Resonances in MBL

(+ other thoughts about MBL)

Bryan Clark
with Benjamin Villalonga

Outline:

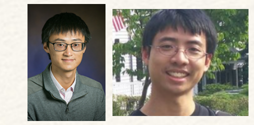
Beyond MBL

- Mobility Emulsions (1 slide) *(continuing the conversation of Dima's talk)*
- Transitions in Random Tensor Networks (1 slide)
- Transitions in Circuits with Measurement (1 slide)

MBL Transition

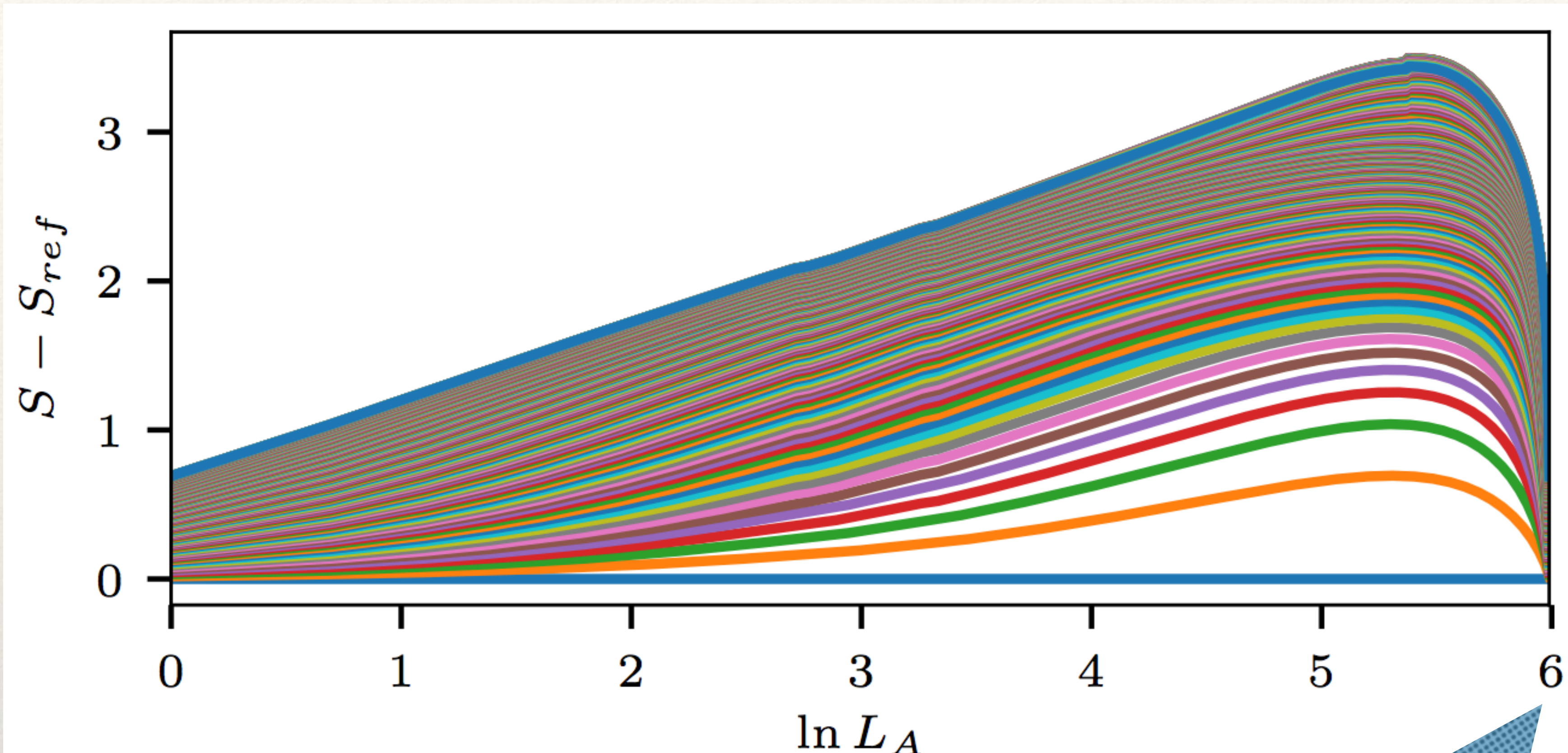
- Background: Understanding of fully MBL (1 slide)
- Background: What we know about the MBL transition… (2 slides)
- Resonances and Statistics of Mutual Information in the Transition Region

Beyond the MBL phase…

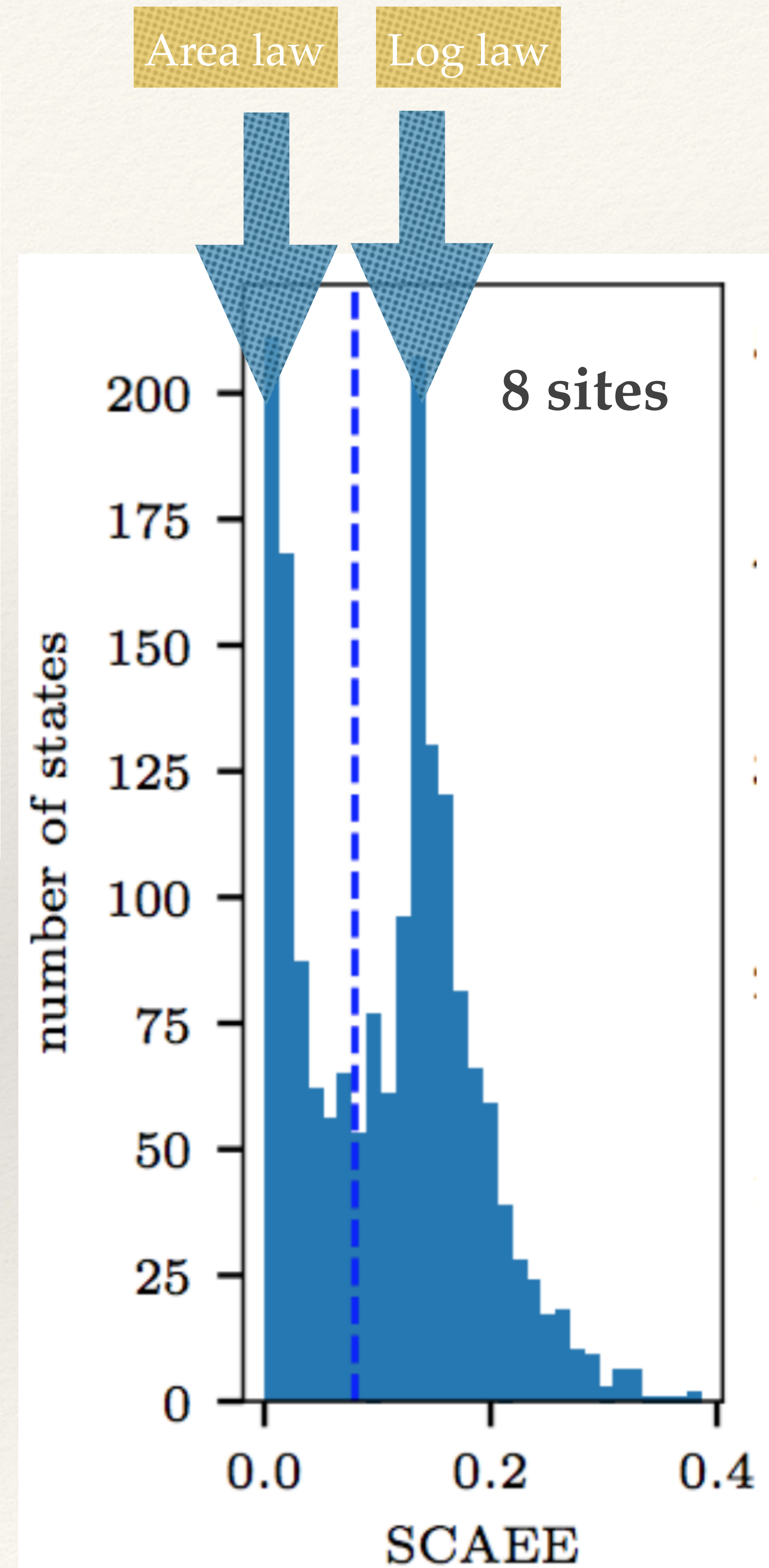


Yu-Luo-Clark (arxiv: 1803.02838)

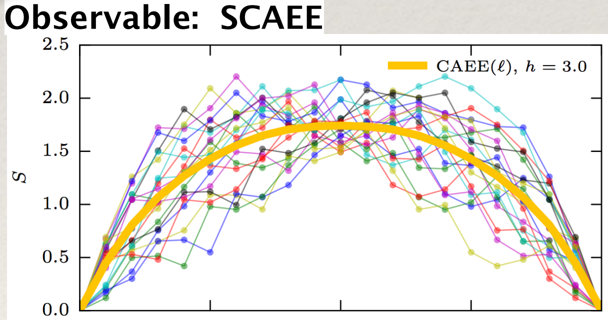
$$H = -t\sum_{i\sigma}(c^{\dagger}_{i\sigma}c_{i+1\sigma} + h.c.) + \sum_{i} U n_{i\uparrow}n_{i\downarrow} + \sum_{i} h_i S^z_i$$



400 sites



Observable: SCAEE



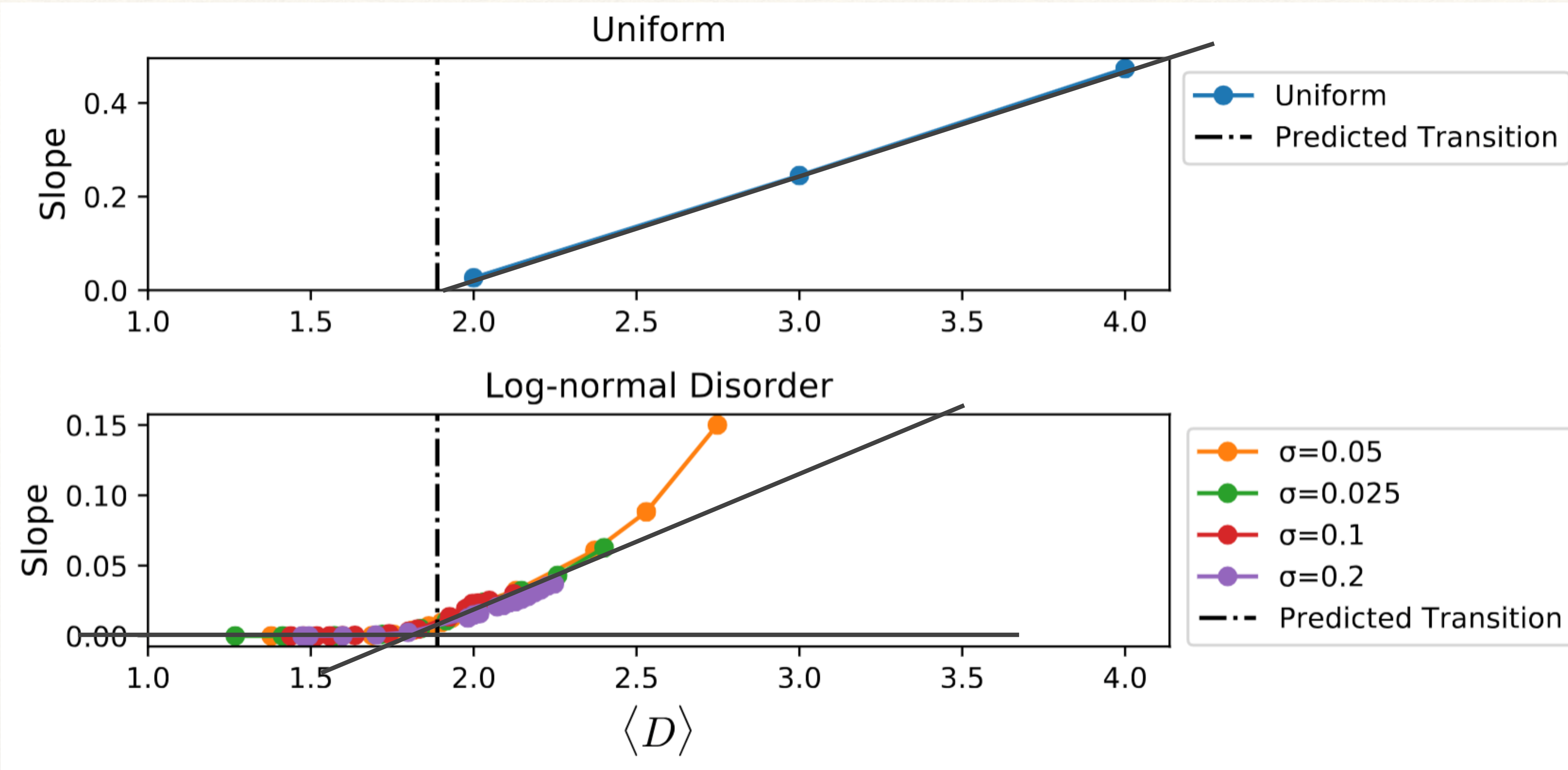
Yu – Luitz – Clark: arxiv: 1606.01260

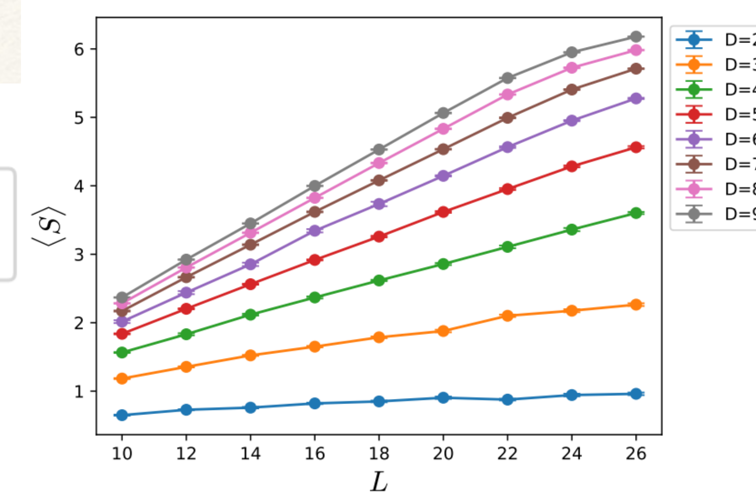
No MBL in SU(2) Symmetric Models

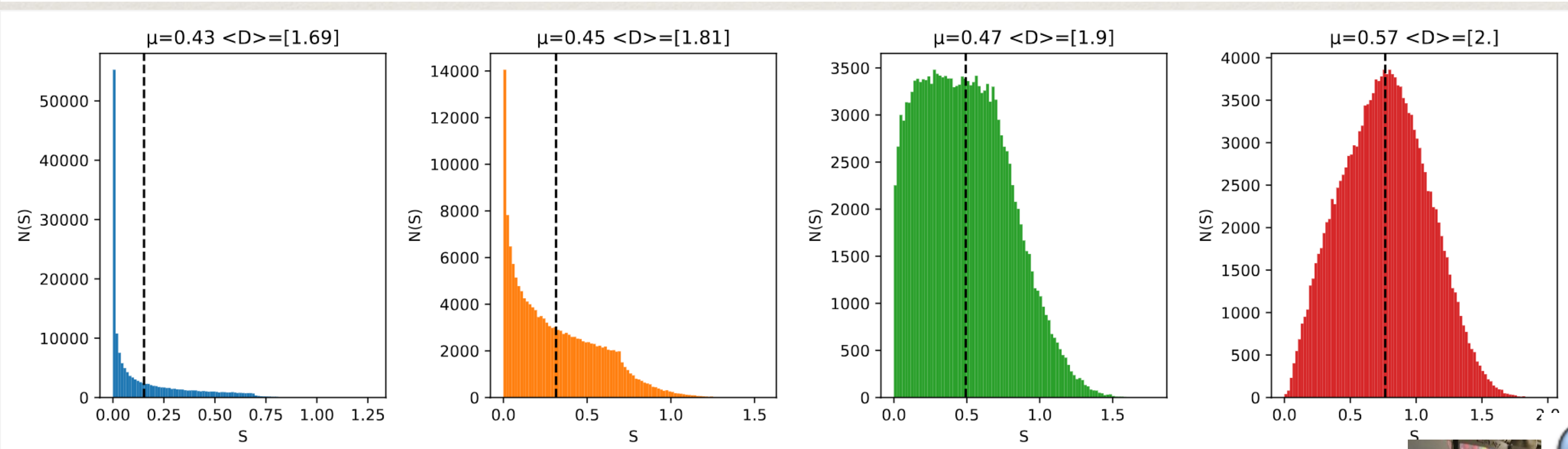
Potter-Vasseur PRB 42 (224206)

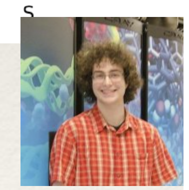
Protopov – Ho- Abanin: PRB 61, 041122

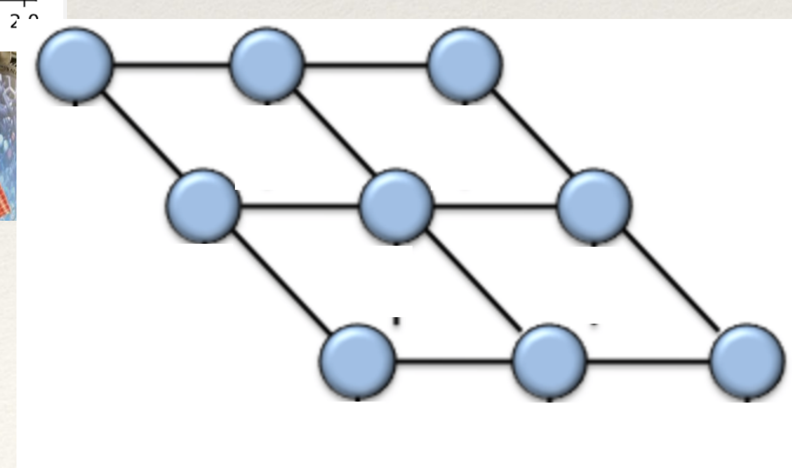
Bond-dimension transition in random PEPS





