Resonances and Renormalizing Tensor Networks in Many Body Localization

Perimeter/Urbana Conference

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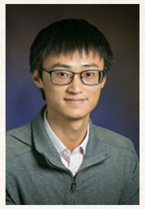
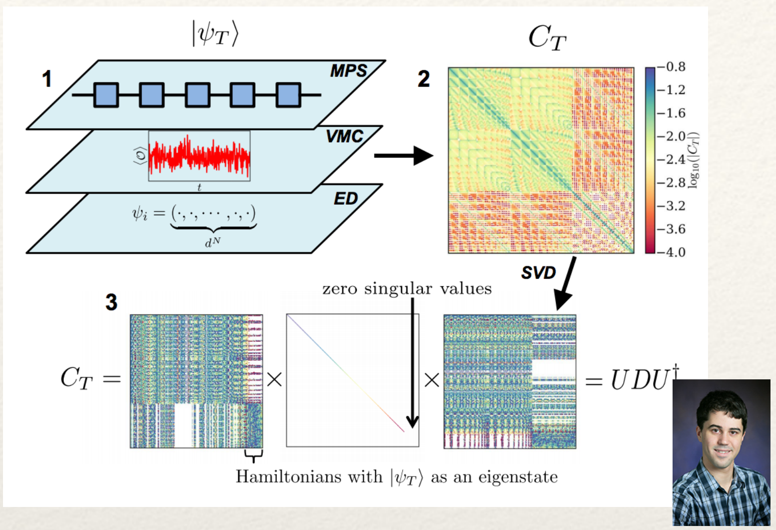


Xiongjie Yu

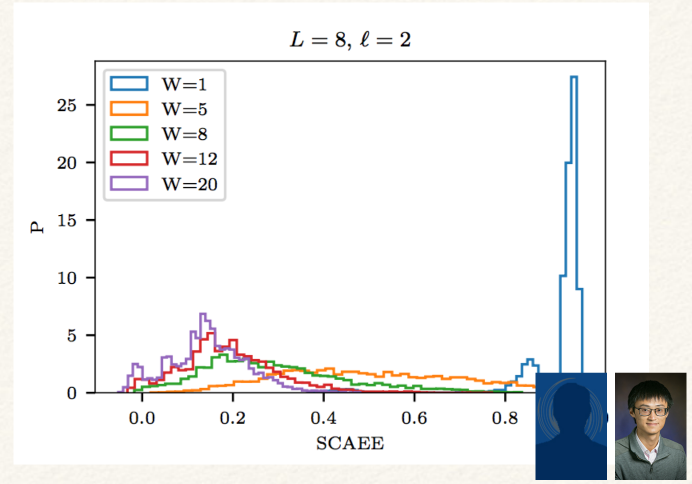
David Pekker

Benjamin Correa

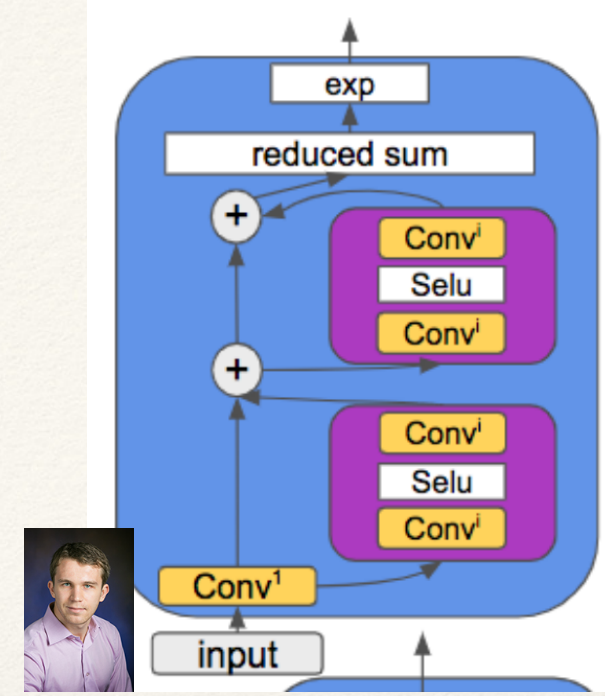
Not this talk



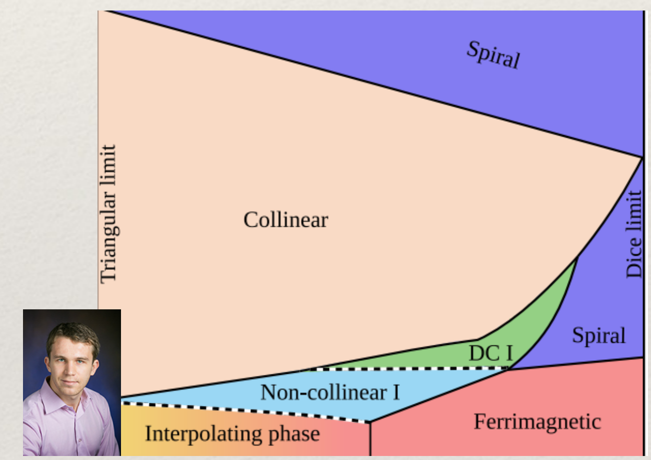
Automatically find parent Hamiltonians



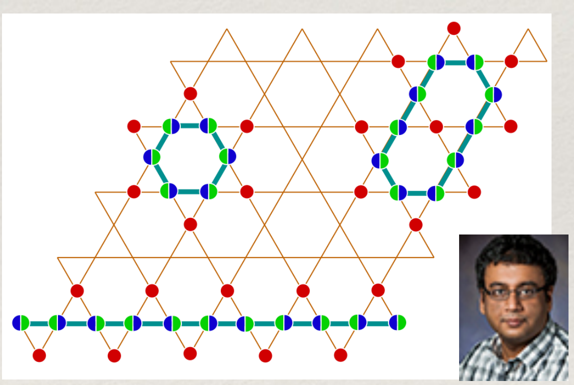
Eta-pairing and entanglement beyond MBL



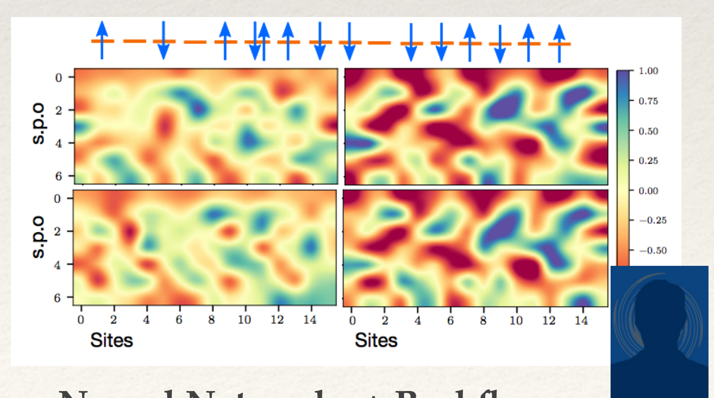
Better machine learning wave-functions



Phase diagram of stuffed honeycomb

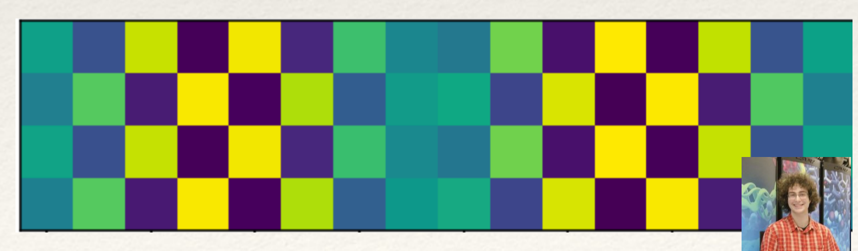


An RVB analogue for quantum colors



Neural Networks + Backflow

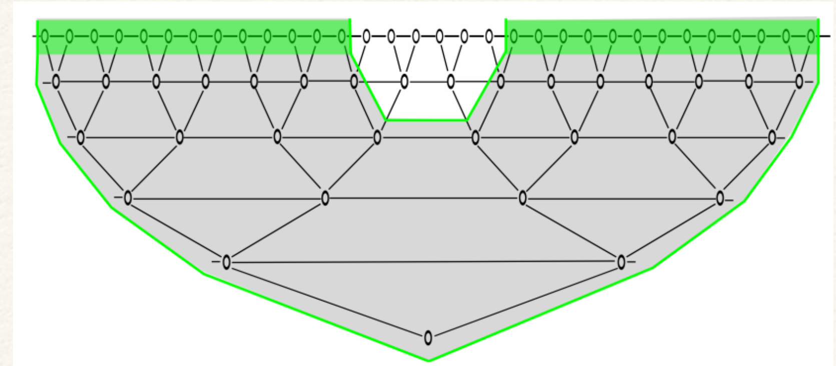
[Di Luo]



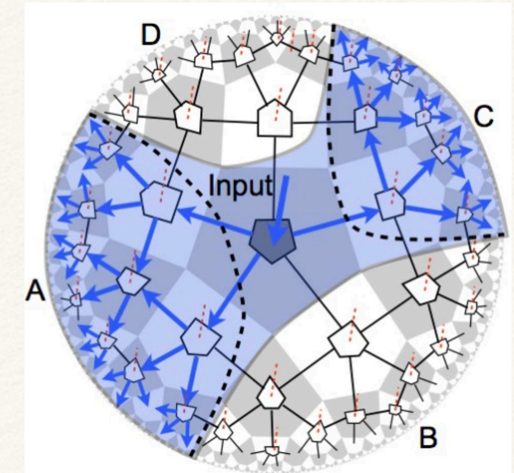
Pair and Charge Density Waves

Uses of `Bulk' Tensor Networks…

Holographic Tensor Networks…



[Swingle]



[Fernando Pastawski, Beni Yoshida, Daniel Harlow, John Preskill]

Transitions in Random Tensor Networks

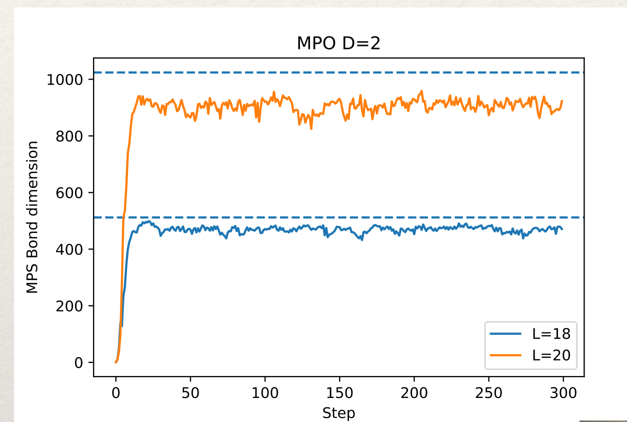
Small D

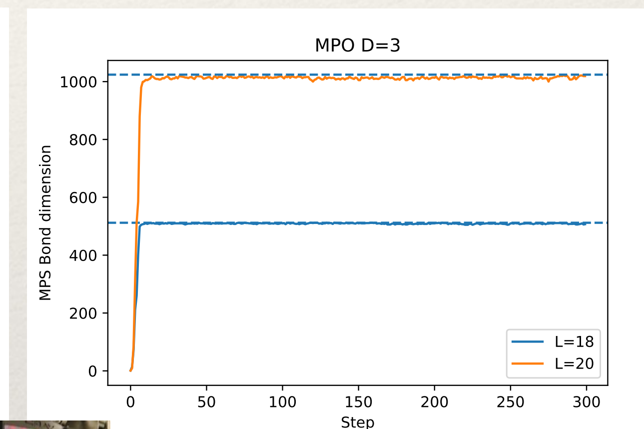
Area Law

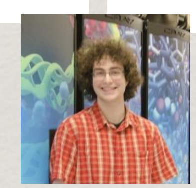
Large D

Volume Law

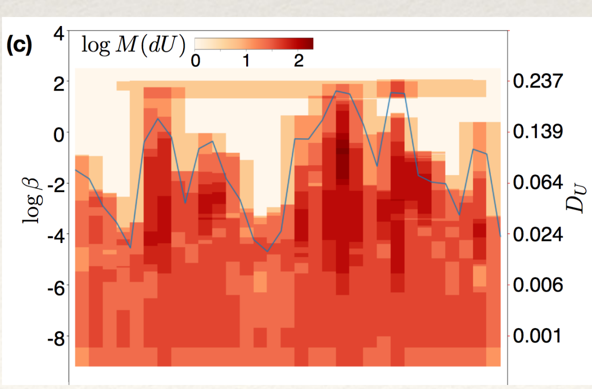
[Romain Vasseur, Andrew C. Potter, Yi-Zhuang You, and Andreas W. W. Ludwig]



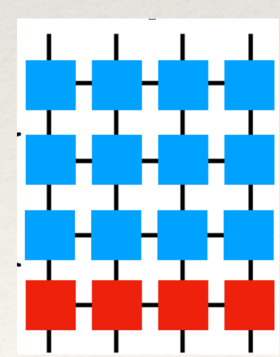




Today:



MBL

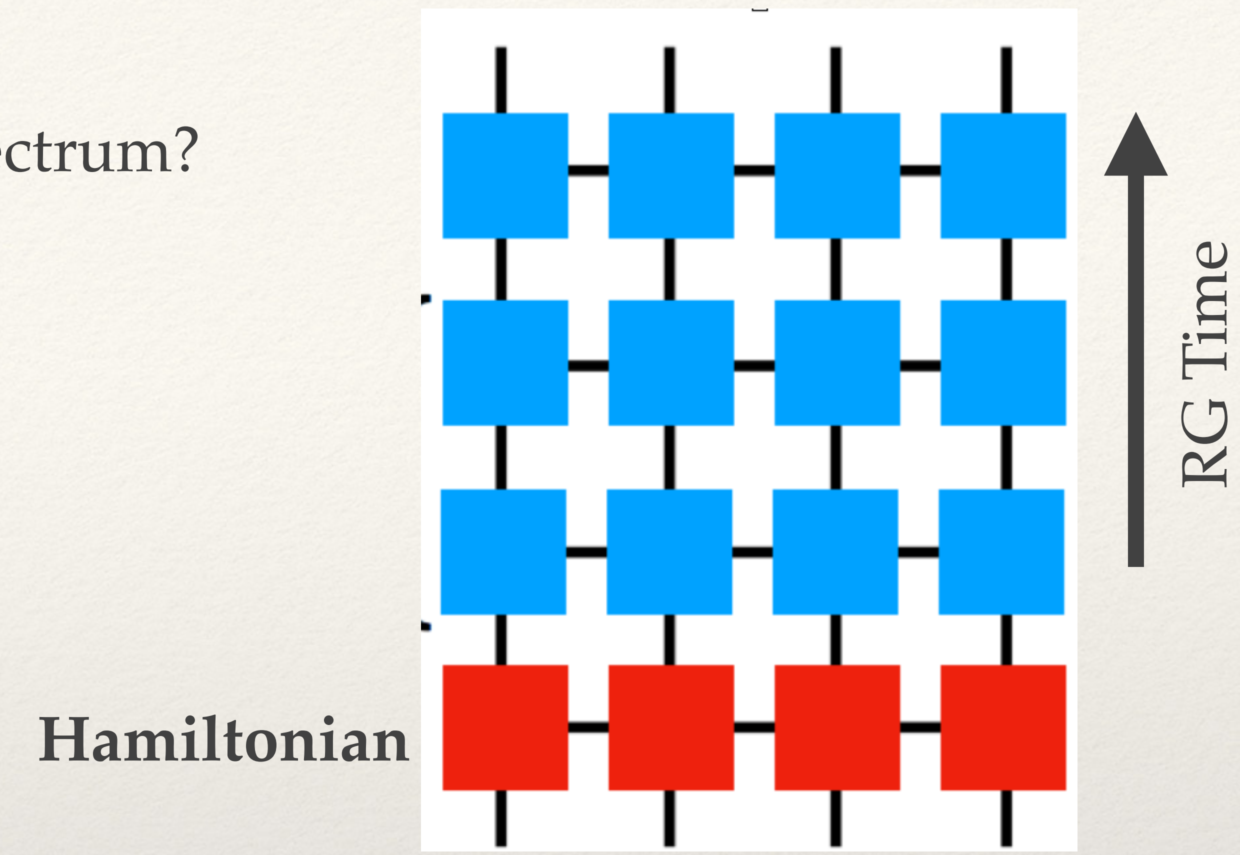


Ergodic

[Pekker, Clark]
[Yu, Pekker, Clark]
Variational approaches:
[Khemani, Sondhi], [Pal, Simons]

Can we build a tensor network, via renormalization, which

- diagonalizes a boundary Hamiltonian?
- diagonalizes chunk of Hamiltonian spectrum?
- generates an eigenstate



and then probe properties of this renormalized tensor network.

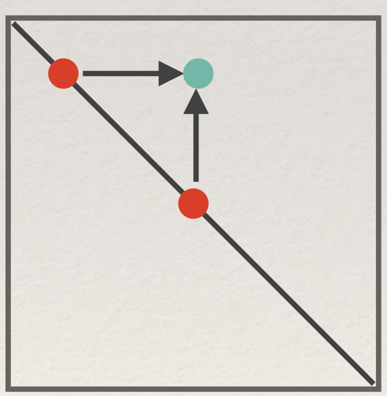
- operator spreading?
- Bulk properties which change between MBL and ergodic phase?

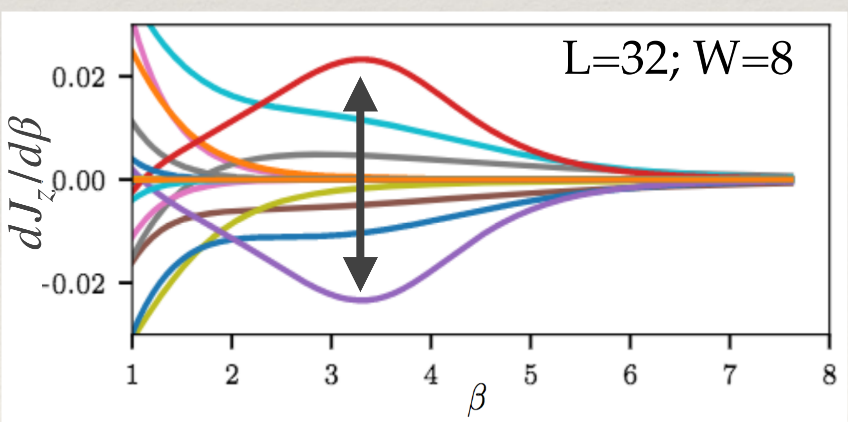
Wegner-Wilson Flow

The Wegner-Wilson Flow is a unitary RG process which diagonalizes a Hamiltonian.

$$H(t) = H_D(t) + H_{OD}(t)$$
$$\eta(t) = [H_D(t), H_{OD}(t)]$$
$$U(\Delta t) = \exp\left(i\Delta t \eta(t)\right)$$
$$H(t+\Delta t) = U(\Delta t) H(t) U^\dagger(\Delta t)$$
$$U(t+\Delta t) = U(\Delta t)U(t)$$

Why RG: It disentangles the largest diagonal energy scales connected by a `non-zero' off-diagonal element first.

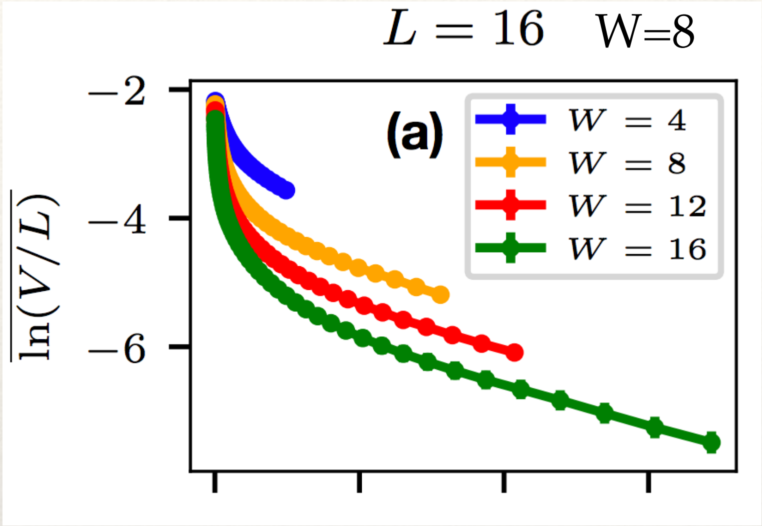




$$H = \sum_i J_i \sigma_z^i + [\dots]$$

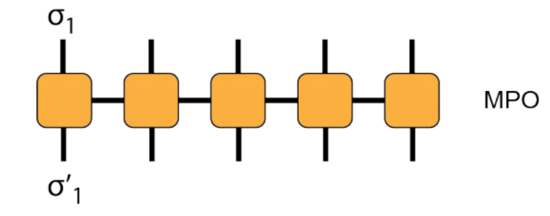
The variance $V(\beta) \equiv (1/N)\sum_{i\neq j} H_{ij}(\beta)$ decreases monotonically with β

