MBL THROUGH MPS

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Dresden - May 15, 2014

Many Body Localization...

It has been suggested that

interactions + (strong) disorder

$$H = \sum_{i=1}^{L} [h_i S_i^z + J \sum \hat{S}_i \cdot \hat{S}_{i+1}]$$

$$h_i \in [-W, W]$$

produce a many-body localized phase at all 'temperatures.'

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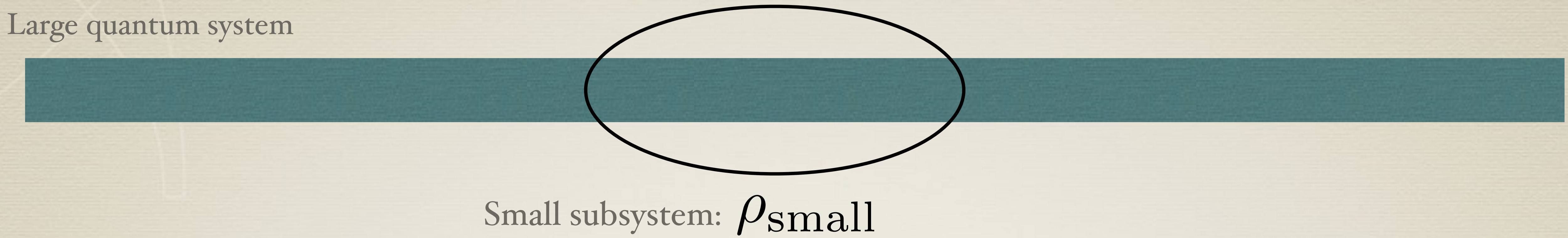
A many body localized phase ...

has the following phenomenological properties:

* doesn't thermalize
* is localized.
* has atypical eigenstates

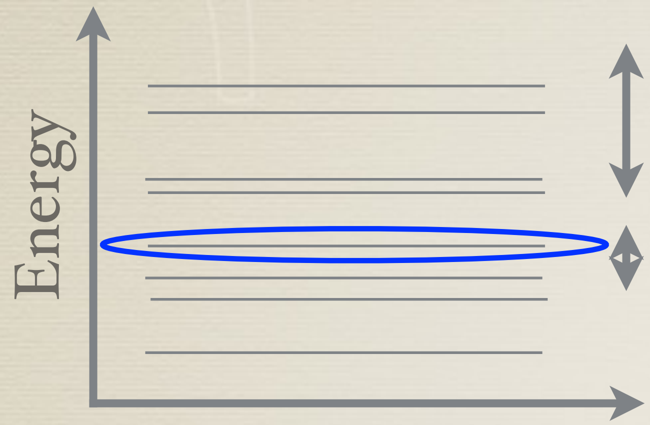
Thermalization



Thermalized if, in equilibrium, $\rho_{\text{small}} = \rho_{\text{thermal}}$ at the temperature that corresponds to the energy density of the system.

What thermalizes?

States which obey the eigenstate thermalization hypothesis...



eigenstates will decohere.

Nearby eigenstates will stay coherent.

$$\Psi = \alpha_0|\Psi_0\rangle + \alpha_1|\Psi_1\rangle + ... + \alpha_n|\Psi_n\rangle$$

$$e^{itH}\Psi = \alpha_0 e^{itE_0}|\Psi_0\rangle + \alpha_1 e^{itE_1}|\Psi_1\rangle + ... + \alpha_n e^{itE_n}|\Psi_n\rangle$$

To thermalize, nearby eigenstates must look the same with respect to local observables.

Eigenstate Thermalization Hypothesis

and what doesn't thermalize?

- States which don't obey the eigenstate thermalization hypothesis...
- Integrable systems are one such system....
  - which don't thermalize because of the extensive number of conserved quantities.
  - Instead the density matrix approaches the generalized Gibbs Ensemble ρ_{GGE}.

We should anticipate then, that MBL phases shouldn't obey ETH and should be 'like' integrable systems.

A many body localized phase ...

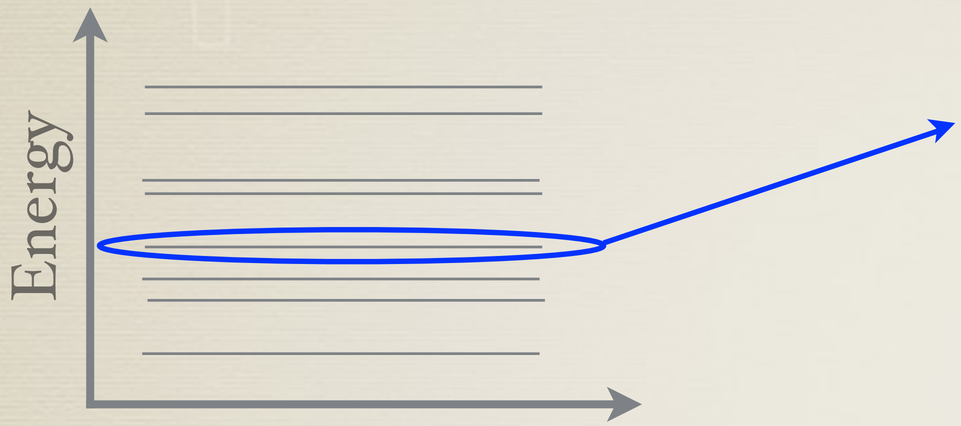
has the following phenomenological properties:

* doesn't thermalize
* is localized.
* has atypical eigenstates

What localizes...

Anderson Insulators!

Noninteracting problem:



Single ≈ orbitals: 'Support' on a few sites

Inverse participation ratio: $\dfrac{\sum_x |\Psi(x)|^4}{\sum_x |\Psi(x)|^2}$

Single particle eigenstates are localized

Interacting systems don't have s.p.o

How should we think of localization in an interacting system?

$$H = t \sum_{\langle i,j \rangle} c_i^\dagger c_j + \sum_i w_i c_i^\dagger c_i$$

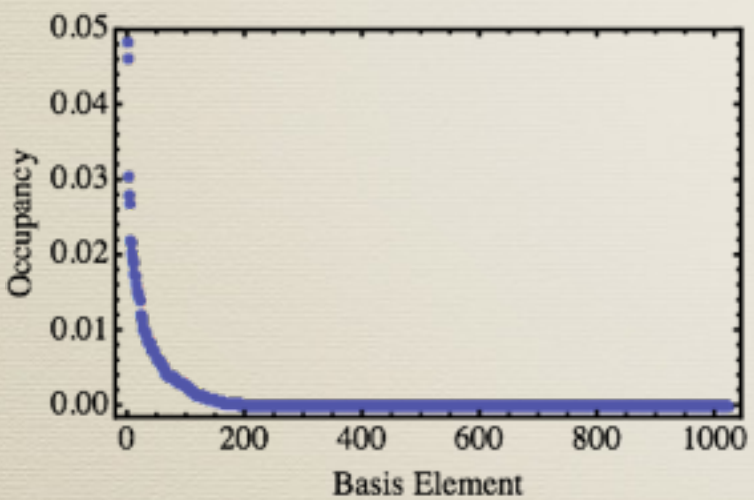
$$w_i \in [-W, W]$$

Localization in Fock space

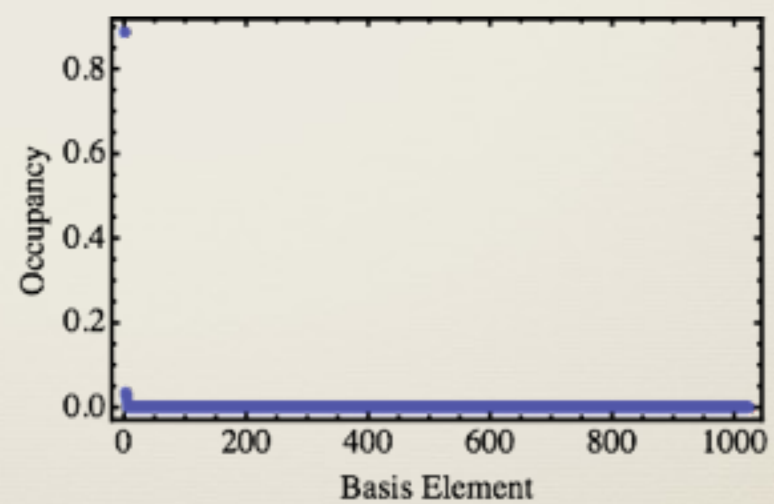
Each many body basis state is on a vertex on the hypercube.

The Hamiltonian connects vertices through edges

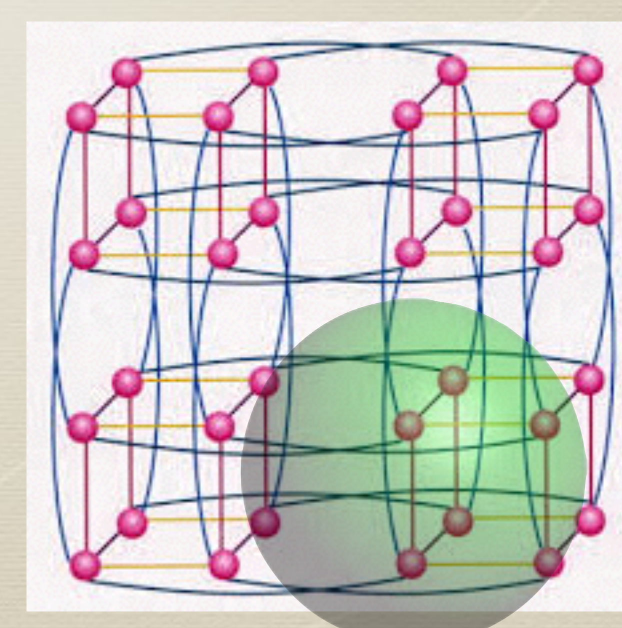
Localization means eigenstates have 'support' on only a few sites.



Low disorder



High disorder



Suggested by Bosca, Aleiner, Altschuler (2001) for many body localization

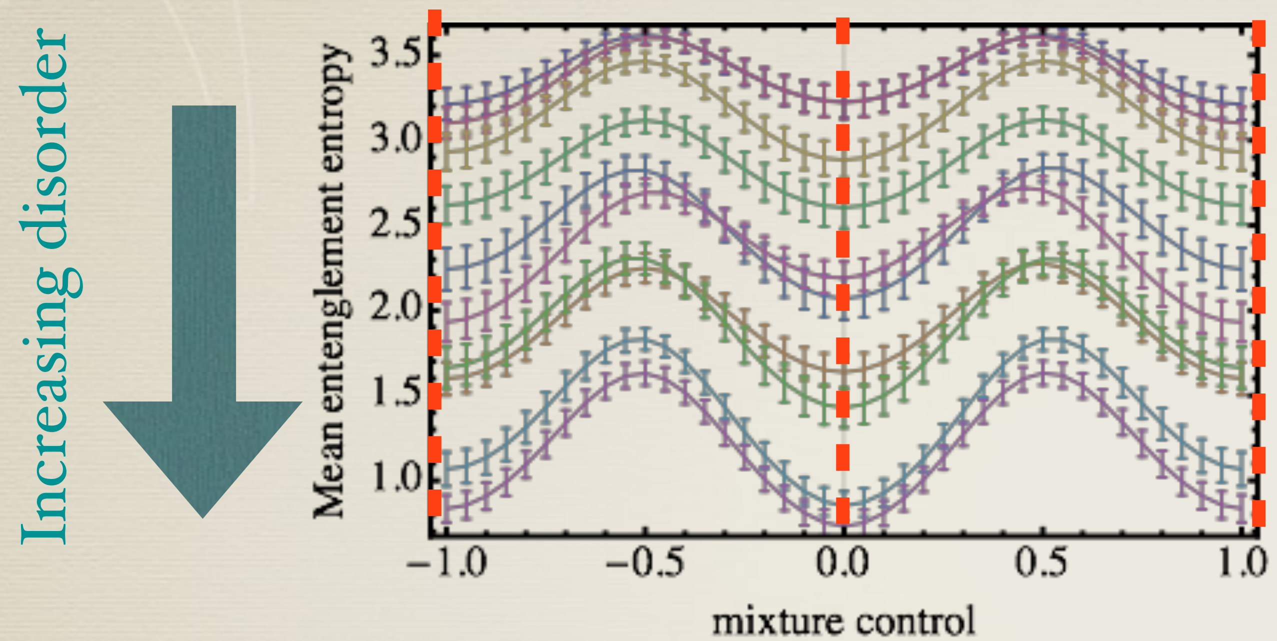
A many body localized phase ...

has the following phenomenological properties:

* doesn't thermalize
* is localized.
* has atypical eigenstates

Atypical Eigenstates

Low Entanglement (obeys area law)



α_0

N=12 Delta=(0.05,10.5)
Average over 10 disorder realizations and 5 eigenstates

Notice that there is interesting quantum effects happening at non-zero 'temperature'

Strange Spectral Statistics....