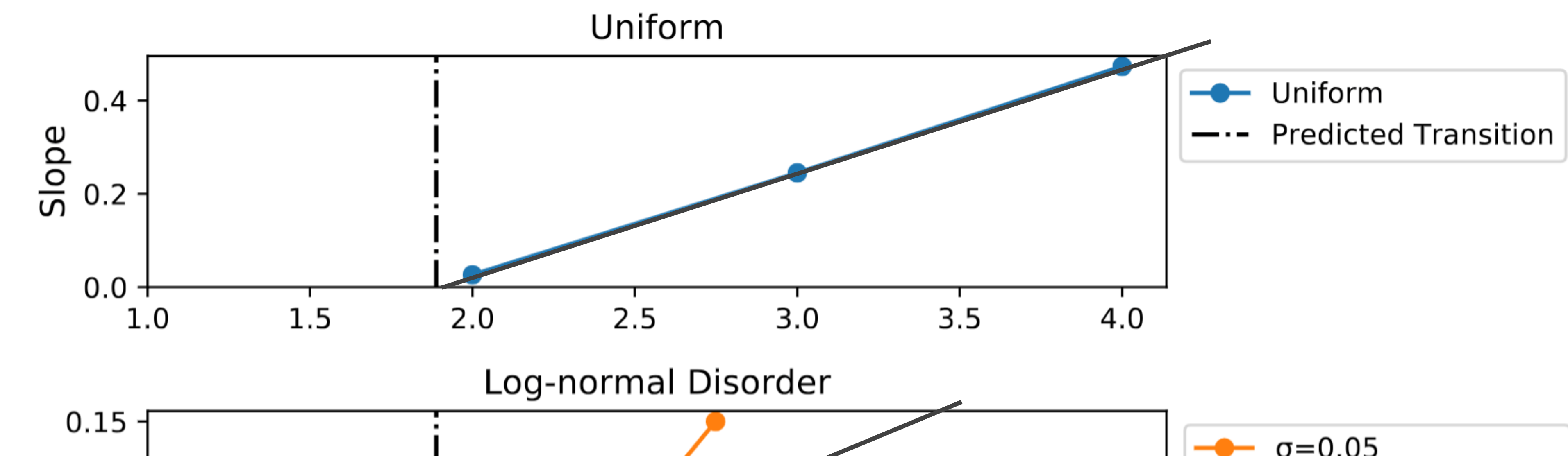


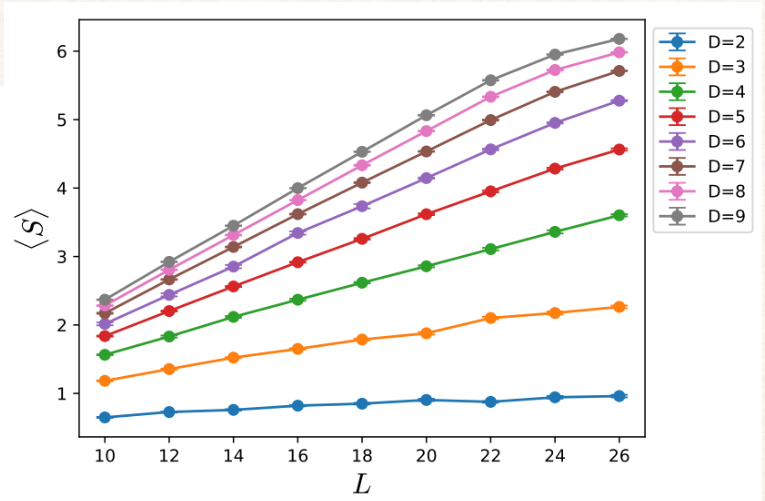


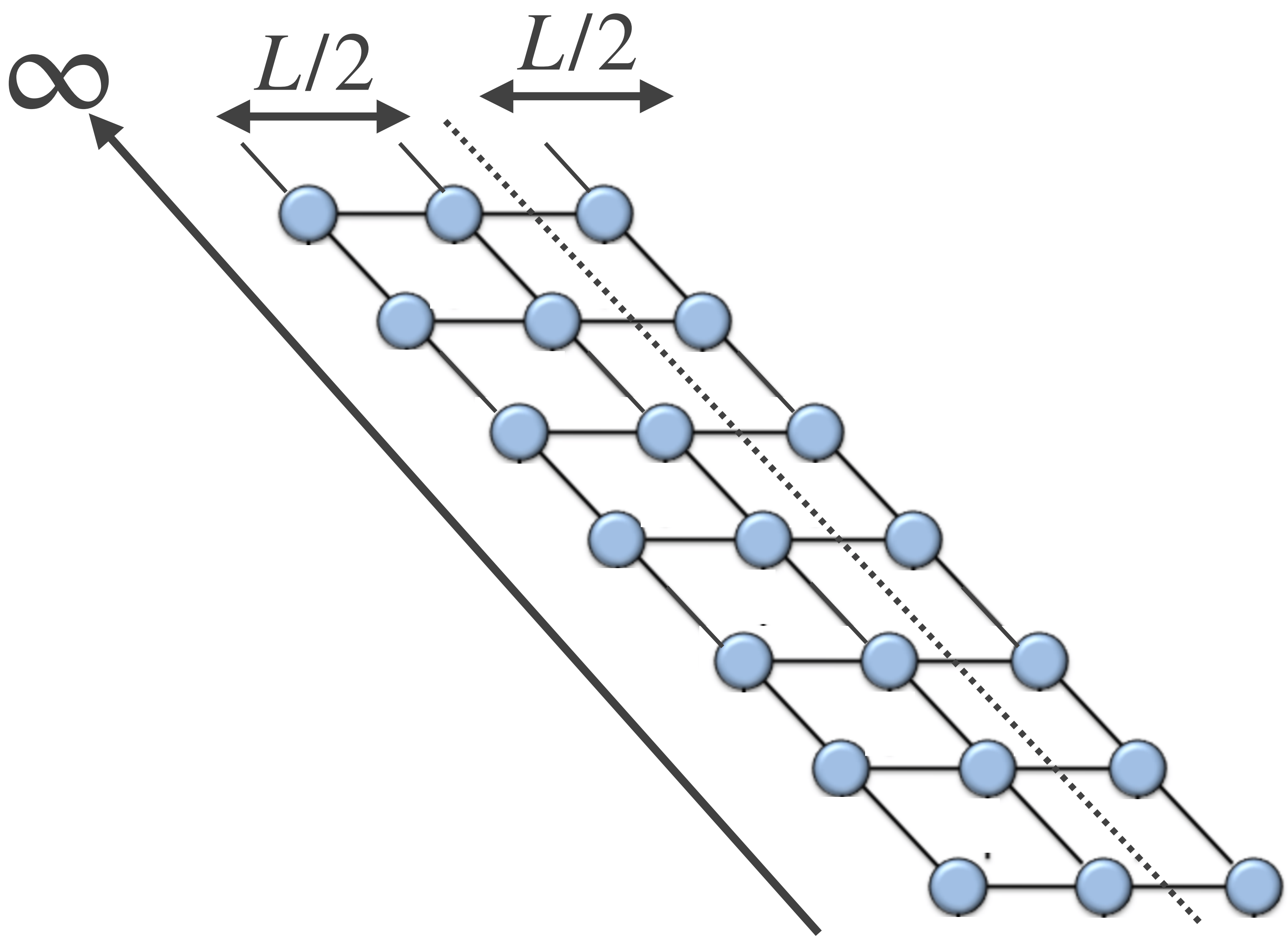


Building on: **Vasseur-Potter-You-Ludwig arxiv: 1807.07082**

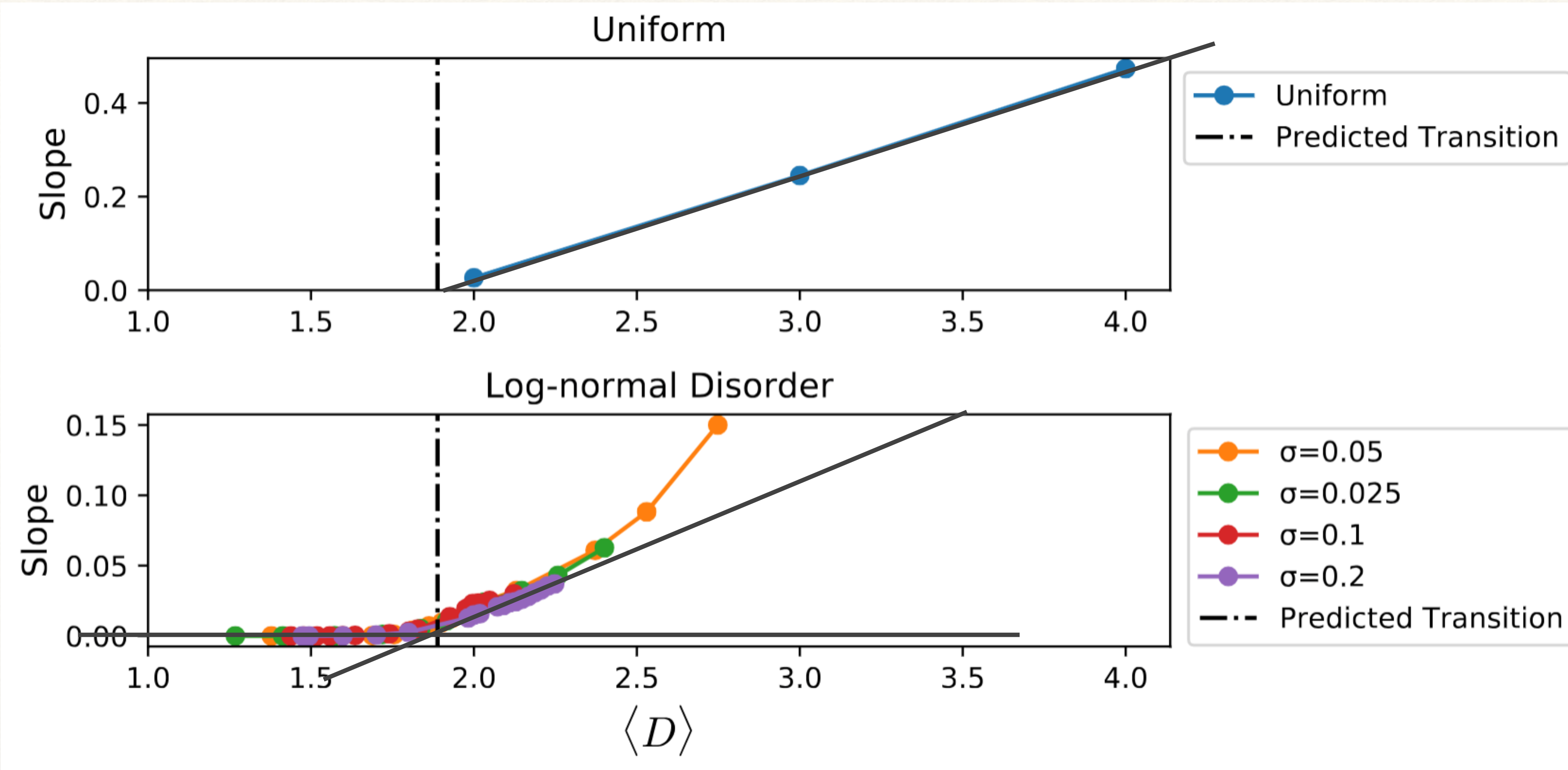
Bond-dimension transition in random PEPS

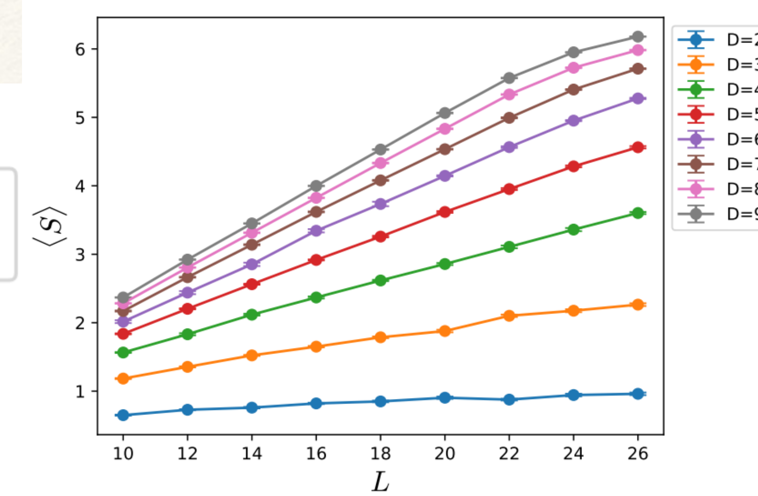


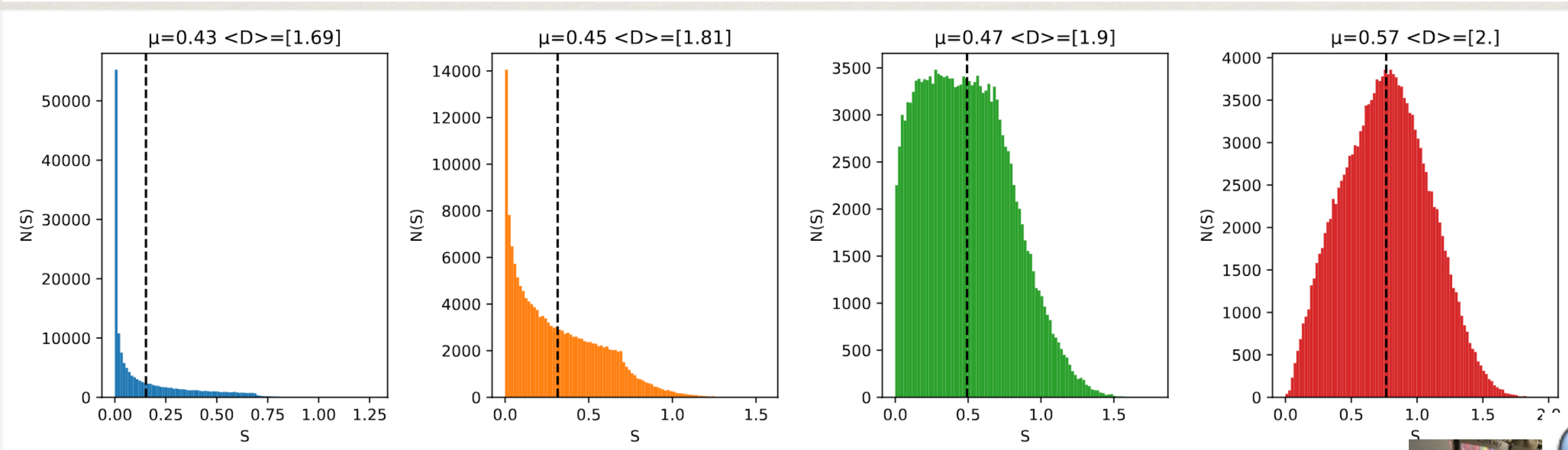


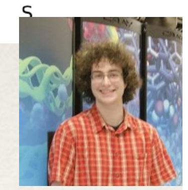


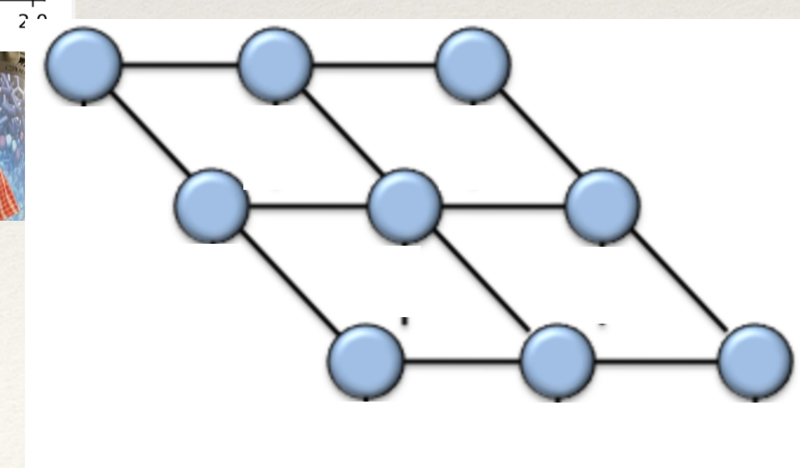
Bond-dimension transition in random PEPS





