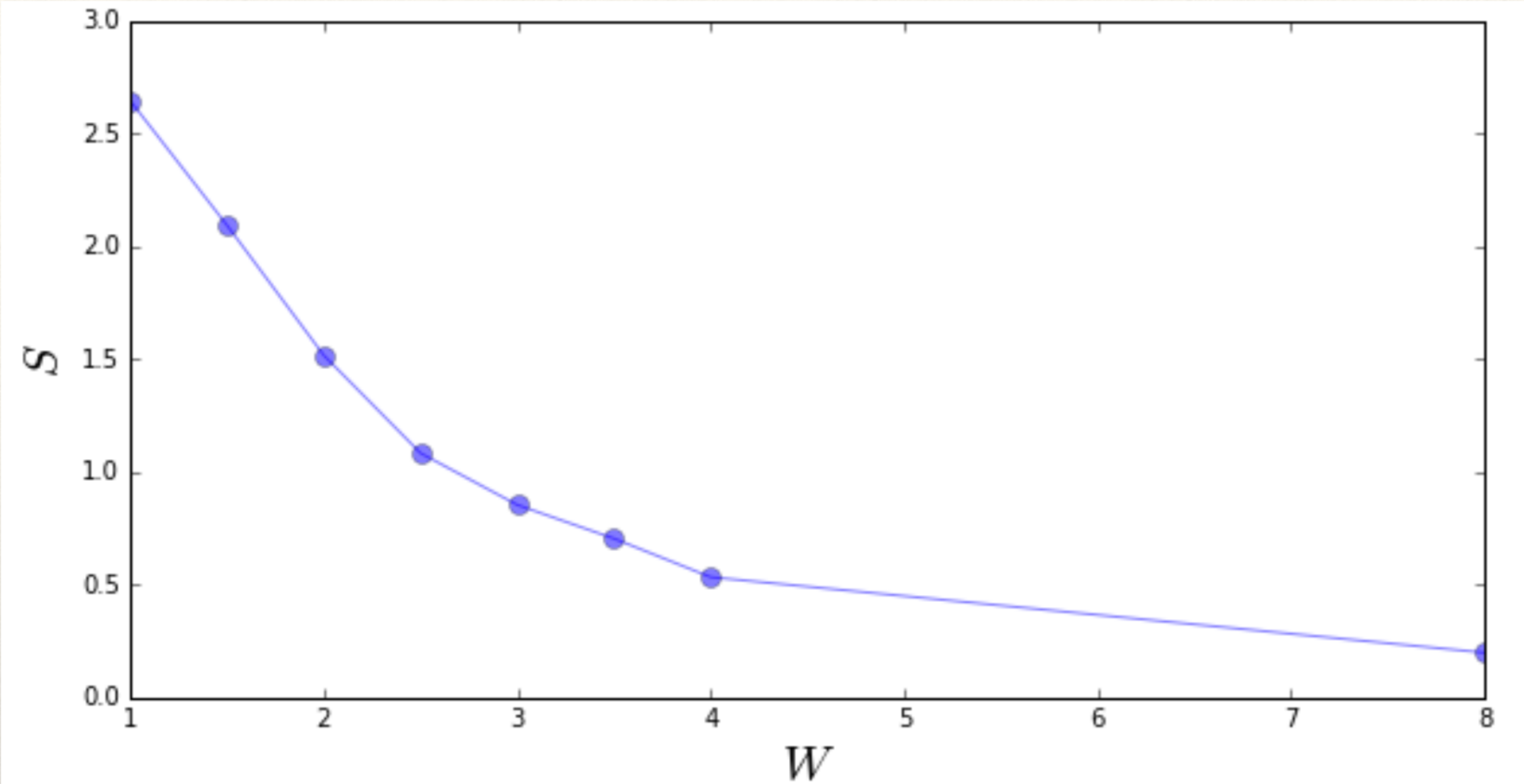


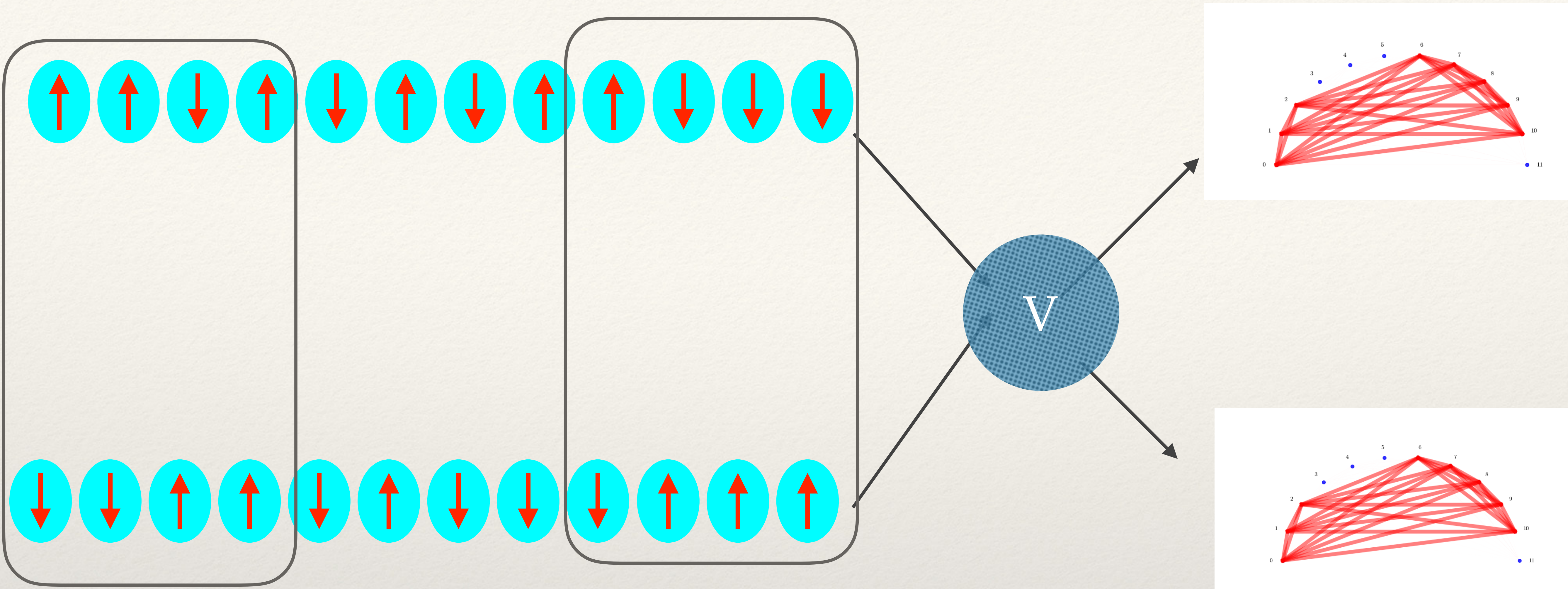


Entanglement increases (kindof sloppily) at the transition.



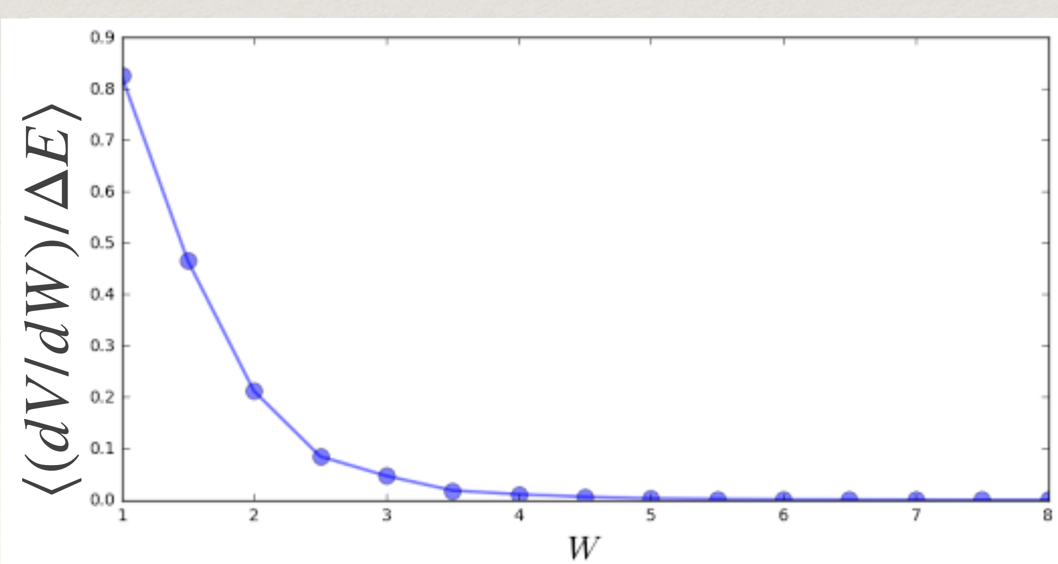
Naive picture: Each collision generates a singlet, which increases the entanglement, which eventually makes a volume-law state of singlet spaghetti.

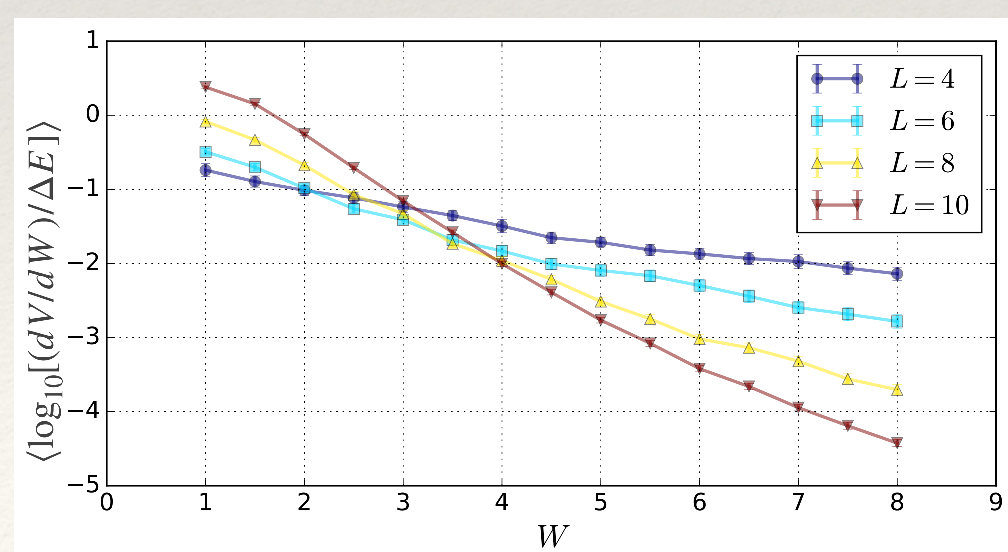
Q: What's driving this?