Poisson vs. Gaussian orthogonal ensemble

How should we understand these strange phenomena?

We need a unifying understanding of MBL?

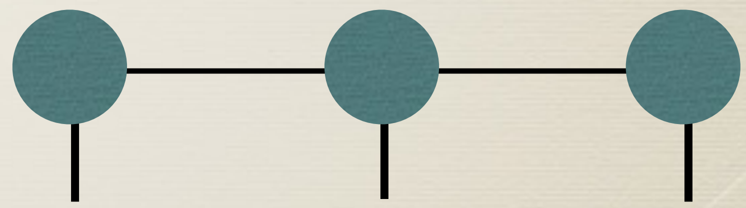
A wave function is an object that eats a configuration of spins and generates a number: $\Psi(\uparrow,\downarrow,\uparrow) = 0.3$

A product state eats a configuration of spins and generates a number by taking the product of complex numbers.

$$\Psi(\uparrow,\downarrow,\uparrow) = M^{1,\uparrow} M^{2,\downarrow} M^{3,\uparrow}$$

A matrix product state (MPS) eats a configuration of spins and generates a number by taking the product of matrices/vectors.

$$\Psi(\uparrow,\downarrow,\uparrow) = M_i^{1,\uparrow} M_{ij}^{2,\downarrow} M_j^{3,\uparrow}$$



MPS are spatially local: A change in the matrix of one site decays exponentially.

MPS are spatially local: They are connected to product states by constant depth unitary circuits.

A new basis

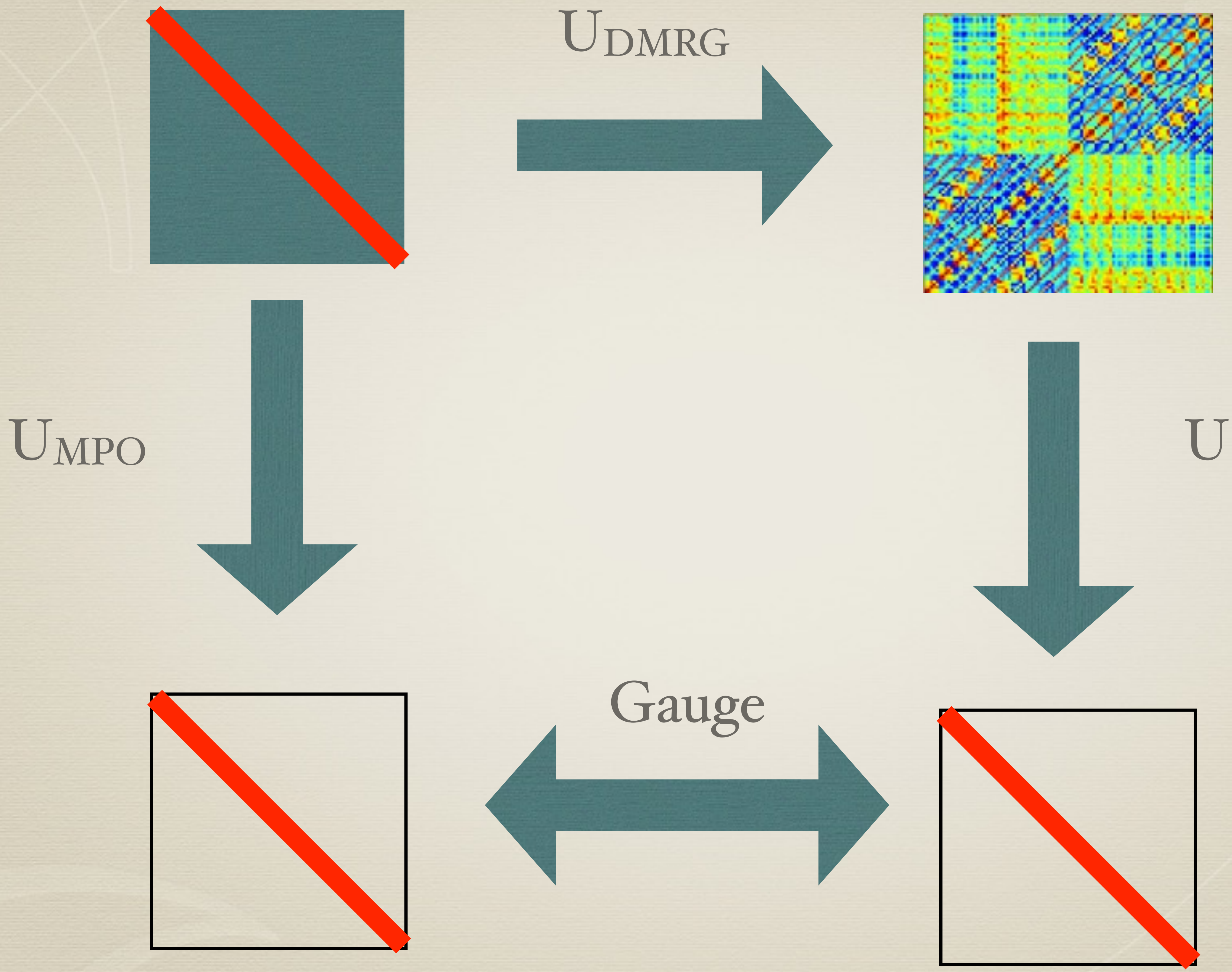
What is the right basis to think of many-body localization in?

The typical basis

In the product state basis, there is no indication that anything qualitative happens as the disorder gets larger.

$$H = \sum_{i=1}^{L} [h_i S_i^z + J \sum \hat{S}_i \cdot \hat{S}_{i+1}]$$

$$h_i \in [-W, W]$$


UDMRG
UMPO
U
Gauge

A Schmidt Decomposed Basis

$$|\Psi_G\rangle \equiv \sum_i |a_i\rangle|b_i\rangle$$

Basis: $\{|a_1\rangle|b_1\rangle, |a_2\rangle|b_2\rangle, \ldots |a_9\rangle|b_9\rangle\}$

$$|\Psi_G\rangle \equiv \sum_i |\alpha_i\rangle|\beta_i\rangle$$

$\{|\alpha_1\rangle|\beta_1\rangle, |\alpha_2\rangle|\beta_2\rangle, \ldots |\alpha_9\rangle|\beta_9\rangle\}$

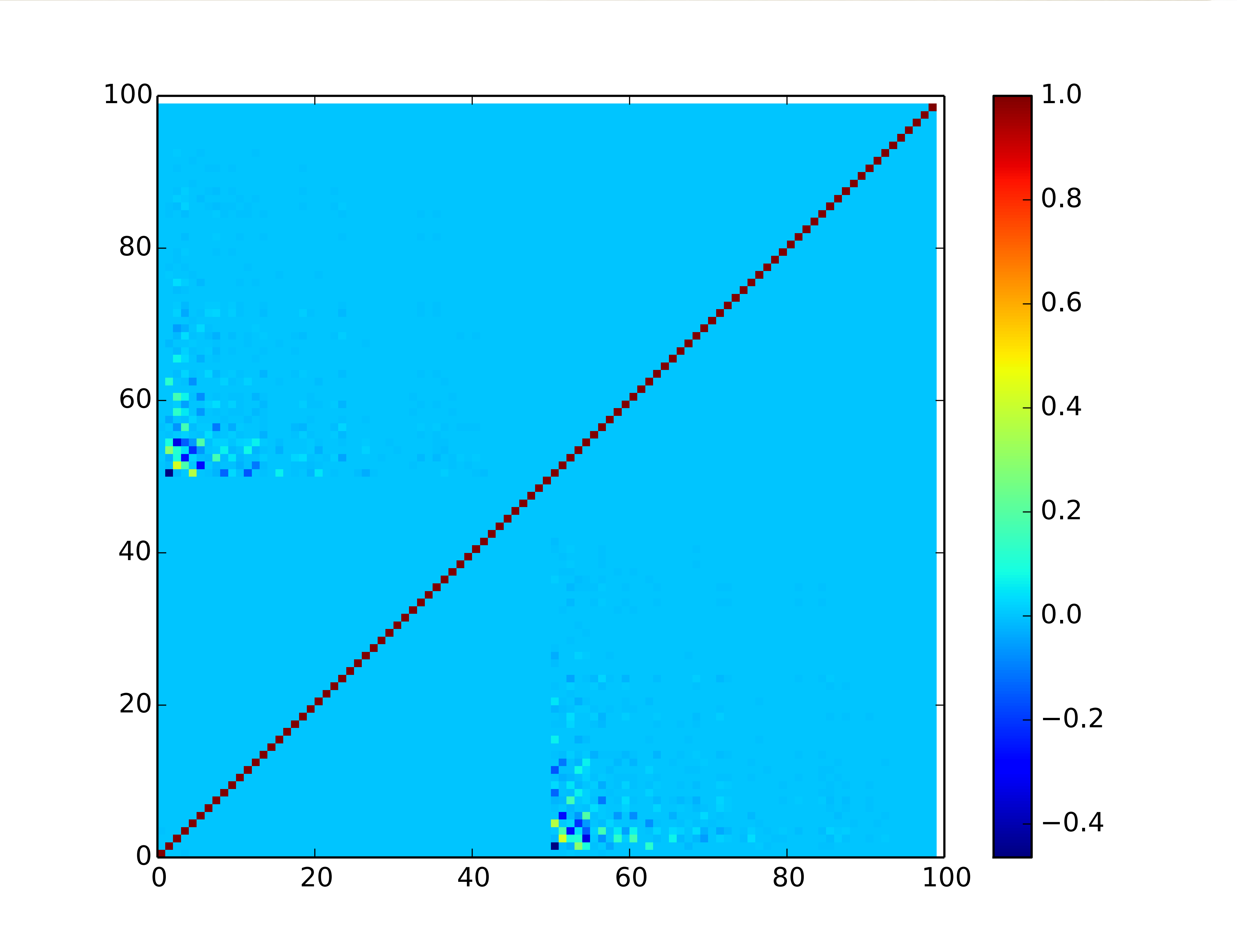
This basis is local MPS.