as $V(\beta) = \exp[-\beta(E_i - E_j)^2]$

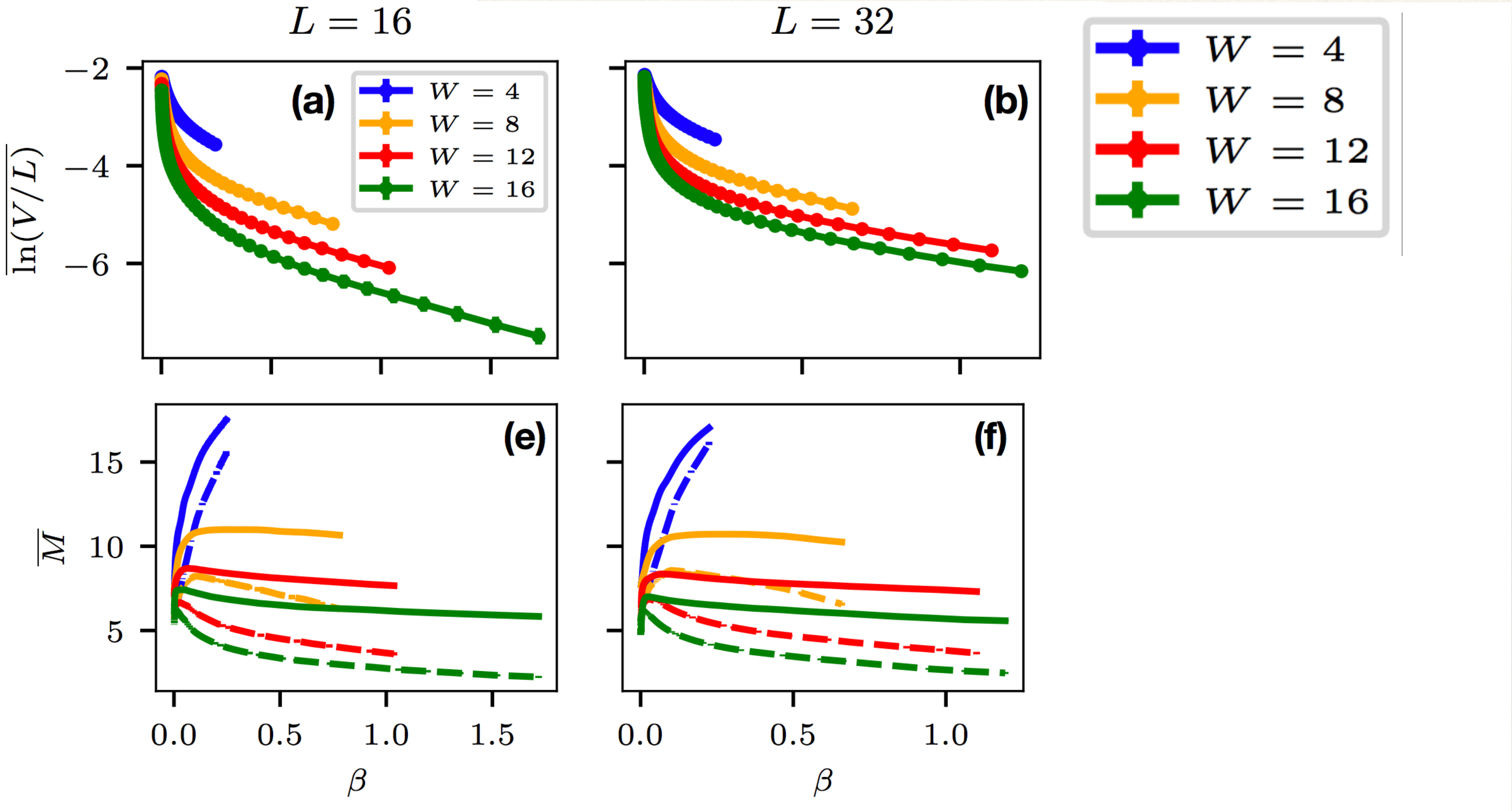


Turn the unitary into a tensor network

Take Hamiltonian as MPO

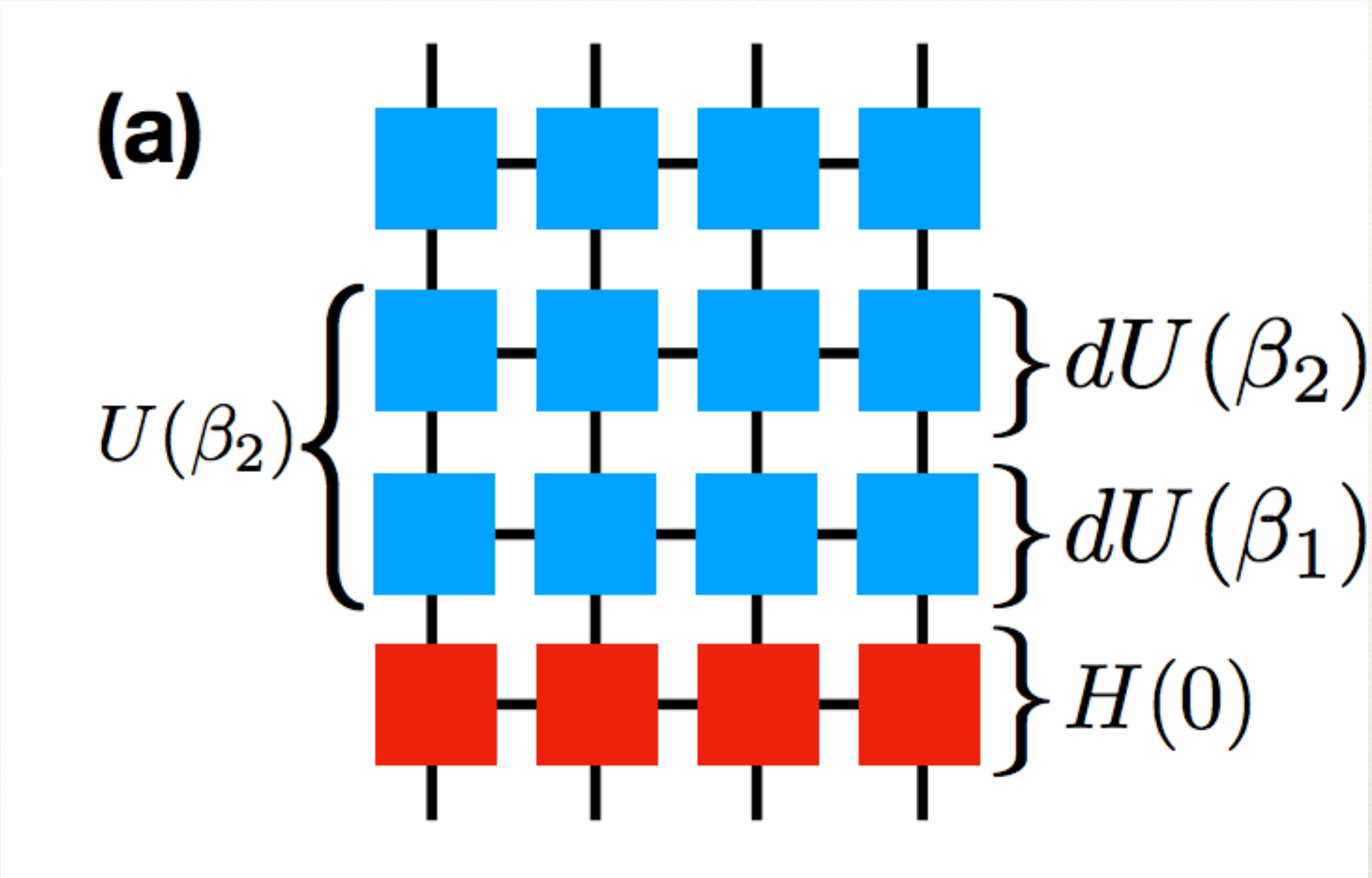


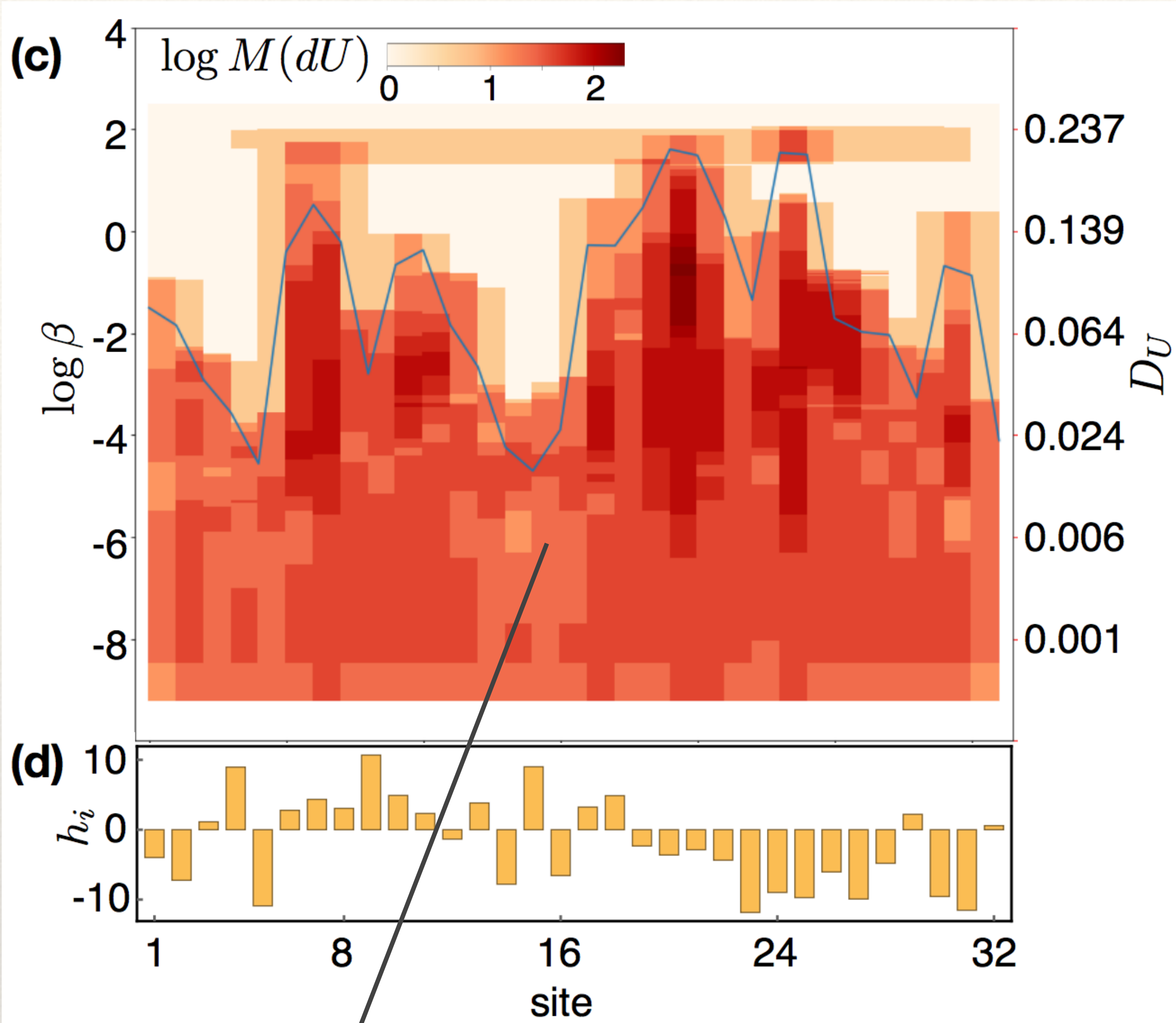
$$
\begin{aligned}
&H(\beta+\Delta\beta)\\
&= H + \Delta\beta[\eta, H] + \frac{\Delta\beta^2}{2!}[\eta,[\eta,H]] + \frac{\Delta\beta^3}{3!}[\eta,[\eta,[\eta,H]]] + \cdots\\
&= H + \Delta\beta\left[\eta,\ H + \frac{\Delta\beta}{2}\left[\eta,\ H + \frac{\Delta\beta}{3}\left[\eta,\ H + \cdots\right]\right]\right]
\end{aligned}
$$



Works efficiently in the MBL phase.

Gives l-bits, eigenstates, etc.


(a)
$U(\beta_2)$
$dU(\beta_2)$
$dU(\beta_1)$
$H(0)$



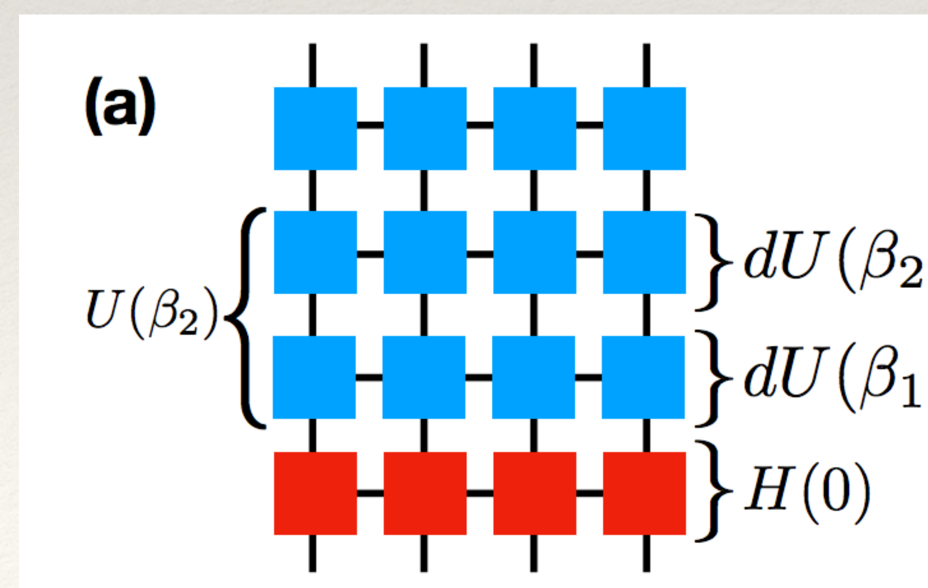
$$D_U(\beta) = \int_0^\beta \sqrt{\frac{Tr(\eta(\tau)\eta^\dagger(\tau))}{\dim(H)L}}d\tau = \int_0^\beta \sqrt{-\frac{1}{2L}\frac{dV(\tau)}{d\tau}}d\tau$$

(Generalized) version of distance used in cMERA [Ryu, et. al]

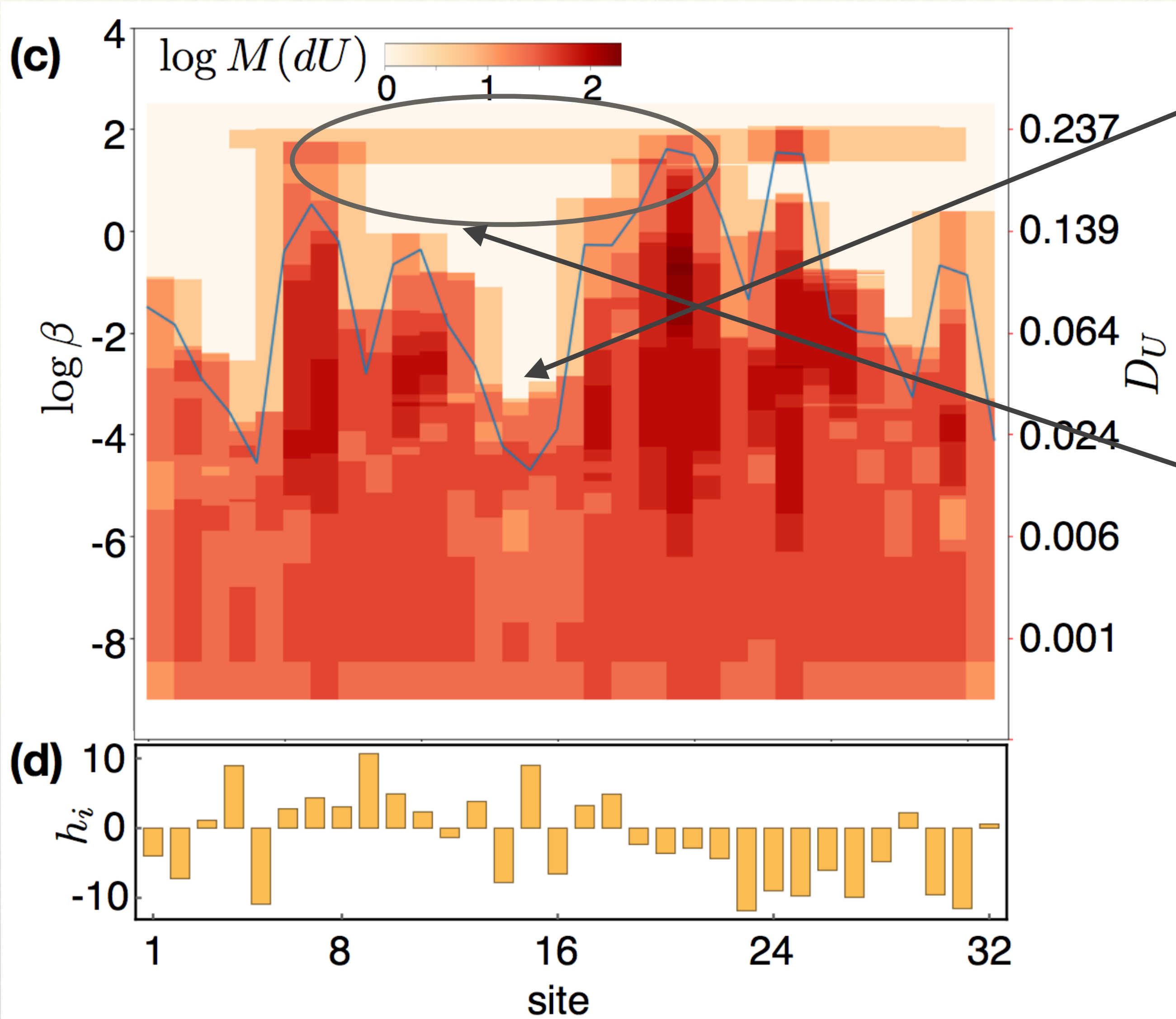
Here it is connected to variance.

RT analogue - geodesics related to entanglement.

Log(Bond Dimension)



Understanding the MBL bulk…

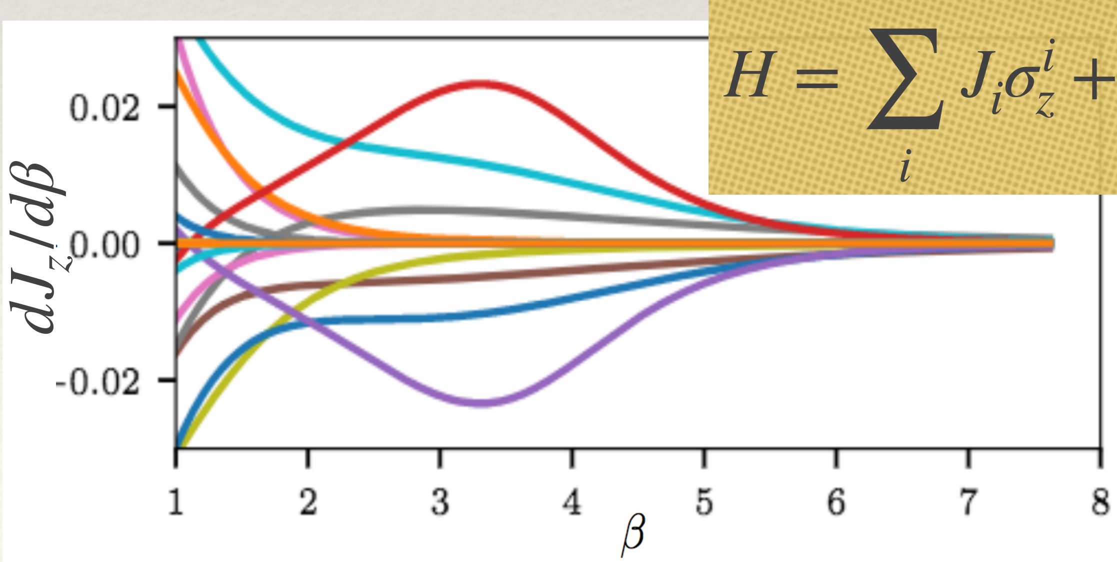


Here these l-bit's are diagonal (i.e. popped out of the system)

They have stopped rotating.

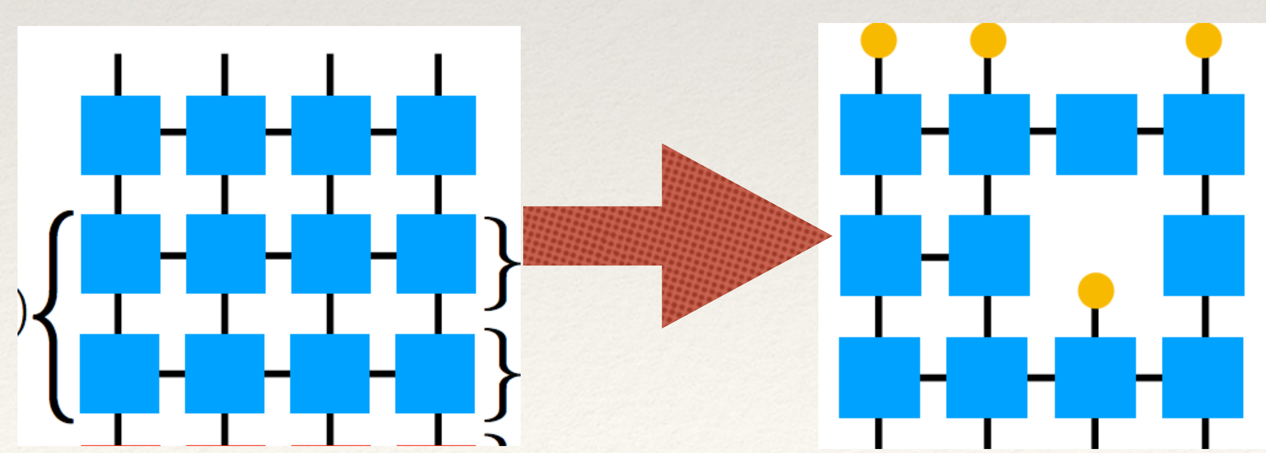
This bar is essentially (up to control-bits) connecting distance l-bits which disentangle at the lowest energy scales.

If I fix the l-bit eigenvalues, we generate a MERA for the state.



$$H = \sum_i J_i \sigma_z^i + [\ldots]$$

Unitary **State**



As $E \sim 1/\beta^2$ gives an energy scale for the l-bits

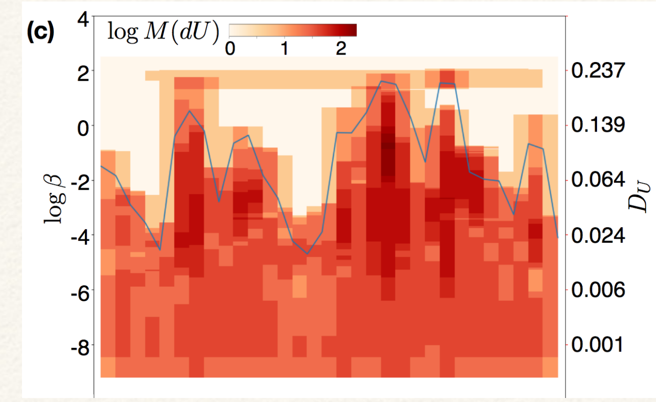
A real space picture…

Unitary distance

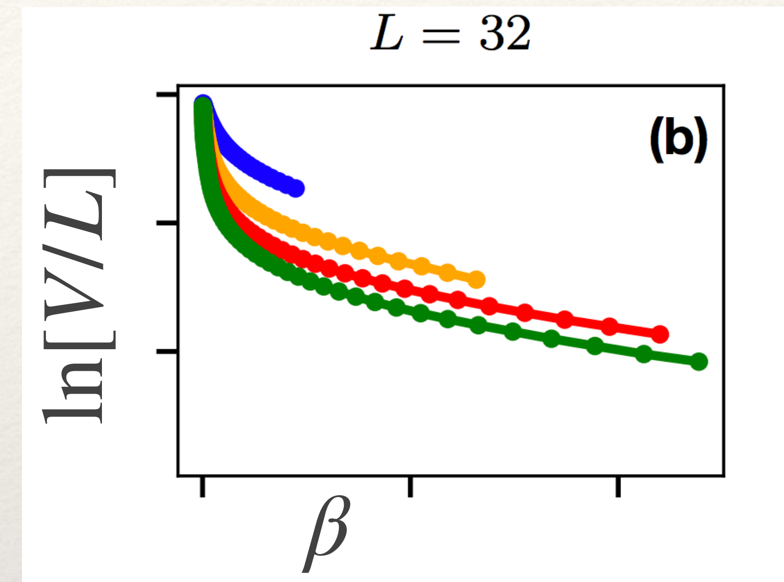
To explore rare regions, we define a distance measure in the RG time…

$$D_U(\beta) = \int_0^\beta \sqrt{\frac{Tr(\eta(\tau)\eta^\dagger(\tau))}{\dim(H)L}} d\tau = \int_0^\beta \sqrt{-\frac{1}{2L}\frac{dV(\tau)}{d\tau}} d\tau$$

where $U(\Delta t) = \exp[i\eta(t)\Delta t]$



(Generalized) version of distance used in cMERA.
Here it is connected to variance.