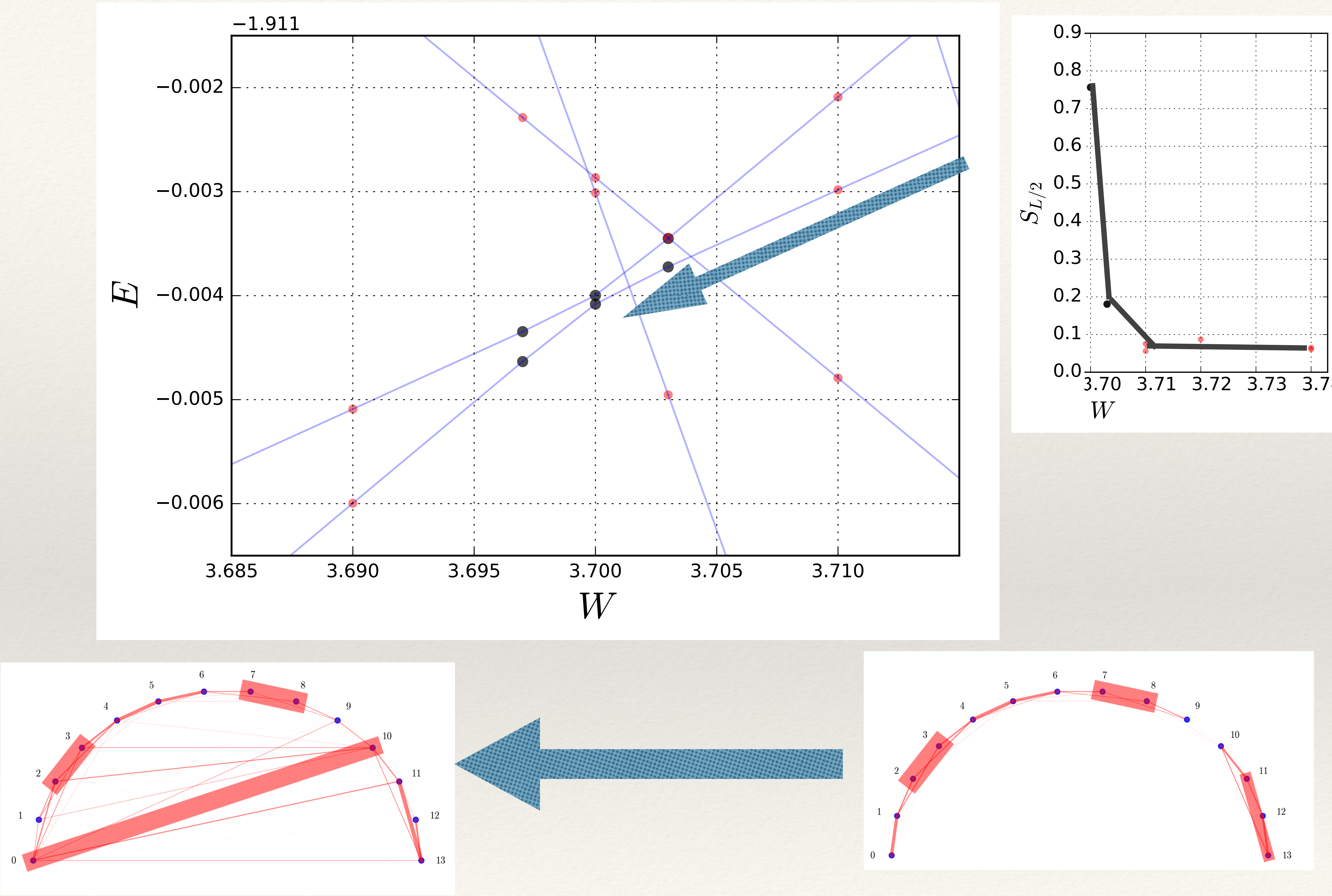


Collisions happen both in MBL and ergodic.

What should we look at to see how likely a collision is? $V \approx \Delta E$







−1.911
−0.002
−0.003
−0.004
−0.005
−0.006
E
3.685
3.690
3.695
3.700
3.705
3.710
W
0.9
0.8
0.7
0.6
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0.4
0.3
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0.0
$S_{L/2}$
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3.71
3.72
3.73
3.74
W
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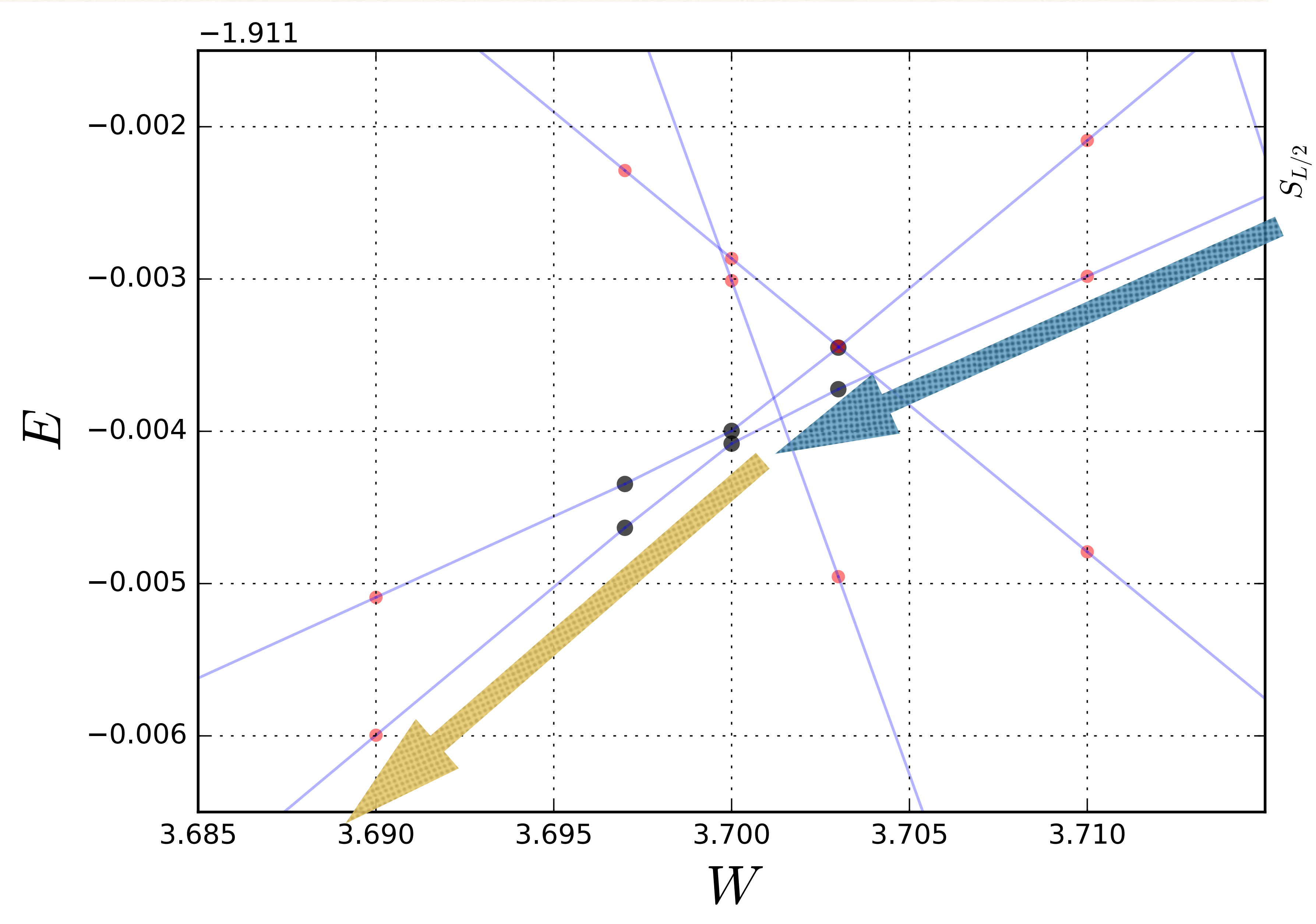
0 1 2 3 4 5 6 7 8 9 10 11 12 13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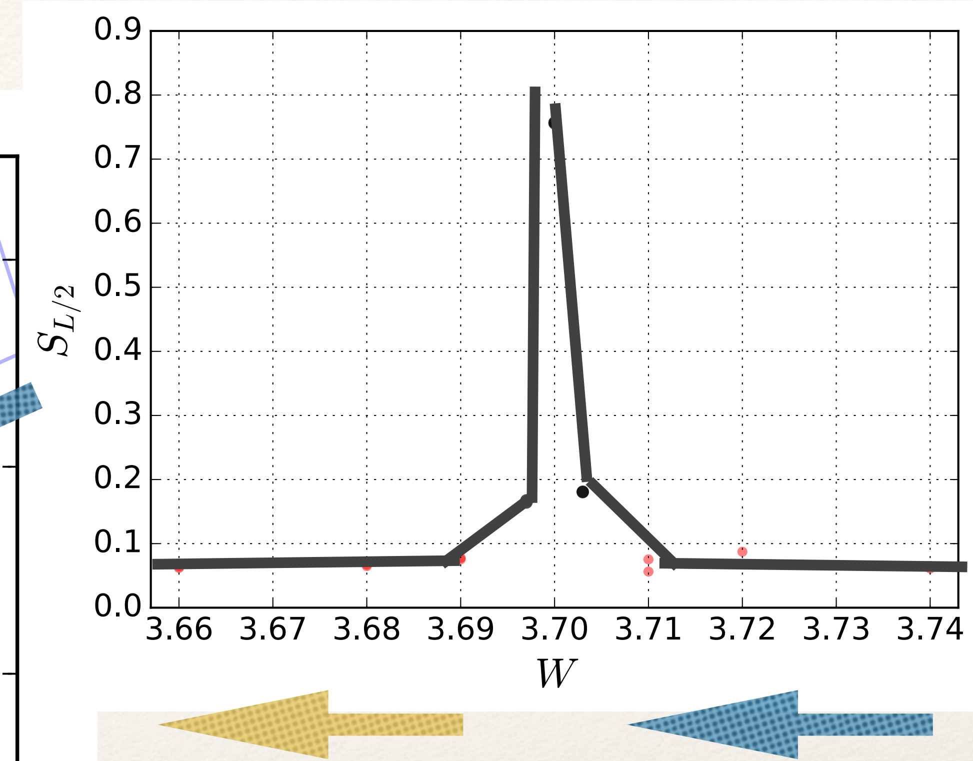
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3.69
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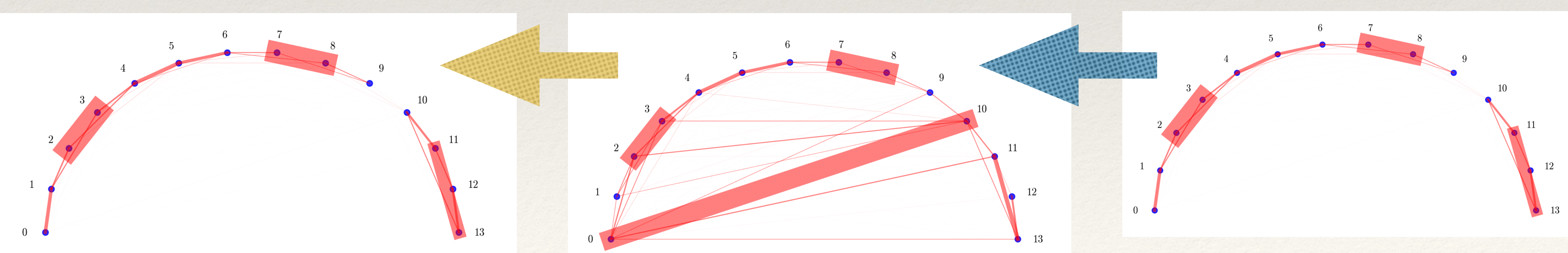
3.72
3.73
3.74
W


0
1
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13

Q: How then do you get volume law entanglement.