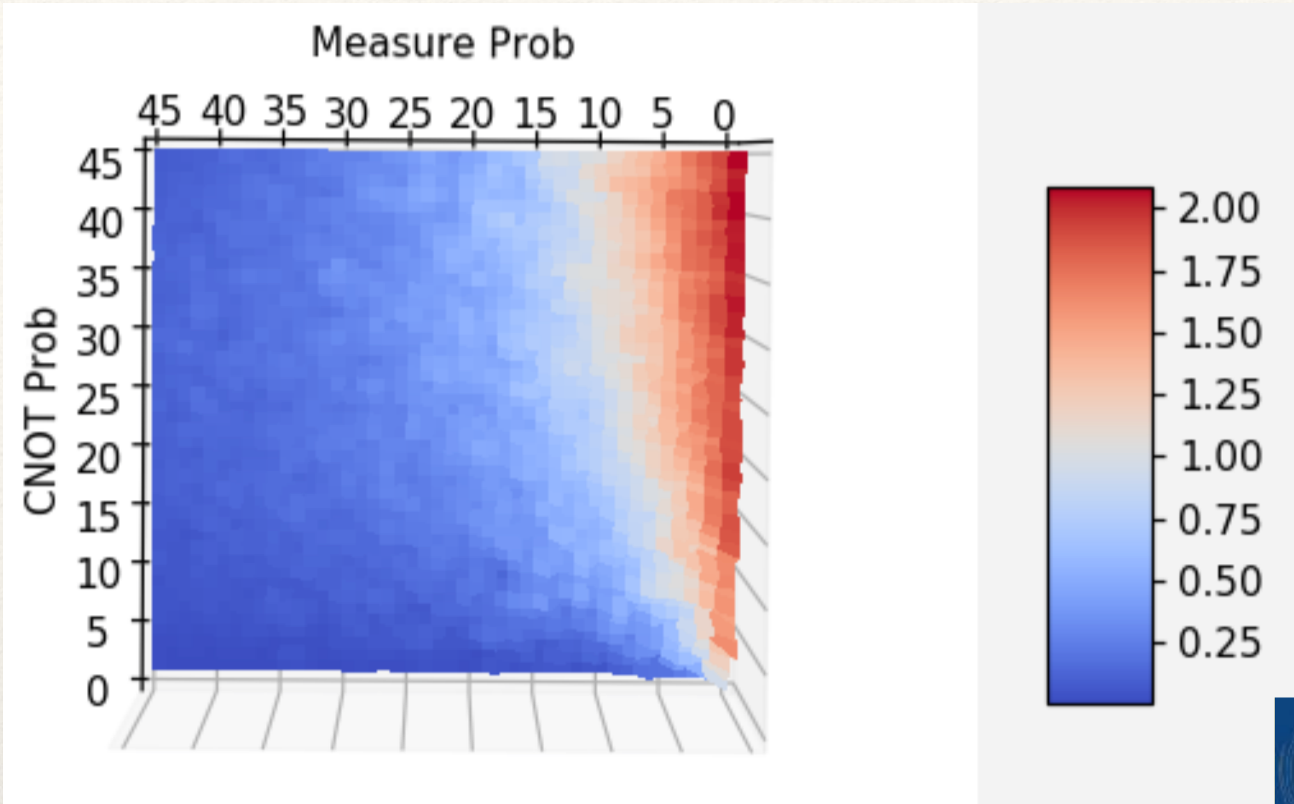






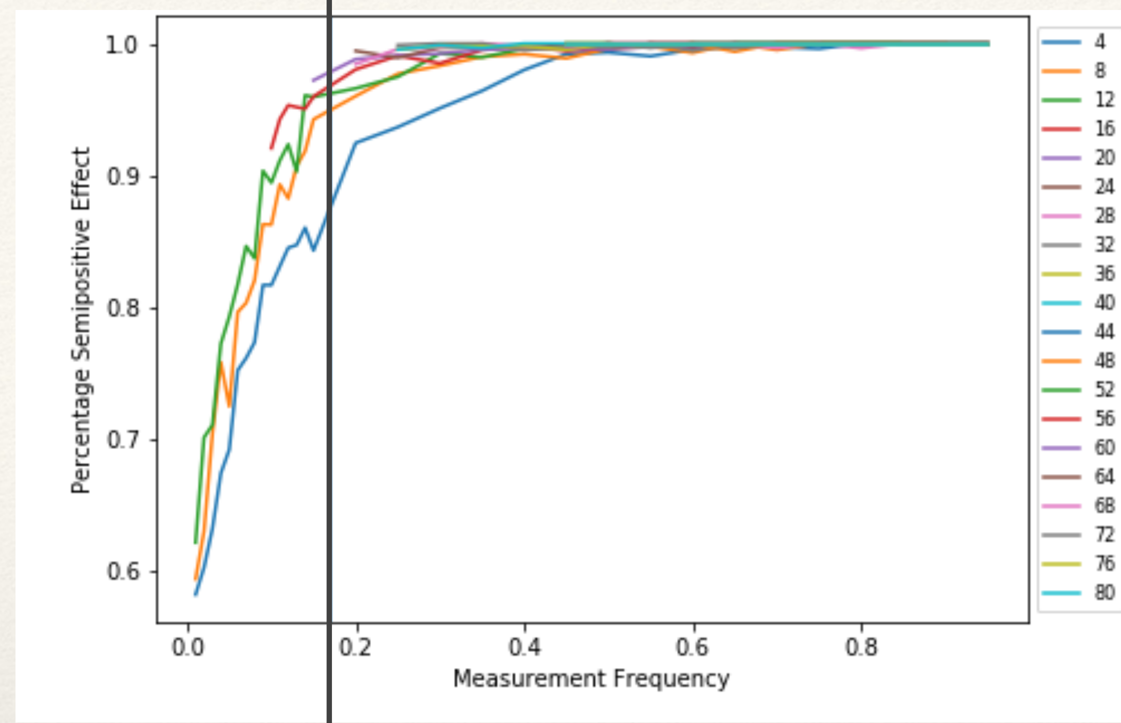
Building on: **Vasseur-Potter-You-Ludwig arxiv: 1807.07082**

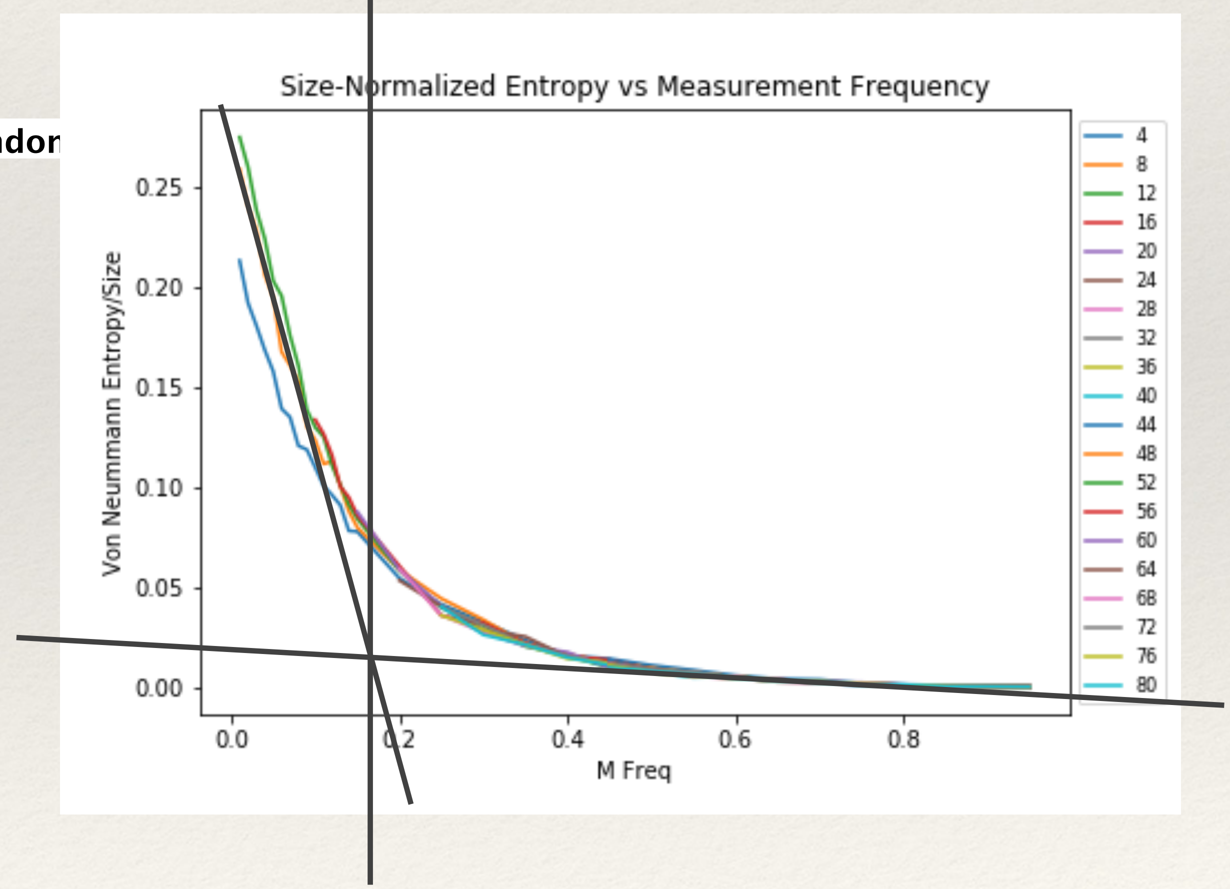
Entanglement Transition in Long Range Gates + Measurements



Volume law - volume law transitions

Kyle Herndon



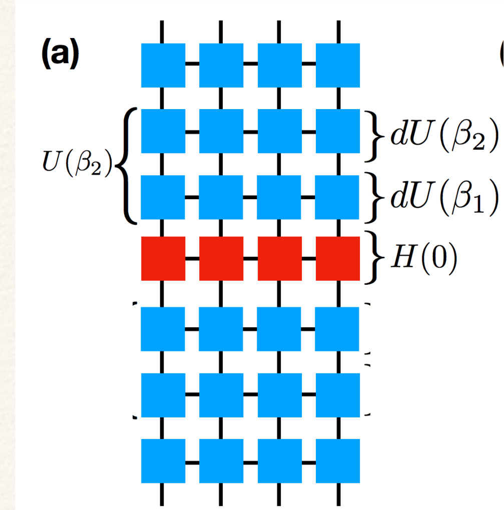


Building on:

Li-Chen-Fisher: arxiv: 1808.06134
Chan - Nandkishore - Pretko - Smith: arxiv: 1808.05949
Skinner - Ruhman -Nahum arxiv: 1808.05953

Fully Many-Body Localization

Small bond-dimension diagonalizing unitary tensor network



$$H_D = UHU^\dagger$$

Pekker-Clark arxiv:1410.2224

Chandran, Carrasquilla, Kim, Abanin, Vidal- arxiv: 1410.0687

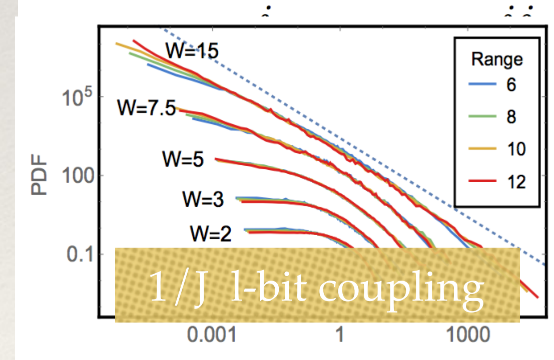
Algorithms: [Khemani-Sondhi] [Pal-Simons] [Yu-Pekker-Clark]

l-bits

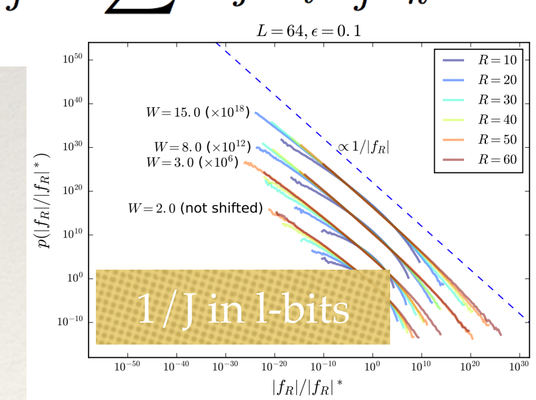
Huse-Nandkishore-Oganesyan arxiv: 1408.4297
Serbyn-Papic-Abanin arxiv:1305.5554

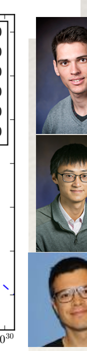
Algorithms: Ros-Mueller-Scardicchio arxiv:1406.2175
Chandran, Kim, Vidal, Abanin arxiv: 1410.0687
Pekker-Clark arxiv:1410.2224

$$H_{\text{l-bit}} = \sum_i h_i \tau_i^z + \sum_{ij} V_{ij} \tau_i^z \tau_j^z + \sum V_{ijk} \tau_i^z \tau_j^z \tau_k^z + \ldots$$



Pekker-Clark-Ogansyen-Refael arxiv:1607.07884



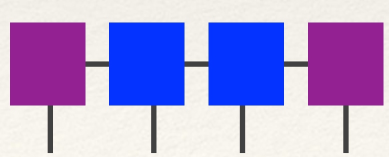


Villalonga-Yu-Luitz-Clark arxiv:1710.05036

`Simple construction of each eigenstate'

$G_1, G_2, G_3, \ldots, G_n$

$E_1, E_2, E_3, \ldots, E_n$



Pekker-Clark arxiv:1410.2224

Algorithm: SIMPS/ES-DMRG [Yu-Pekker-Clark] arxiv: 1509.01244

At the MBL transition…

Phenomenological RG:

41 Anna Goremykina, Romain Vasseur, and Maksym Serbyn. Analytically solvable renormalization group for the many-body localization transition. *Physical Review Letters*, 122(4):040601, 2019.

42 Philipp T Dumitrescu, Romain Vasseur, and Andrew C Potter. Scaling theory of entanglement at the many-body localization transition. *Physical review letters*, 119(11):110604, 2017.

43 Ehud Altman and Ronen Vosk. Universal dynamics and renormalization in many-body-localized systems. *Annu. Rev. Condens. Matter Phys.*, 6(1):383–409, 2015.

44 Ronen Vosk and Ehud Altman. Many-body localization in one dimension as a dynamical renormalization group fixed point. *Phys. Rev. Lett.*, 110:067204, Feb 2013.

45 Romain Vasseur, Andrew C Potter, and SA Parameswaran. Quantum criticality of hot random spin chains. *Physical review letters*, 114(21):217201, 2015.

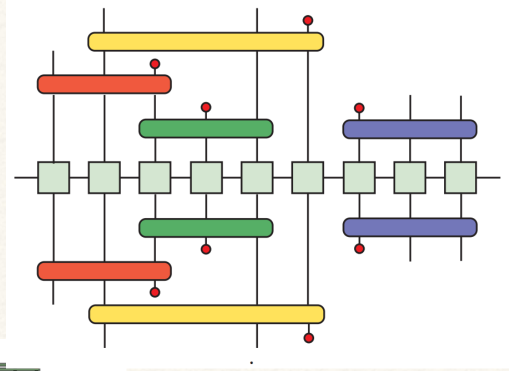
46 SA Parameswaran, Andrew C Potter, and Romain Vasseur. Eigenstate phase transitions and the emergence of universal dynamics in highly excited states. *Annalen der Physik*, 529(7):1600302, 2017.

47 Ronen Vosk, David A. Huse, and Ehud Altman. Theory of the many-body localization transition in one dimensional systems, 2015, 1412.3117.

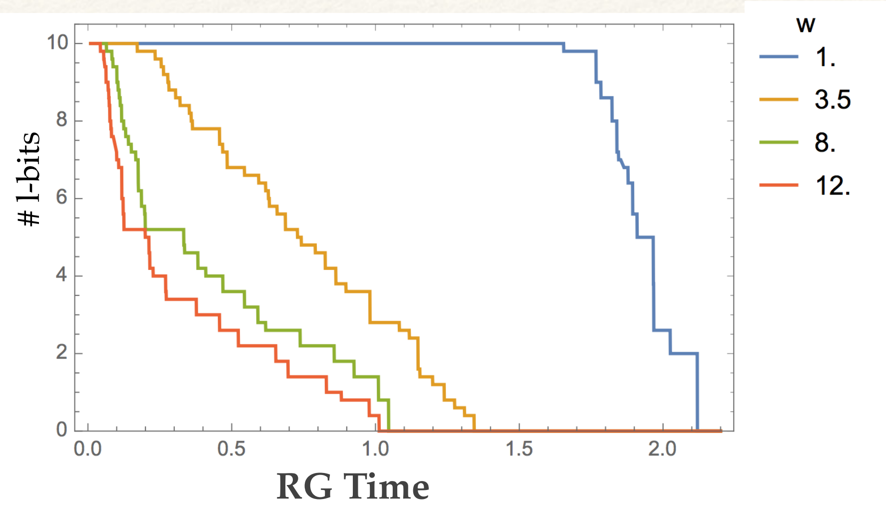
48 Kartiek Agarwal, Ehud Altman, Eugene Demler, Sarang Gopalakrishnan, David A Huse, and Michael Knap. Rare-region effects and dynamics near the many-body localization transition. *Annalen der Physik*, 529(7):1600326, 2017.