(Can be generalized for more basis elements)

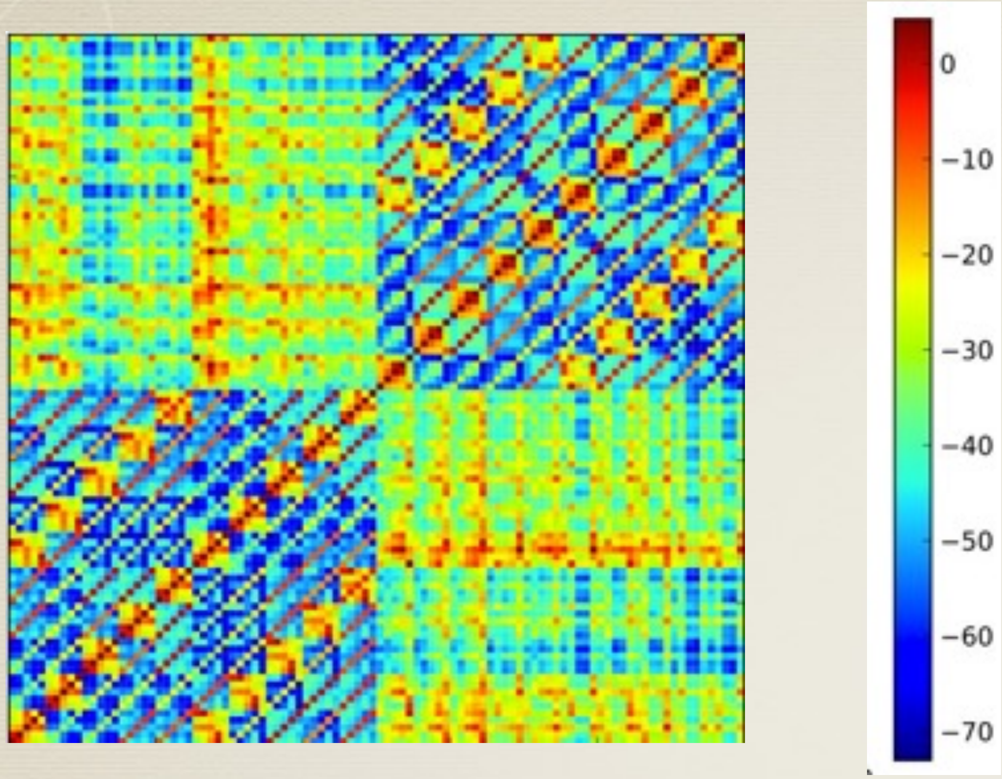
Am I a basis? $\langle \Psi_i | \Psi_j \rangle = \delta_{ij}$

Not exactly .... but pretty close.

Overlap Matrix

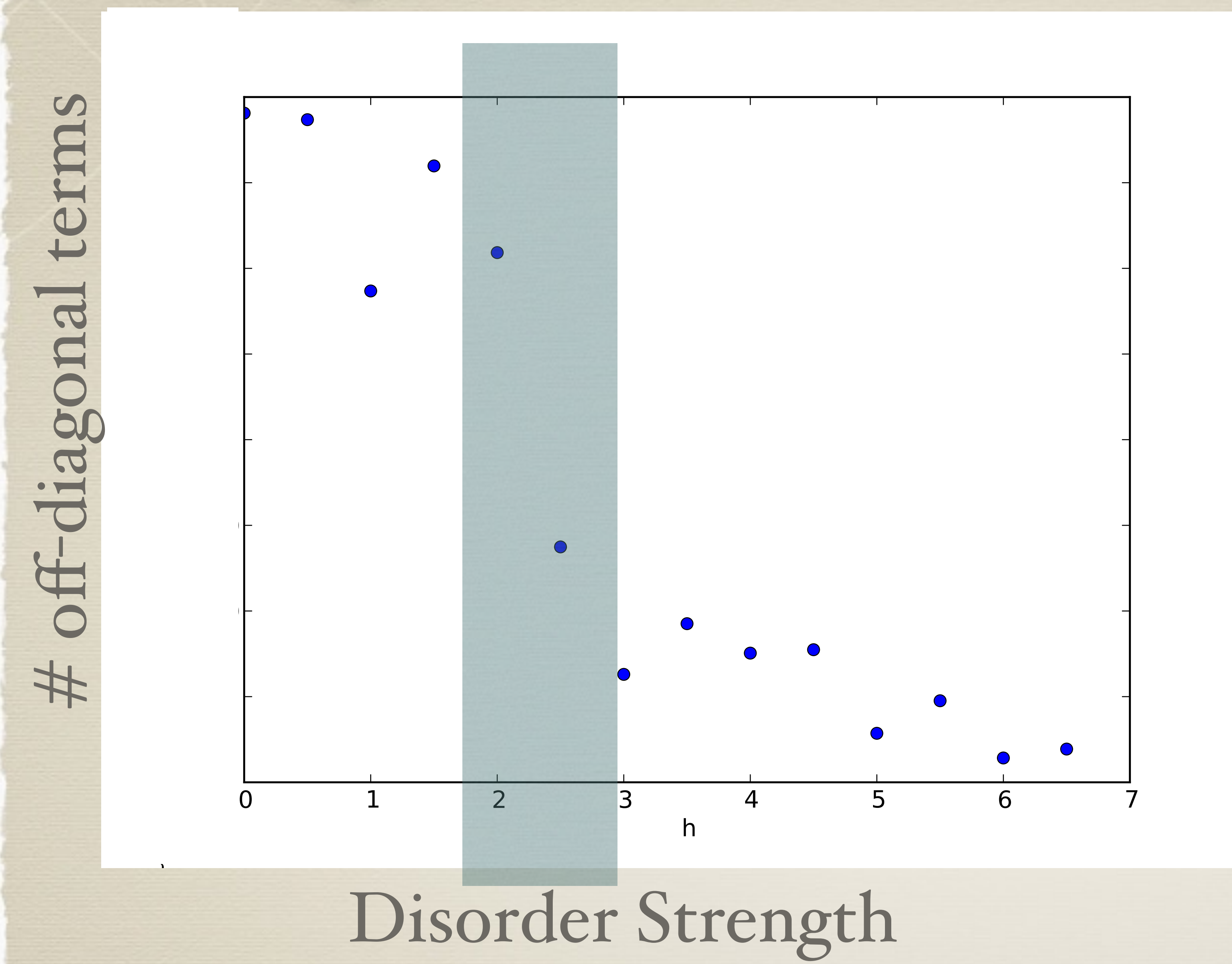


Hamiltonian H



We want a basis where the Hamiltonian becomes ‘essentially’ block diagonal.

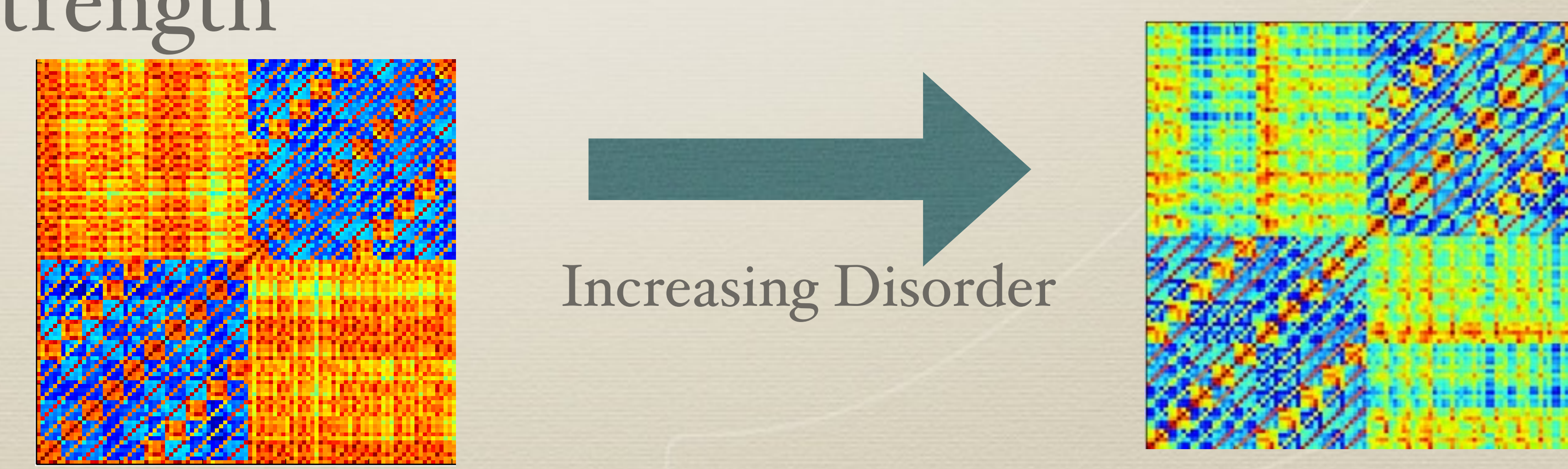
Seeing the transition ...



Apply rotation into blocks...

Check the number of off-diagonal terms.

Almost a ground state property....



Getting the eigenstates

$\hat{H}_{\text{eff}}[i]$

$\Psi_0[i]$
$\Psi_1[i]$
$\Psi_2[i]$
$\vdots$
$\Psi_n[i]$

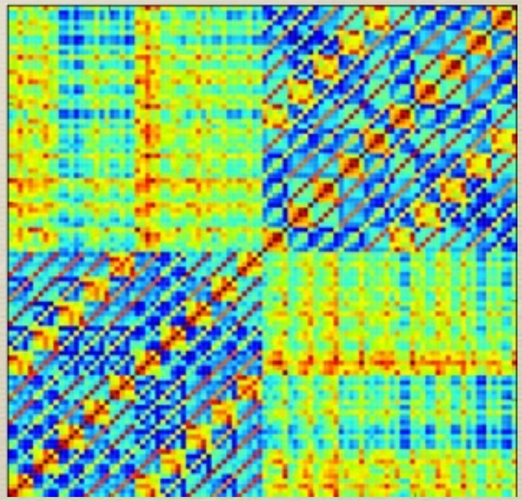
All MPS

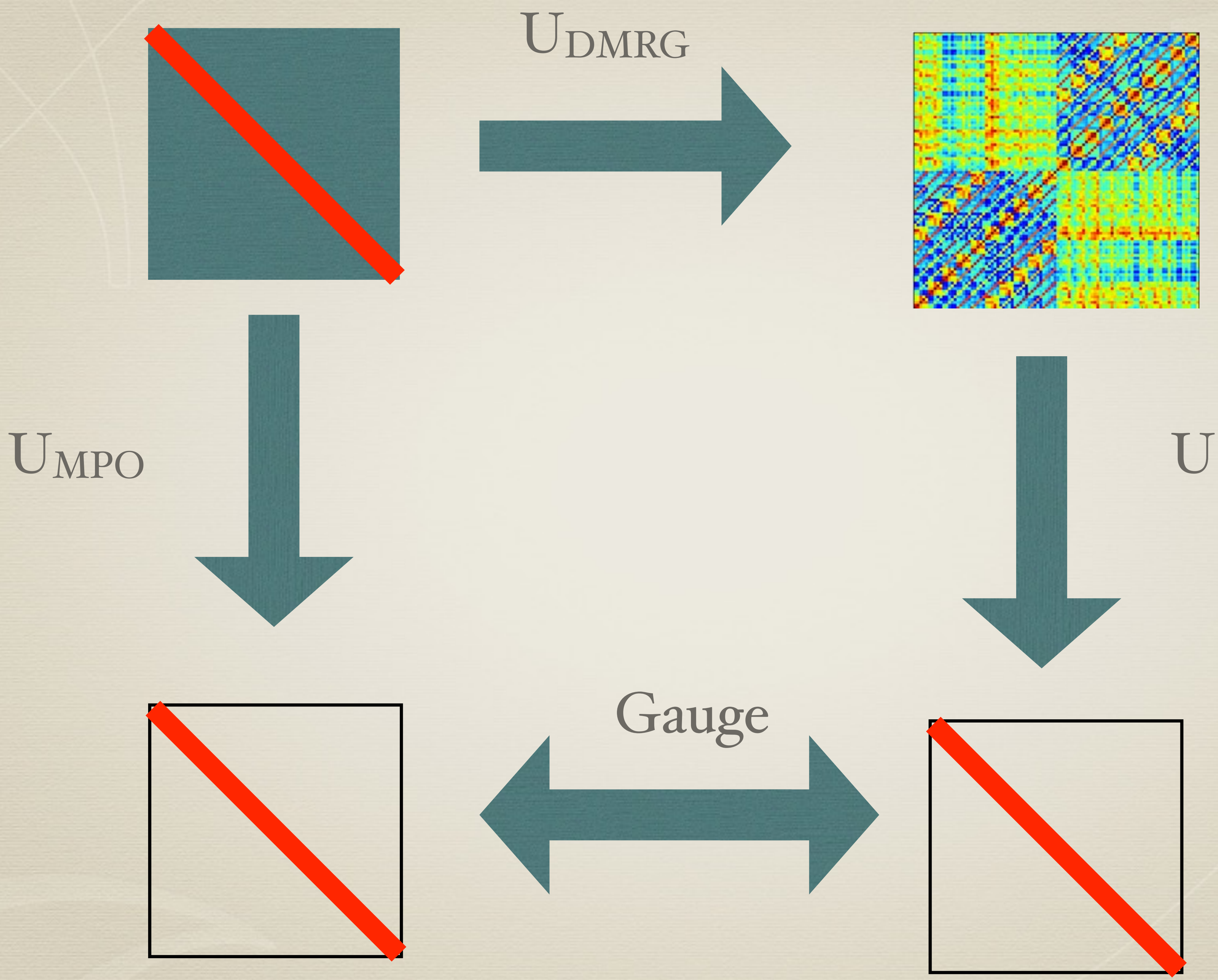
$\hat{H}_{\text{eff}}[j]$

$\Psi_0[j]$
$\Psi_1[j]$
$\Psi_2[j]$
$\vdots$
$\Psi_n[j]$

'Single-site' quasi-particles on sites i and j.