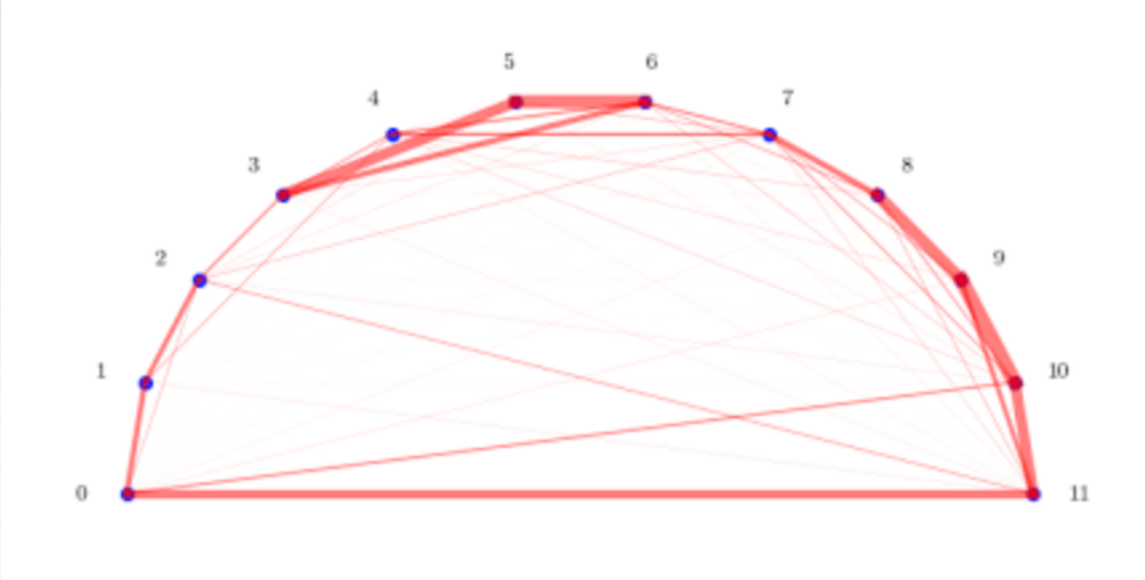
You must be continuously generating `long-range singlets'

You must be continuously colliding.

(Also what you need to get GOE level statistics)

How are we going to see this?

Deeper in the ergodic phase, everything starts to be a mess.



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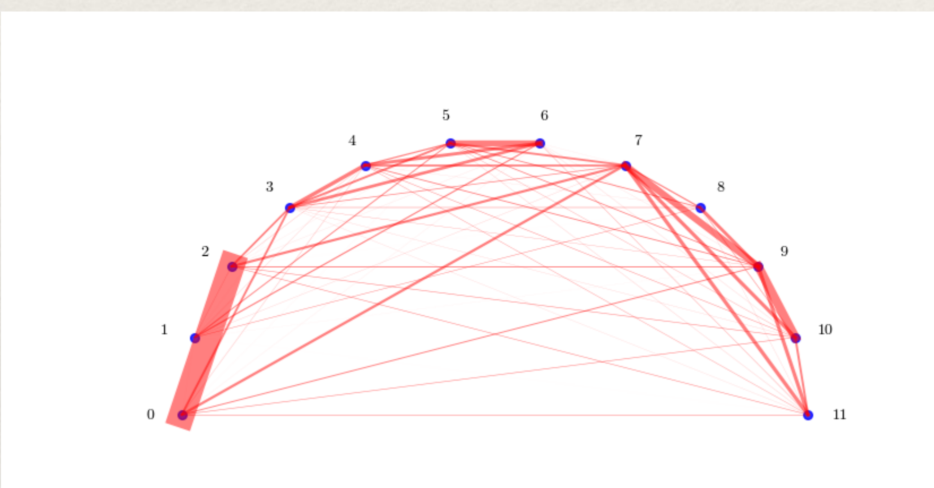
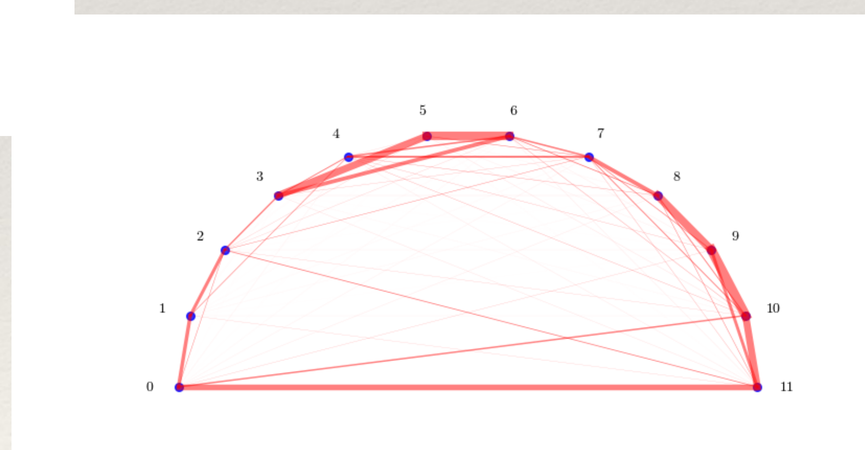
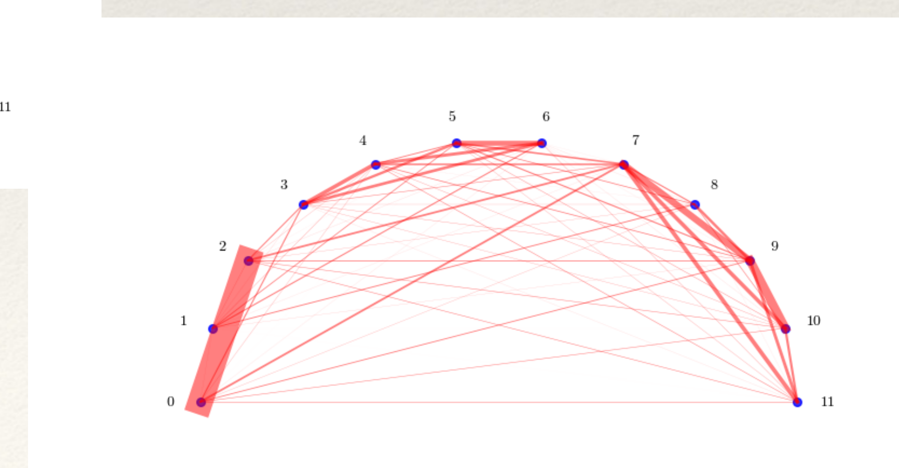






The relative picture…

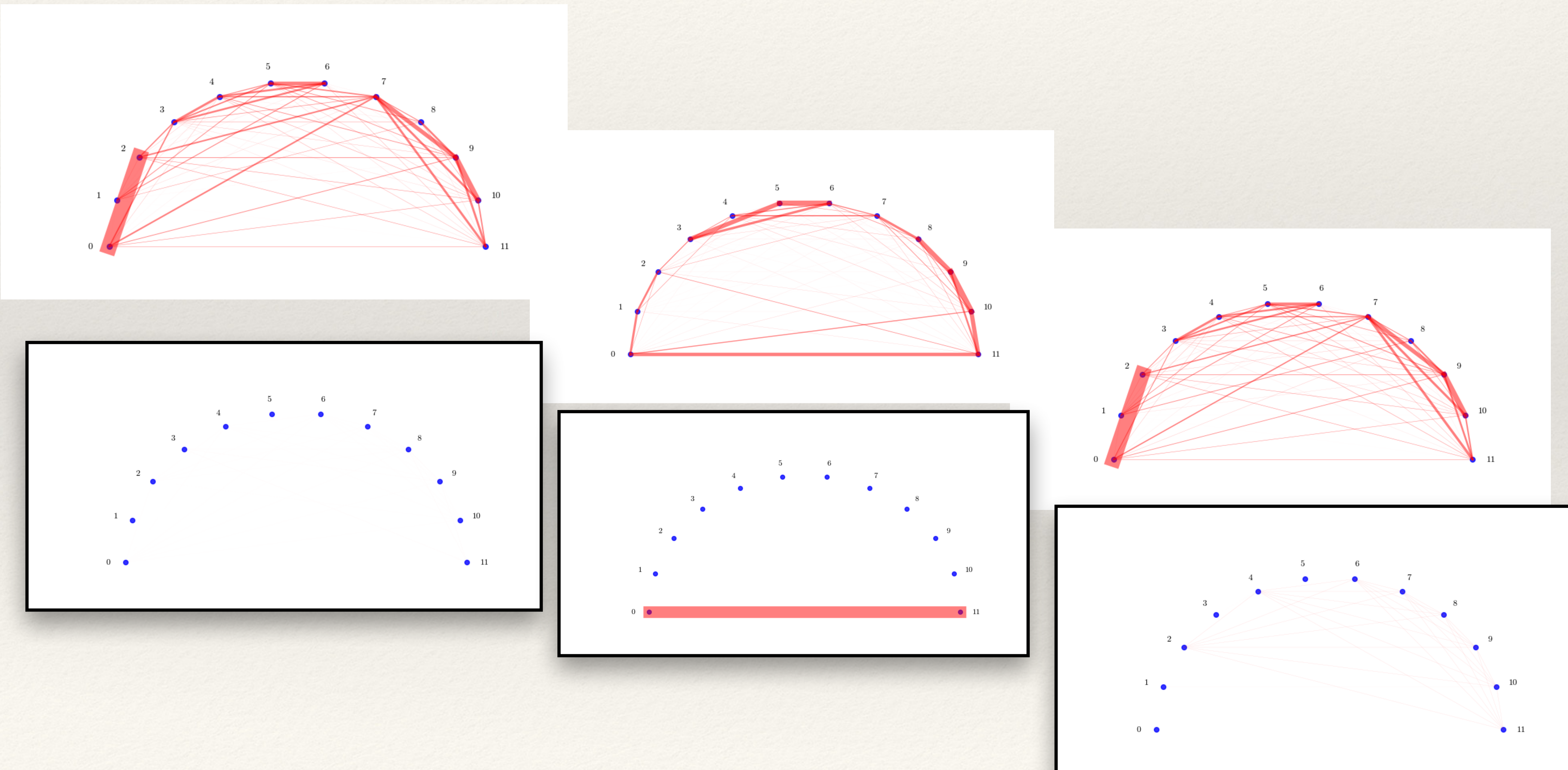
We will work with the relative Hamiltonian…