At the MBL transition…

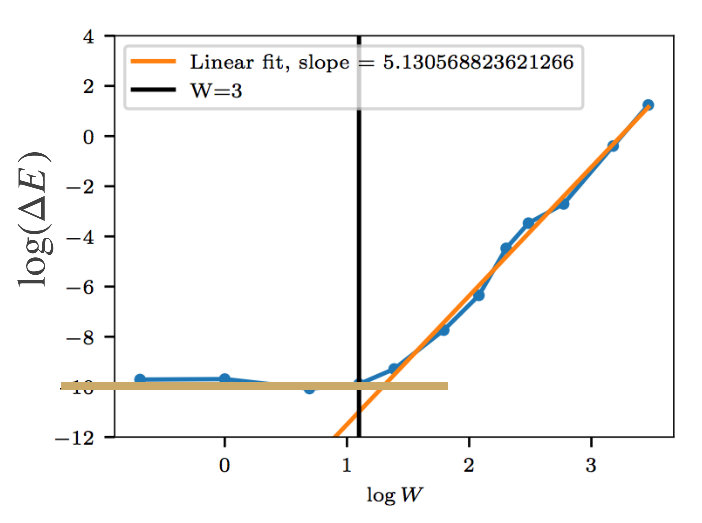
Numerical RG:



Wegner-Wilson Flow



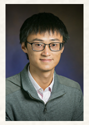
Uniform integrating out of l-bits under RG



RG integrates out final energy scale:

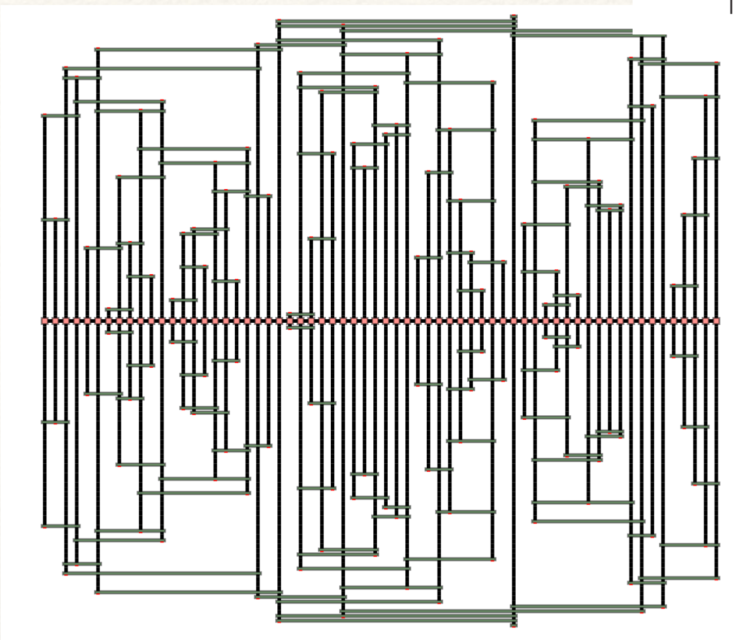
$$\Delta E \propto 2^{-L}$$

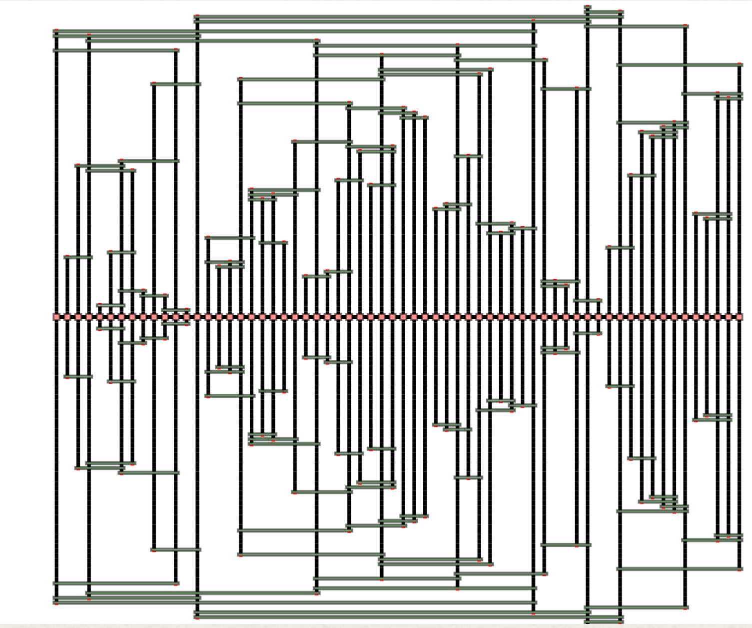
$$\Delta E \propto W^{-\alpha(L)}$$

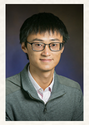


Yu-Pekker-Clark

MERA





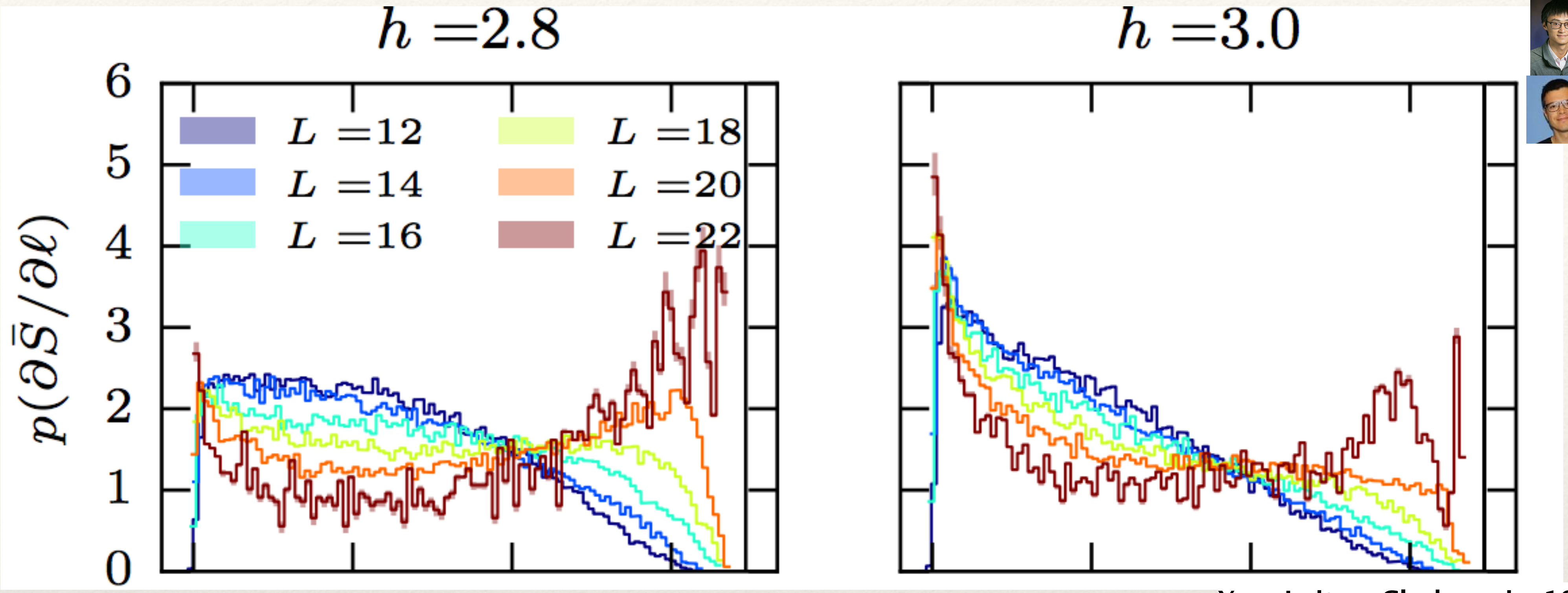


Yu-Pekker-Clark

At the MBL transition…

Numerical Data

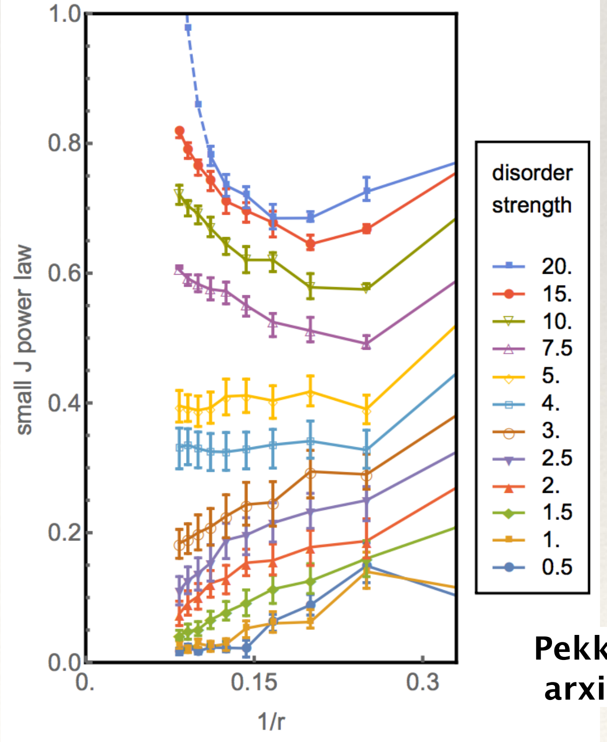
Bimodality in Entanglement…



Yu – Luitz – Clark: arxiv: 1606.01260

Similarly Khemani – Lim – Sheng – Huse: arxiv: 1607.05756

Scale Invariance in l-bit couplings.



Pekker–Clark–Ogansyen–Refael
arxiv:1607.07884

Today: Resonances at the Transition + Statistics of Mutual Information

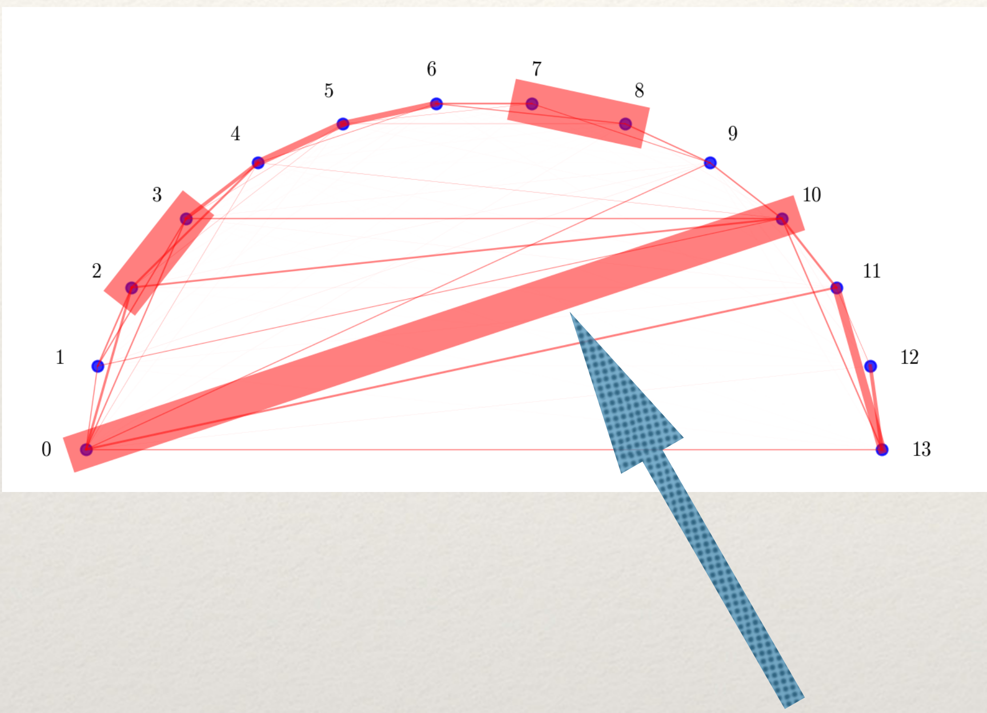
Resonances in MBL

Mutual Information

$$I_{AB} \equiv S_A + S_B - S_{AB}$$

$$0 < I_{AB} < 2\ln 2$$

Singlet by Monogamy of Entanglement

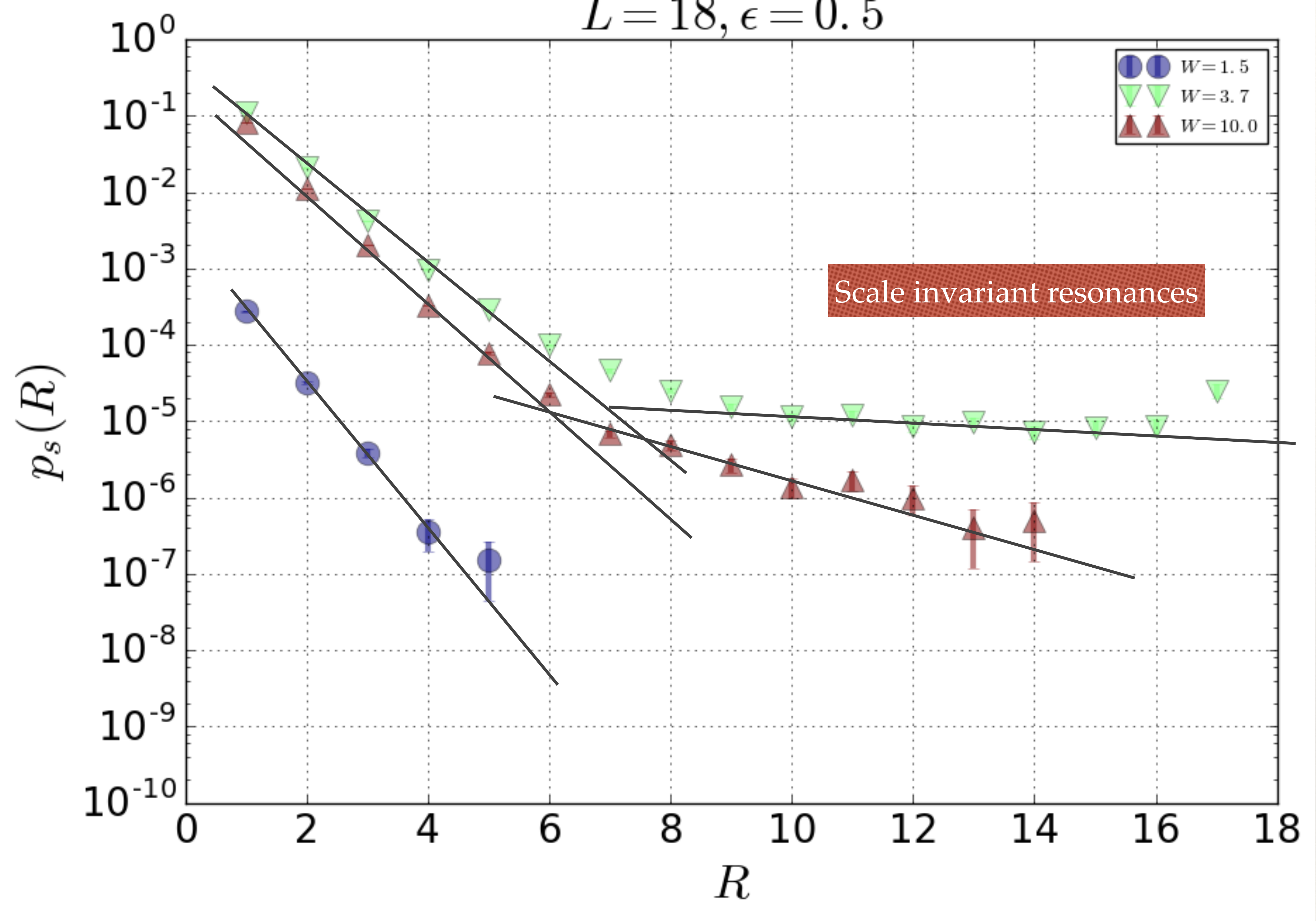


Resonance

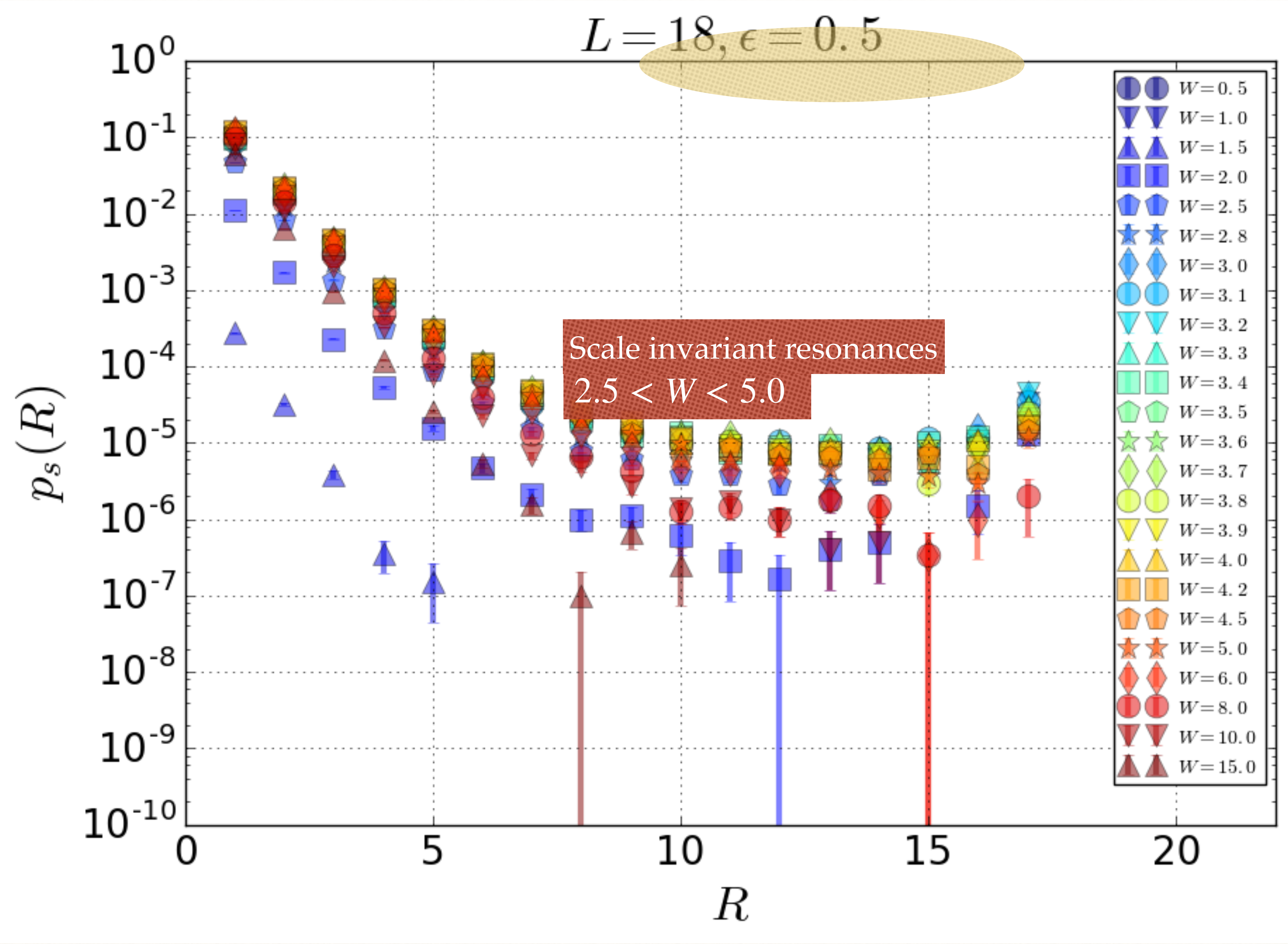
Q: Range dependence?