This tells us about the structure of the block diagonal Hamiltonian but not as much the eigenstates.





U_{DMRG}
U_{MPO}
U
Gauge

$$H = \sum_{i=1}^{L} [h_i S_i^z]$$

$$h_i \in [-W, W]$$

Ground State: ↑ ↓ ↓ ↓ ↑

Excited State: ↑ ↓ ↓ ↑ ↑

Highly Excited State: ↓ ↑ ↑ ↓ ↓

The product basis are eigenstates if we turn interactions off.

We need a suitable generalization of the product state basis for interacting systems.

$$H = \sum_{i=1}^{L} [h_i S_i^z + J \sum \hat{S}_i \cdot \hat{S}_{i+1}]$$

$$h_i \in [-W, W]$$

Ground State: $A^{1,\sigma_1} A^{2,\sigma_2} A^{3,\sigma_3} A^{4,\sigma_4} A^{5,\sigma_5}$

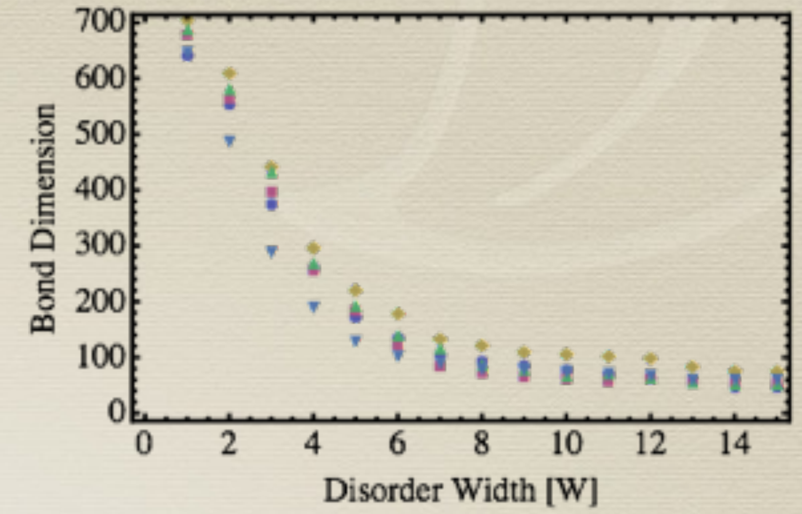
Excited State: $A^{1,\sigma_1} B^{2,\sigma_2} A^{3,\sigma_3} A^{4,\sigma_4} A^{5,\sigma_5}$

Highly Excited State: $B^{1,\sigma_1} B^{2,\sigma_2} A^{3,\sigma_3} B^{4,\sigma_4} A^{5,\sigma_5}$

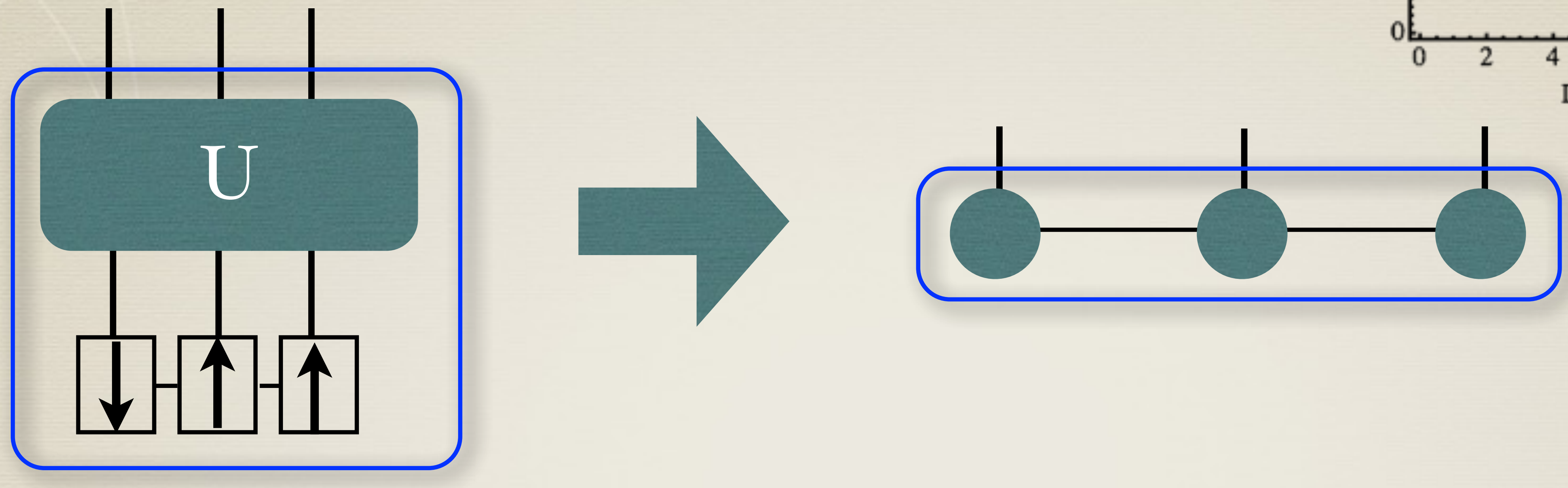
Creation operators change a matrix in the MPS.

How do we choose the A^i and B^i matrices?

Choosing A and B



$$U H_{\mathrm{bd}} U^{\dagger} = H$$



Given a product state, we produce a MPS using U.

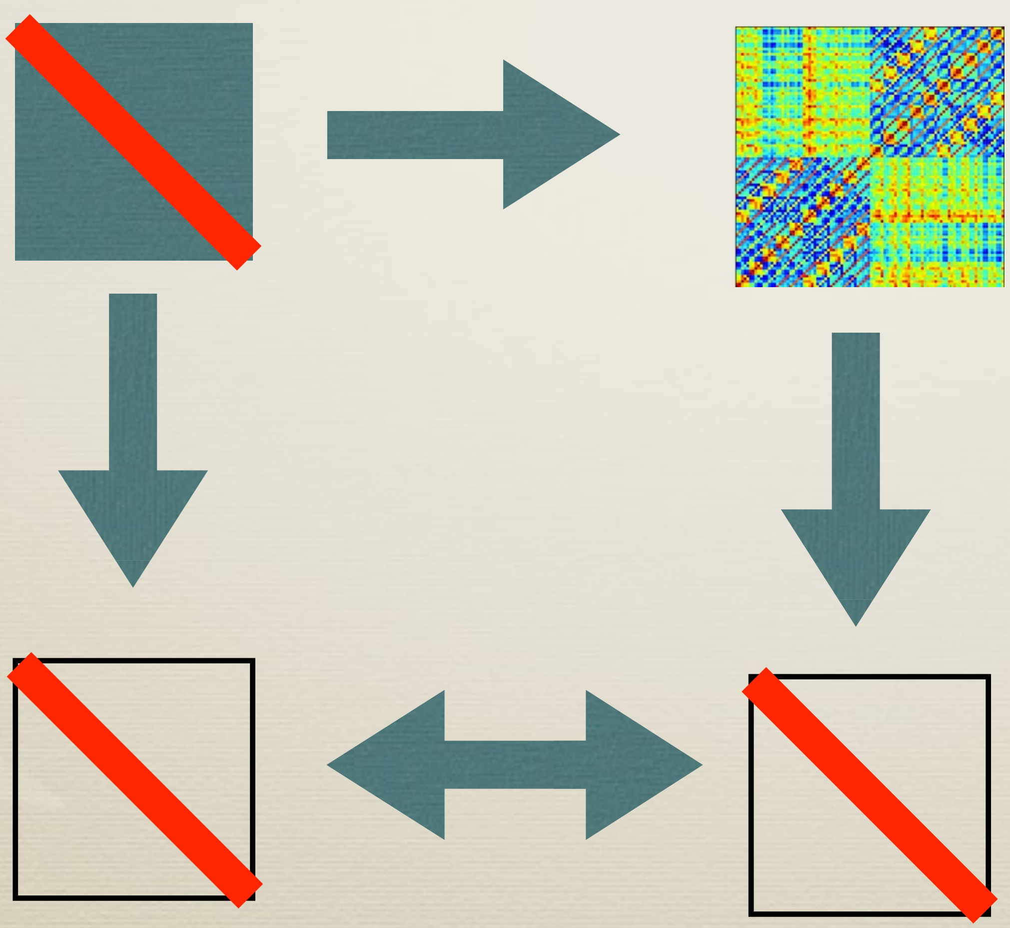
We can read Ai and Bi off of U (represented as a MPO)

Gauge transformation transform to other eigenstates

Creation operators create MPS Ai or Bi

How should we understand these strange phenomena?

We need a unifying understanding of MBL?