$$U^{\dagger}(W)$$

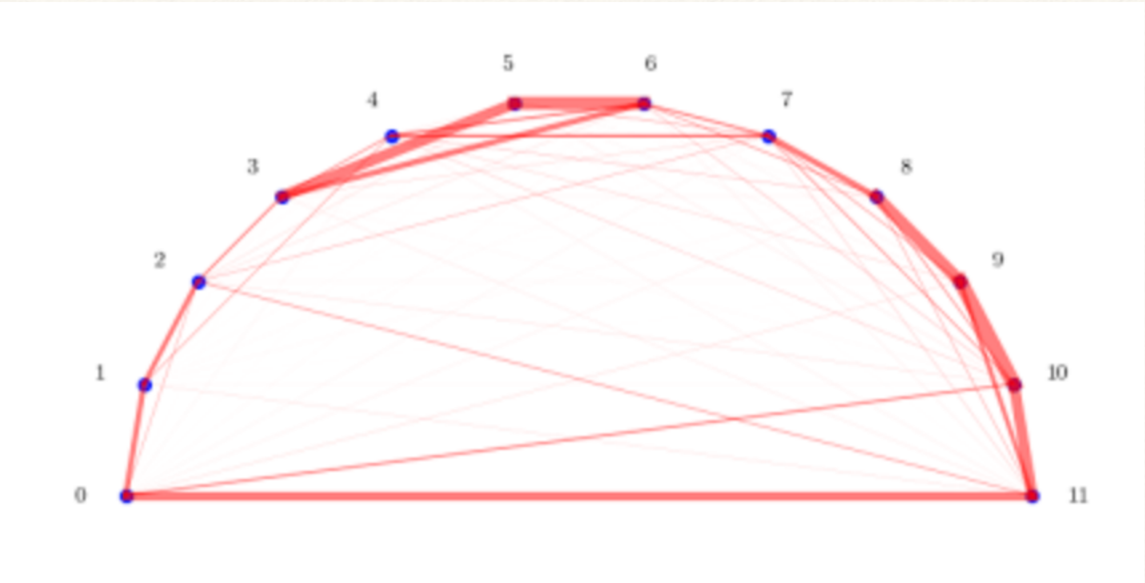
$$H(W-\delta)$$

$$U(W)$$

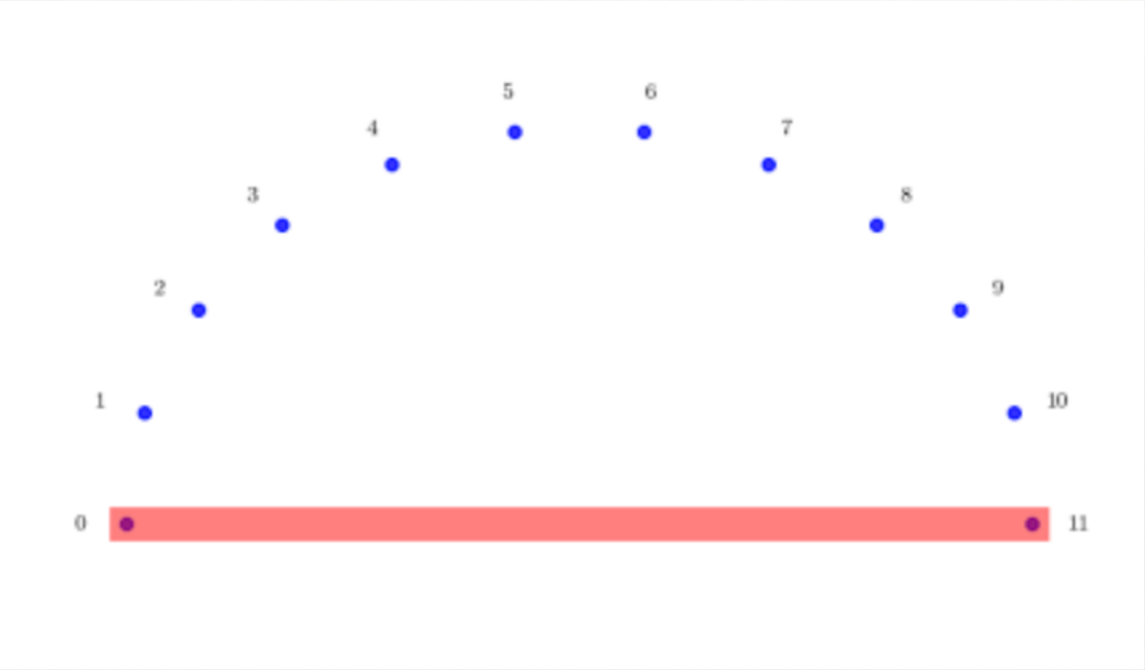
where the unitary is generated by the Wegner-Wilson flow.



W=2.5

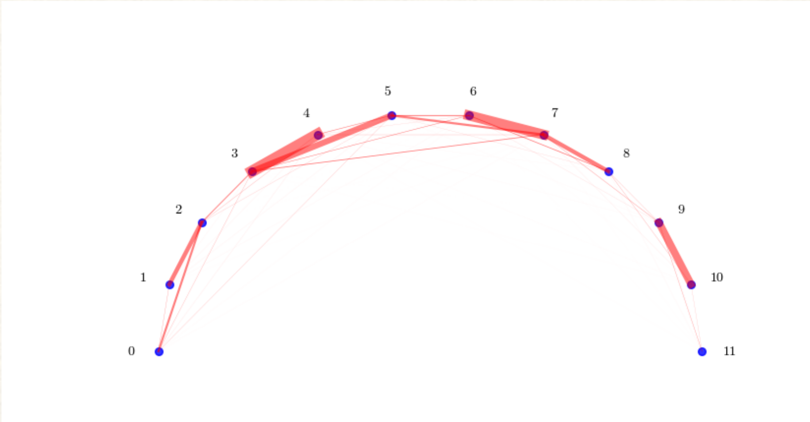


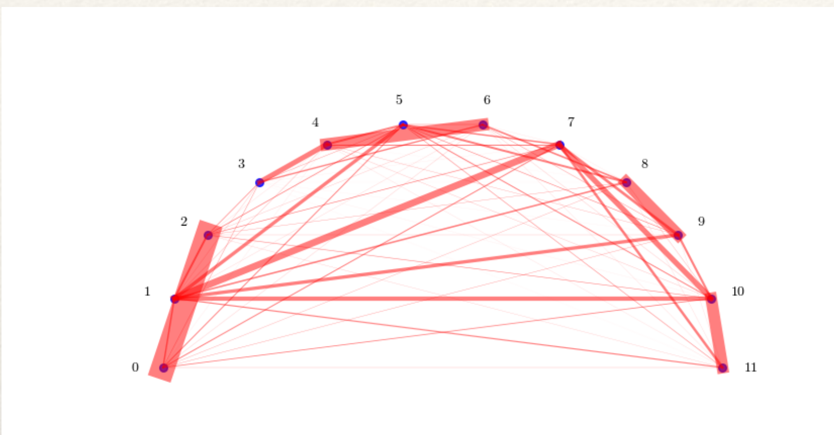
Long Range Resonance (in real space)

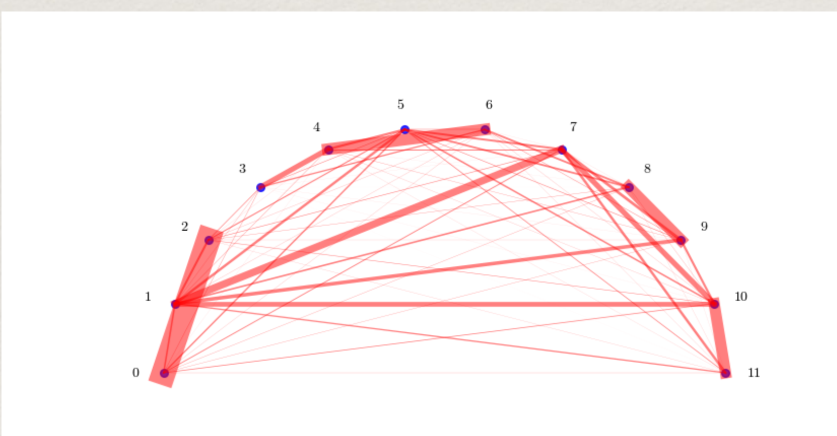


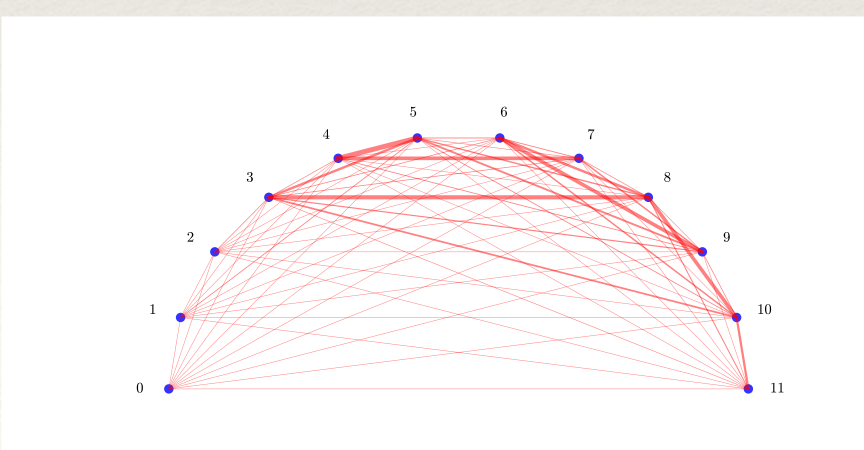
Same Long Range Resonance (in relative space)