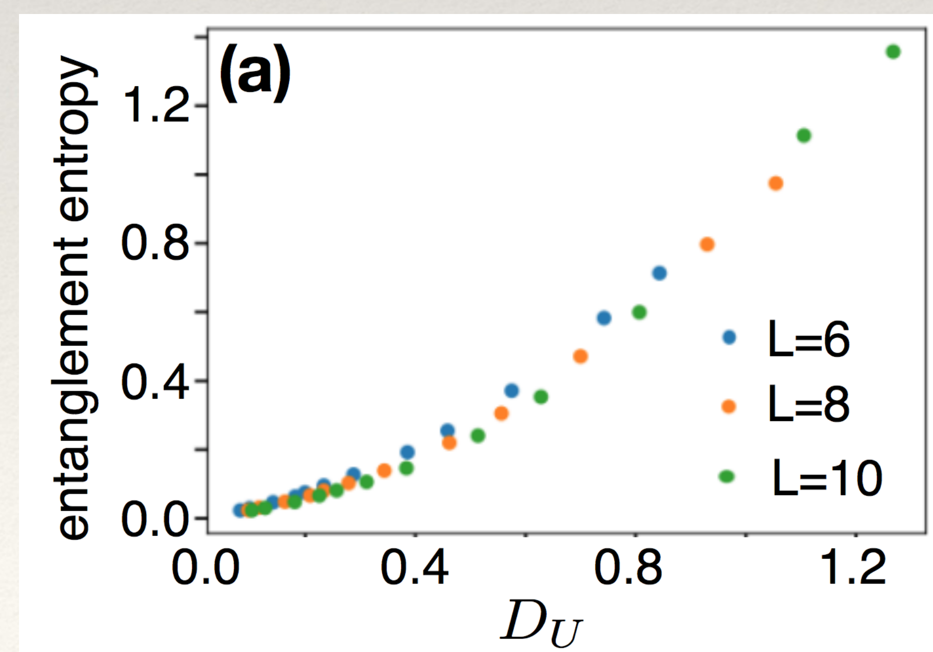


There is a RT theorem analogue.

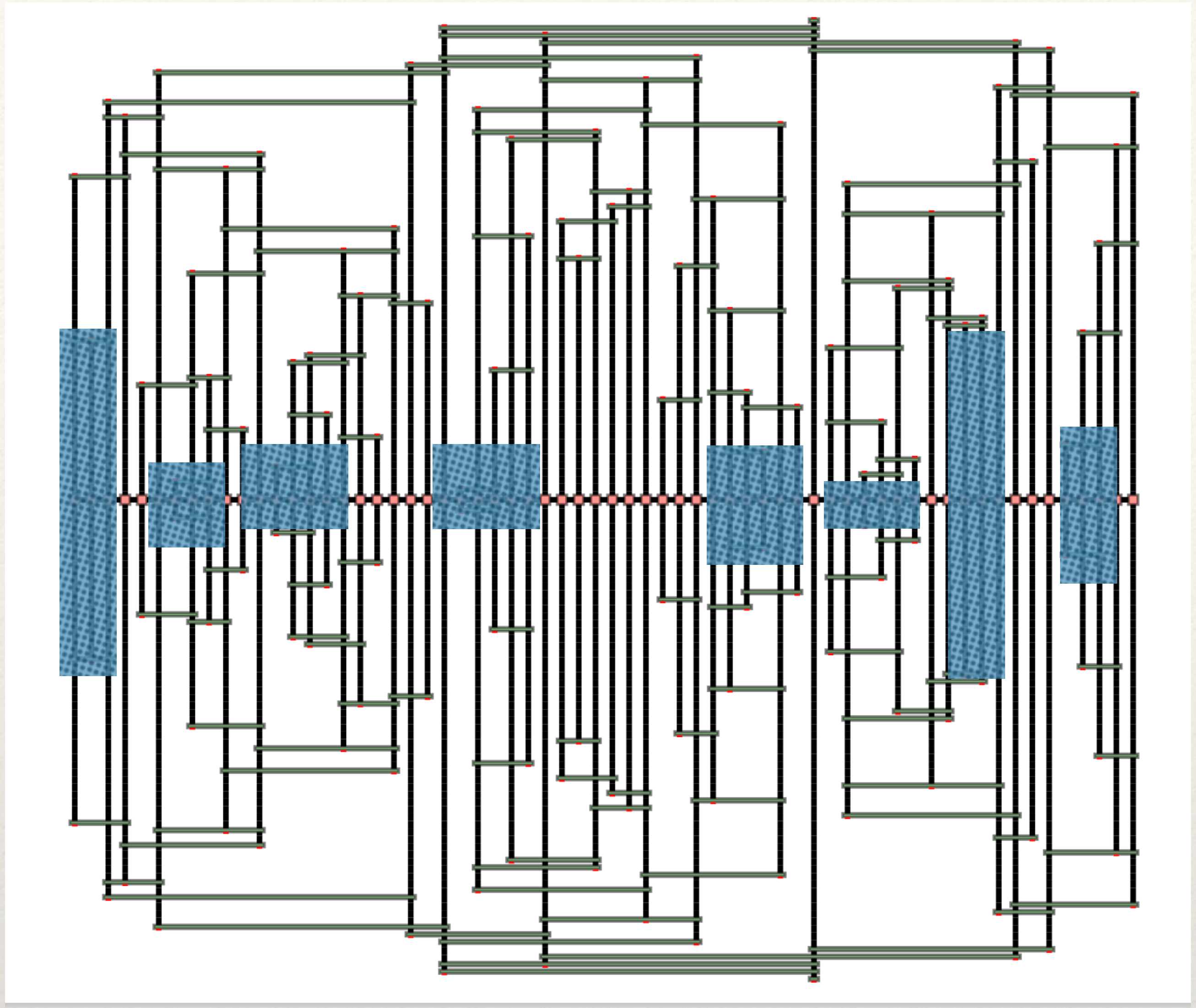
Most the geodesics are through the ceiling

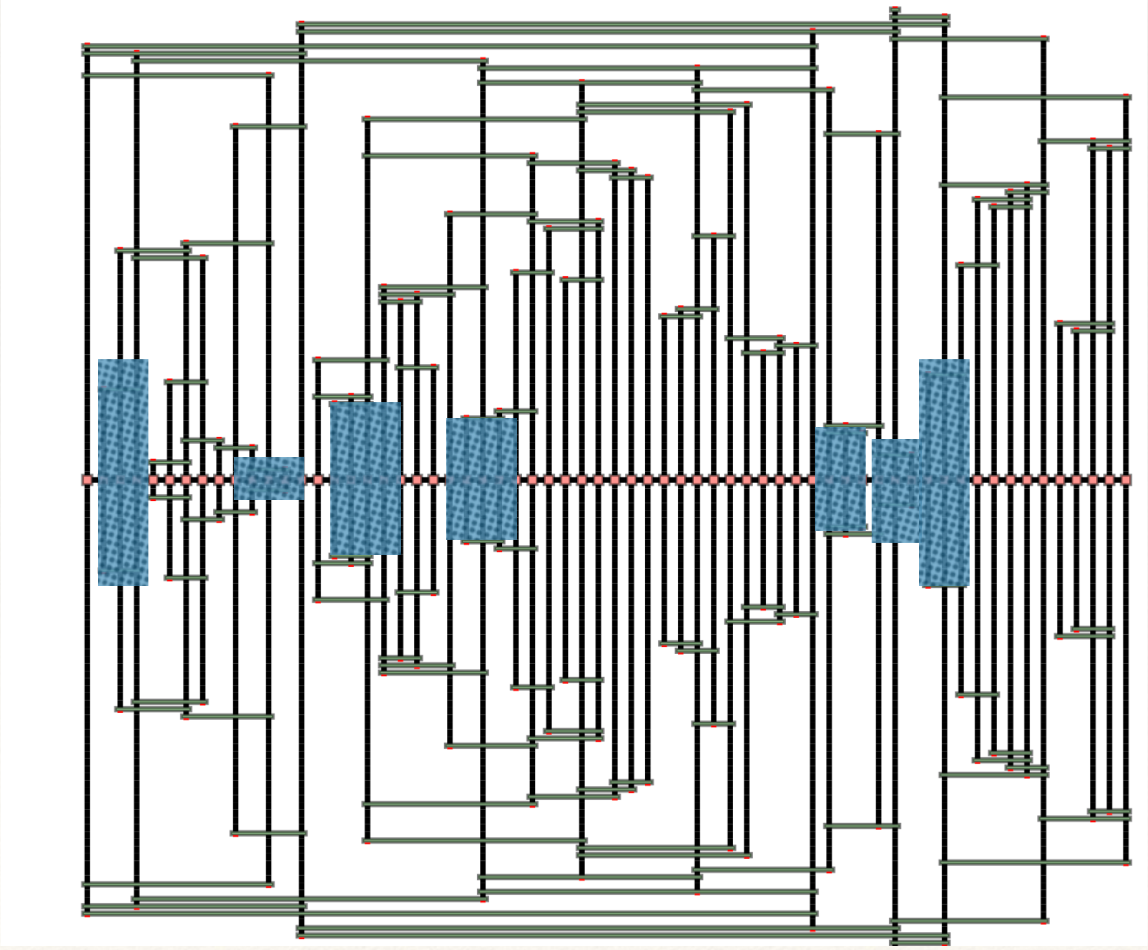
$\langle S \rangle \propto D_u^2$

Average over all eigenstates.



An alternative approach....

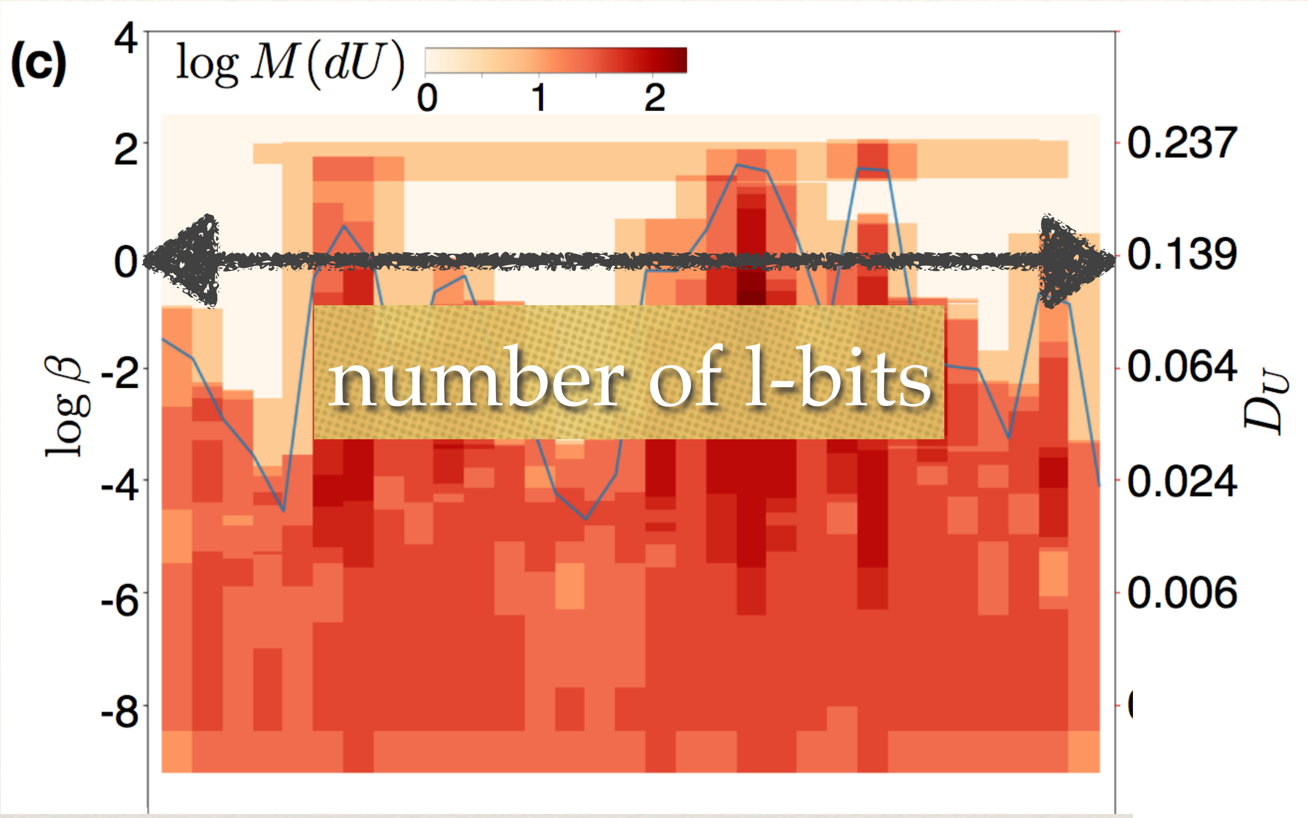




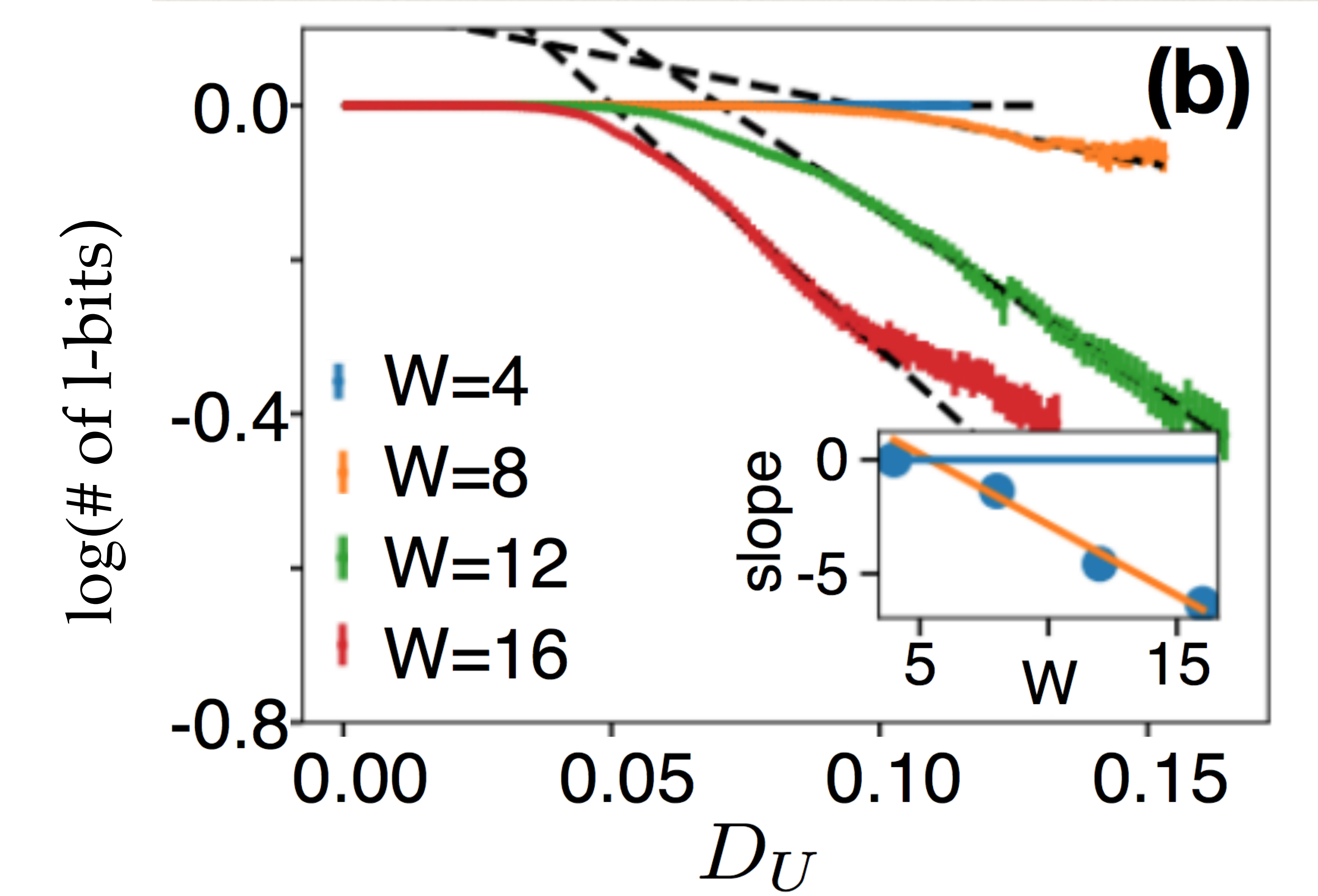
Understanding the MBL bulk…

At what rate do the l-bits pop out of MBL bulk?

Number of l-bits remaining decay exponentially



Evidence of rare regions at all scale!



At what rate do the l-bits pop out of …. ergodic bulk?

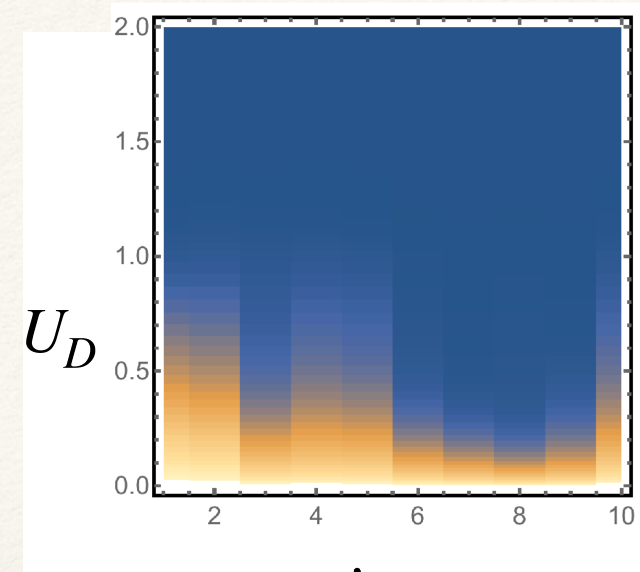
At what rate do the l-bits pop out of …. critical bulk?

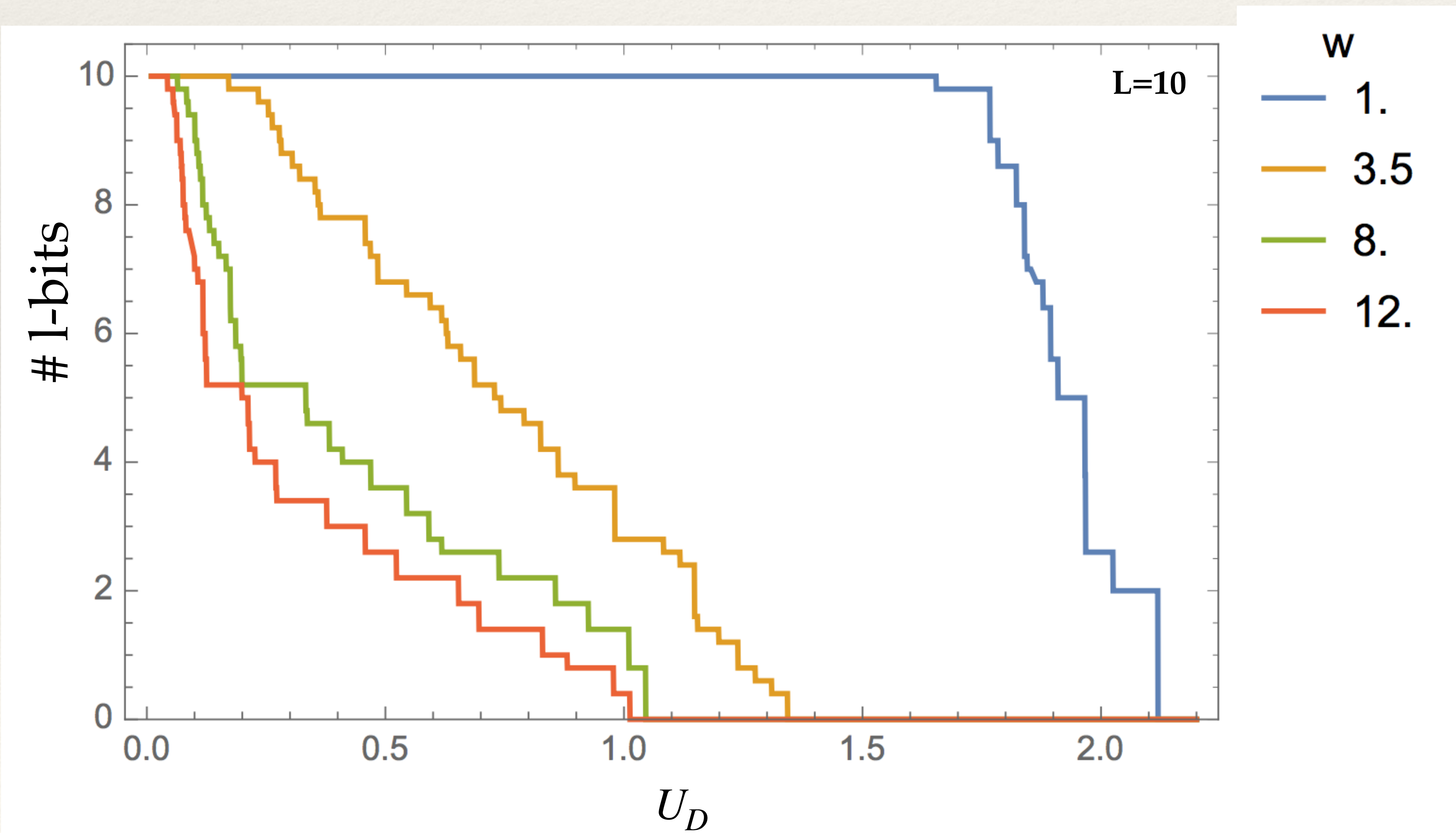
Ergodic: All the `l-bits' pop out at the end.

Transition: `l-bits' pop uniformly.

MBL: `l-bits' pop exponentially quickly.