1. Block Diagonal
2. Eigenstates are MPS

Thermalization

Each block has unrelated eigenstates

↓

Similar energies have different properties

↓

Violation of ETH

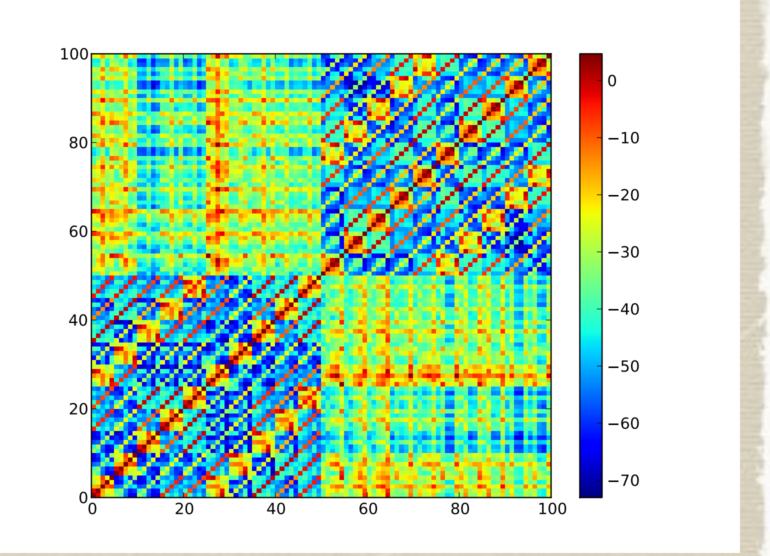
Extensive number of blocks

↓

Extensive number of conservation laws

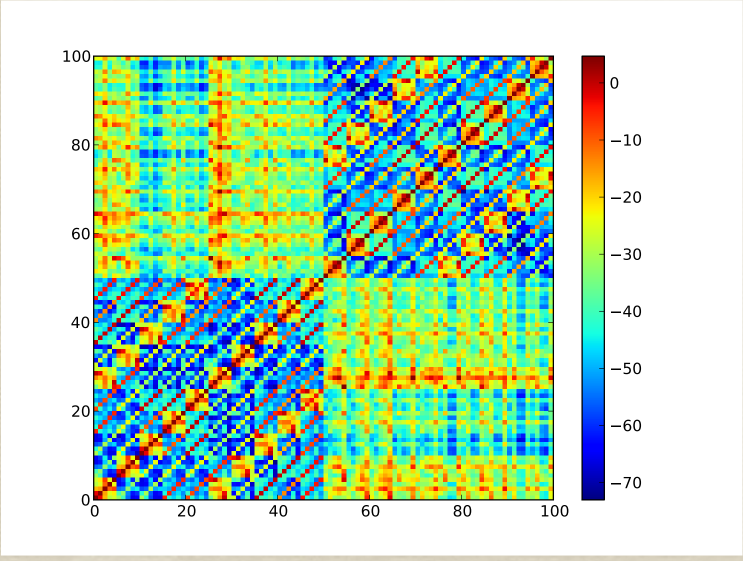
↓

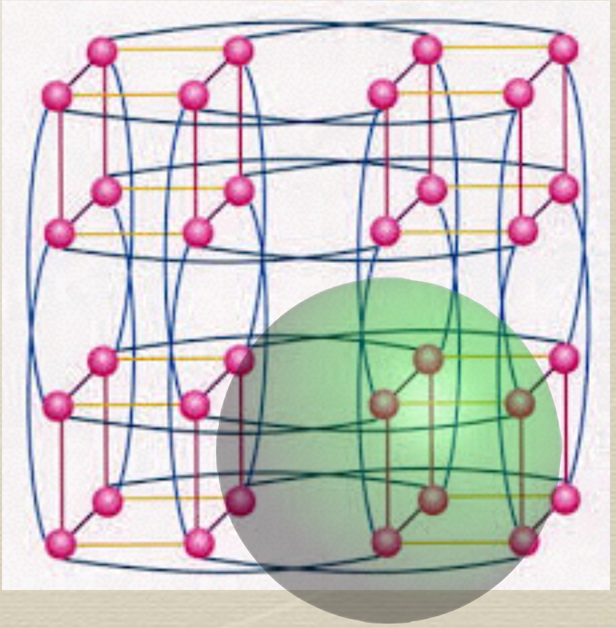
'Integrable'



Localization in Fock space

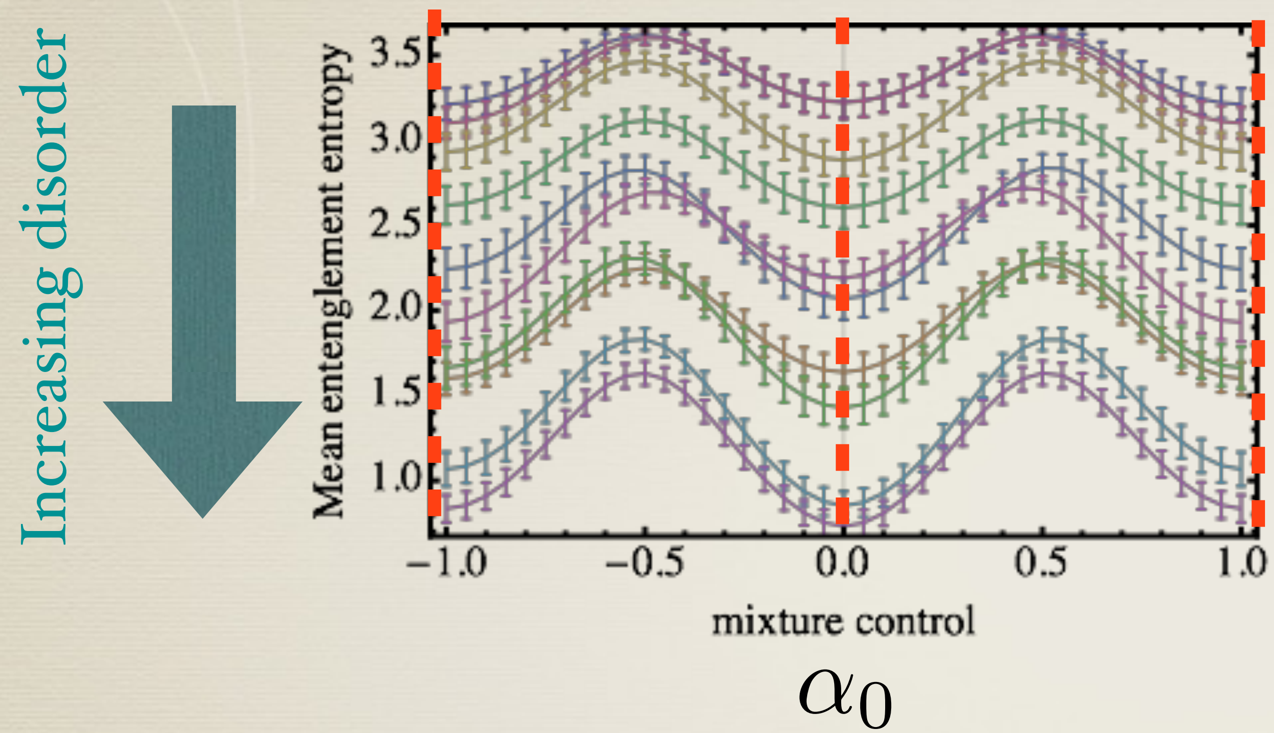
An eigenstate only lives on a small fraction of Hilbert space since it only contains weight on its 'block' (mainly)





Atypical Eigenstates

Low Entanglement (obeys area law)



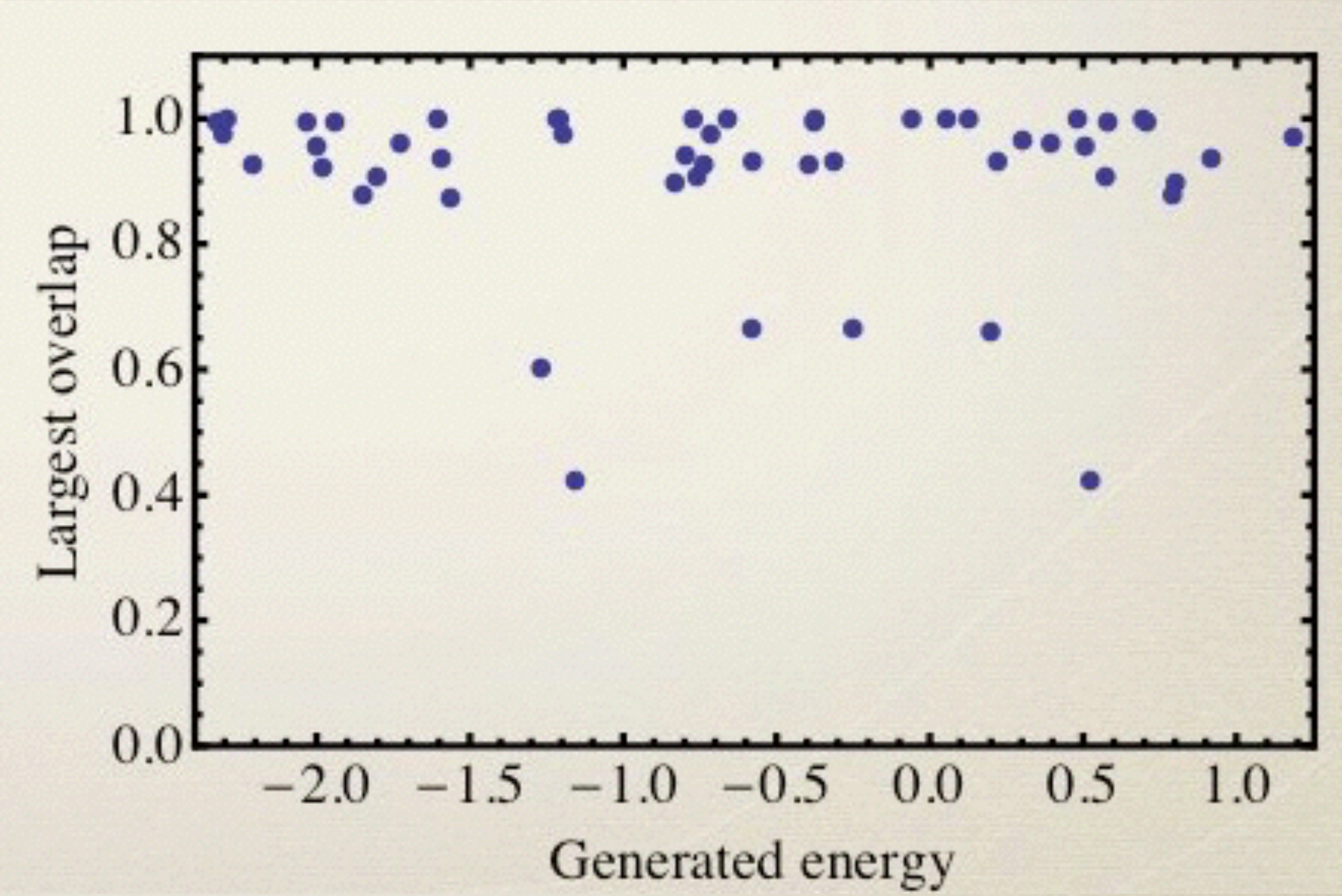
Structure implies MPS states of low bond dimension for all eigenstates

Different blocks don't talk.

Strange Spectral Statistics....

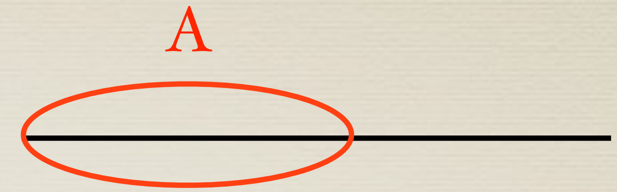
No level repulsion.!

Can we use these eigenstates to check MBL?



Diagonalizing in our little block produce PGE (pretty-good eigenstates). Can we get great great eigenstates?

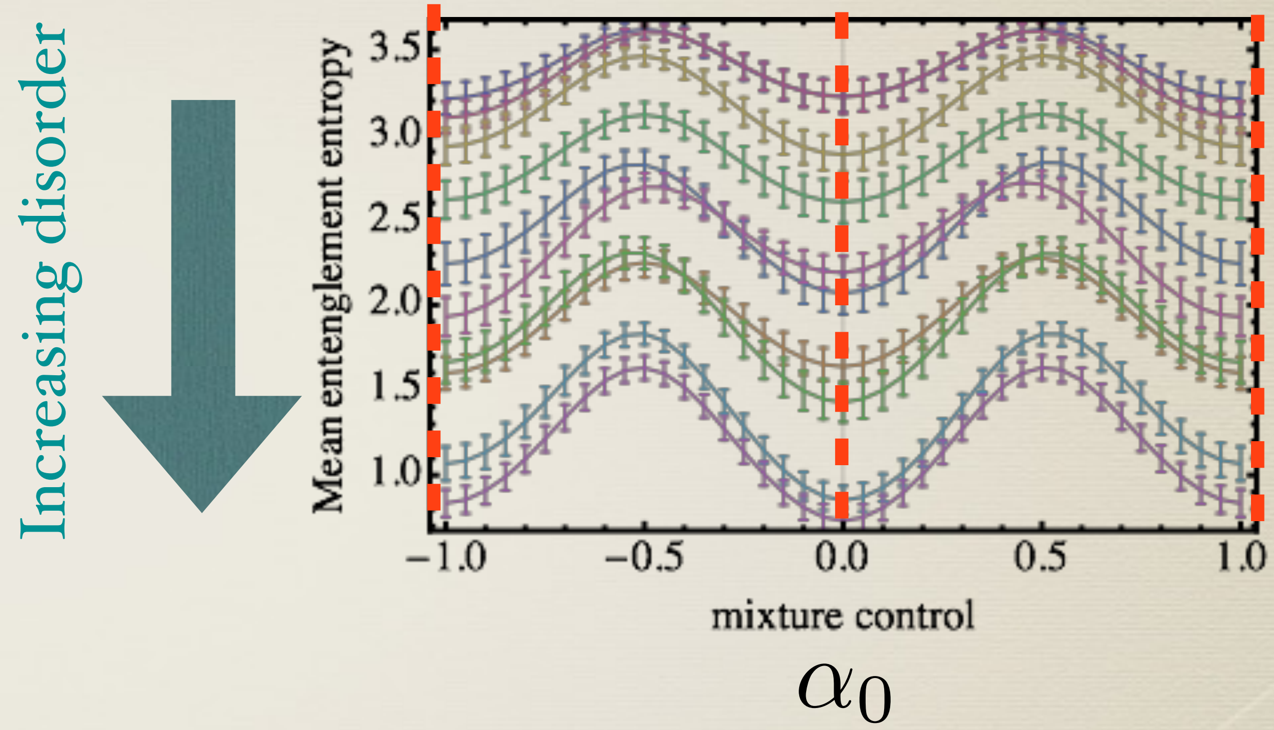
Eigenstate Filter



Need a property to filter out an eigenstate.