Real Space

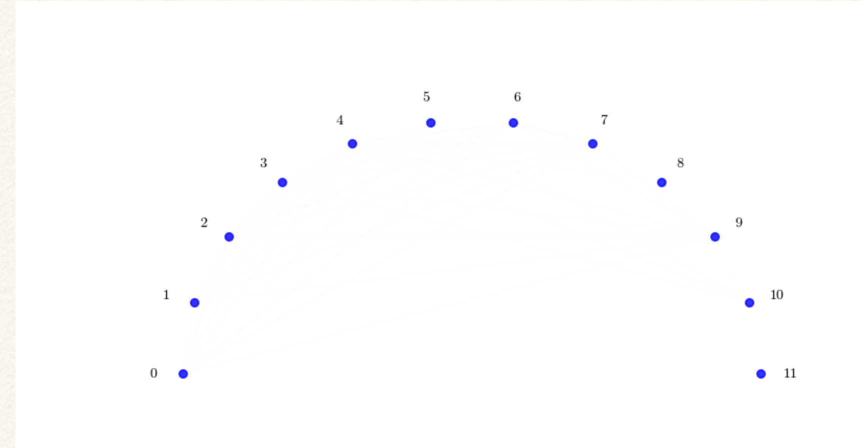


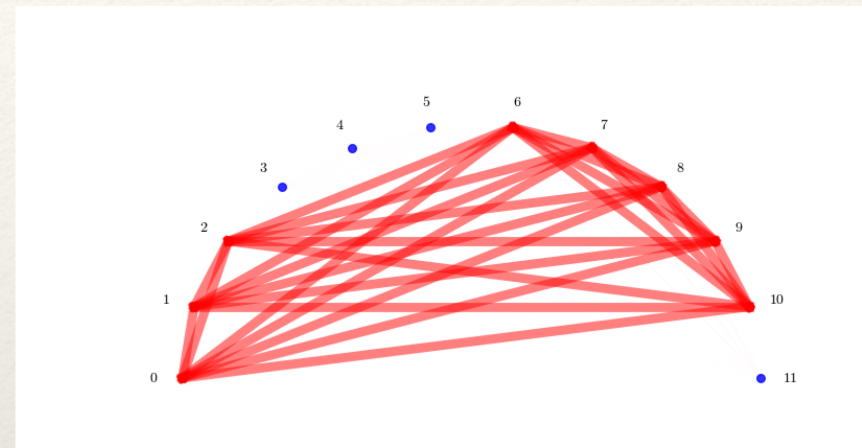


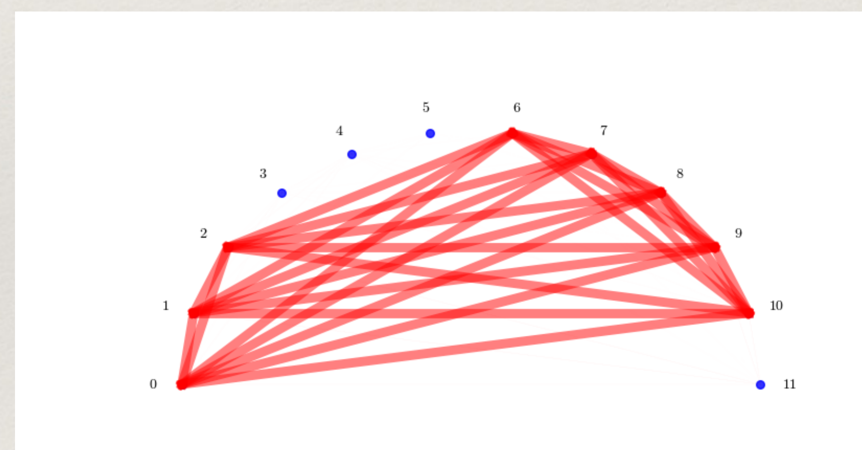


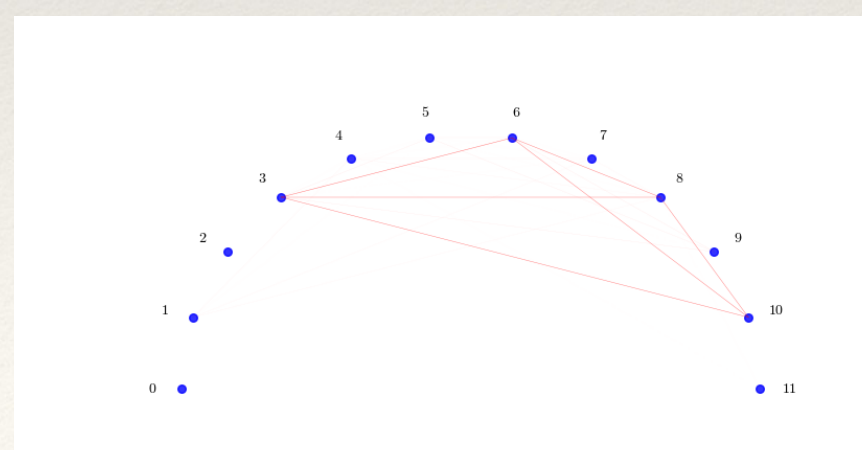


Relative Space



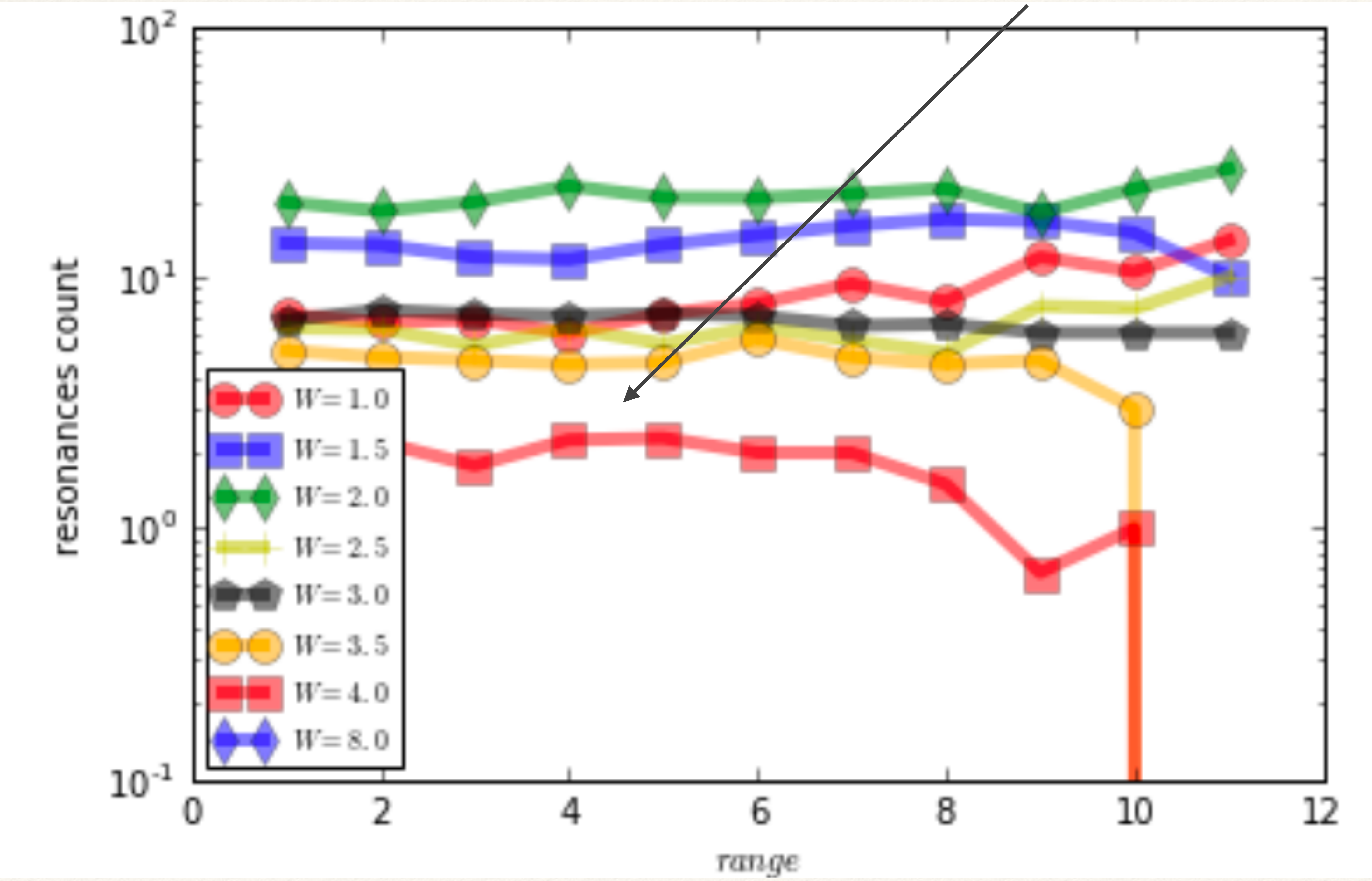






In relative space, resonances are always scale invariant.

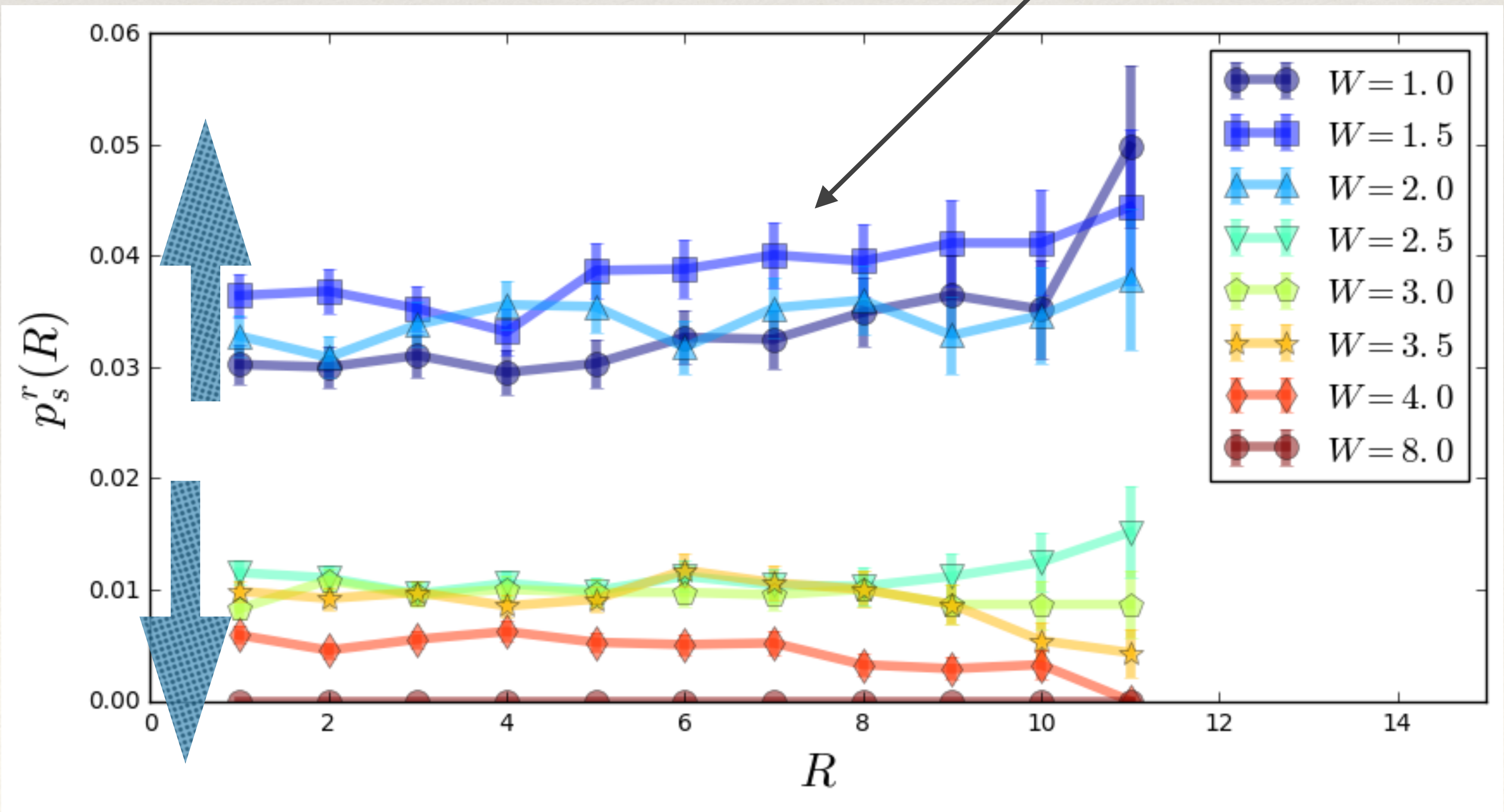
Continuous adiabatic singlet creation.