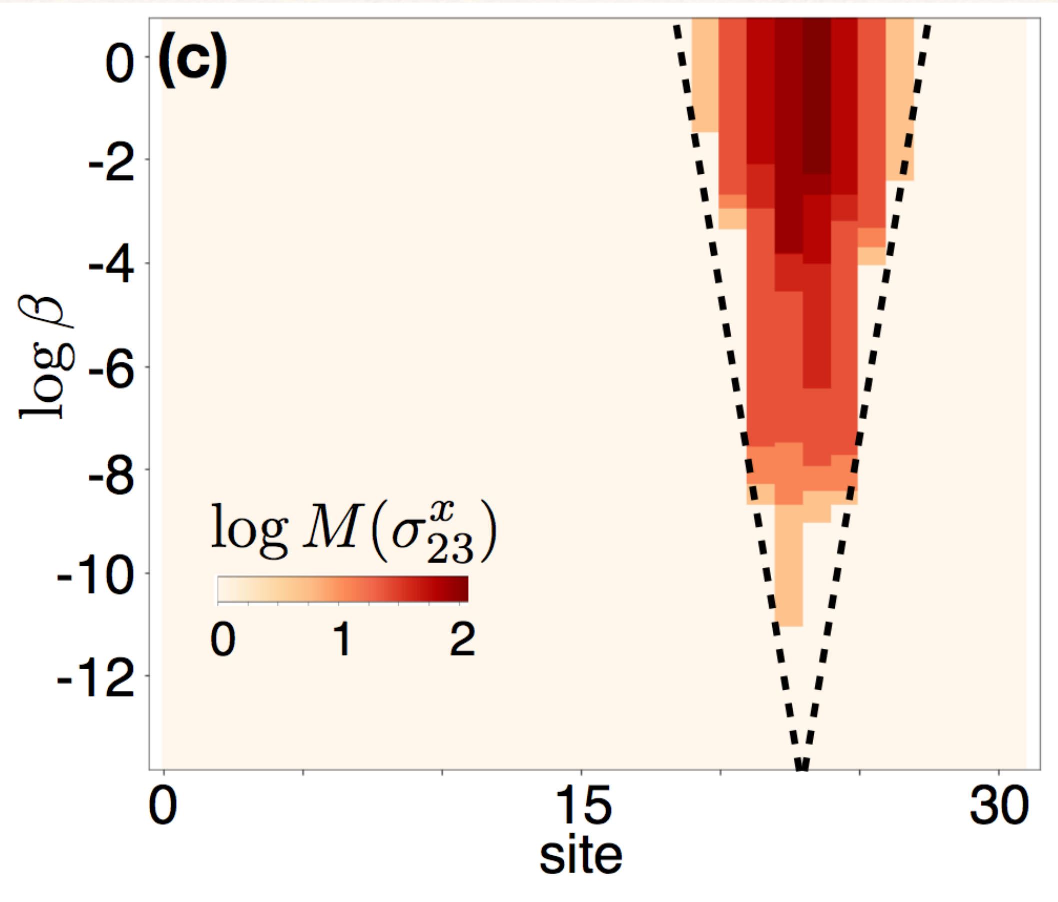
W=3.5



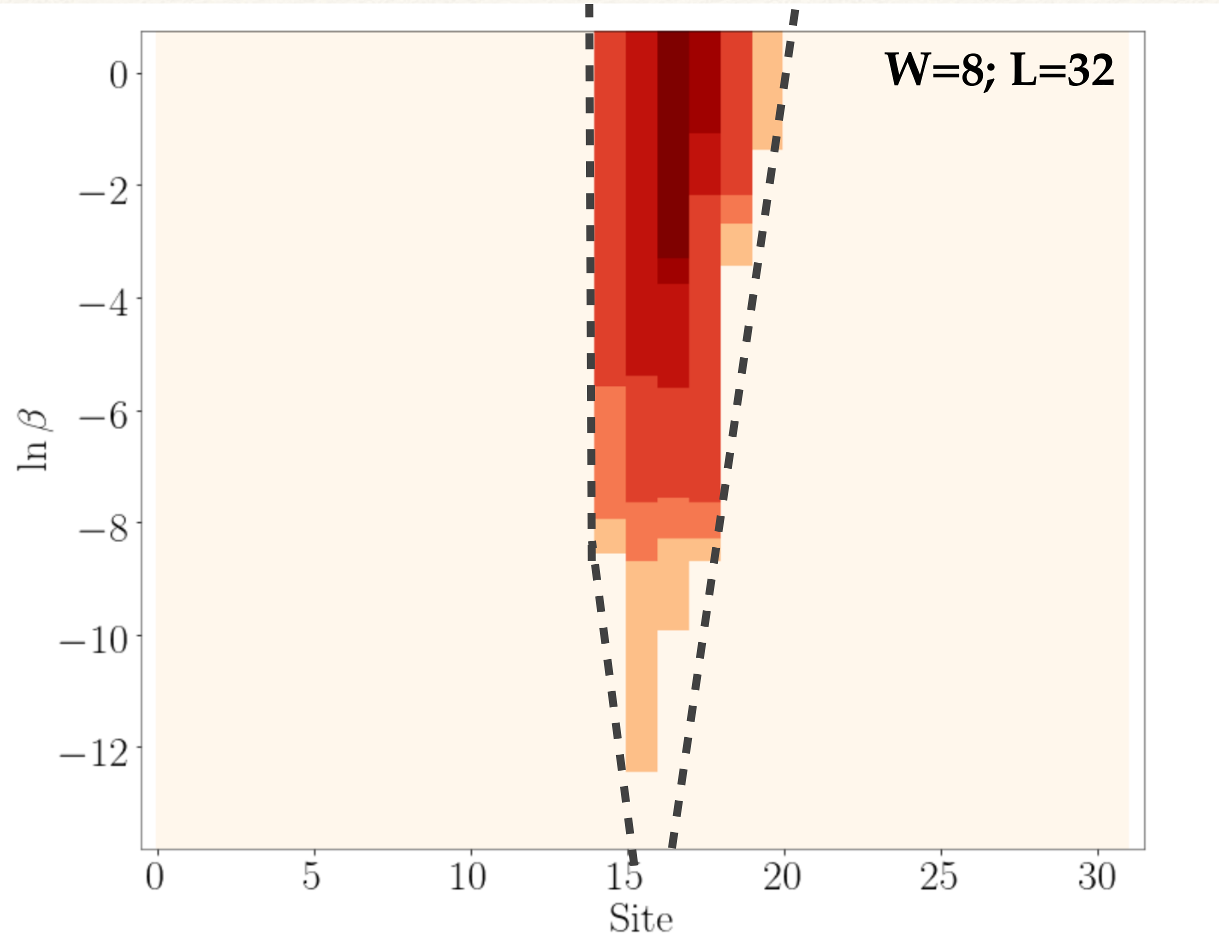


In the MBL bulk, how fast do operators spread?

In the MBL phase



Operator light cone spread as $\log(\beta)$

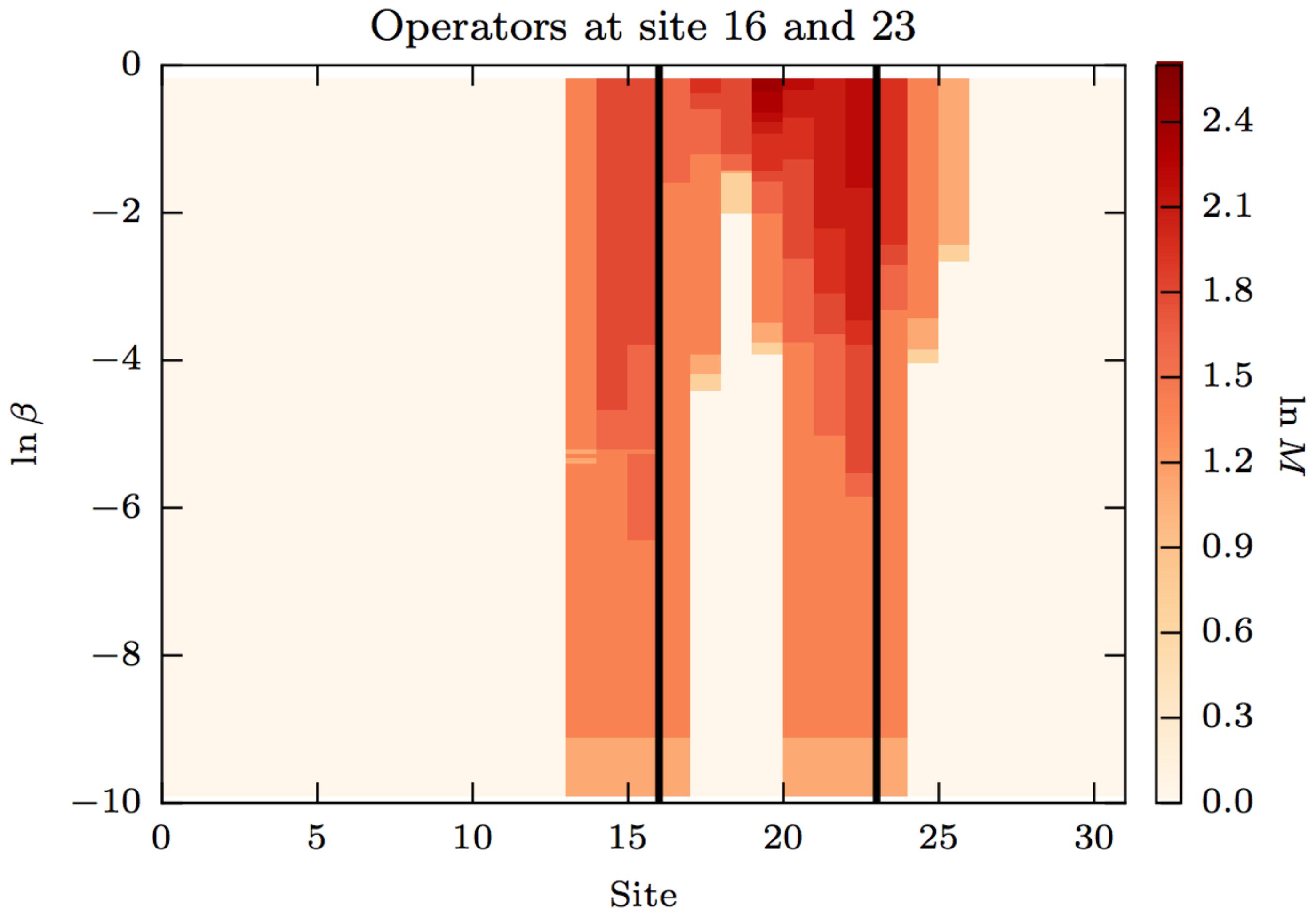


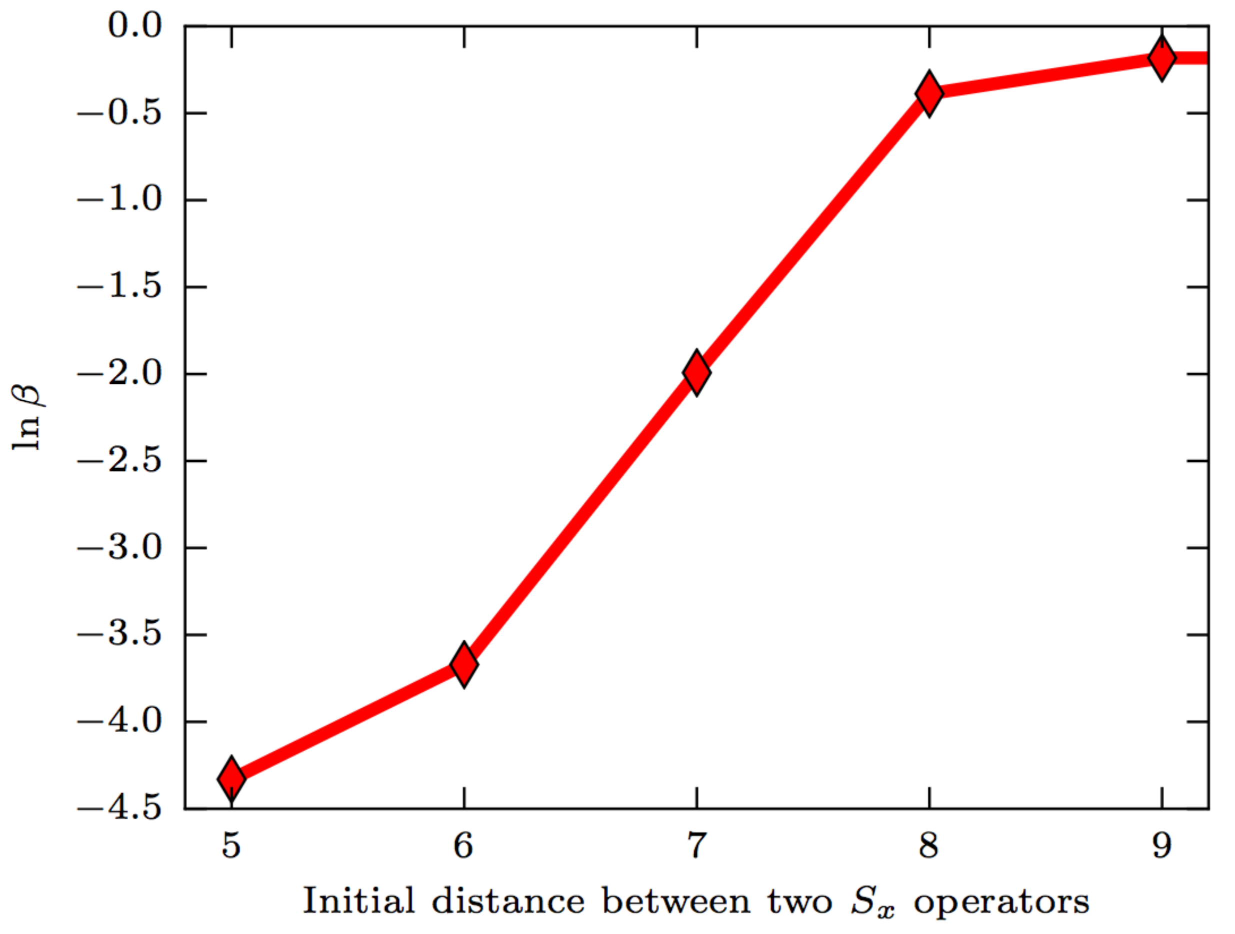
until they hit a wall.

Energy scale of coupled l-bits: e^{-L_0} where L_0 is cutoff

In the MBL bulk, how fast do operators spread?

You can also see it by looking at how long it takes light cones from two operators to collide.



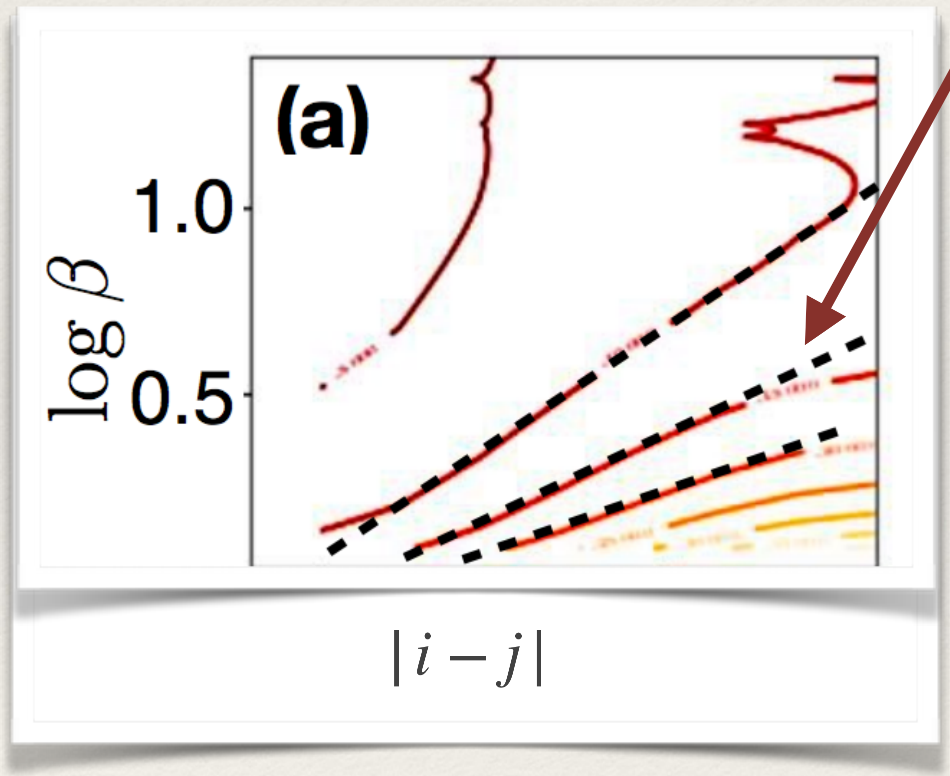


In the MBL bulk, how fast do operators spread?

You can also see it by looking at contours of the spread of the operator in its Pauli expansion.

$$U(\beta)\hat{O}U^{\dagger}(\beta) = [\ldots] + \sum_{ij} V_{ij}(\beta)\sigma_i\sigma_j$$

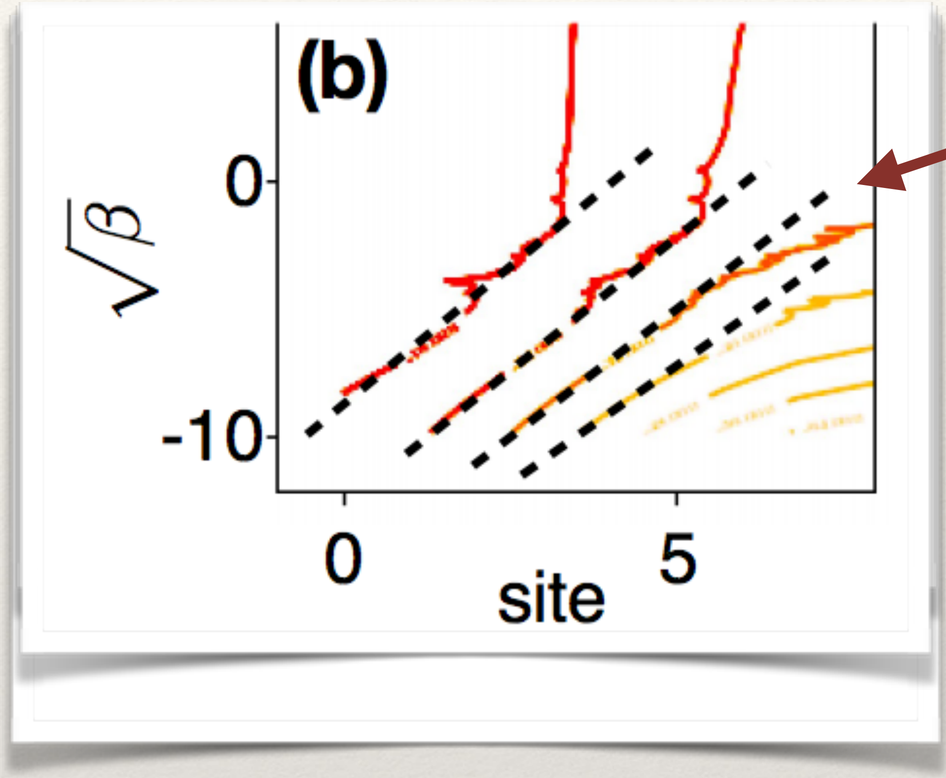
Contours of V



In the ergodic bulk, how fast do operators spread?

operator light cone spread as $\sqrt{\beta}$

$$U(\beta)\hat{O}U^{\dagger}(\beta) = [\ldots] + \sum_{ij} V_{ij}(\beta)\sigma_i\sigma_j$$



Contours of V

l-bit energy scales as 1/E

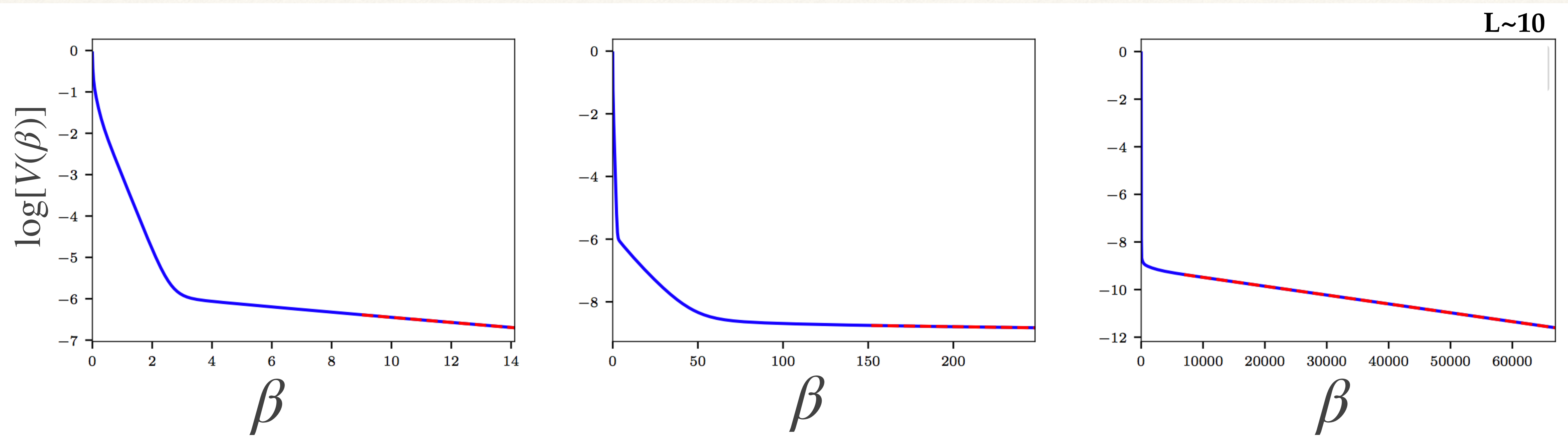
In the critical bulk, how fast do operators spread? *(speculation)*

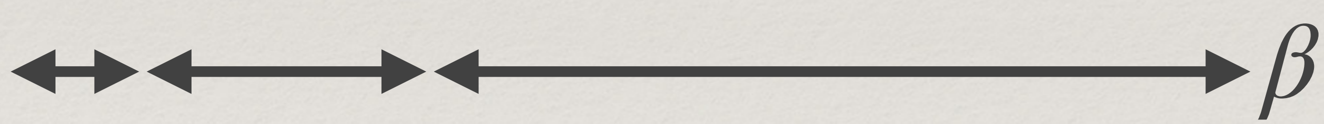
Log without any ceiling.

The bulk at infinite beta tells us about the transition.

Variance shrinks as $V(\beta) = \exp[-\beta(\Delta E)^2]$

Unitary Distance shrinks as $\Delta E \exp[-\beta/2(\Delta E)^2]$



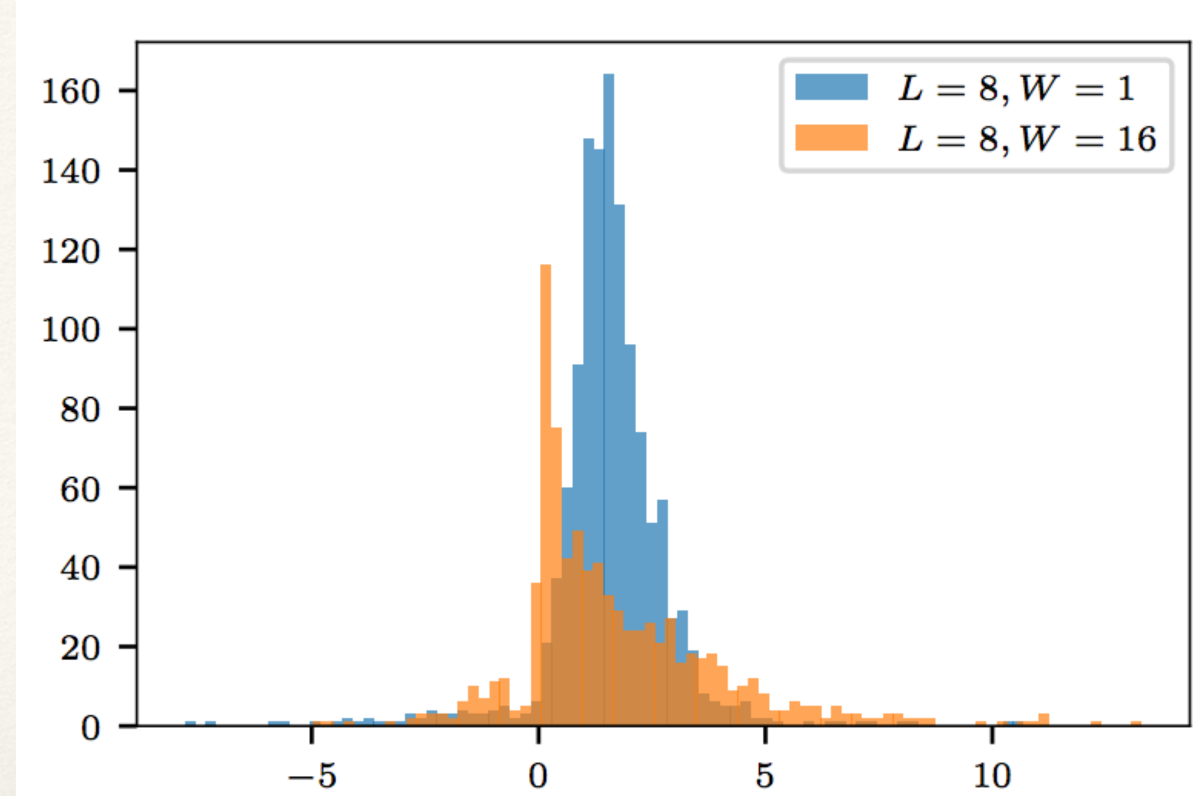


Different Epochs in the bulk

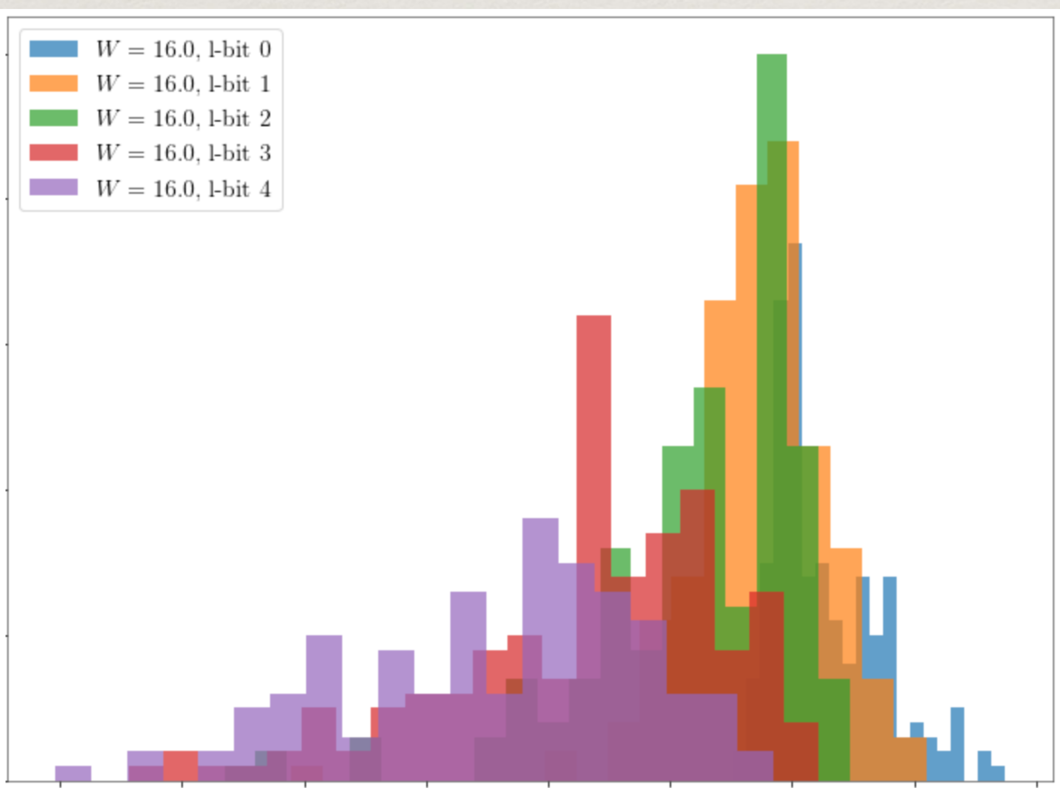
In the bulk, can you see the level spacing?