Q: How many?

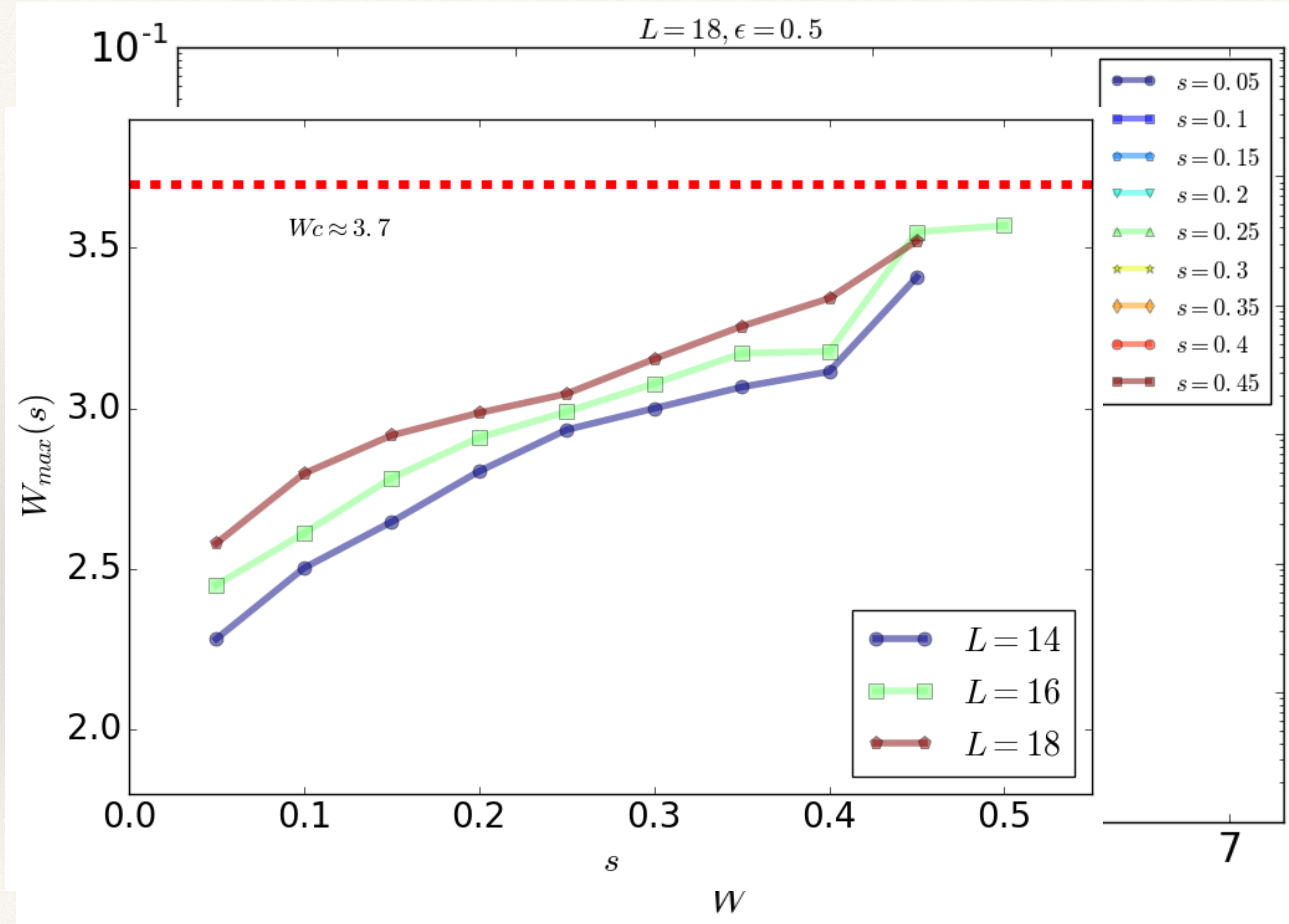
De Tomasi – Bera – Bardarson – Pollmann PRL 118(1) 016804



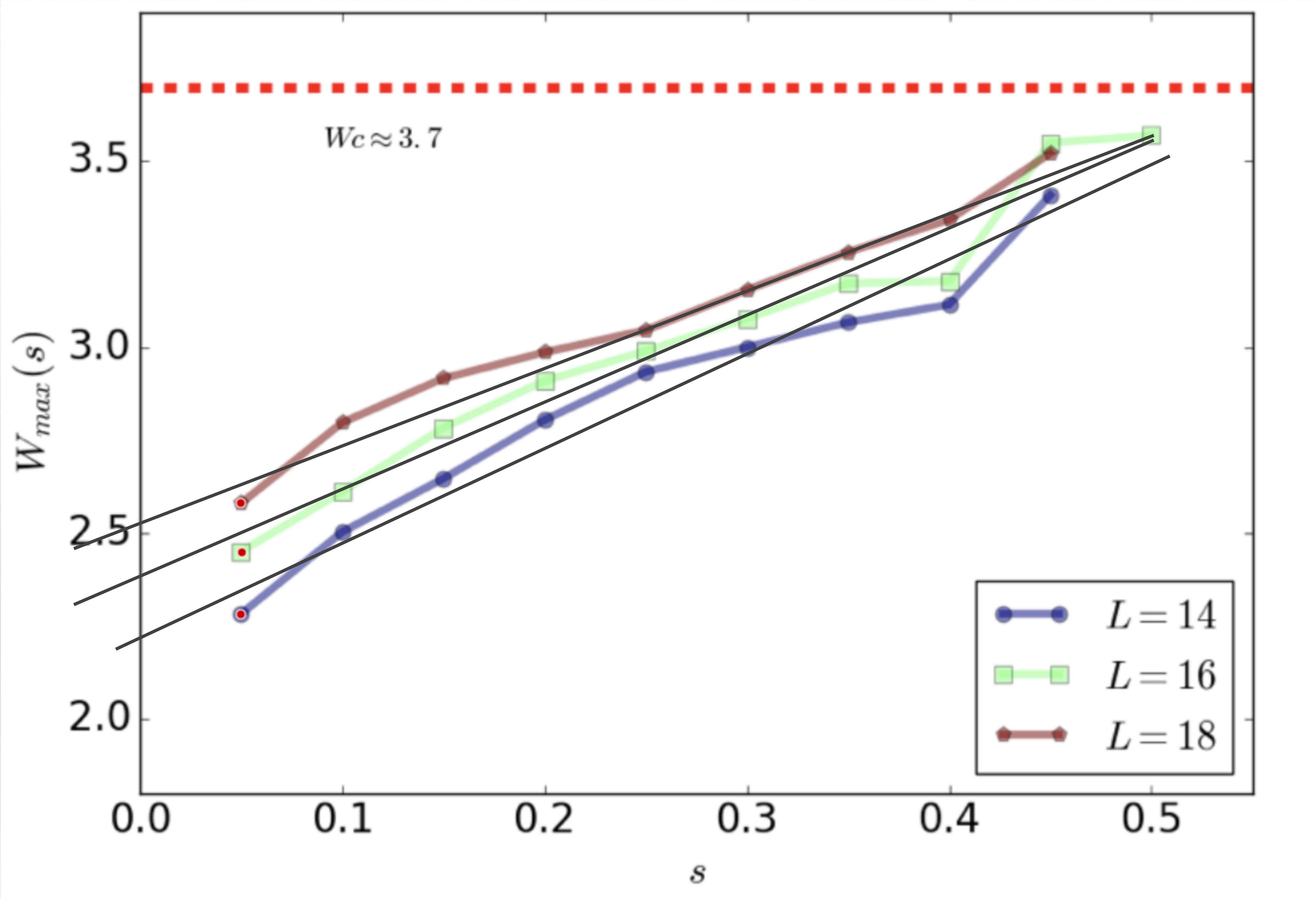
Threshold=0.45



Threshold=0.45



$L = 18, \epsilon = 0.5$
10^{-1}
$s = 0.05$
$s = 0.1$
$s = 0.15$
$s = 0.2$
$s = 0.25$
$s = 0.3$
$s = 0.35$
$s = 0.4$
$s = 0.45$
$Wc \approx 3.7$
$W_{max}(s)$
3.5
3.0
2.5
2.0
$L = 14$
$L = 16$
$L = 18$
0.0
0.1
0.2
0.3
0.4
0.5
7
s
W

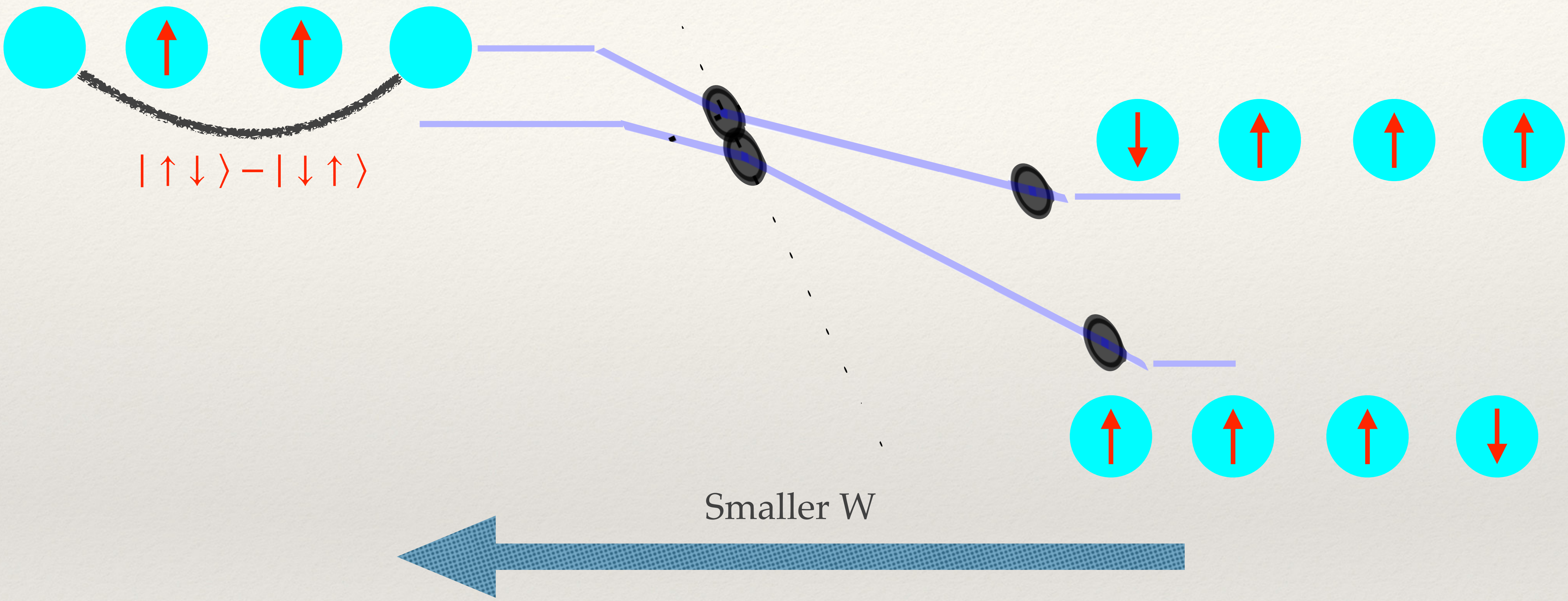


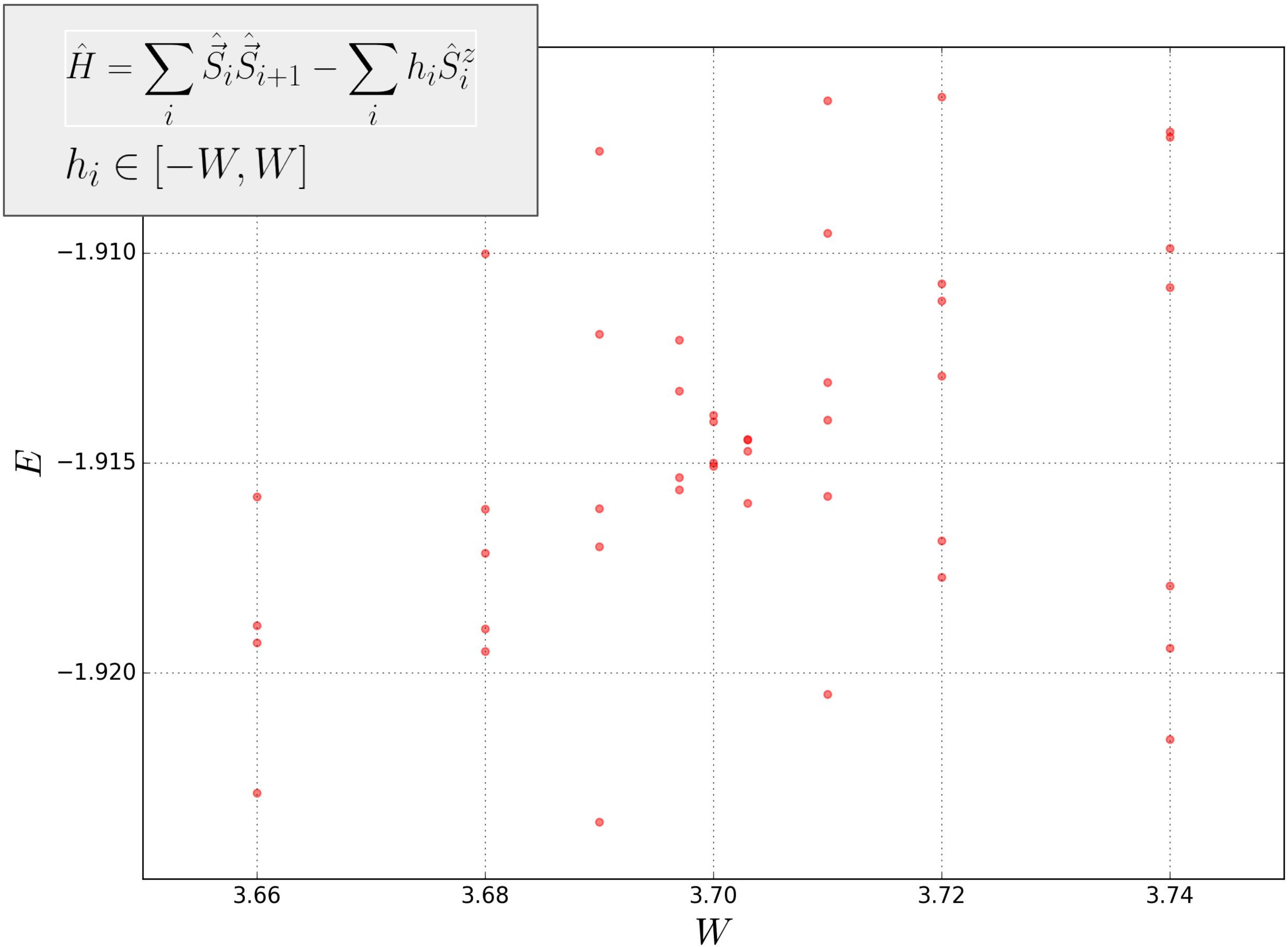
$Wc \approx 3.7$
3.5
3.0
2.5
2.0
0.0
0.1
0.2
0.3
0.4
0.5
$W_{max}(s)$
s
$L = 14$
$L = 16$
$L = 18$

Q: What generates resonances?

Q: Do the resonances drive the transition to volume law entanglement?

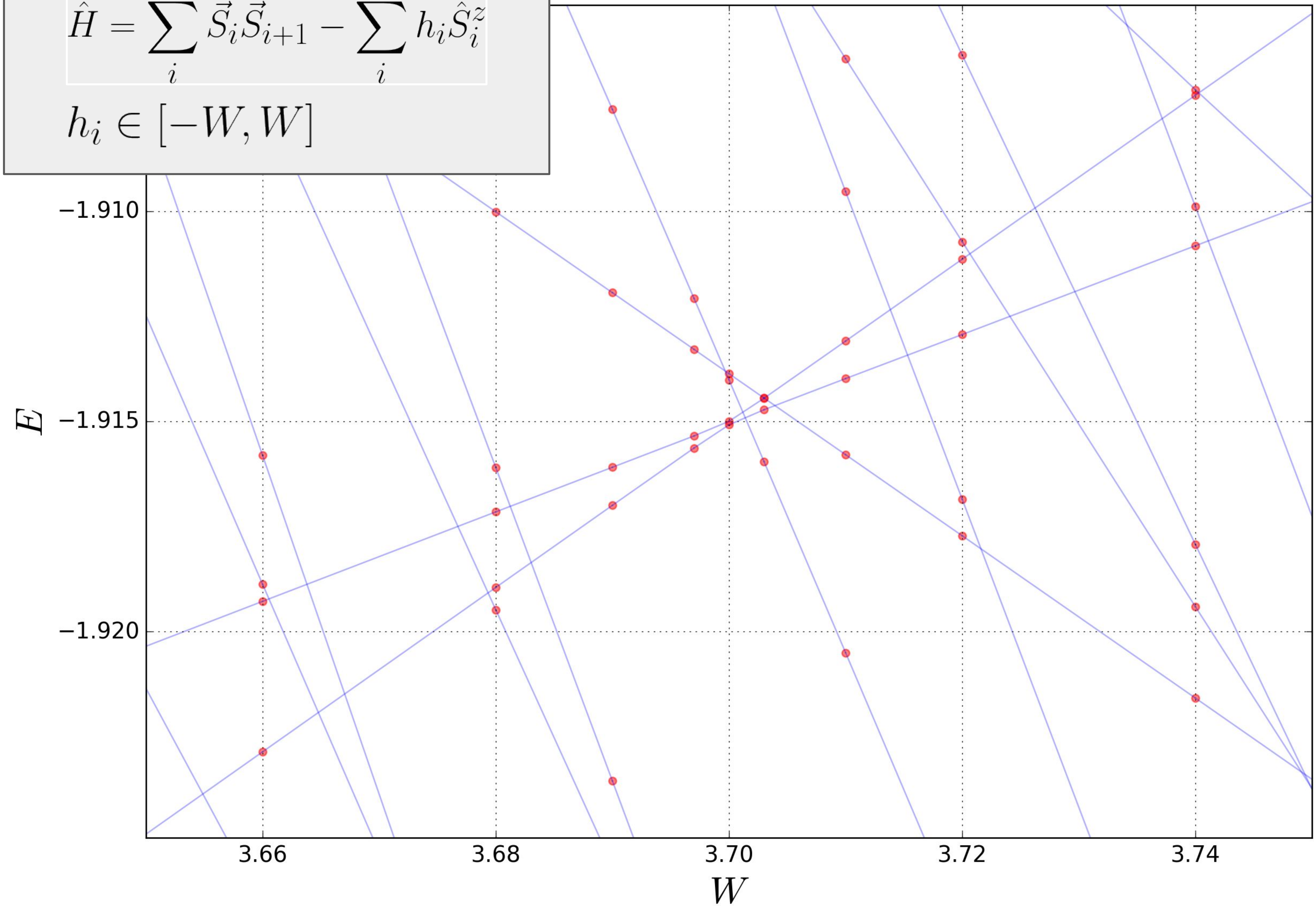
Q: What generates resonances?