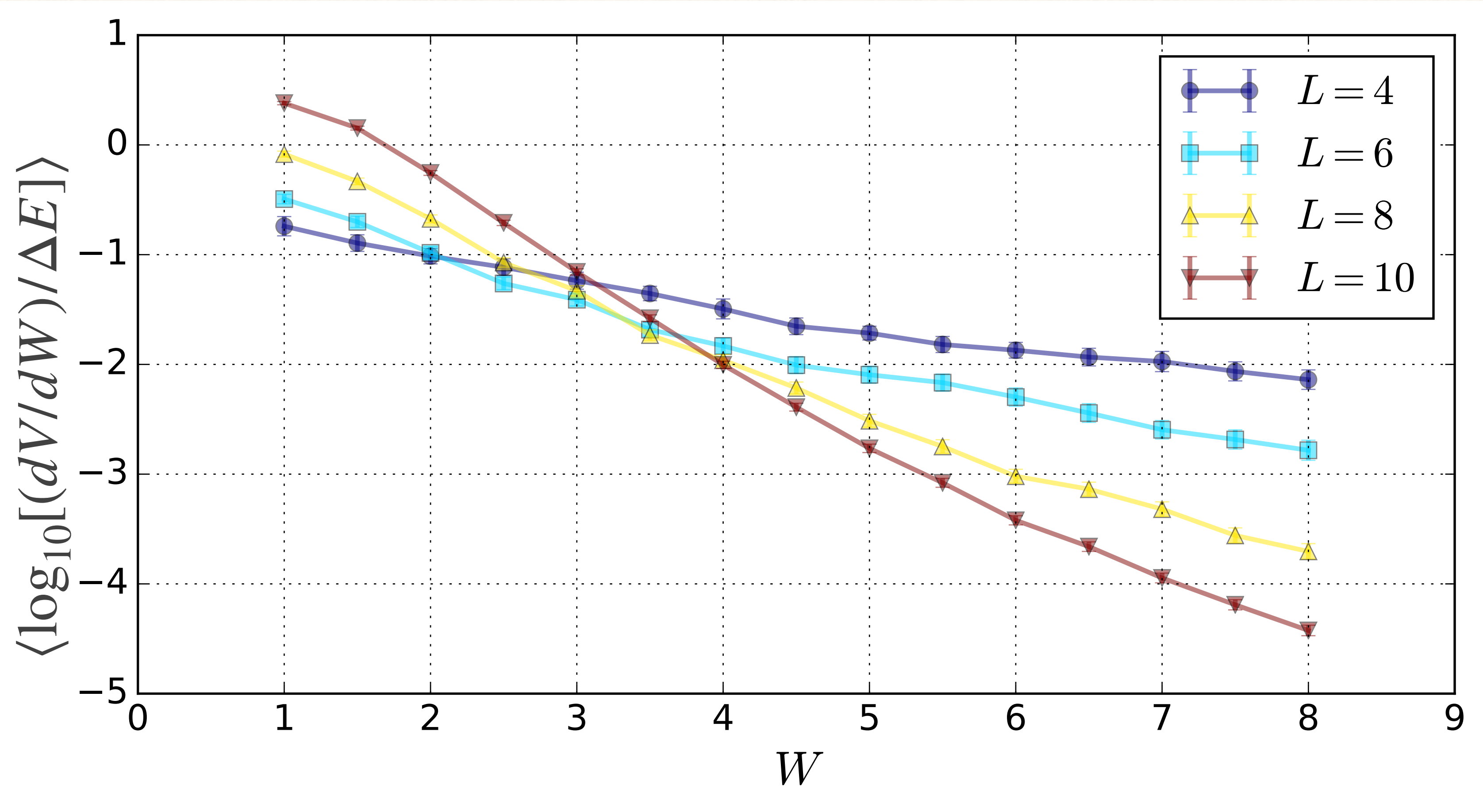
In relative space, resonances are always scale invariant.

They increase in probability in the ergodic phase.

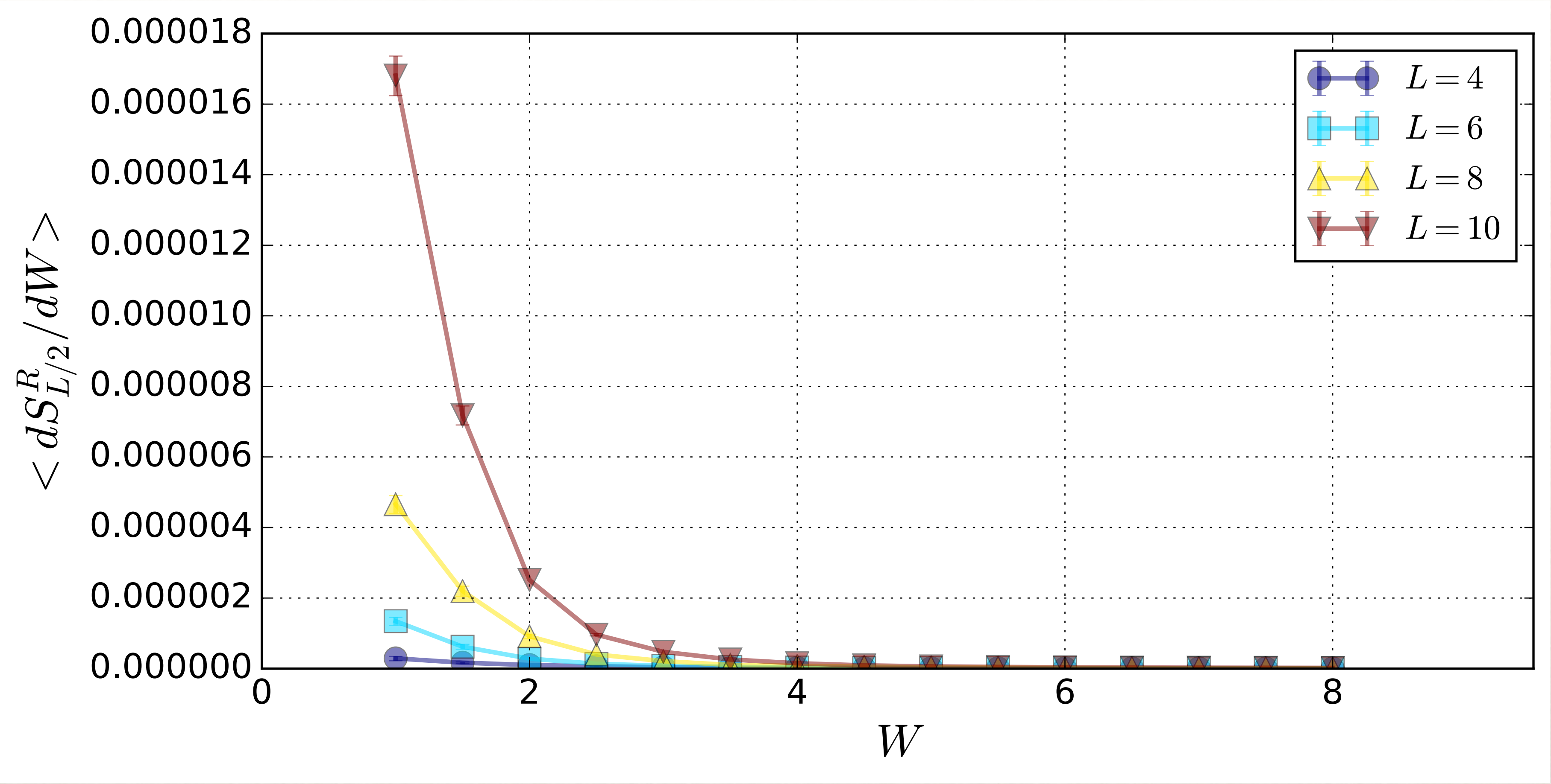
Continuous adiabatic singlet creation.



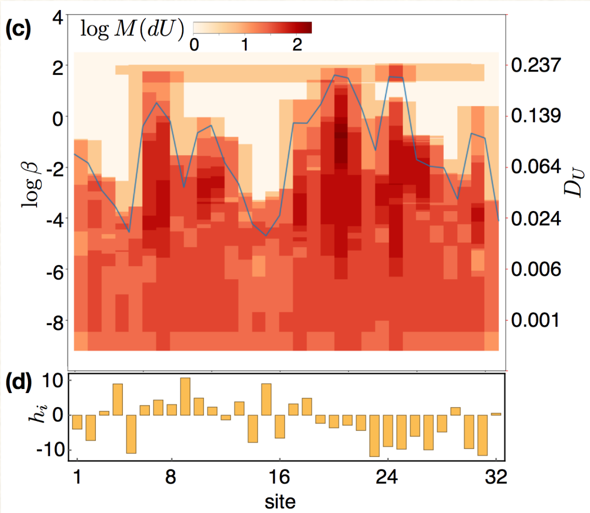
Q: What's driving this?

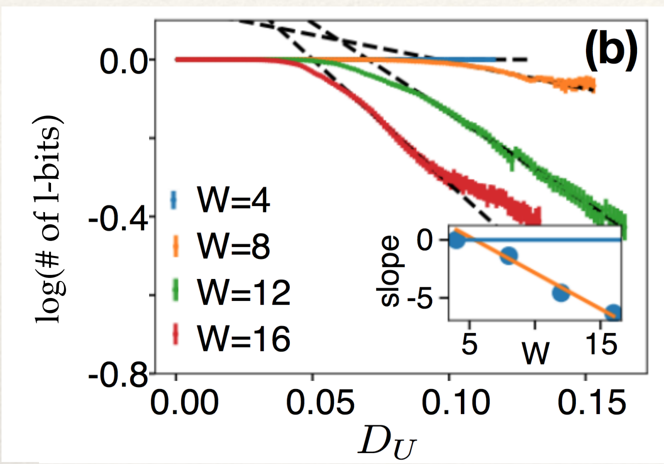


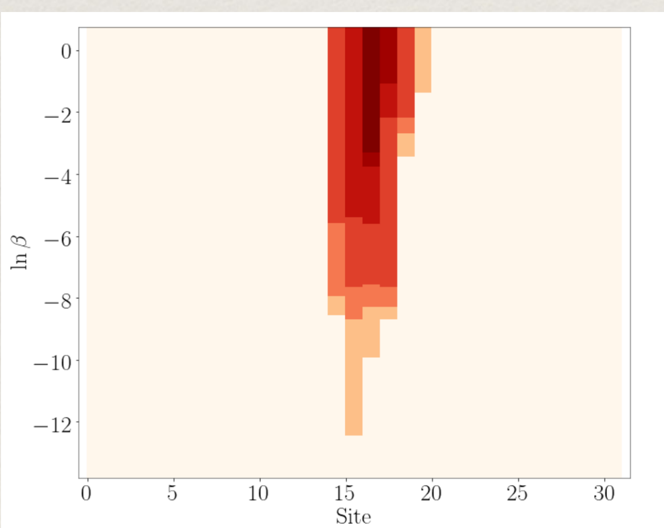
The relative entanglement is significantly cleaner…