Energy must get down to `inter-level spacing'.

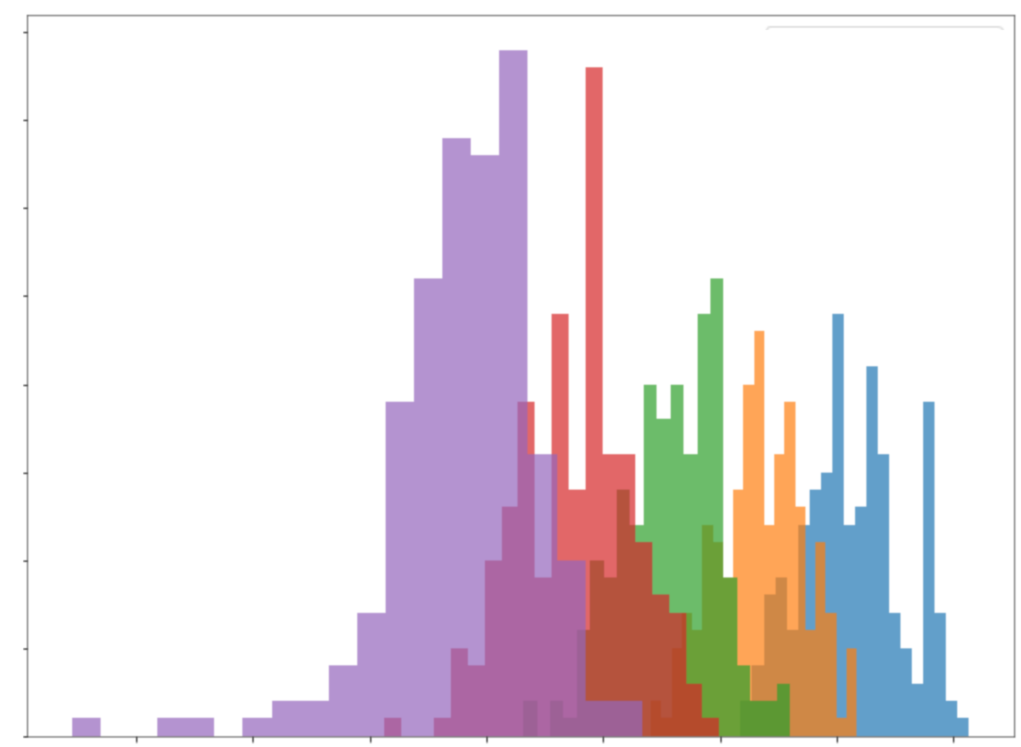
- Ergodic: RG energy scale drops exponentially
- Critical: RG energy scale drops as 1/e
- MBL: RG energy scale drops as poisson?



$\log(\Delta E_{i+1}/\Delta E_{i+2})$



MBL $\ln \Delta E$



$\ln \Delta E$

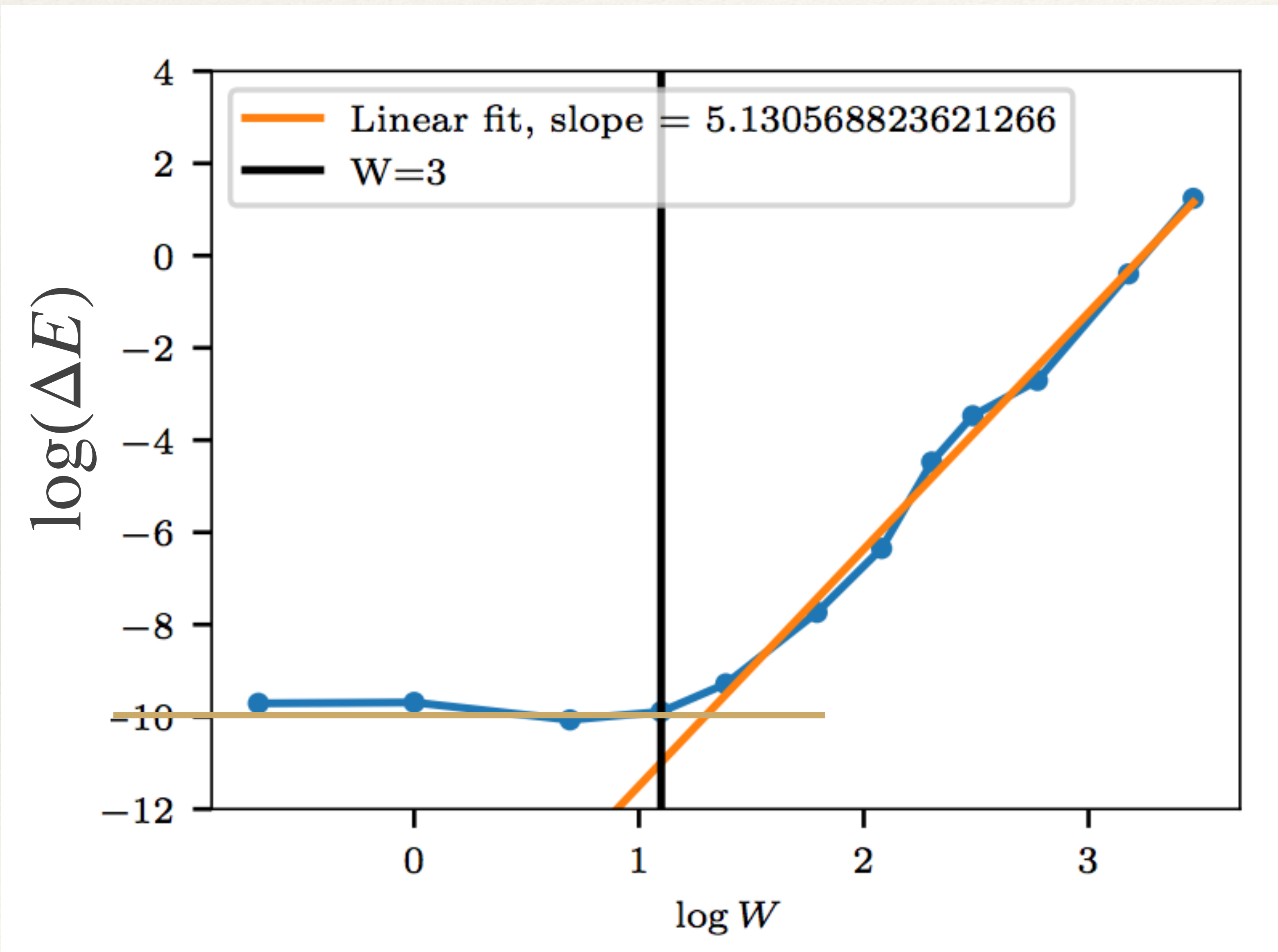
Ergodic

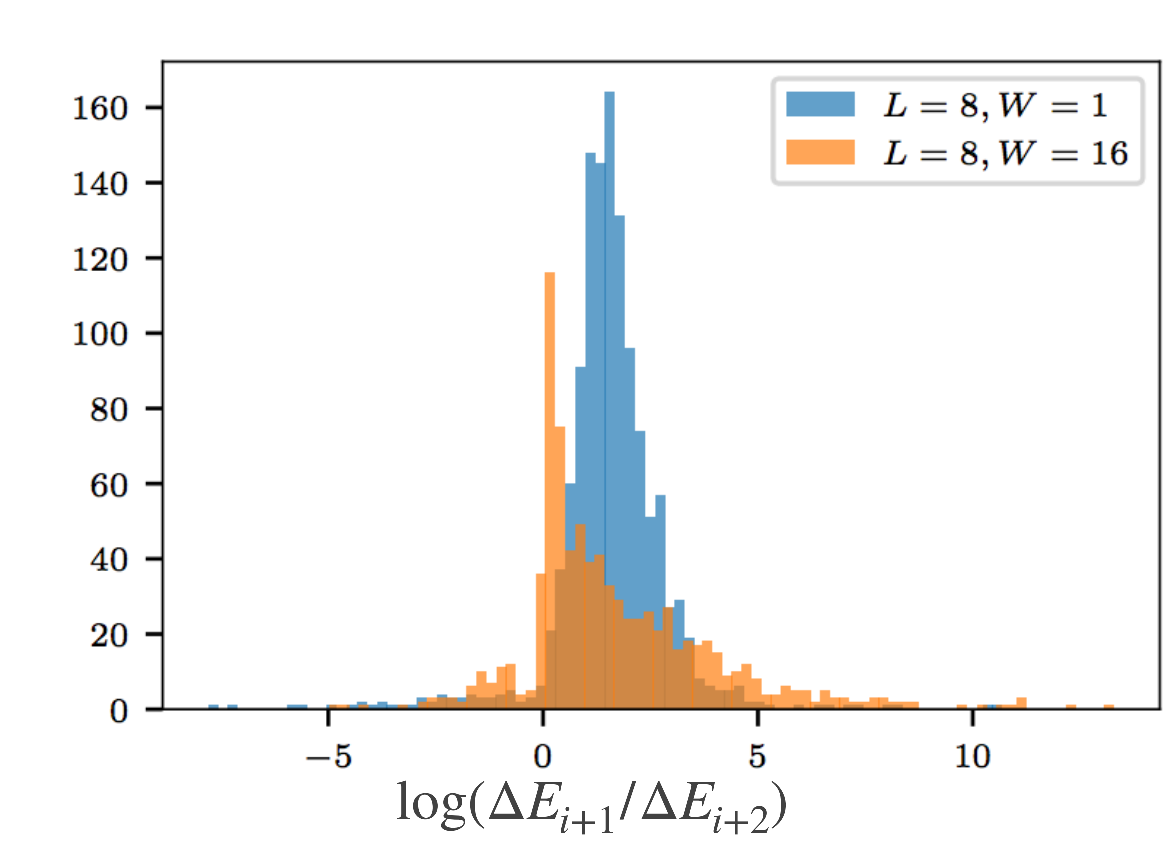
In the bulk, can you see the level spacing?

Lowest energy scale of RG tells you about MBL vs. ergodic.

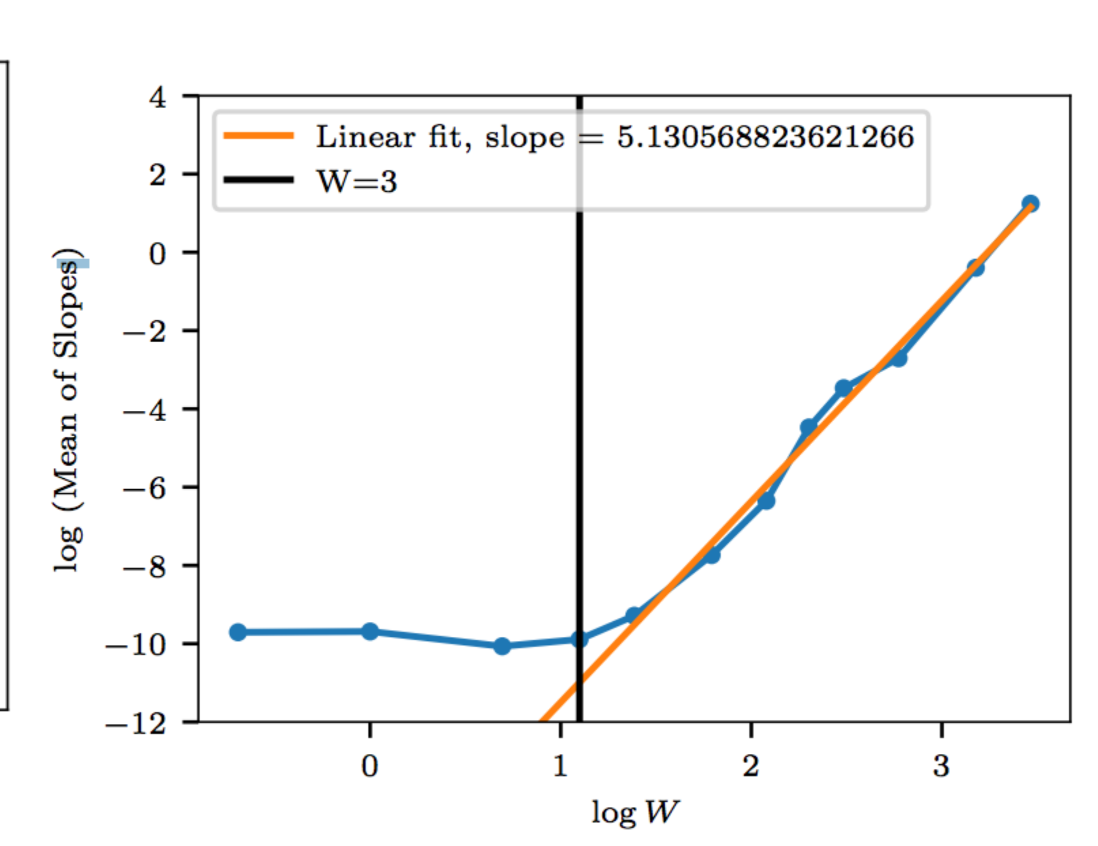
Different Epochs in the bulk

MBL vs. ergodic distinguished by the `infinite' distance in the bulk.





L = 8, W = 1
L = 8, W = 16
log(ΔE_{i+1}/ΔE_{i+2})



Linear fit, slope = 5.130568823621266
W=3
log (Mean of Slopes)
log W

Lesson: The MBL and Ergodic Bulk are very different.

We should be able to learn something about the transition from how they change.

Can we just measure the fidelity of U?

$$I + dU(W) \equiv U^{\dagger}(W)U(W+\delta)$$

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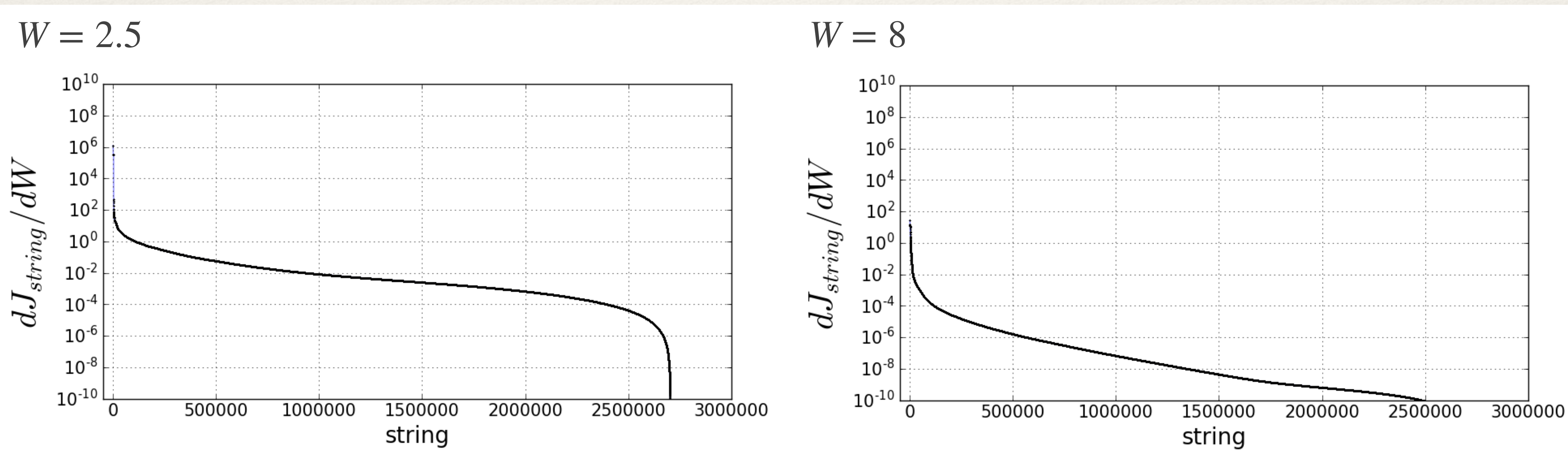
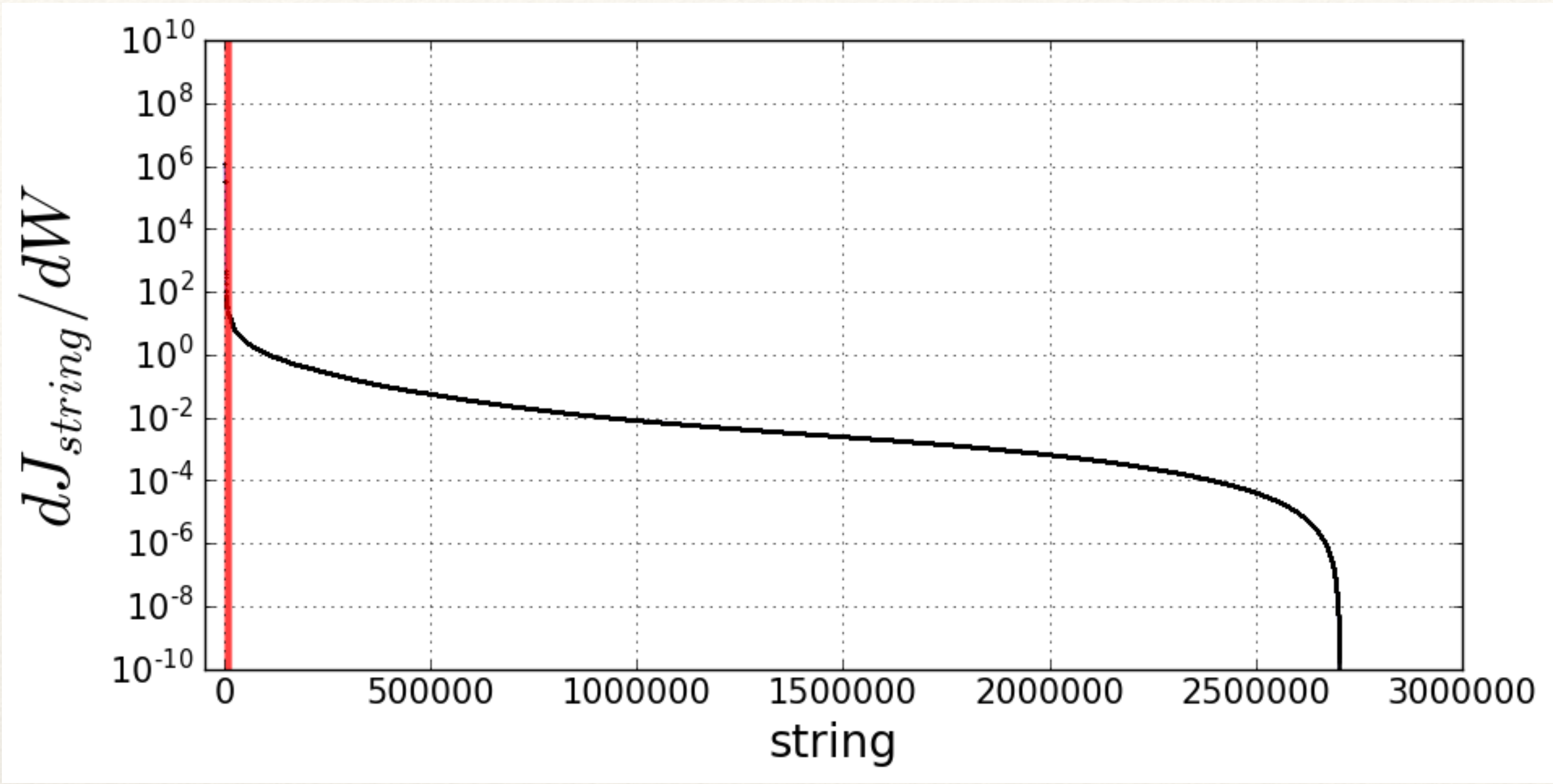
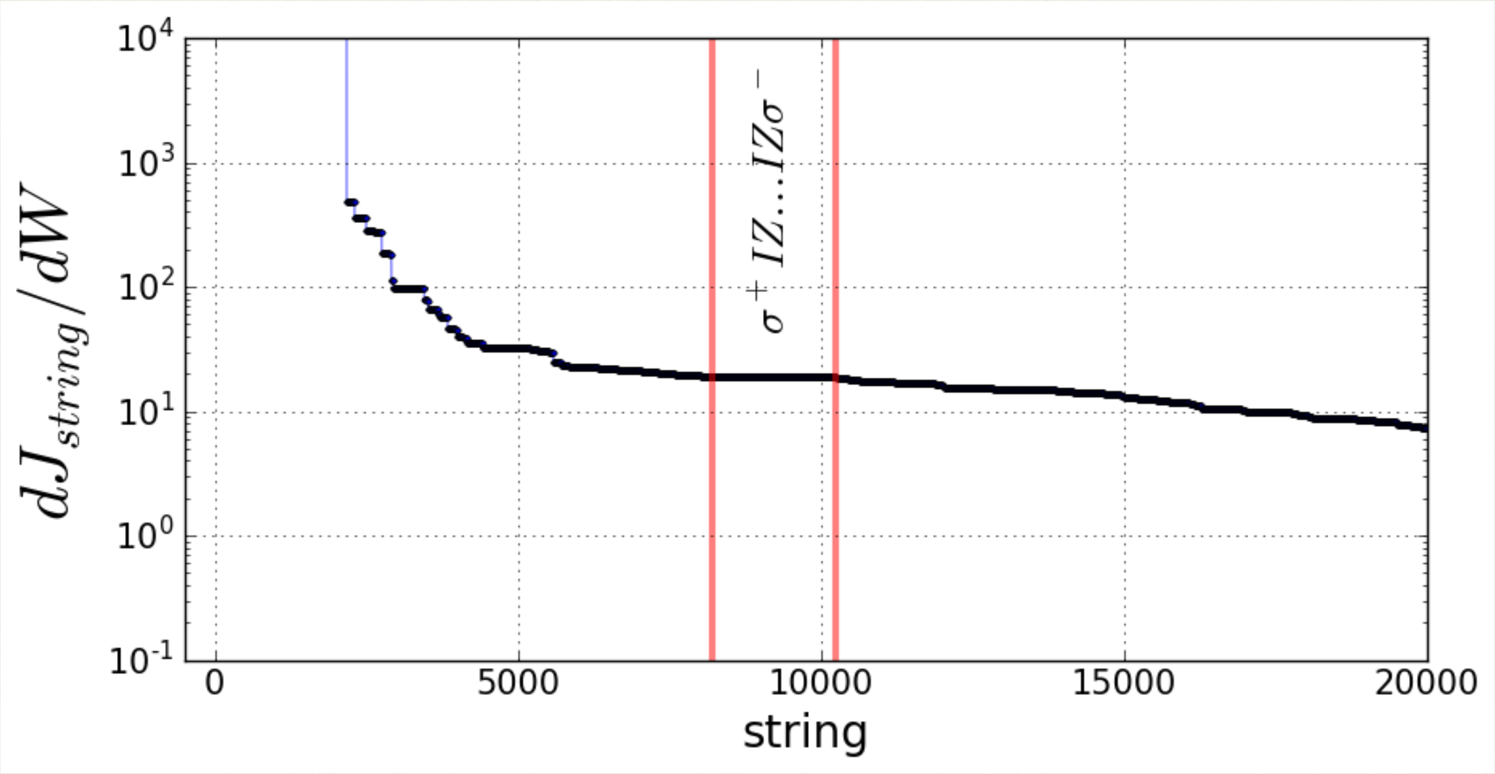
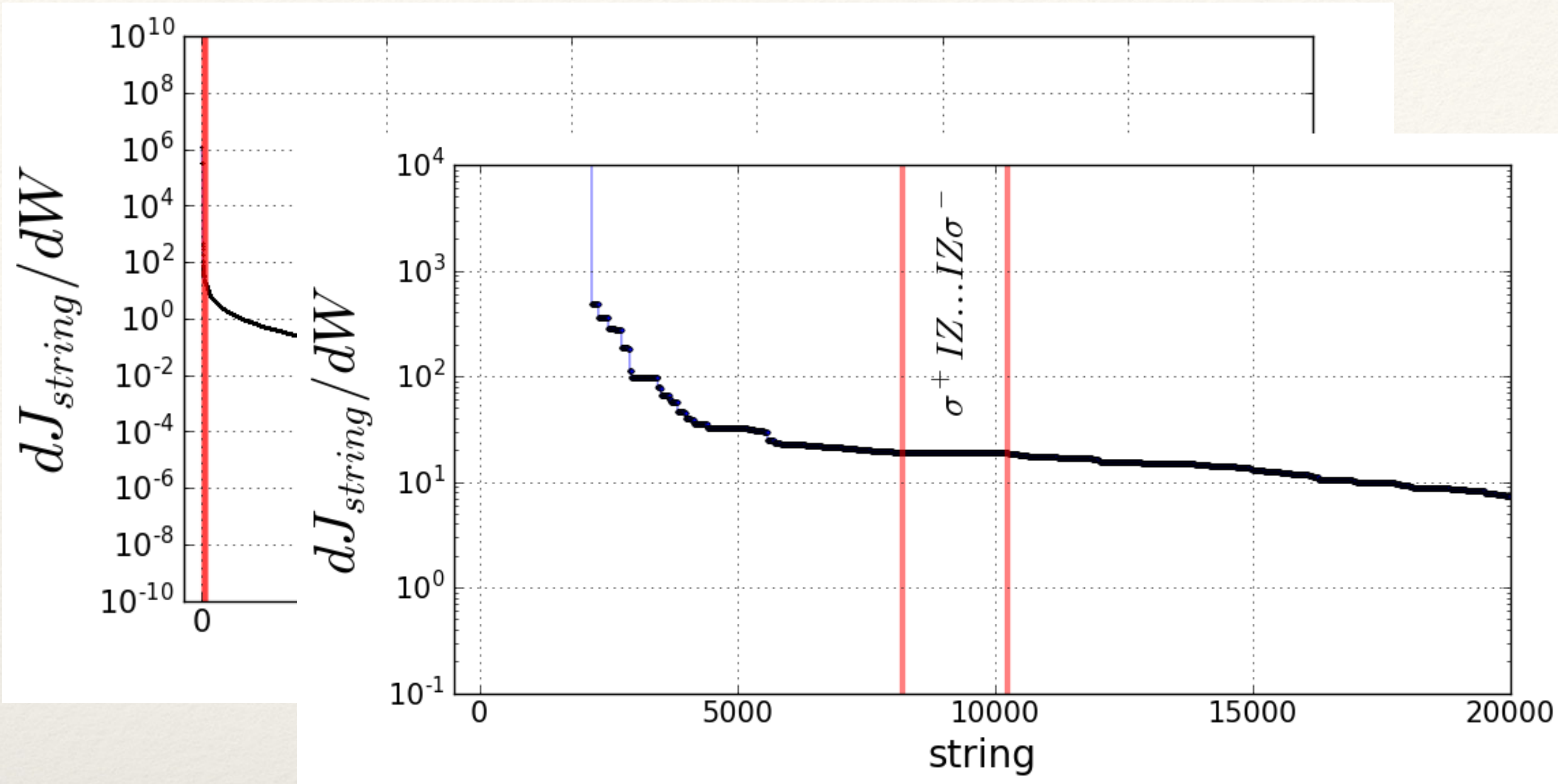