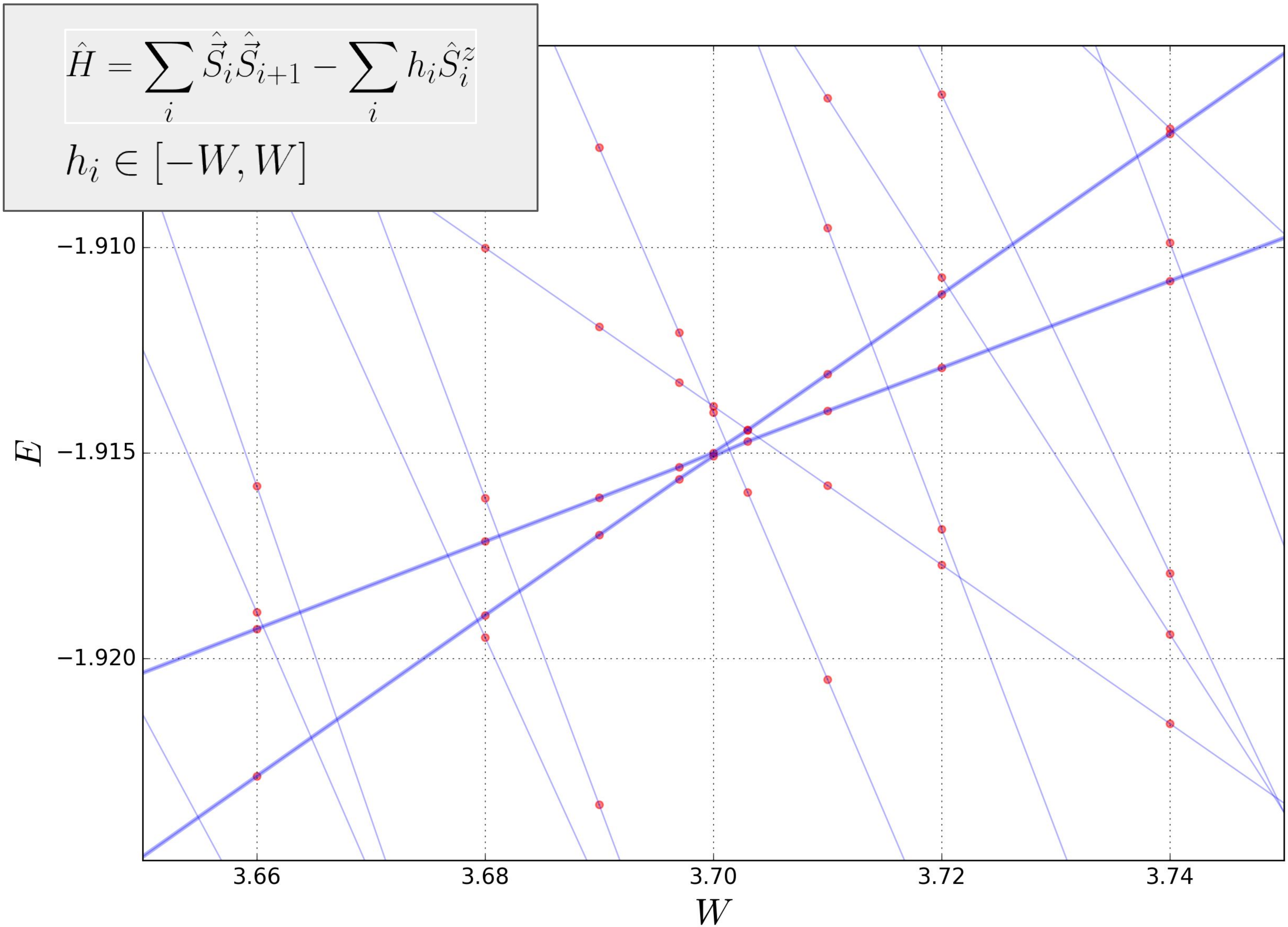



$\hat{H} = \sum_i \hat{\vec{S}}_i \hat{\vec{S}}_{i+1} - \sum_i h_i \hat{S}_i^z$
$h_i \in [-W, W]$
−1.910
−1.915
−1.920
E
3.66
3.68
3.70
3.72
3.74
W

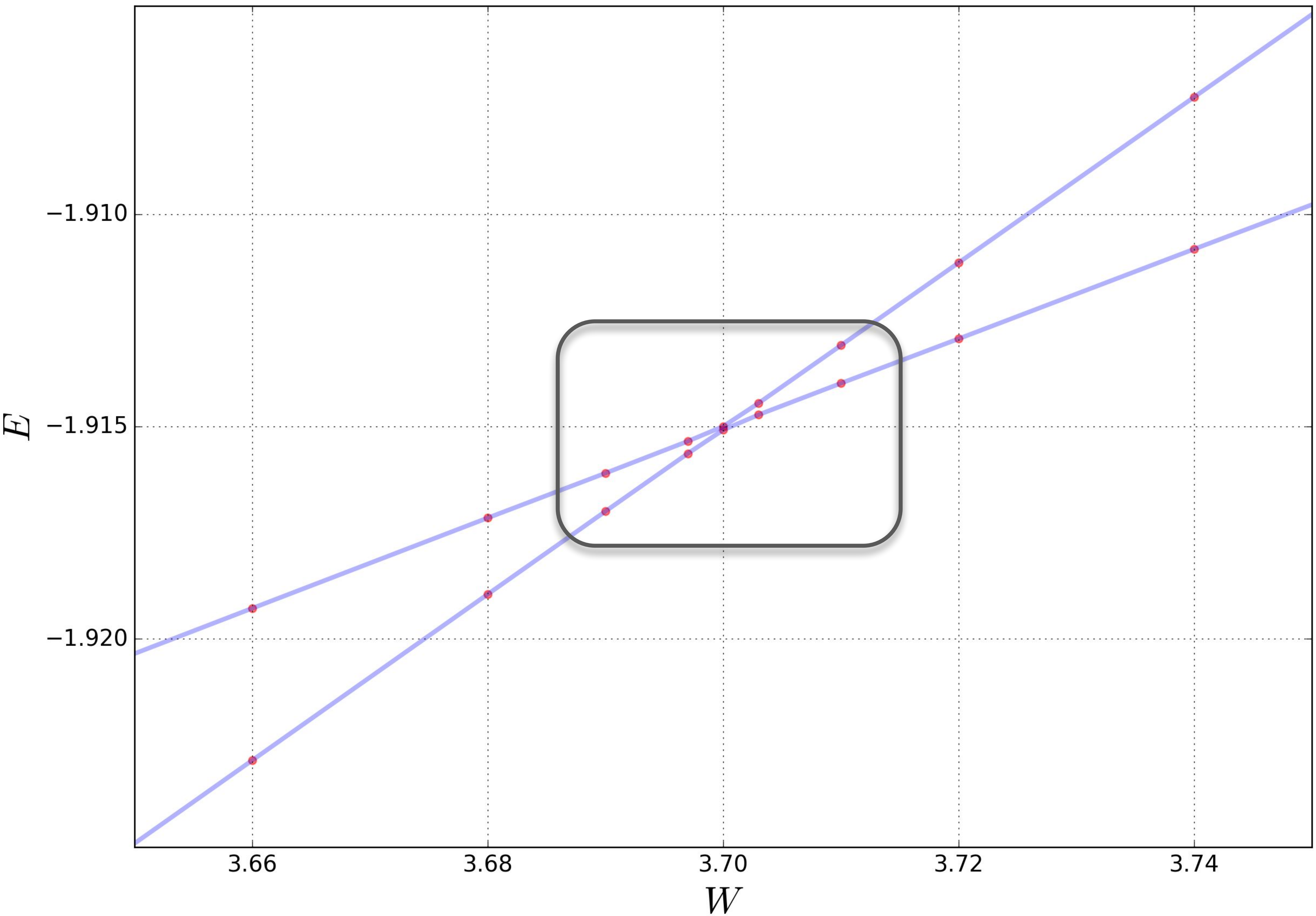
$$\hat{H} = \sum_i \hat{\vec{S}}_i \hat{\vec{S}}_{i+1} - \sum_i h_i \hat{S}_i^z$$

$$h_i \in [-W, W]$$





$\hat{H} = \sum_i \hat{\vec{S}}_i \hat{\vec{S}}_{i+1} - \sum_i h_i \hat{S}_i^z$
$h_i \in [-W, W]$
−1.910
−1.915
−1.920
E
3.66
3.68
3.70
3.72
3.74
W


−1.910
−1.915
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E
3.66
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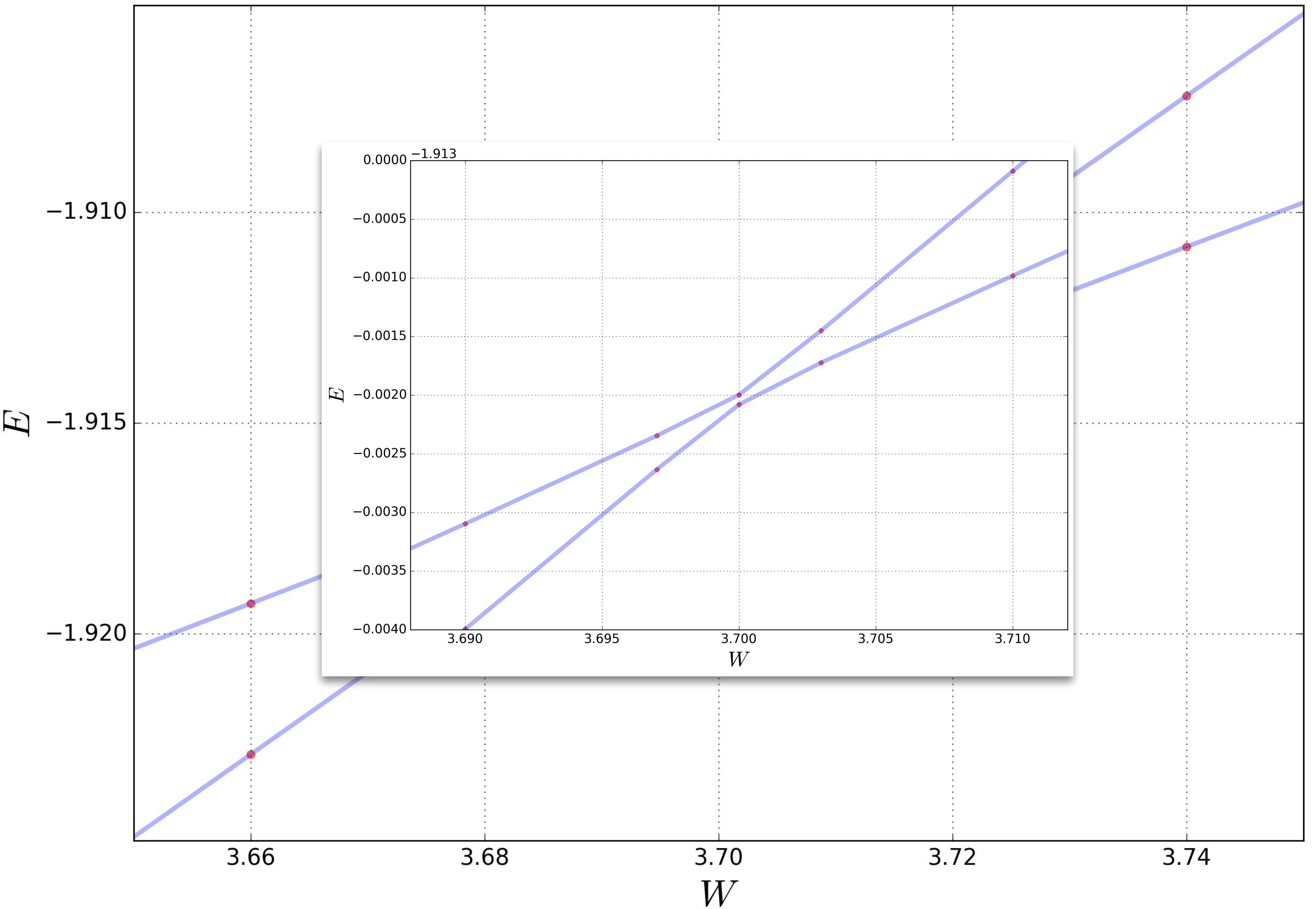
W



−1.910
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E
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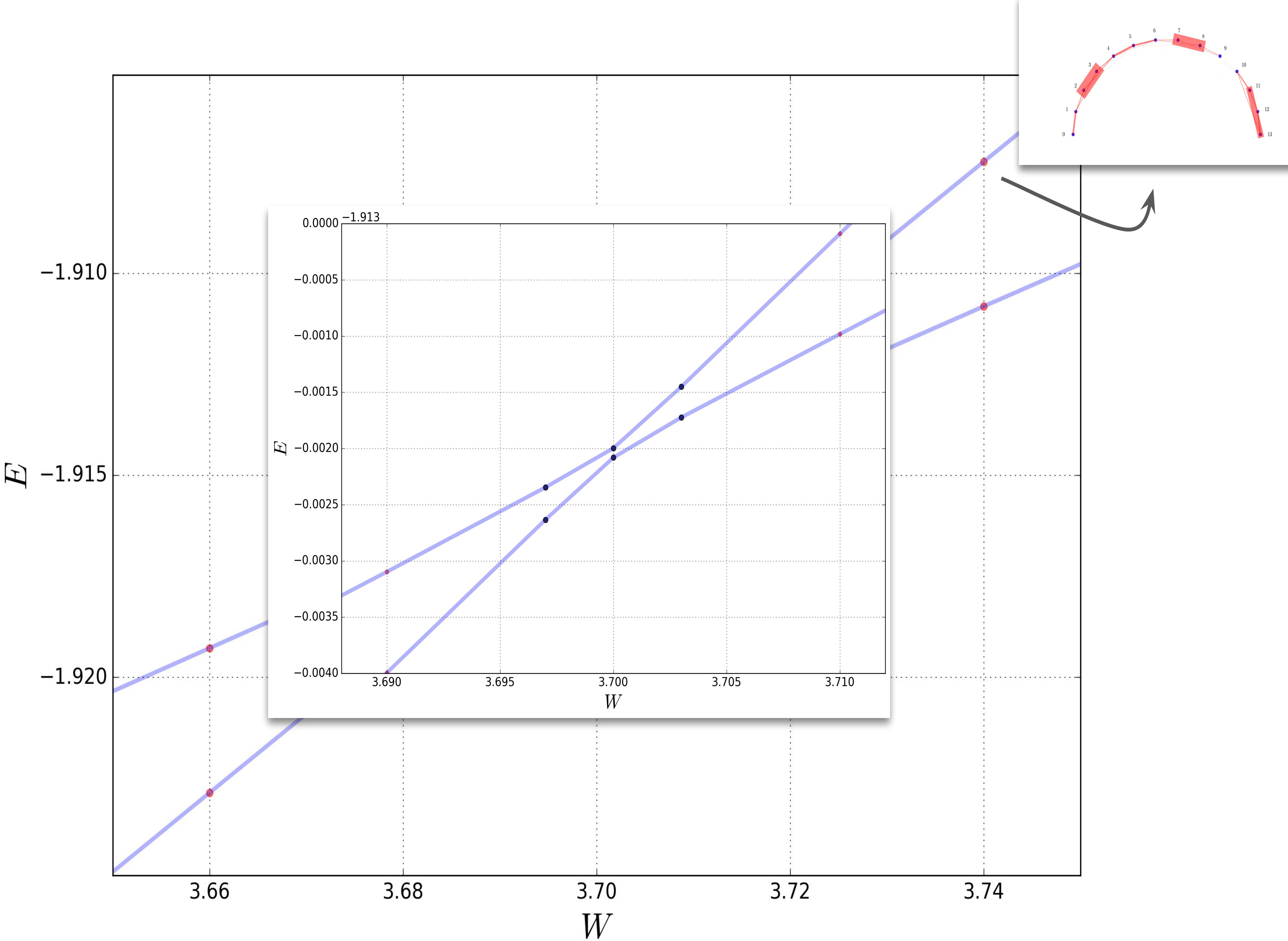
−0.0010
−0.0015
−0.0020
−0.0025
−0.0030
−0.0035
−0.0040
3.690
3.695
3.700
3.705
3.710