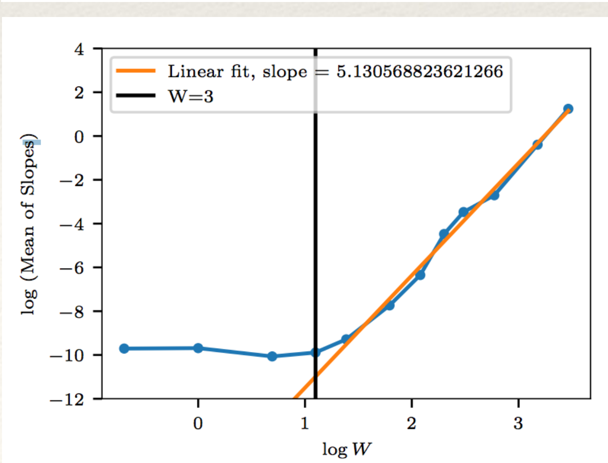


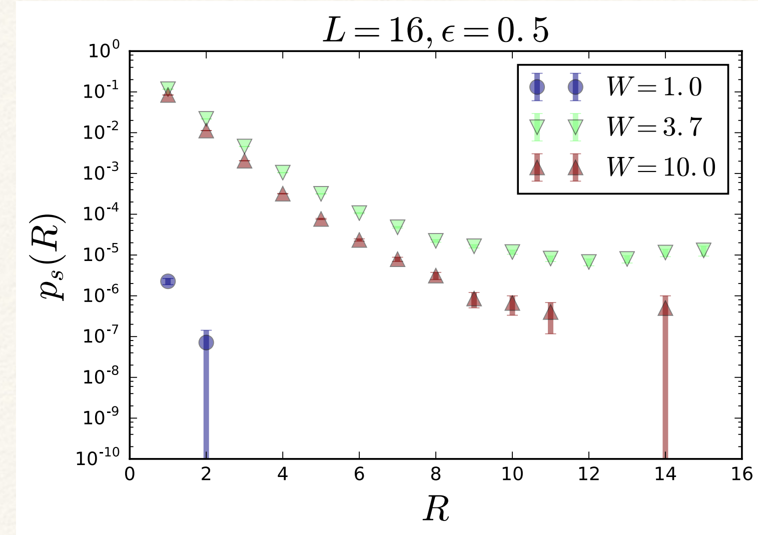
Conclusions

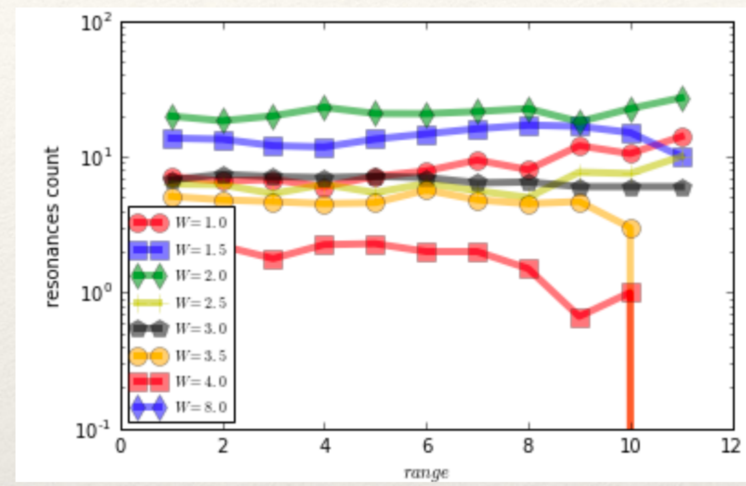


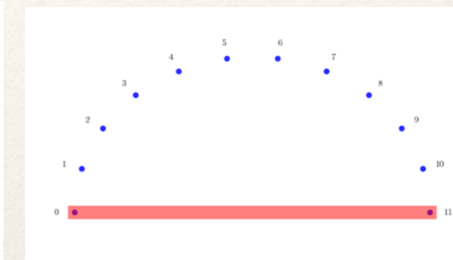


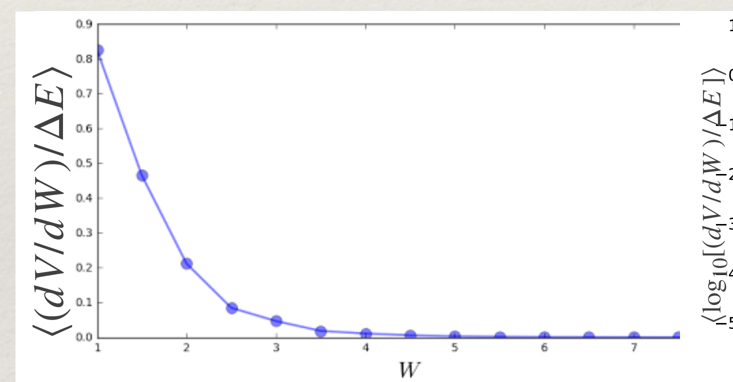


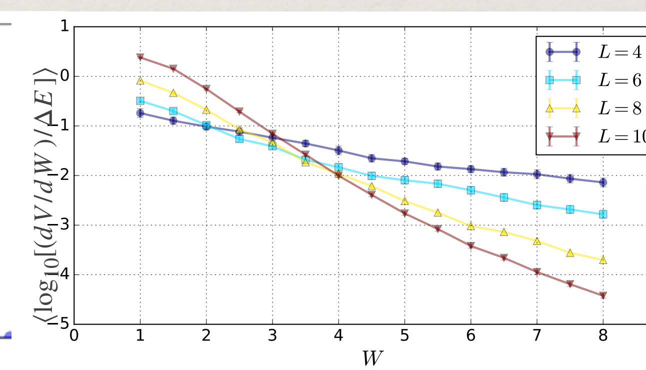


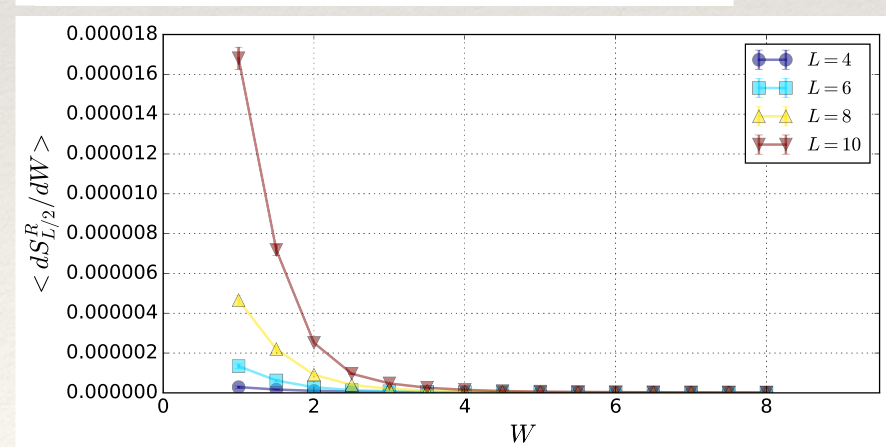


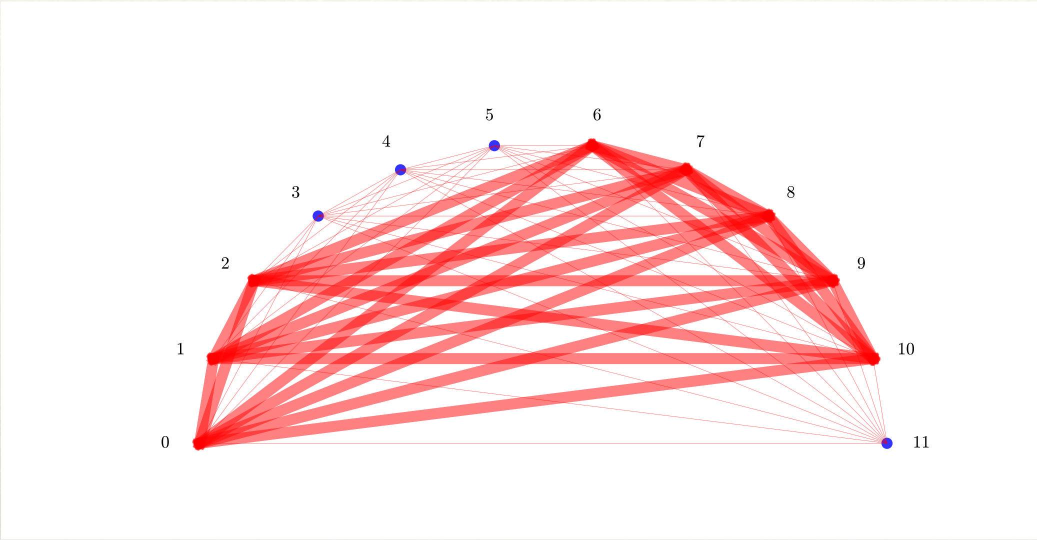




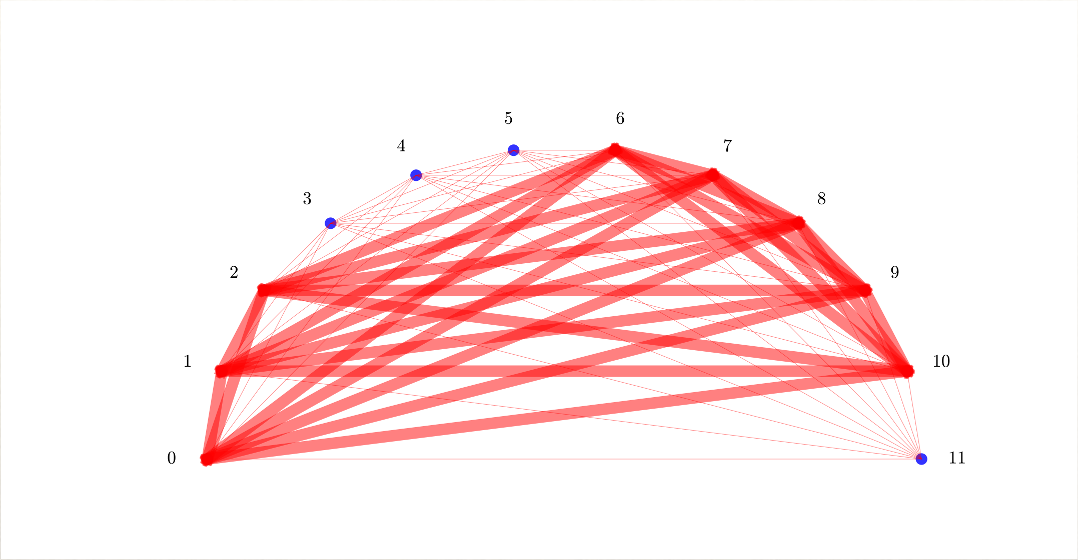








0
1
2
3
4
5
6
7
8
9
10
11



0
1
2
3
4
5
6
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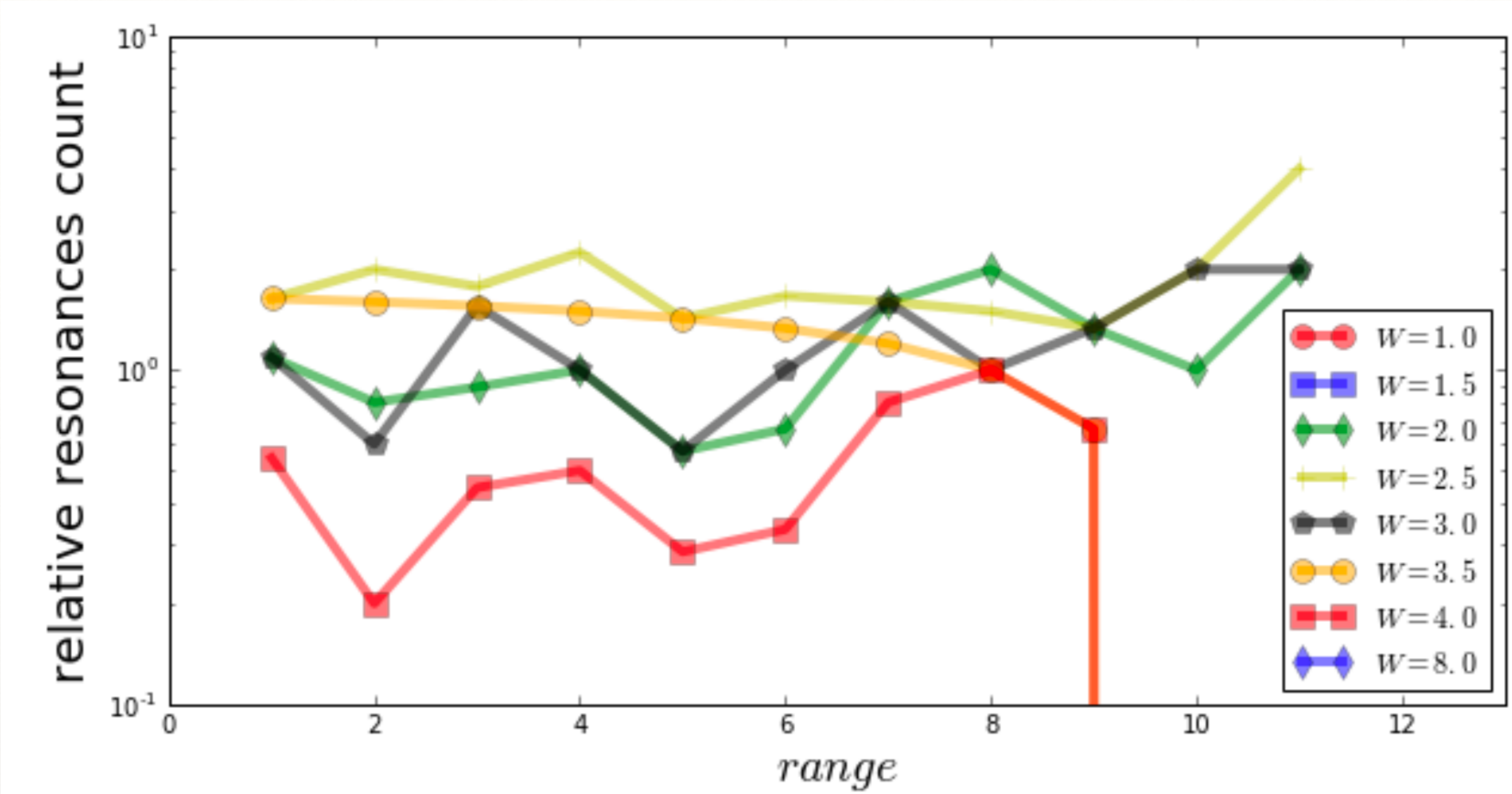
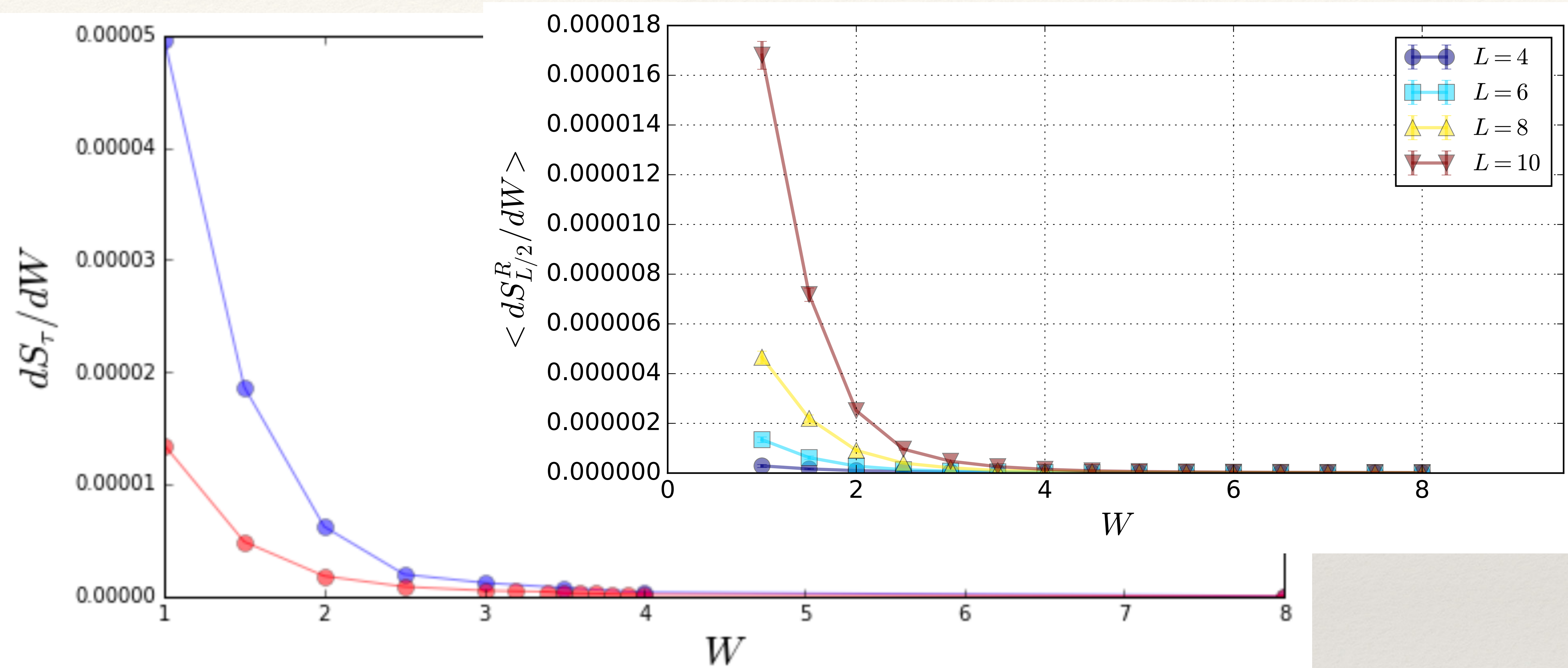