Entanglement: In a many-body localized phase, the eigenstates have 'low' entanglement: area law*

$$S = Tr[\rho_A \log \rho_A]$$



Key insight: Although eigenstates have low entanglement, superposition of eigenstates have larger entanglement.

$$\Psi = \alpha_0 \Psi_0 + \sqrt{1-\alpha_0^2}\Psi_1$$

We need a low entanglement filter!

Want the lowest entangled state close to a given energy.

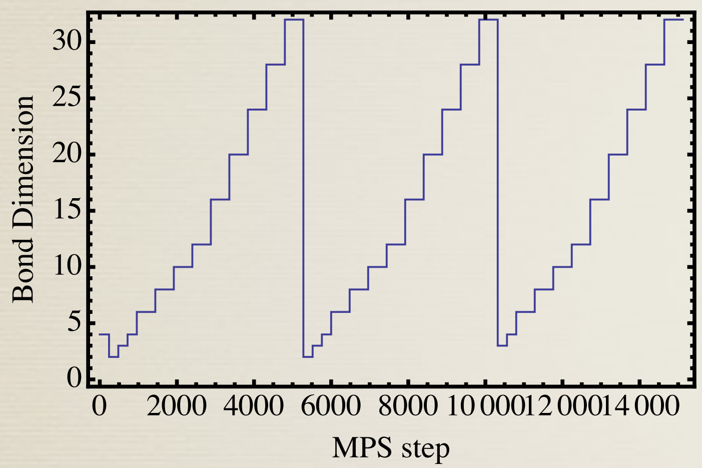
N=12 Delta=(0.05,10.5)
Average over 10 disorder realizations and 5 eigenstates

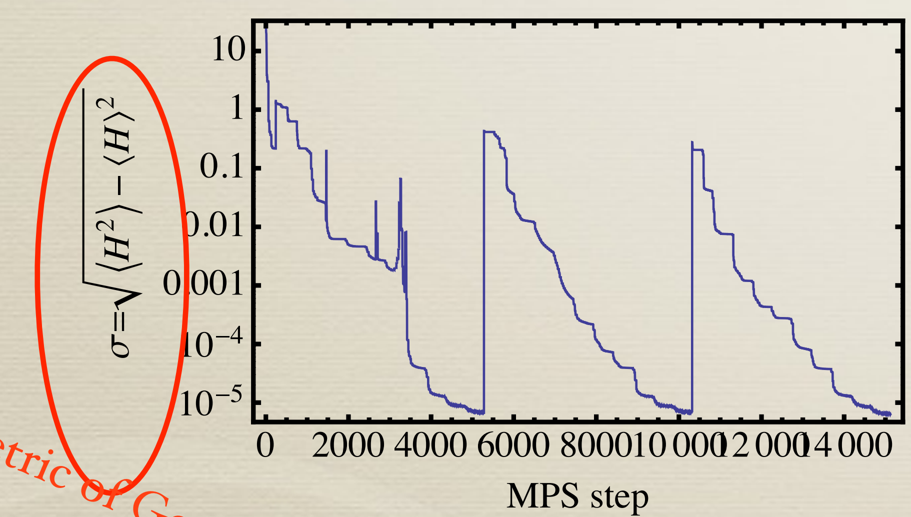
* Bela and Chetan; Brian Swingle; Abanin, et. al

Getting a MPS

MPS are a good representation. How do we get them?

DMRG on $(H - E)^2$
+
artificially drop bond-dimension during run.

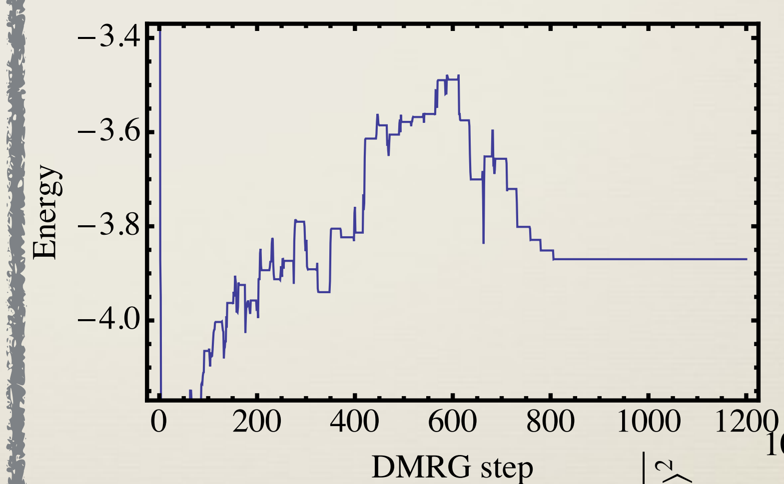


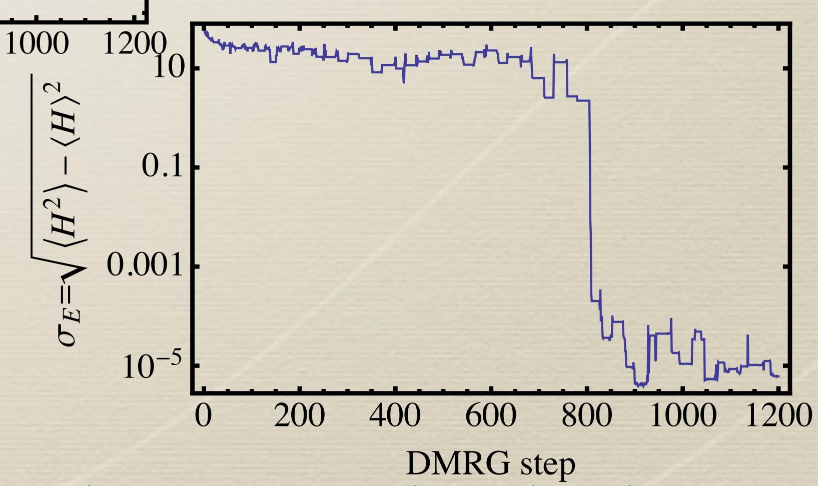


Disorder 10; 24 Sites
Level Spacing: $1.5*10^{-5}$

Typical DMRG: For a site produce an effective Hamiltonian H' and solve for the ground state of H'

Modified DMRG: For a site produce an effective Hamiltonian H' and choose the eigenstate of H' closest to the current energy of your state.





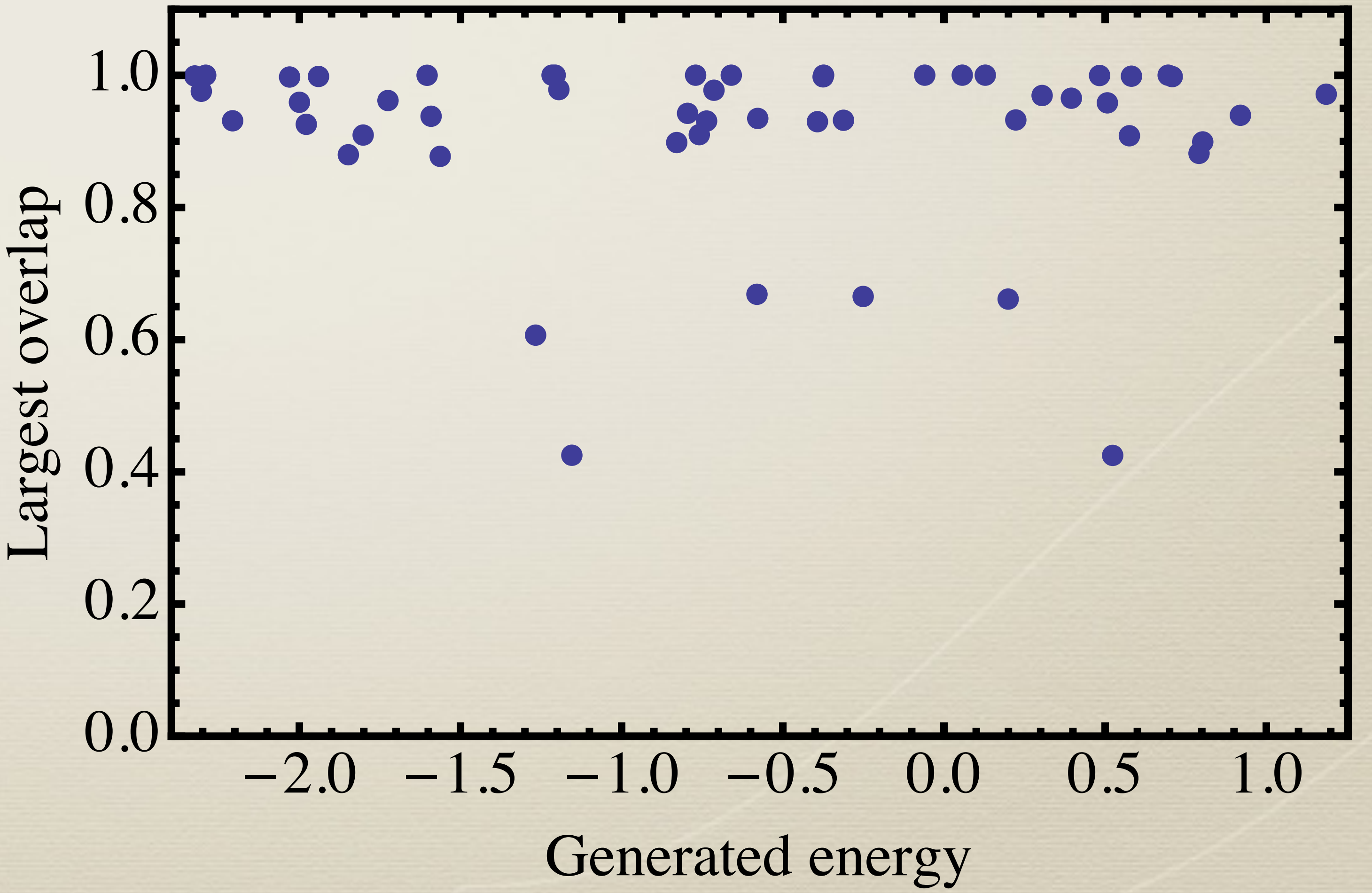
Disorder 20; 24 Sites
Level Spacing: $3*10^{-5}$

* Also time evolution variants of these; easier to 'analyze' but general experience is diagonalization approach tends to be more accurate.