−1.910
−1.915
−1.920
E
3.66
3.68
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W
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0.0000
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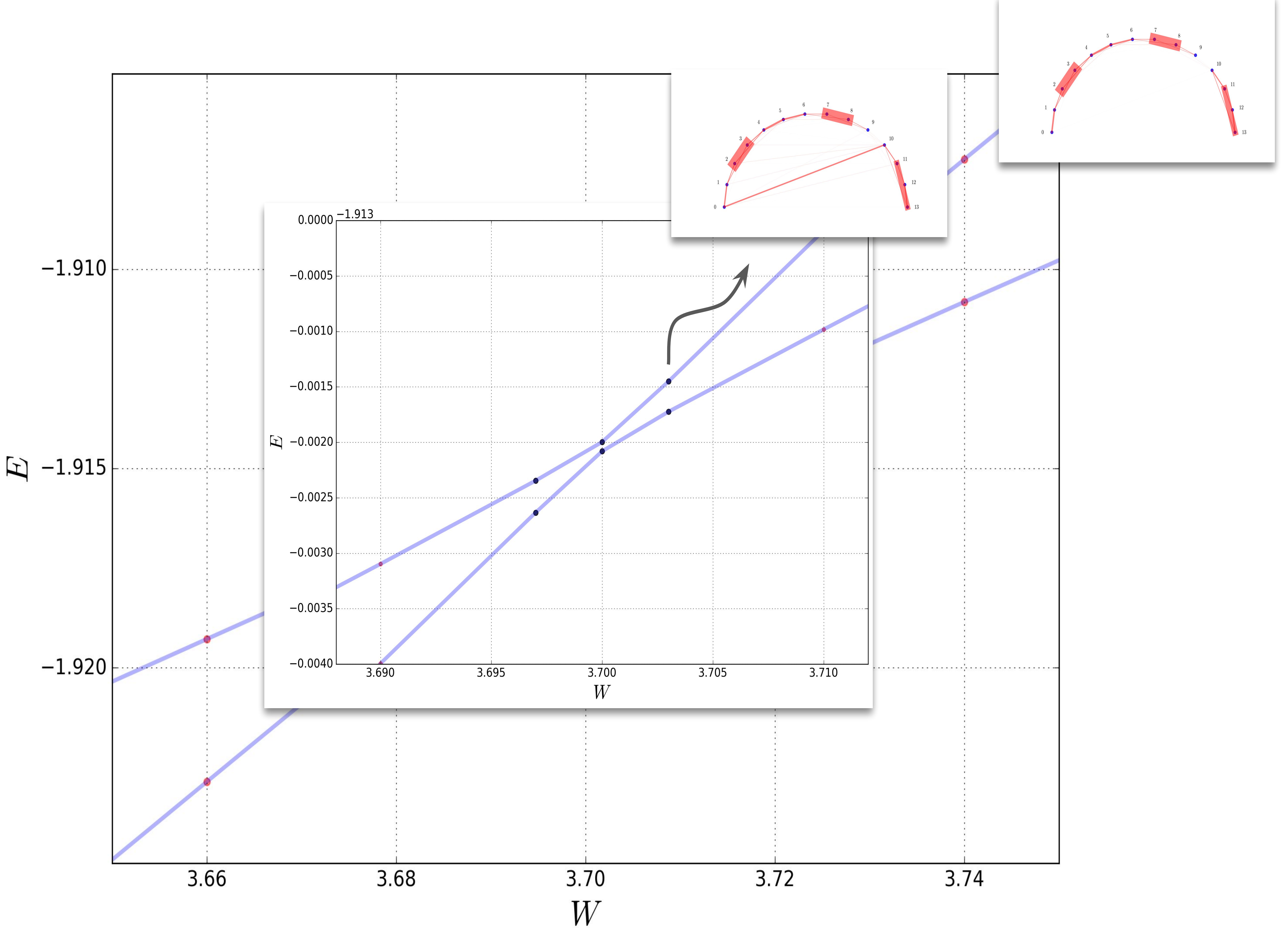
−0.0035
−0.0040
E
3.690
3.695
3.700
3.705
3.710
W



−1.910
−1.915
−1.920
E
3.66
3.68
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3.690
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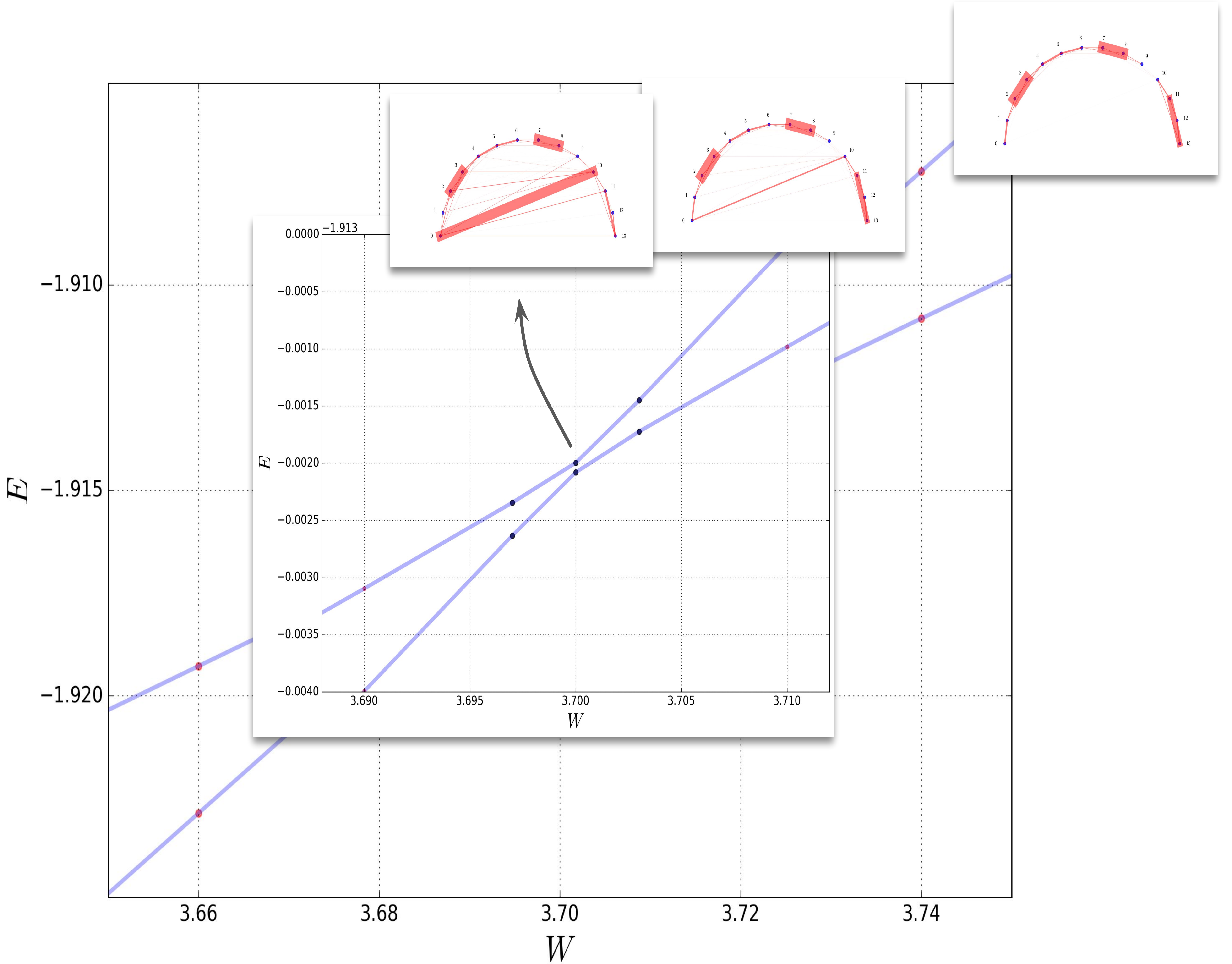

−1.910
−1.915
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3.66
3.68
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E
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−1.913
3.690
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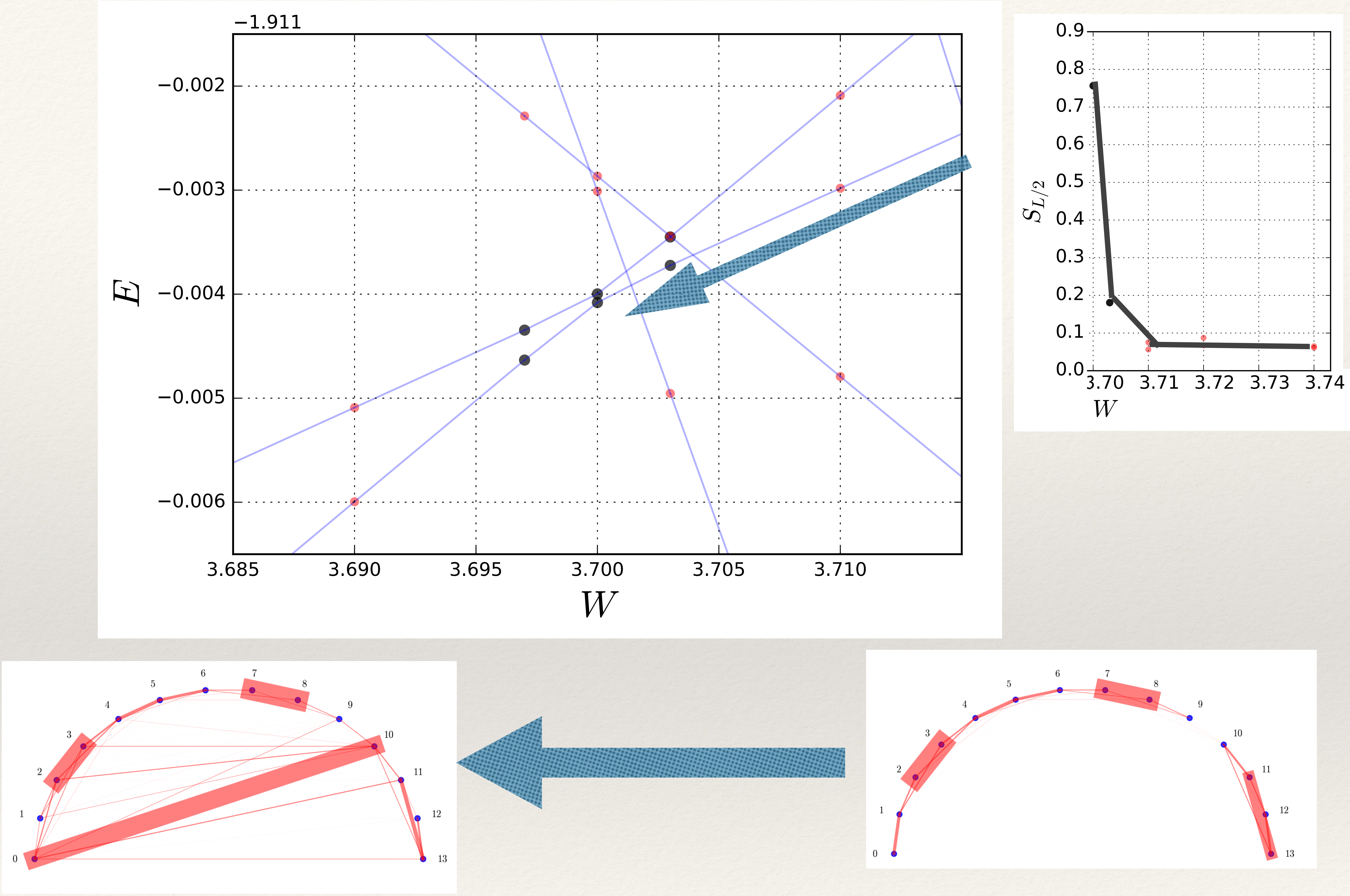
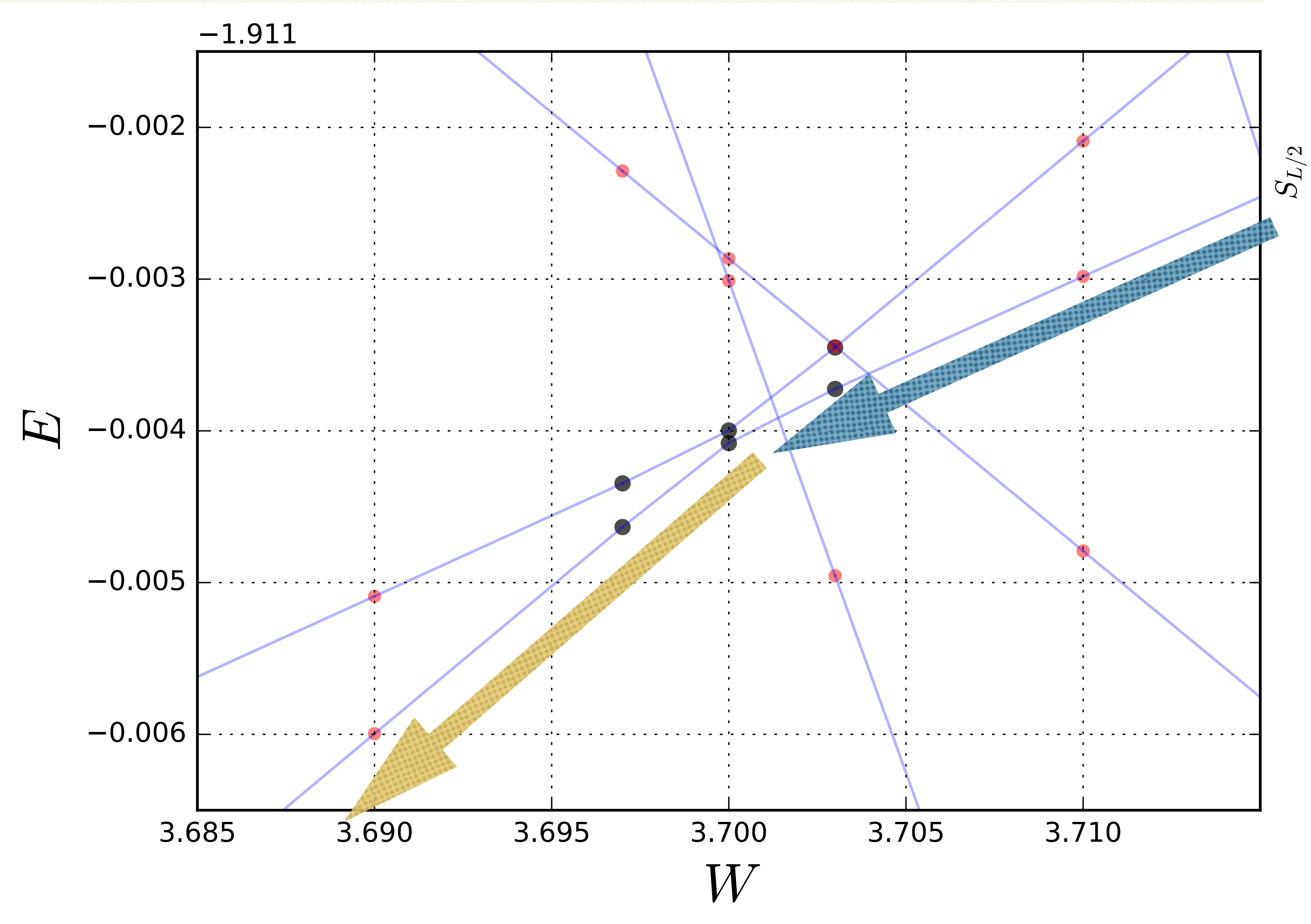
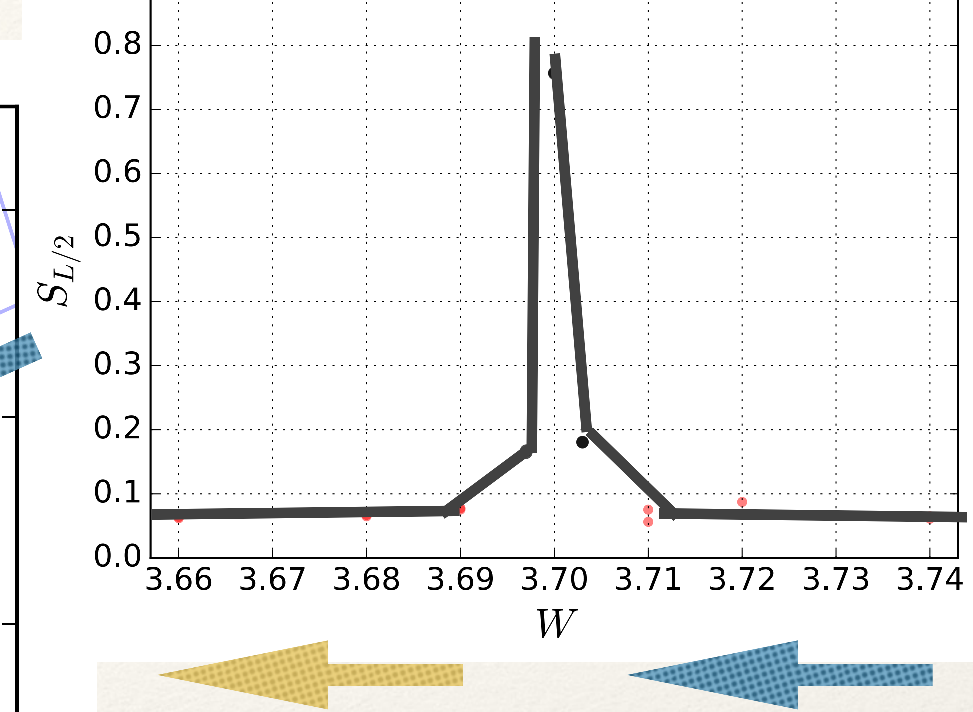
3.710



Collisions drive resonances.