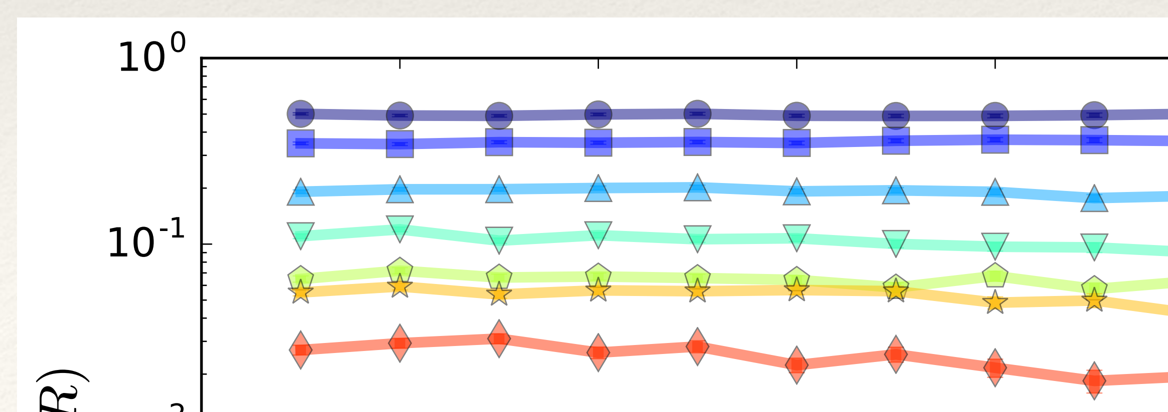
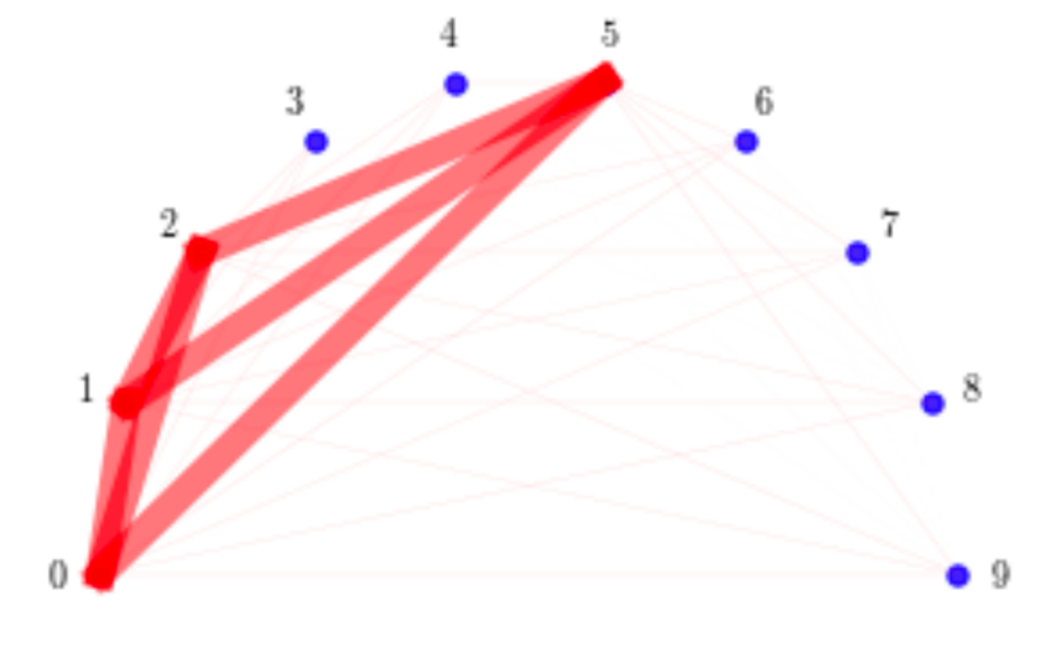
8
9
10
11



relative resonances count
10^1
10^0
10^{-1}
0
2
4
6
8
10
12
range
W = 1.0
W = 1.5
W = 2.0
W = 2.5
W = 3.0
W = 3.5
W = 4.0
W = 8.0


0.00005
0.00004
0.00003
0.00002
0.00001
0.00000
dS_τ/dW
1
2
3
4
5
6
7
8
W
0.000018
0.000016
0.000014
0.000012
0.000010
0.000008
0.000006
0.000004
0.000002
0.000000
$< dS^R_{L/2}/dW >$
0
2
4
6
8
W
$L=4$
$L=6$
$L=8$
$L=10$


10^0
10^{-1}



0
1
2
3
4
5
6
7
8
9

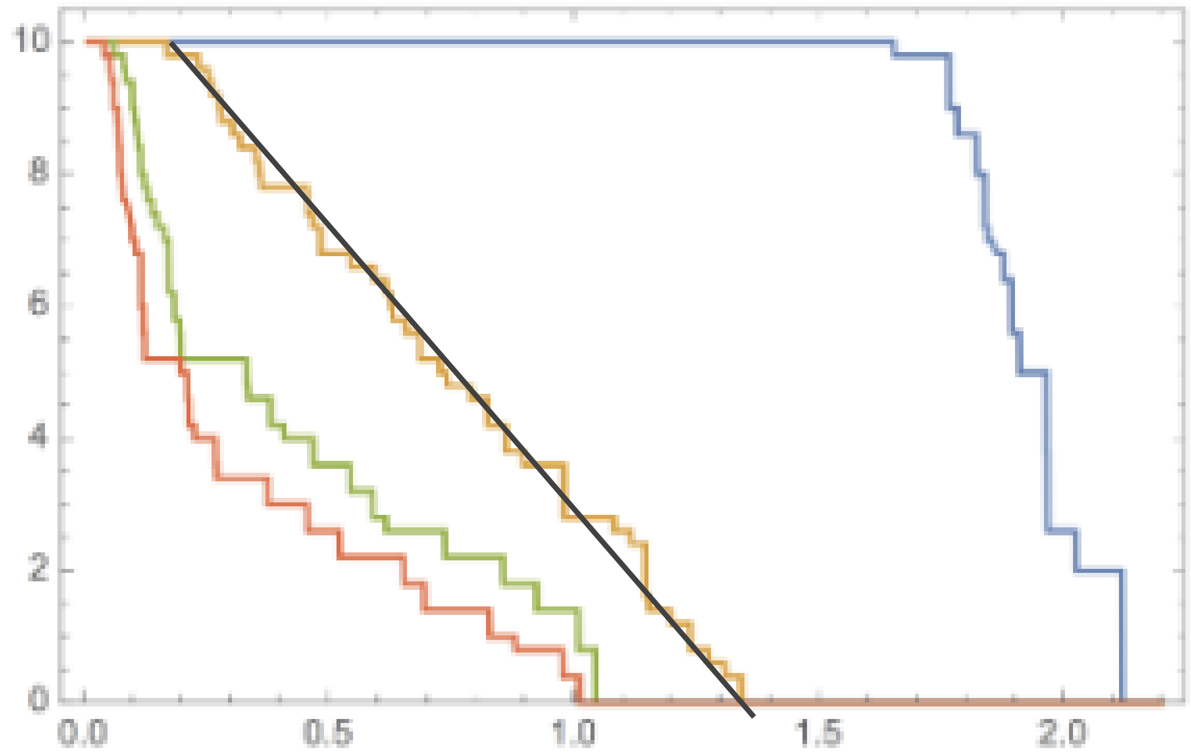


FIG. 21. stuff