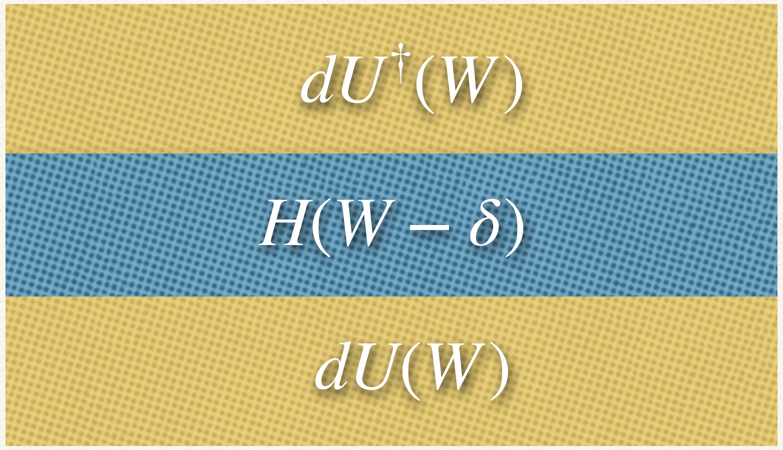
dJ_{string}/dW
string



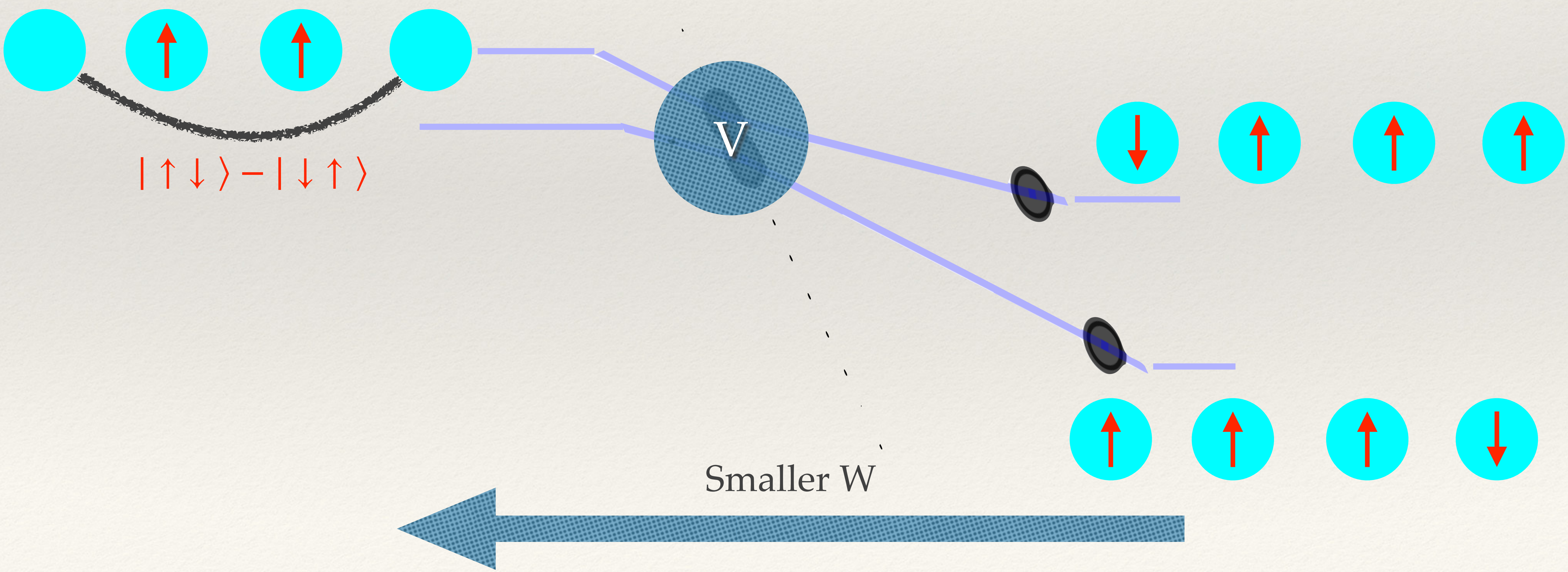
dJ_{string}/dW
string
$\sigma^+ IZ...IZ\sigma^-$



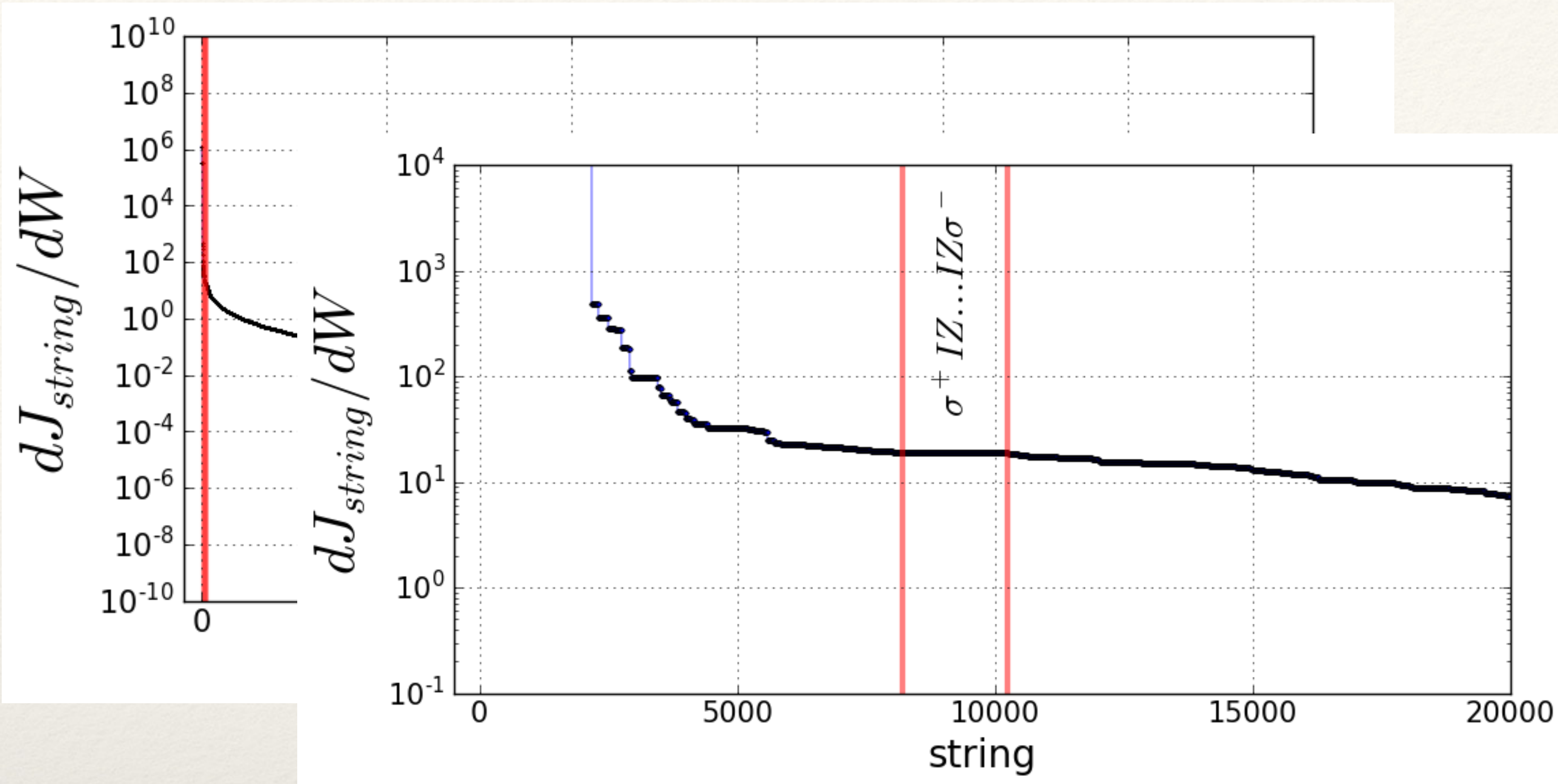
dJ_{string}/dW
string
$\sigma^{+}IZ\ldots IZ\sigma^{-}$