Other Eigenstates

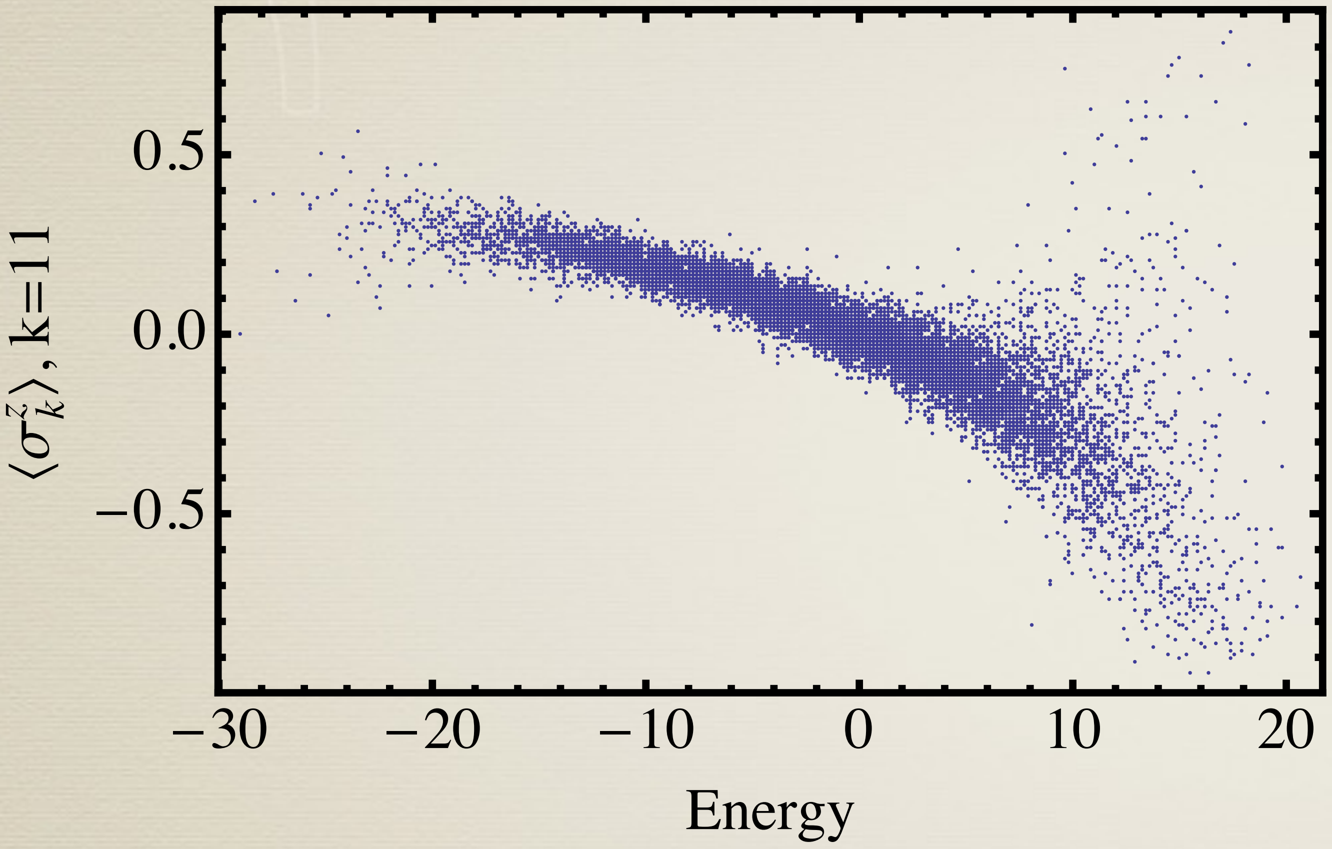
New Approach: For a site produce an effective Hamiltonian H' and choose the eigenstate of H' closest to the current energy

For a site produce an effective Hamiltonian H' and choose another eigenstate.



ETH

Delocalized phase by exact diagonalization (16 sites)

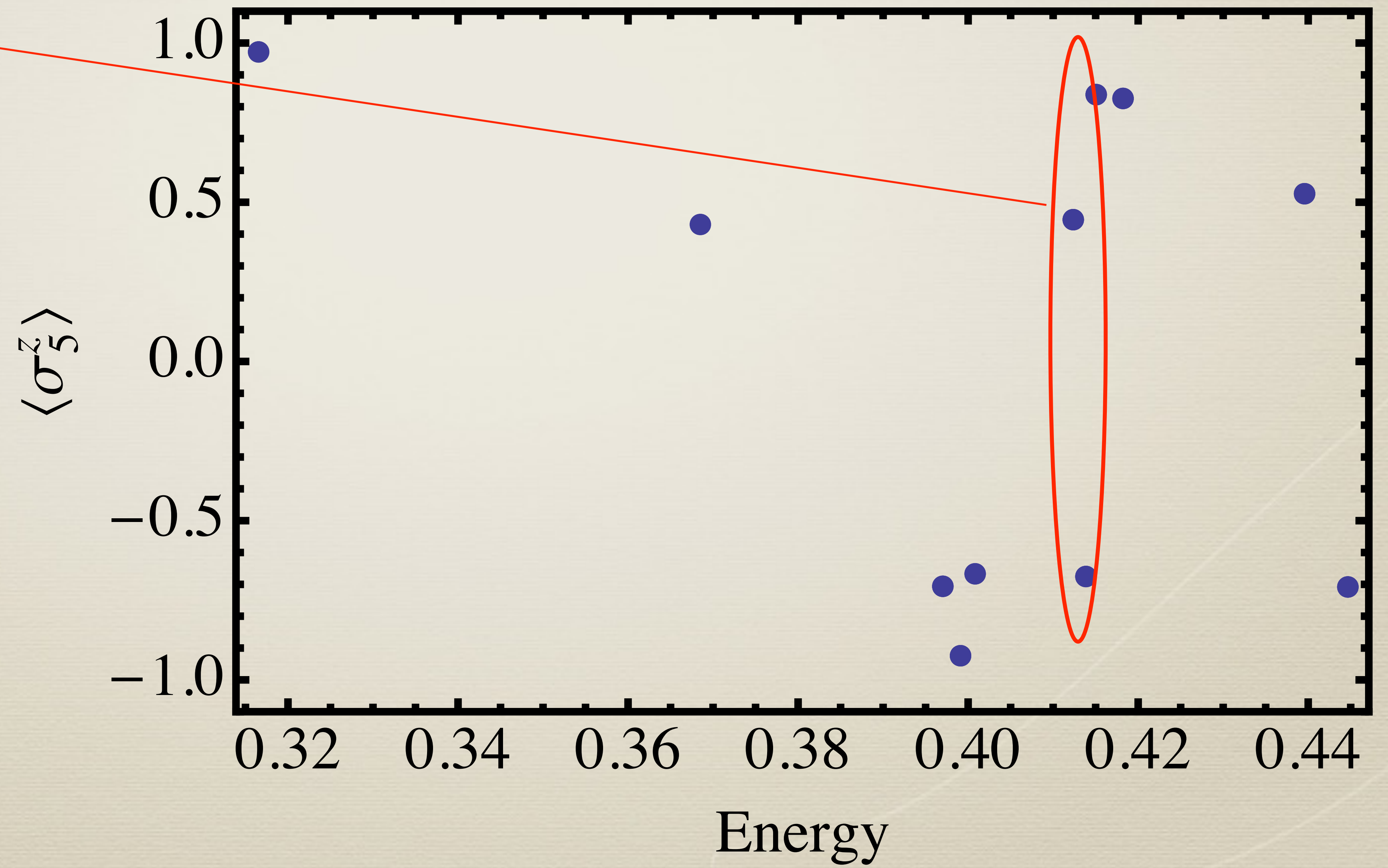


ETH

A first preliminary test of ETH: 24 sites!

Nearly degenerate states have very different local observables.

Failure of ETH!



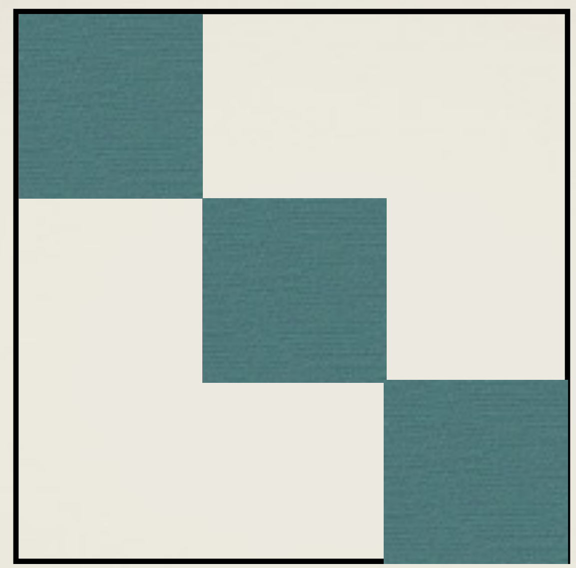
Conclusion

* MBL is a phase where a local unitary transform makes you 'almost' block diagonal with extensive # of blocks.
* MBL is a phase with MPS eigenstates.
  * Blocks don't talk => No ETH => No thermalization
  * => Any eigenstate supports only local blocks (localized)
  * All eigenstates are (essentially) ground states
    * => Area law
    * => Possion level statistics
* We can explicitly produce the block diagonal basis and eigenstates.
* We can find very high quality eigenstates to test MBL on.

A many body localized phase …

Doesn't thermalize: like integrable system

Localized: not support over all basis elements

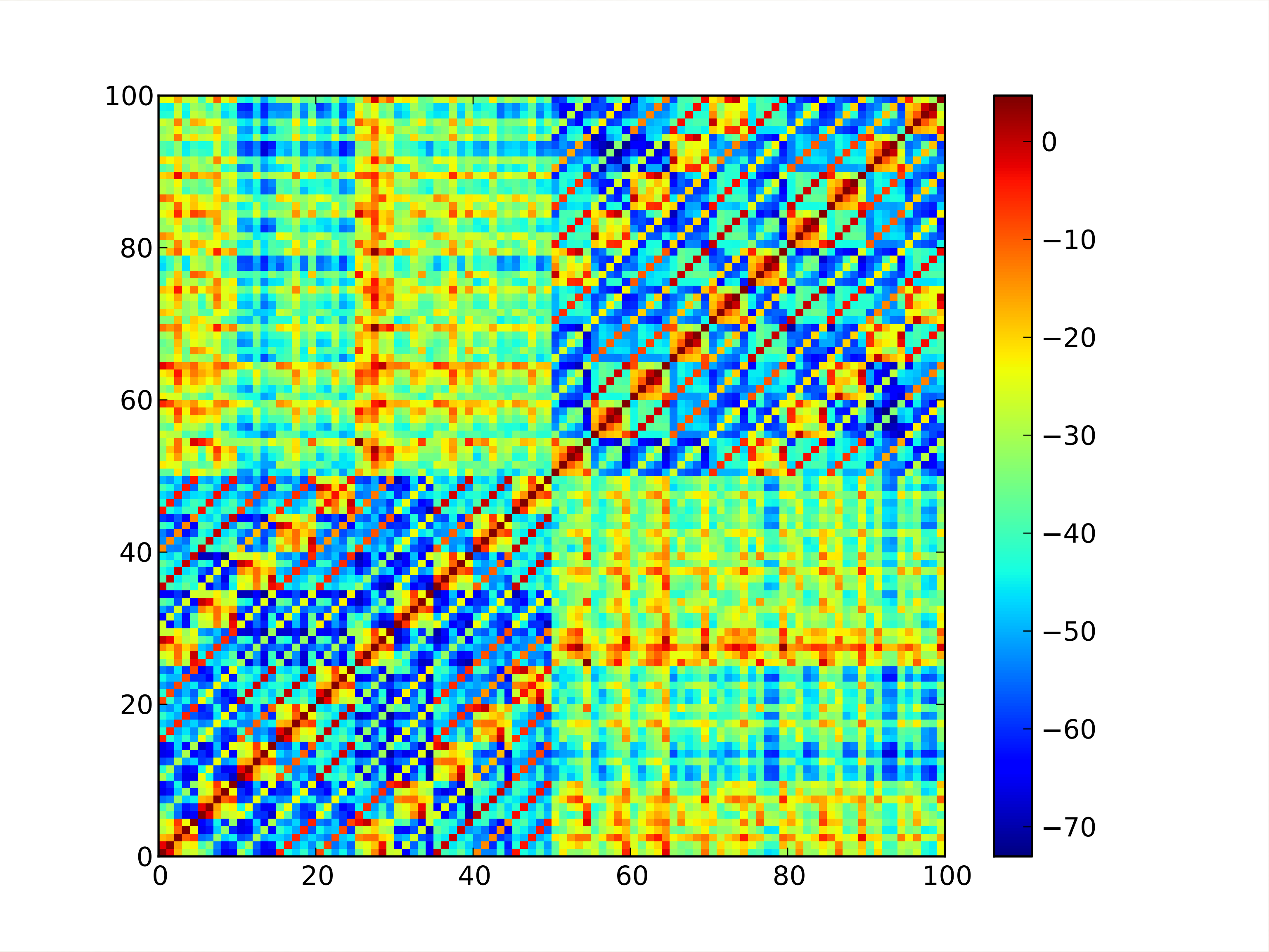


Block diagonal H where many-body basis elements are spatially local.

atypical eigenstates: low entanglement, like the ground state, odd spectral statistics

$$H = \sum_{i=1}^{L} [h_i S_i^z + J \sum \hat{S}_i \cdot \hat{S}_{i+1}]$$

$$h_i \in [-W, W]$$



$|\tilde{\Psi}_G\rangle$

$|\Psi_G\rangle \equiv \sum_i |a_i\rangle|b_i\rangle$ $|\Psi_G\rangle \equiv \sum_i |\alpha_i\rangle|\beta_i\rangle$

Basis: $\{|a_1\rangle|b_1\rangle, |a_2\rangle|b_2\rangle, \ldots |a_9\rangle|b_9\rangle\}$ $\{|\alpha_1\rangle|\beta_1\rangle, |\alpha_2\rangle|\beta_2\rangle, \ldots |\alpha_9\rangle|\beta_9\rangle\}$

l where $2^l = 2D^2$

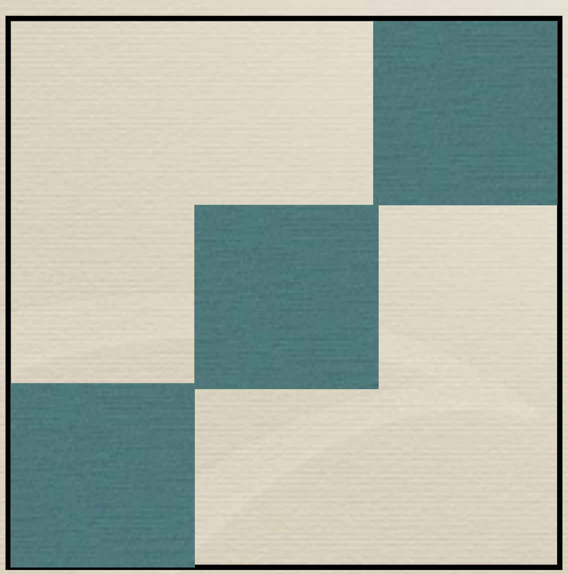
Produce excitations on sites i,j,k,...

These are single particle excitations. How to get a two particle excitation?

I. Produce a quasi-particle on site i

II. Schmidt-decompose on site j, k, etc.

II. Use the Schmidt-vectors as the basis.

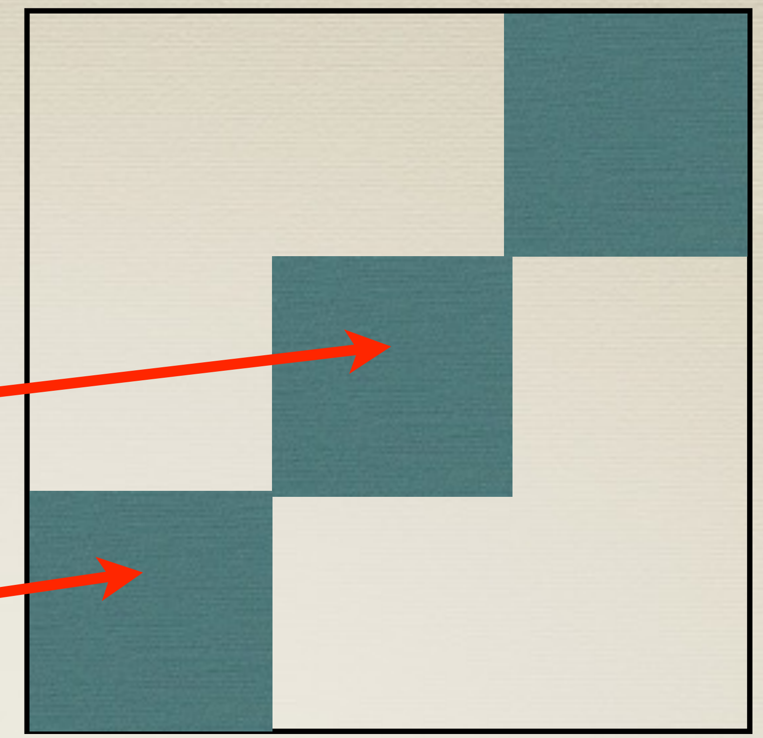


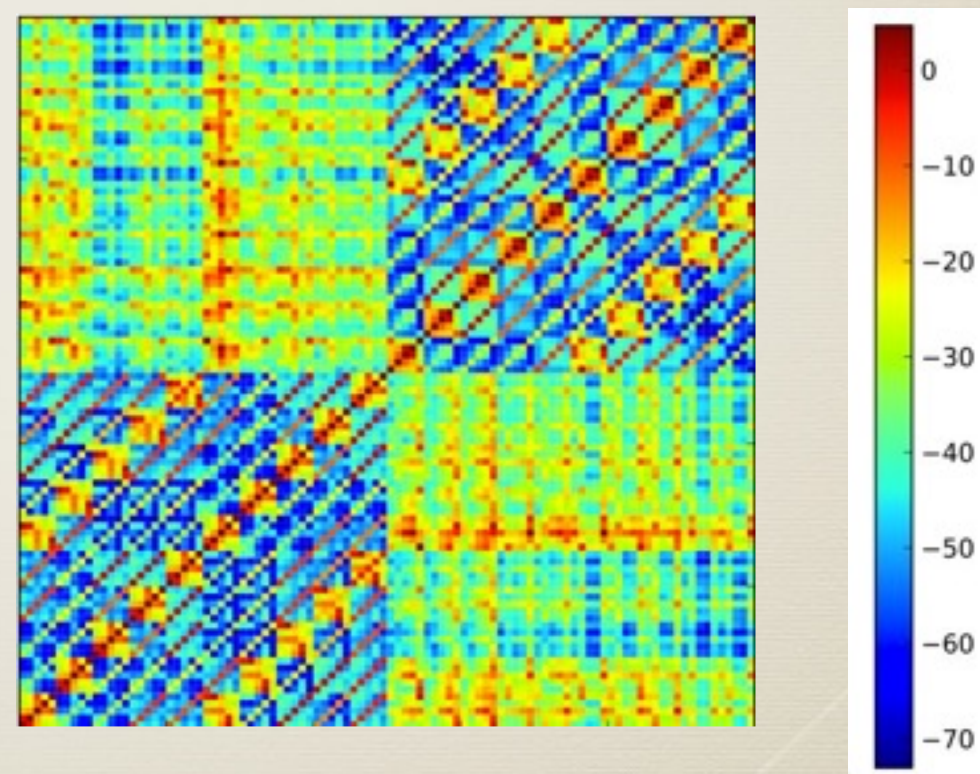
How should we understand these strange phenomena?

A local block diagonal basis is a unifying understanding

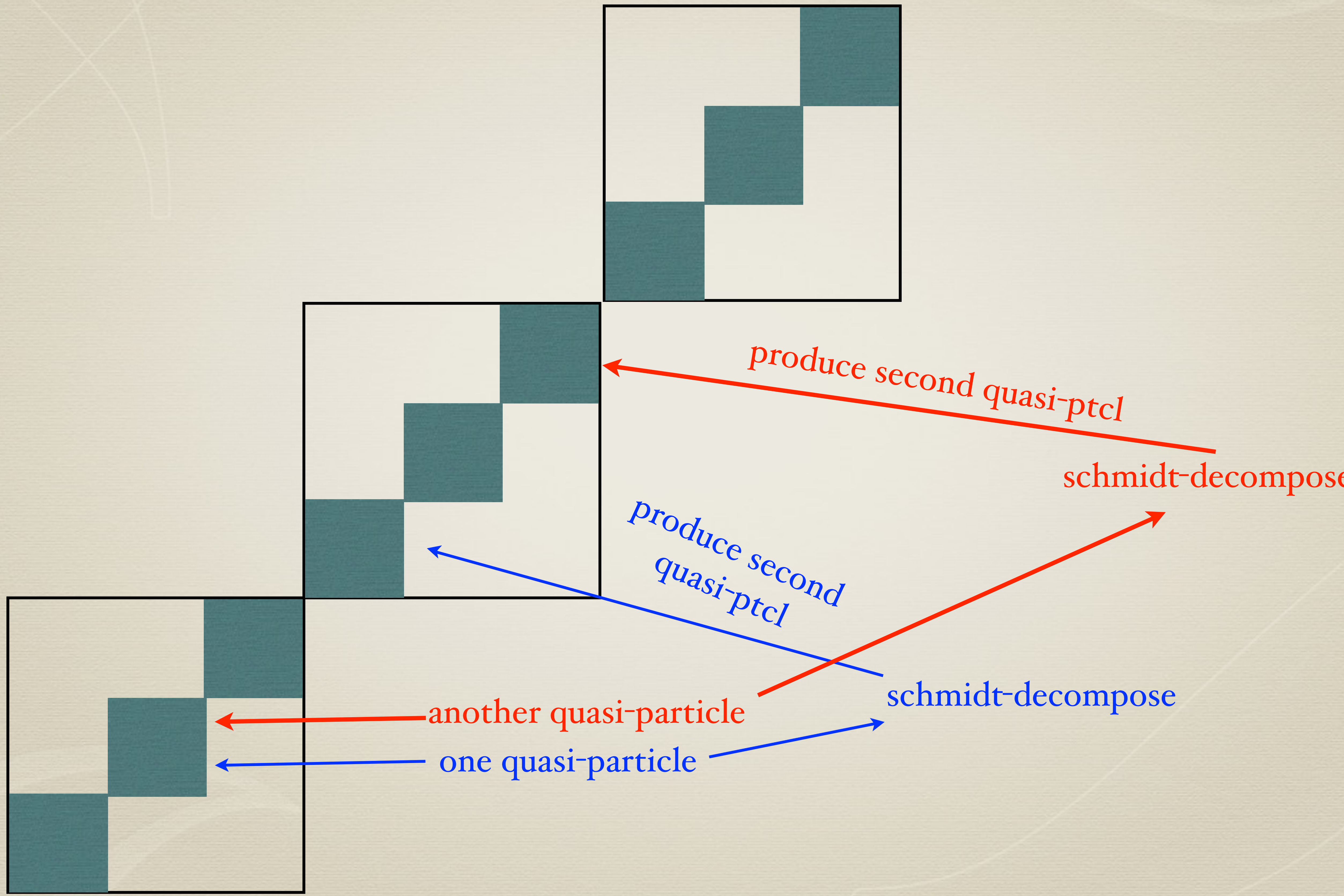
quasi-ptcls from site j
come from this block.

quasi-ptcls from site i
come from this block.





Basis of blocks = Schmidt decomposition of quasi-ptcls


produce second quasi-ptcl
schmidt-decompose
produce second
quasi-ptcl
schmidt-decompose
another quasi-particle
one quasi-particle

How should we understand these strange phenomena?

A local block diagonal basis is a unifying understanding