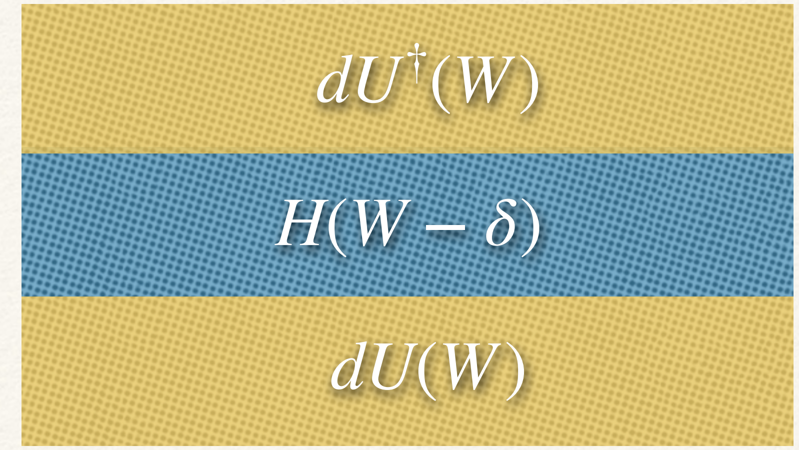


$dU^{\dagger}(W)$
$H(W-\delta)$
$dU(W)$

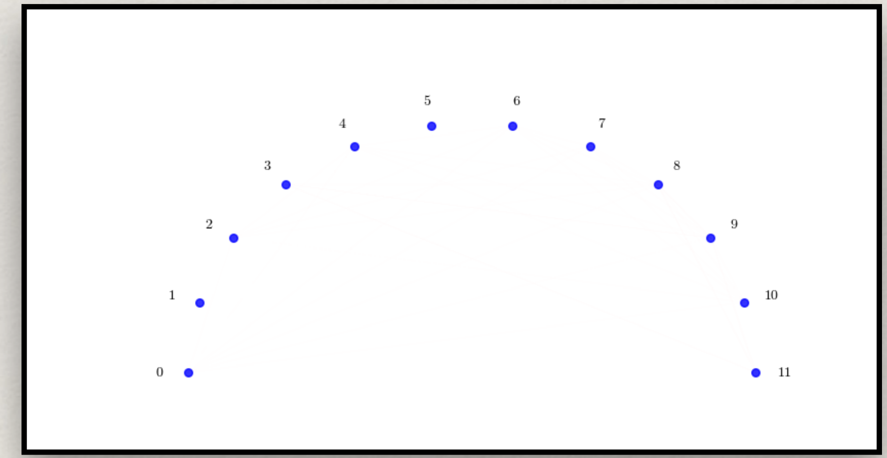


$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$
V
Smaller W

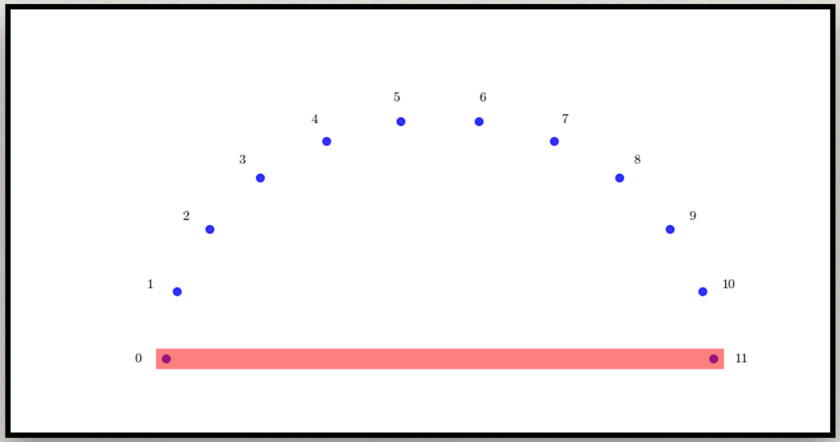




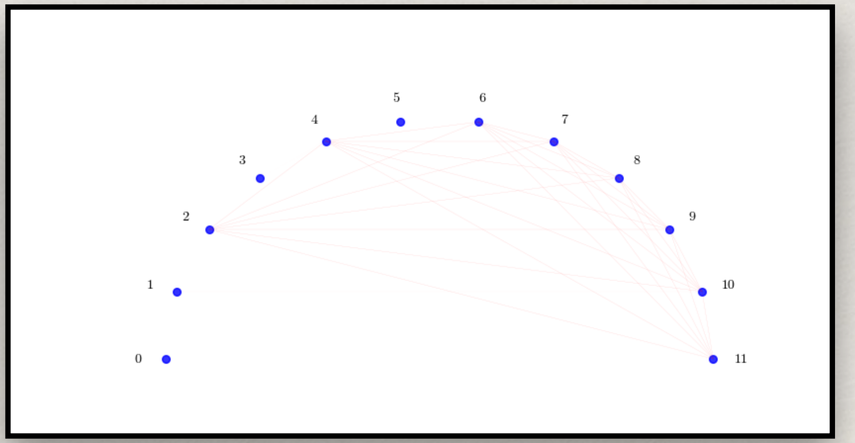
$W = 2.5 - \delta$

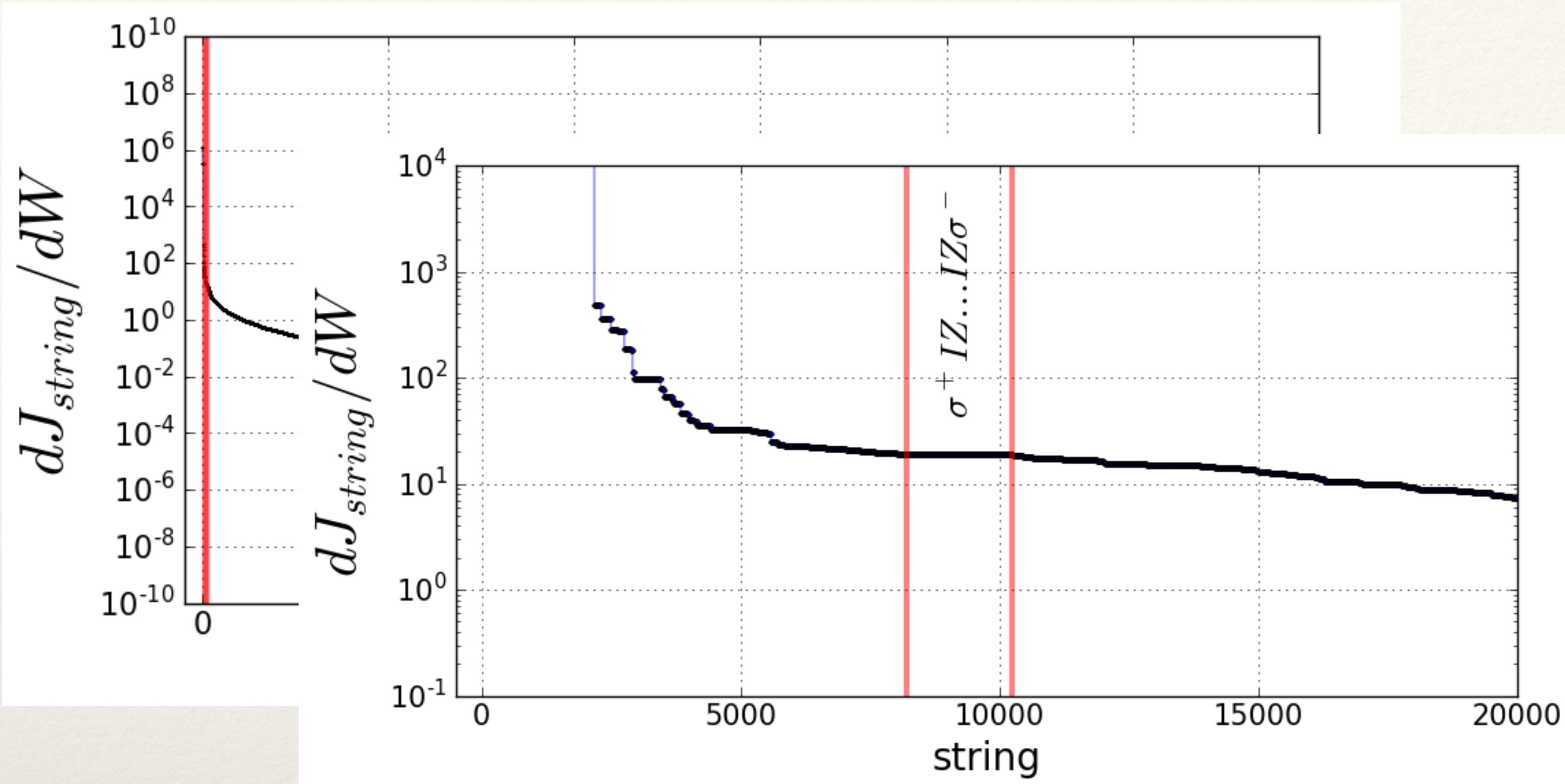


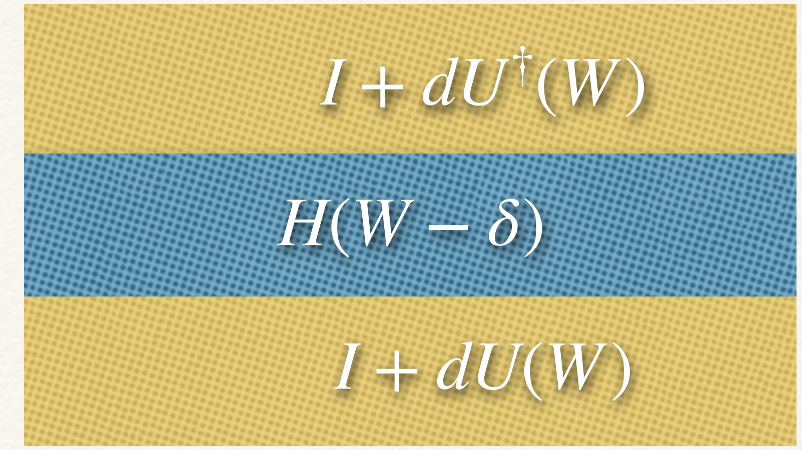
$W = 2.5$



$W = 2.5 + \delta$

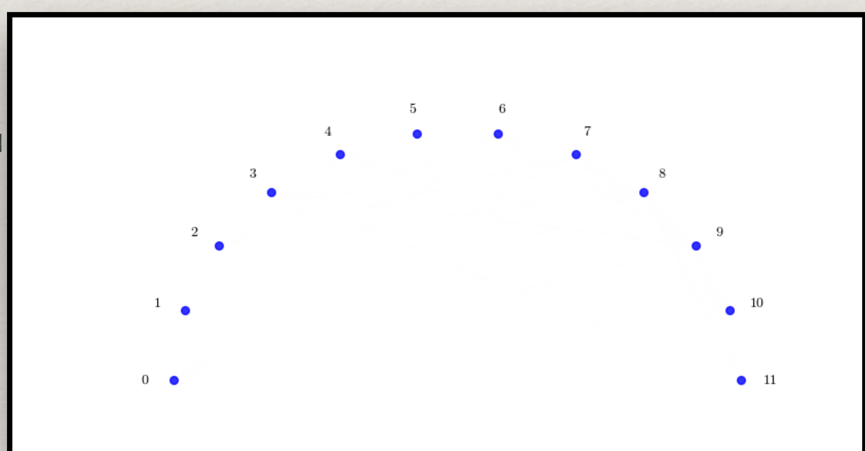




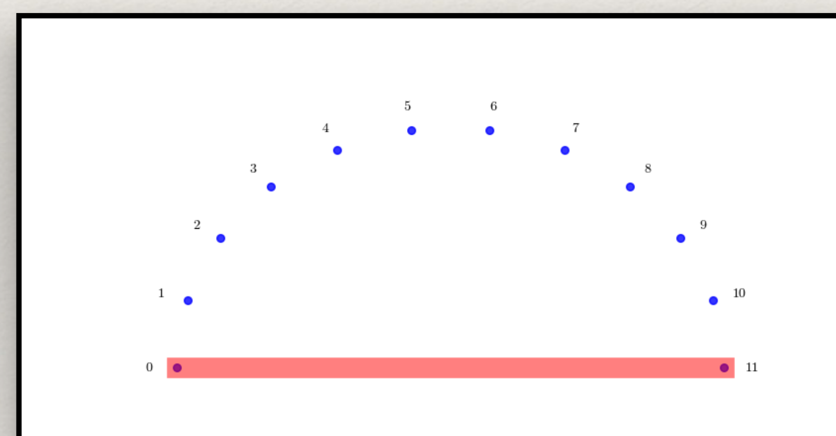


Relative Space

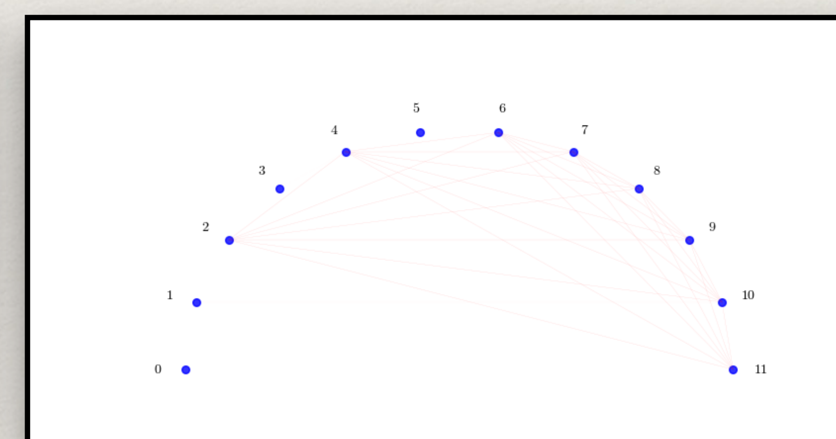
$W = 2.5 - \delta$

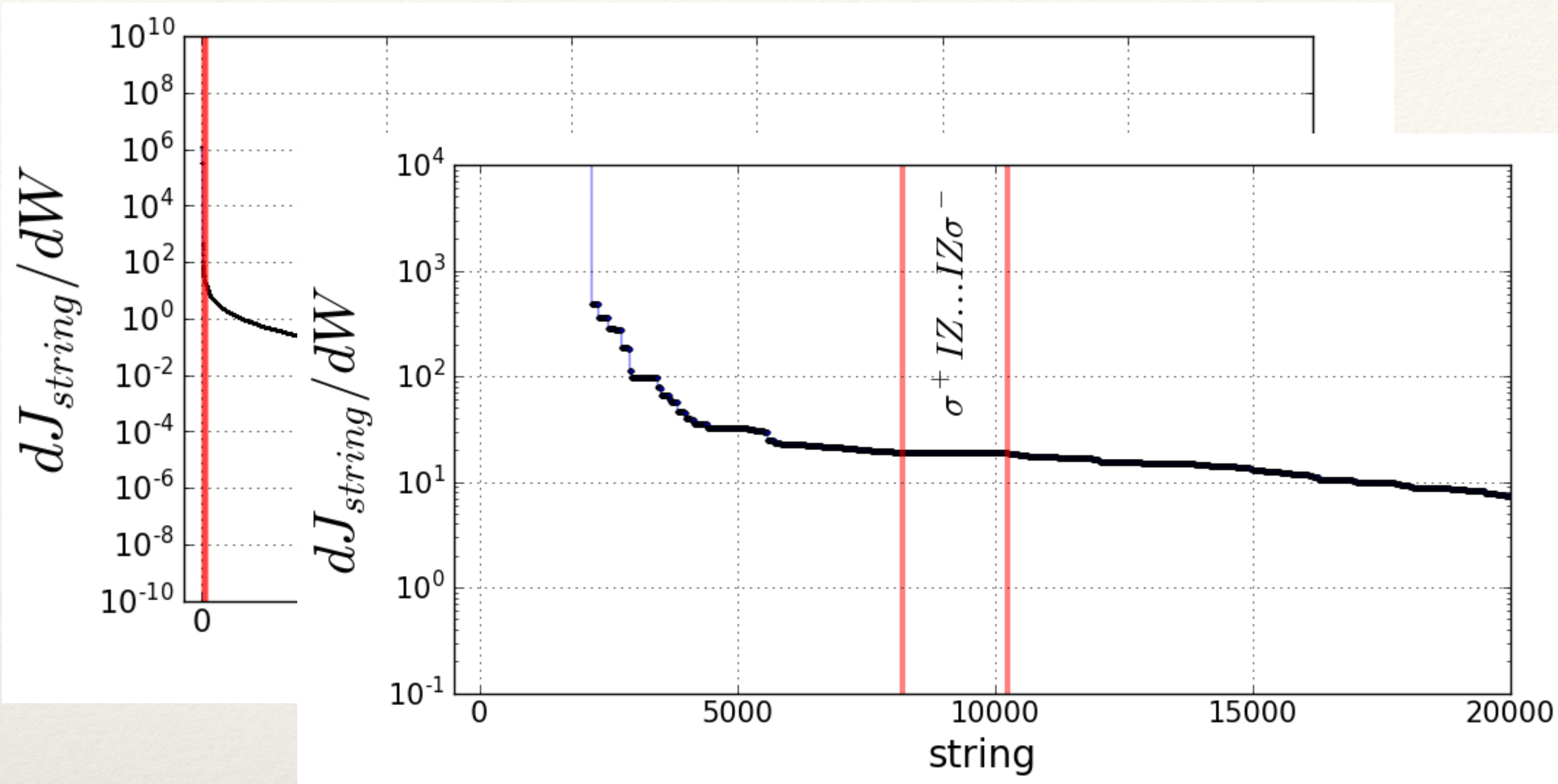


$W = 2.5$

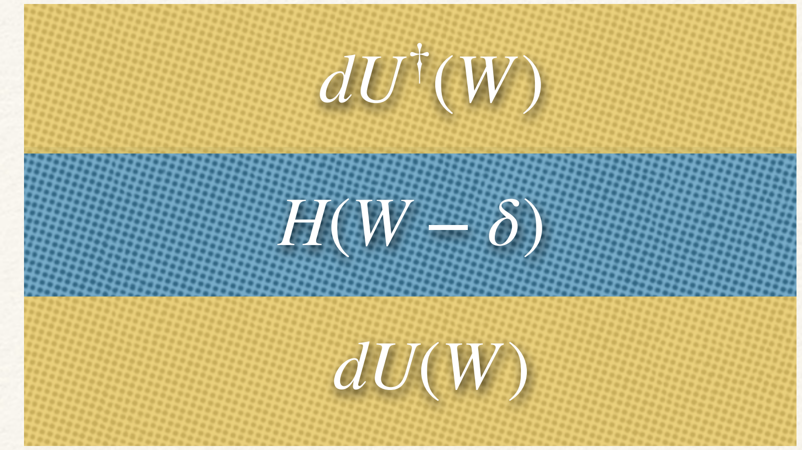


$W = 2.5 + \delta$

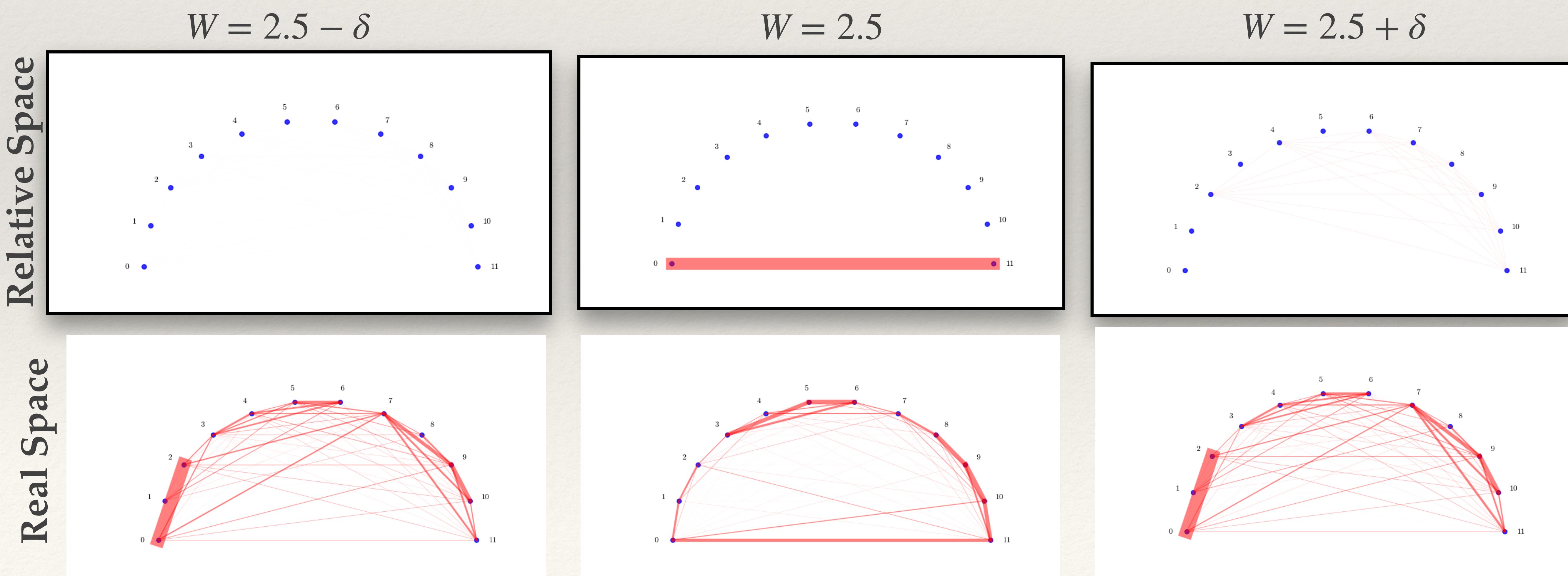




dJ_{string}/dW
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0
dJ_{string}/dW
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
$\sigma^+ IZ \ldots IZ \sigma^-$
0
5000
10000
15000
20000
string

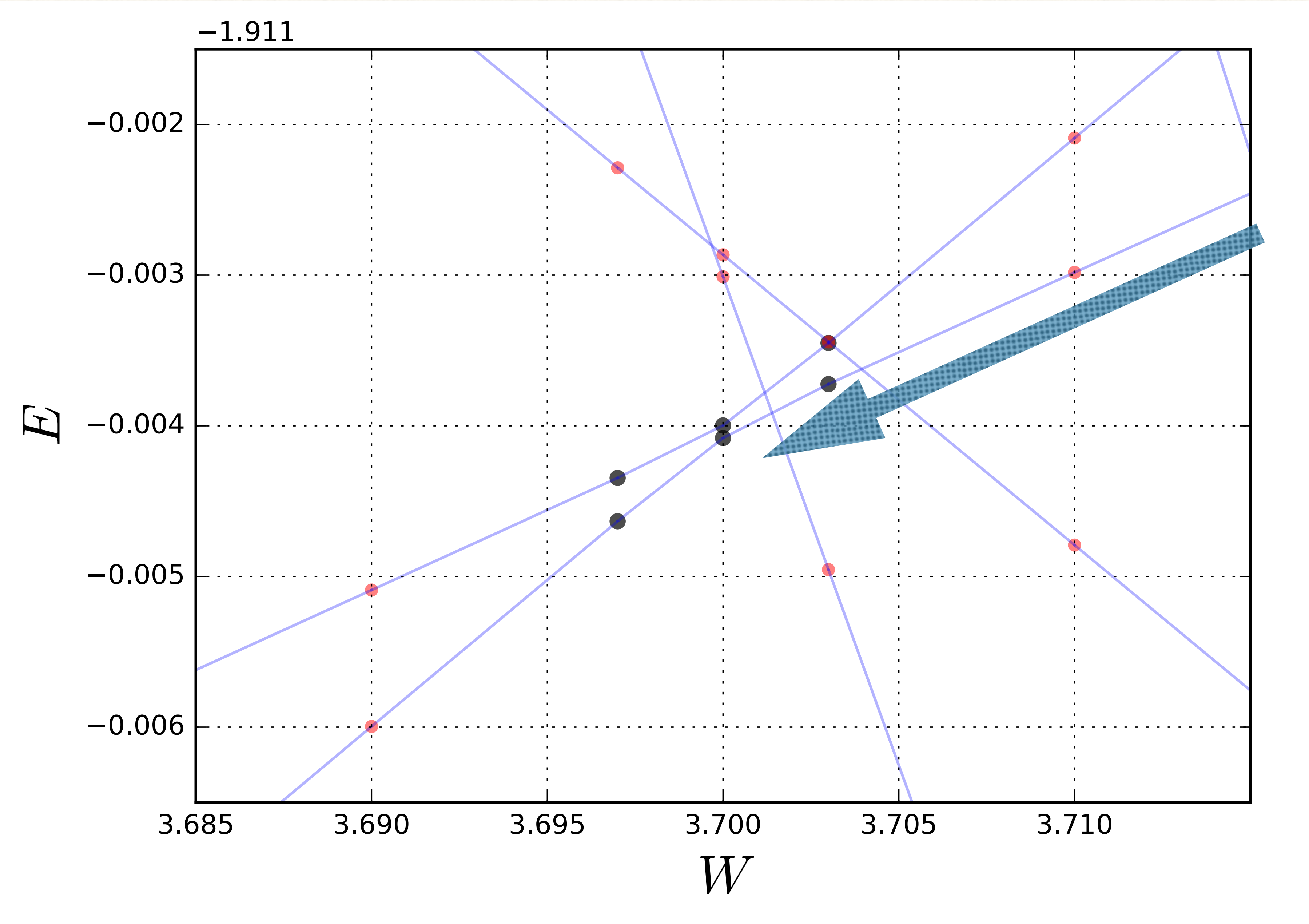


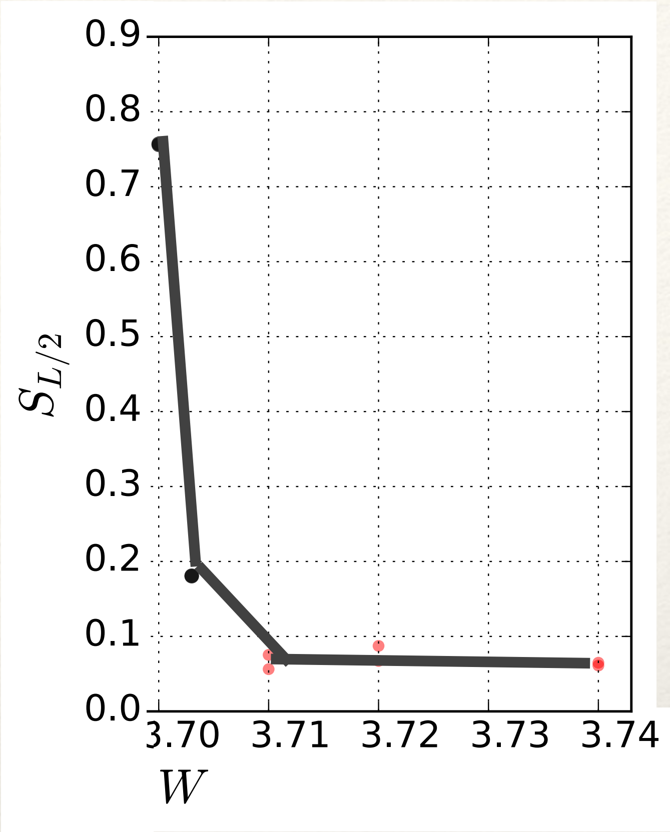
$dU^\dagger(W)$
$H(W-\delta)$
$dU(W)$

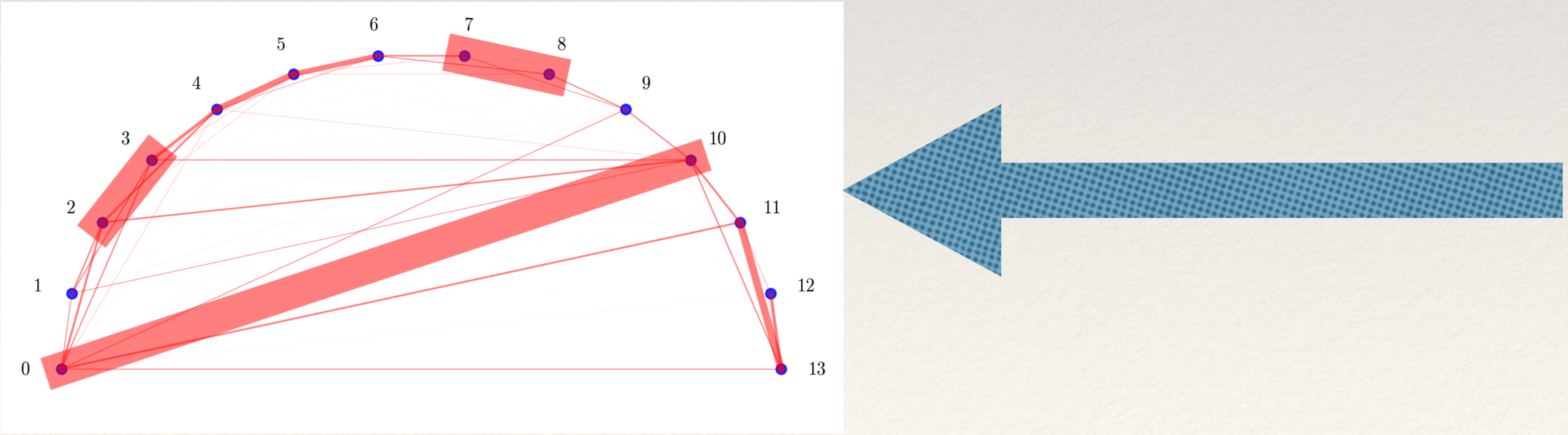


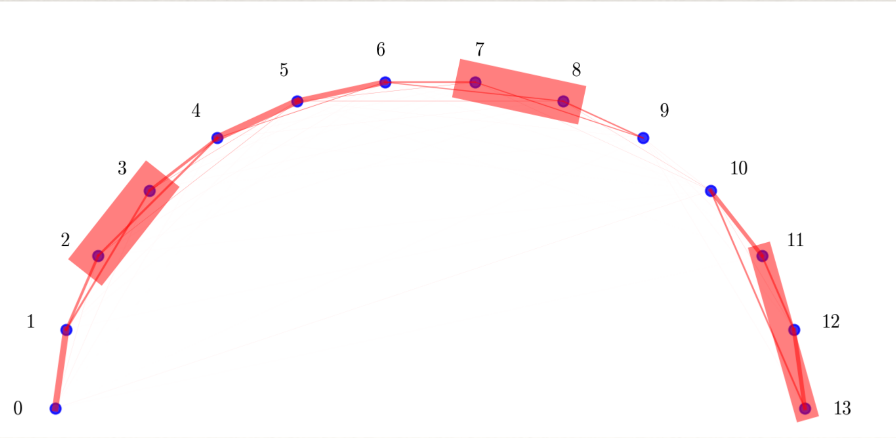
$W = 2.5 - \delta$
$W = 2.5$
$W = 2.5 + \delta$
Relative Space
Real Space

L=14

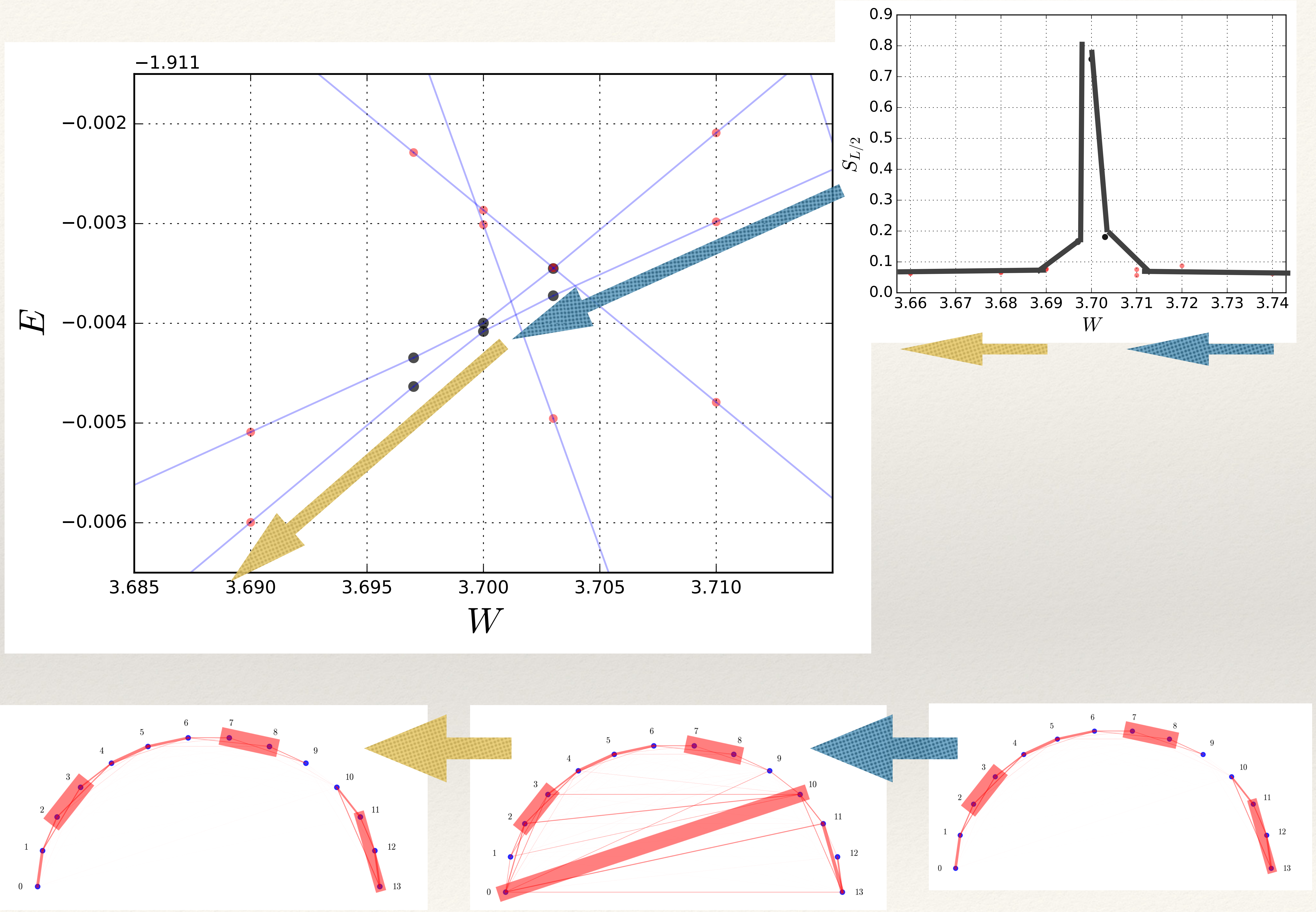








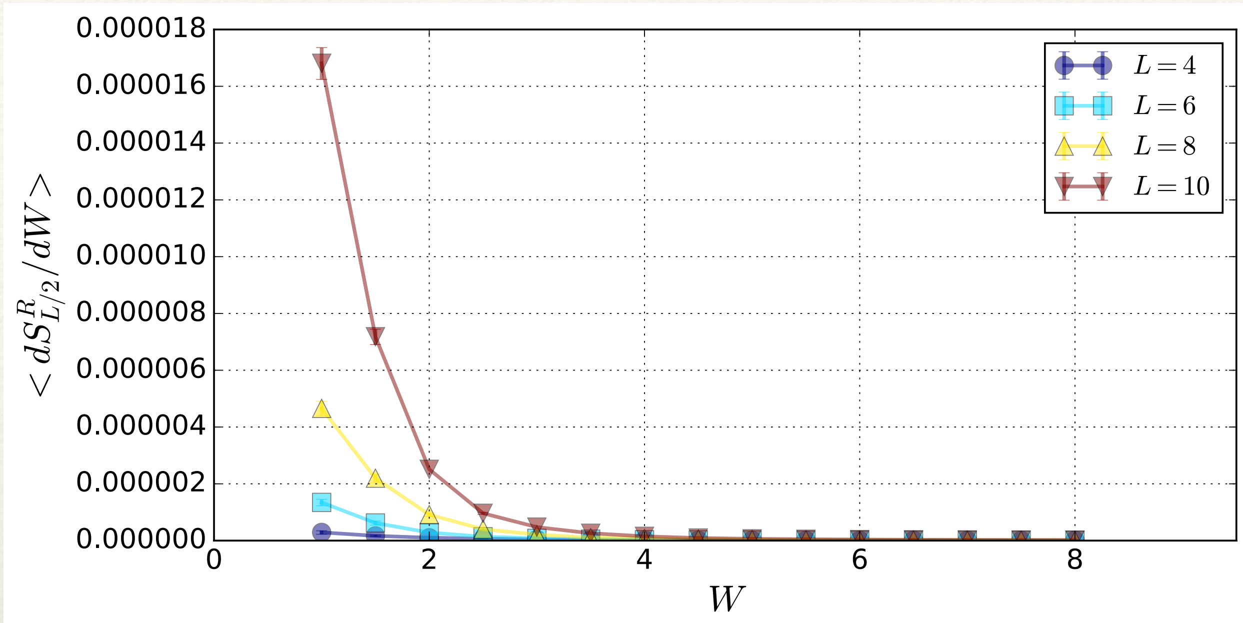
Real Space



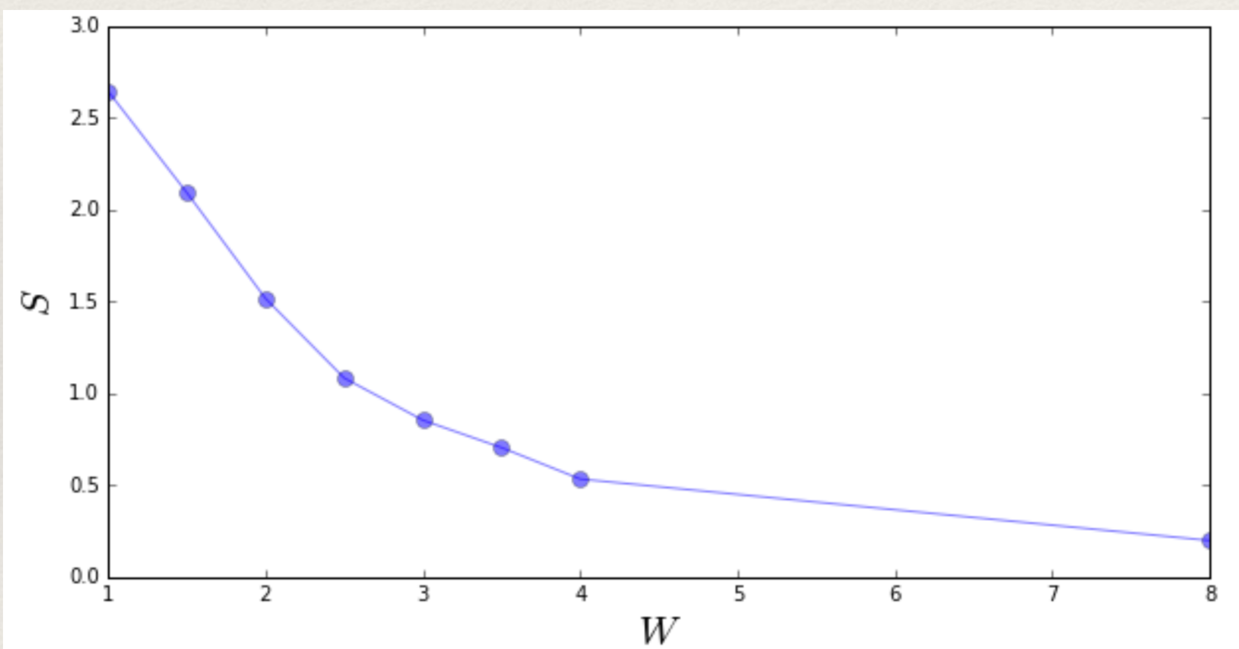
−1.911
−0.002
−0.003
−0.004
−0.005
−0.006
E
3.685
3.690
3.695
3.700
3.705
3.710
W
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$S_{L/2}$
3.66
3.67
3.68
3.69
3.70
3.71
3.72
3.73
3.74
W
0
1
2
3
4
5
6
7
8
9
10
11
12
13

Entanglement

Relative Space



Real Space



In order to have large entanglement you need continuous `resonances'

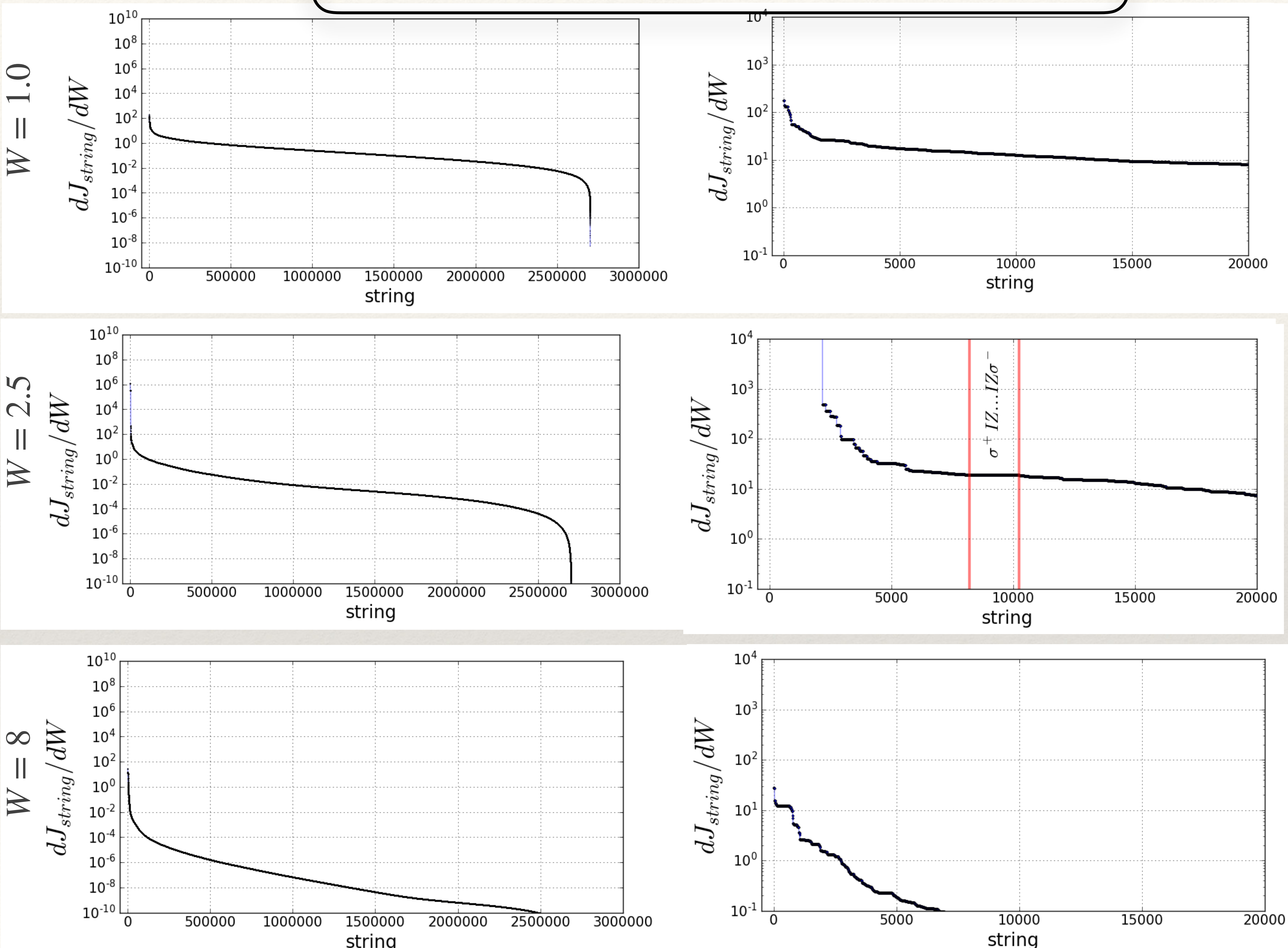
This also gives you level repulsion.

$$M \equiv \langle \Psi_1(W-\delta) \,|\, H(W) \,|\, \Psi_2(W-\delta) \rangle$$

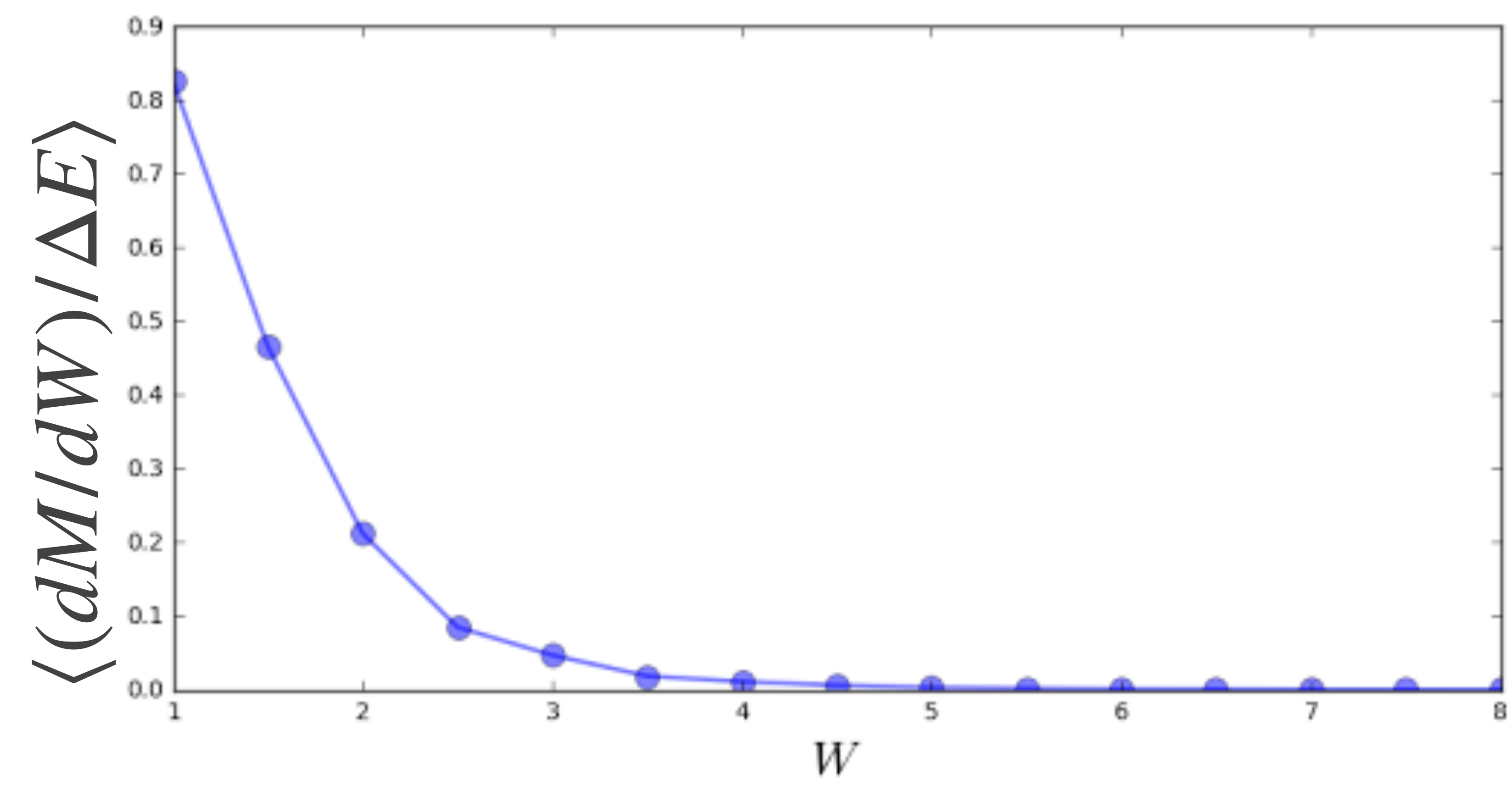
$$\begin{pmatrix} -\Delta E/2 & M \\ M & \Delta E/2 \end{pmatrix}$$

Need $\Delta E \approx M$

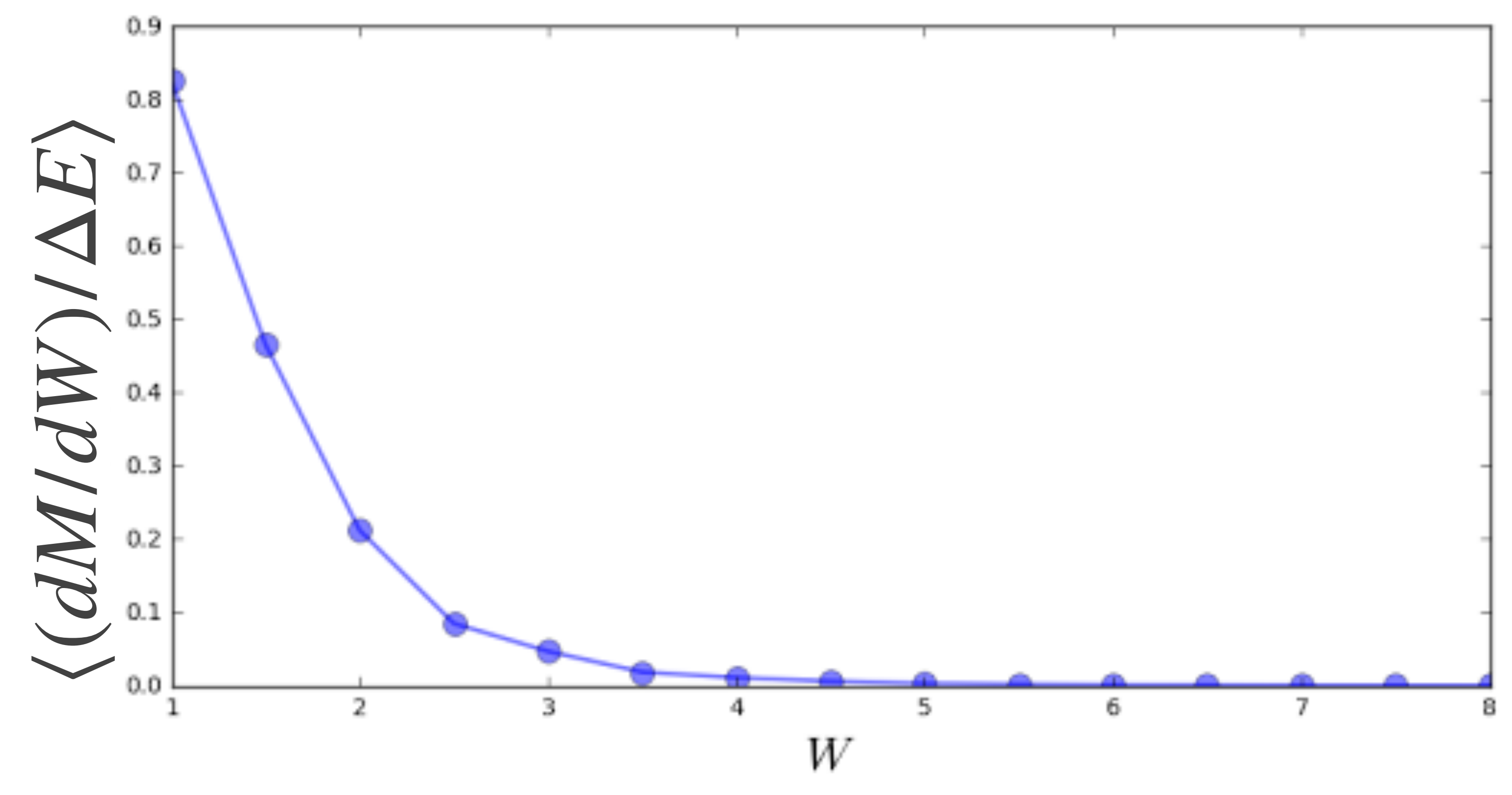
While all MBL unitaries are the same, all ergodic unitaries are different in their own special way.



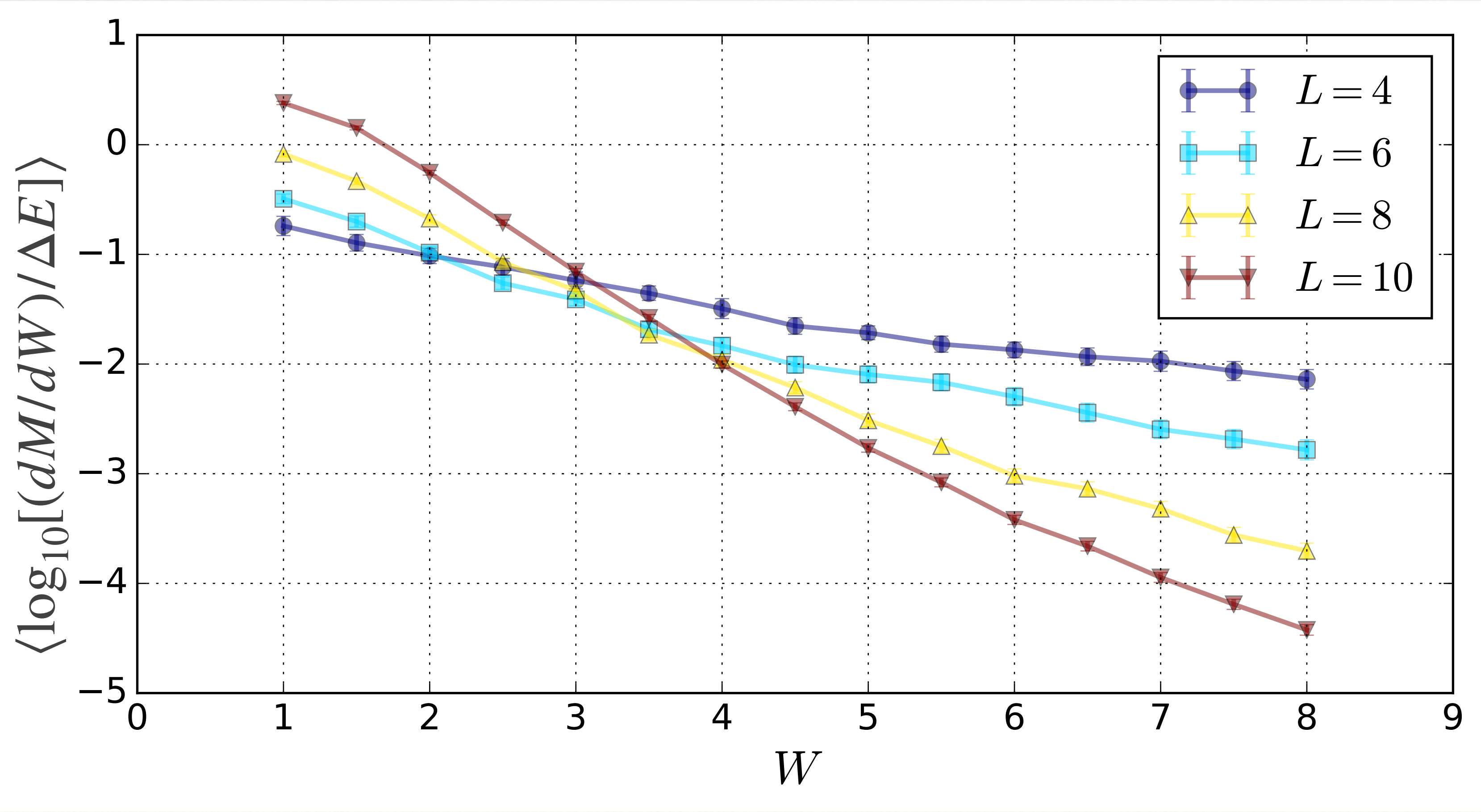
Still non-trivial off-diagonal matrix elements in the ergodic phase.



0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1
2
3
4
5
6
7
8
W
$\langle (dM/dW)/\Delta E \rangle$



0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1
2
3
4
5
6
7
8
$\langle (dM/dW)/\Delta E \rangle$
W



1
0
−1
−2
−3
−4
−5
0
1
2
3
4
5
6
7
8
9
$\langle \log_{10}[(dM/dW)/\Delta E] \rangle$
W
$L=4$
$L=6$
$L=8$
$L=10$

Conclusions