−1.911
−0.002
−0.003
−0.004
−0.005
−0.006
3.685
3.690
3.695
3.700
3.705
3.710
E
W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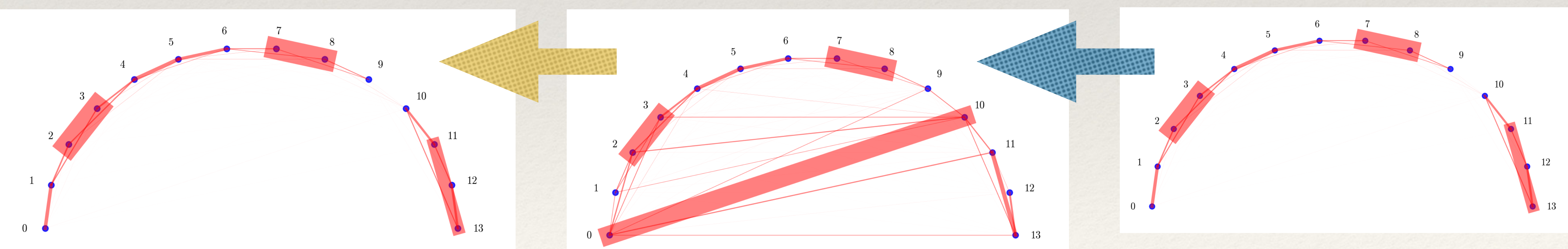
3.69
3.70
3.71
3.72
3.73
3.74
$S_{L/2}$
W


0 1 2 3 4 5 6 7 8 9 10 11 12 13

In order to have large entanglement you need continuous `resonances'

This also gives you level repulsion.

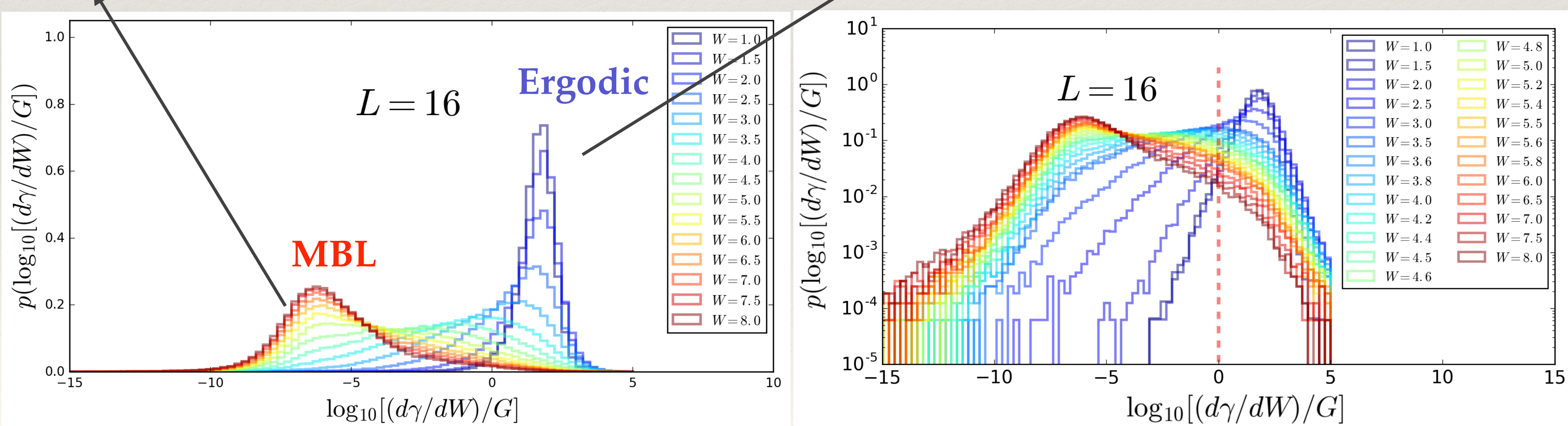
$$H_{\text{two levels}} = \begin{pmatrix} -\Delta E/2 & \gamma \\ \gamma & \Delta E/2 \end{pmatrix}$$

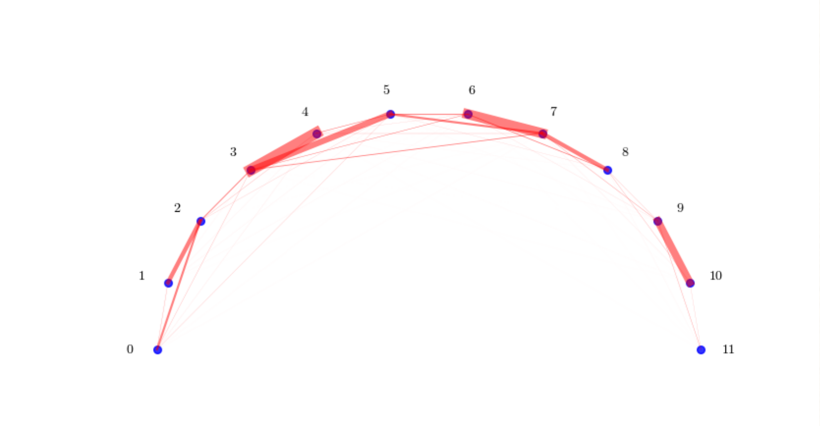
$$\Delta E \equiv \langle \Psi_2(W-\delta)\,|\,H(W)\,|\,\Psi_2(W-\delta)\rangle - \langle \Psi_1(W-\delta)\,|\,H(W)\,|\,\Psi_1(W-\delta)\rangle$$
$$\gamma \equiv \langle \Psi_1(W-\delta)\,|\,H(W)\,|\,\Psi_2(W-\delta)\rangle$$

$$\gamma/\Delta E \geq 1 \implies \text{Collision}$$

Rare Collisions!

Constantly colliding!

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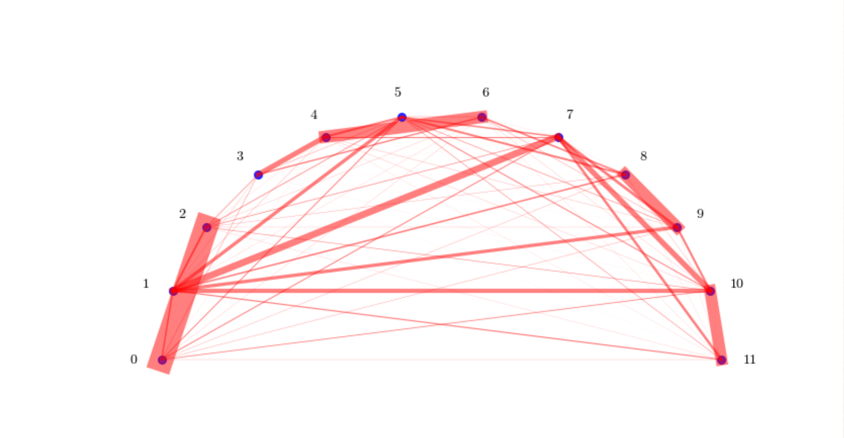
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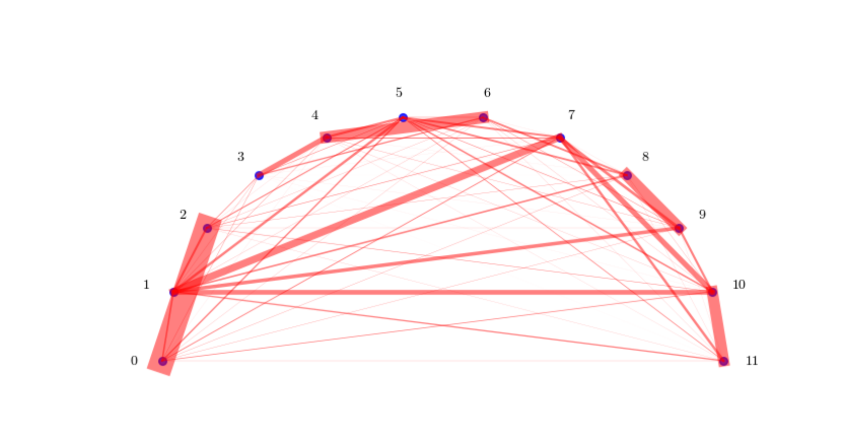
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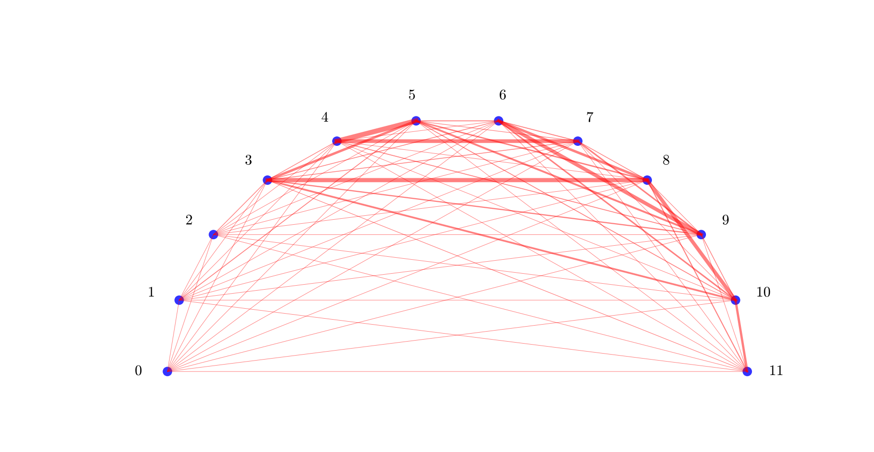
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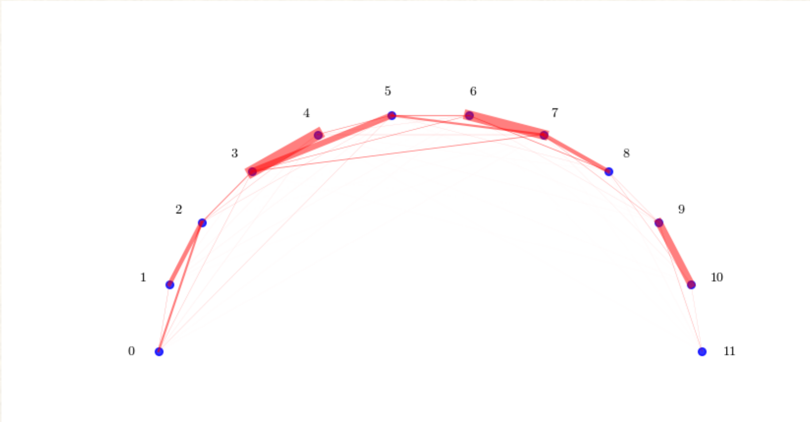
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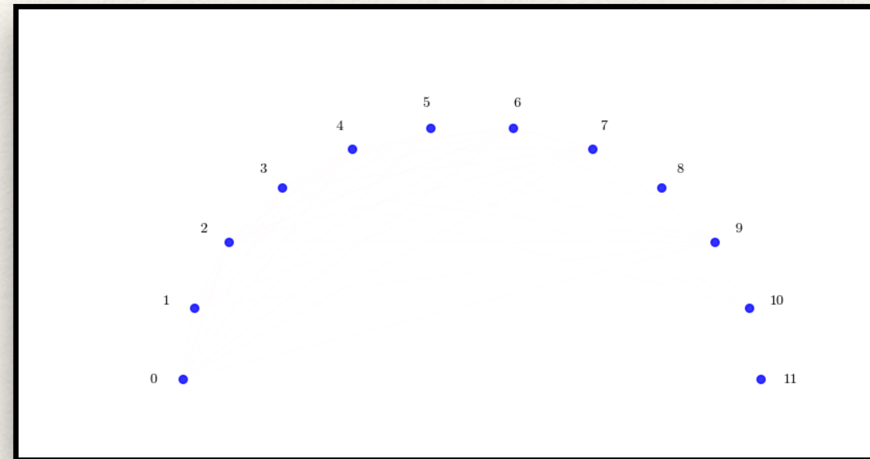
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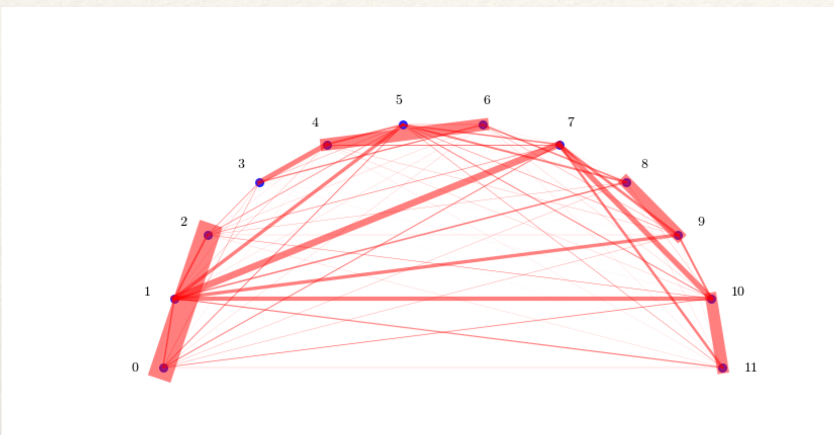
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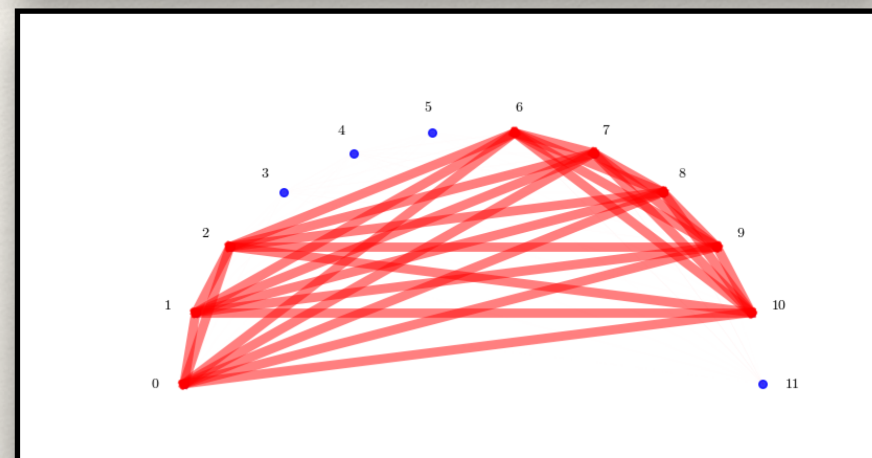
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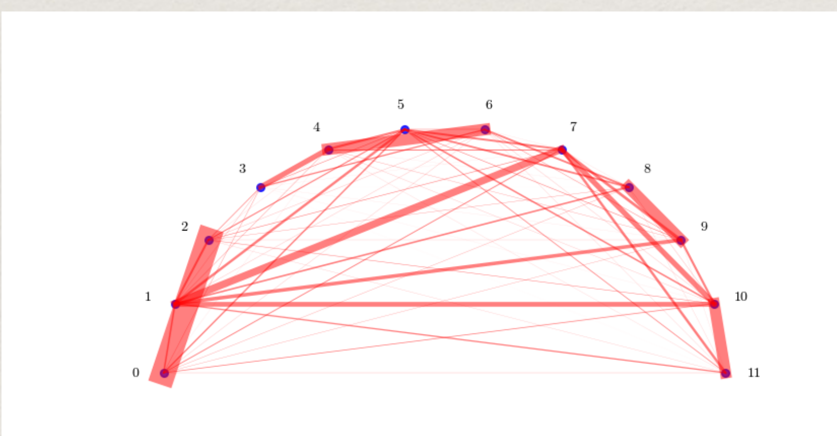
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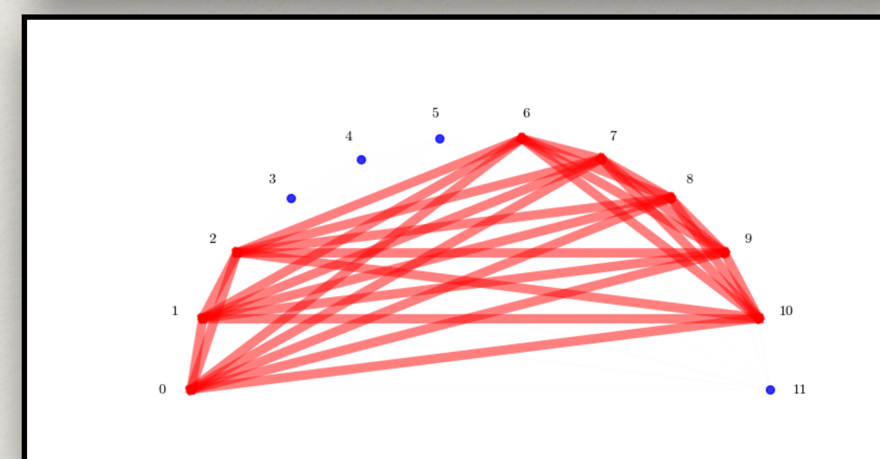
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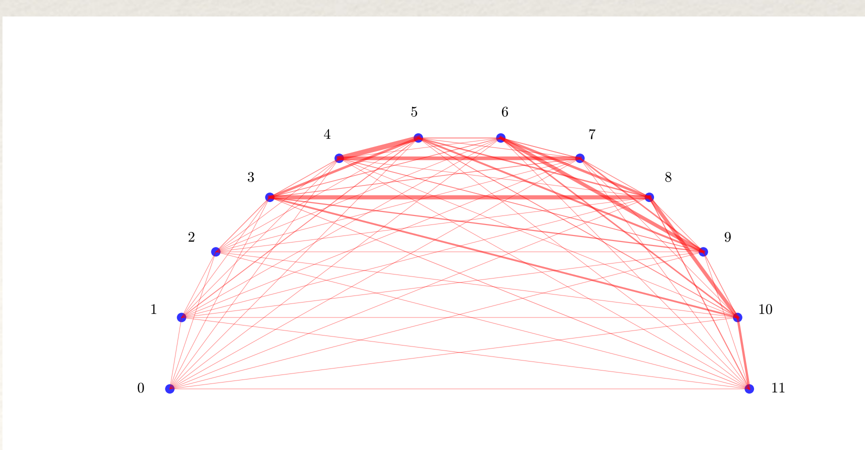
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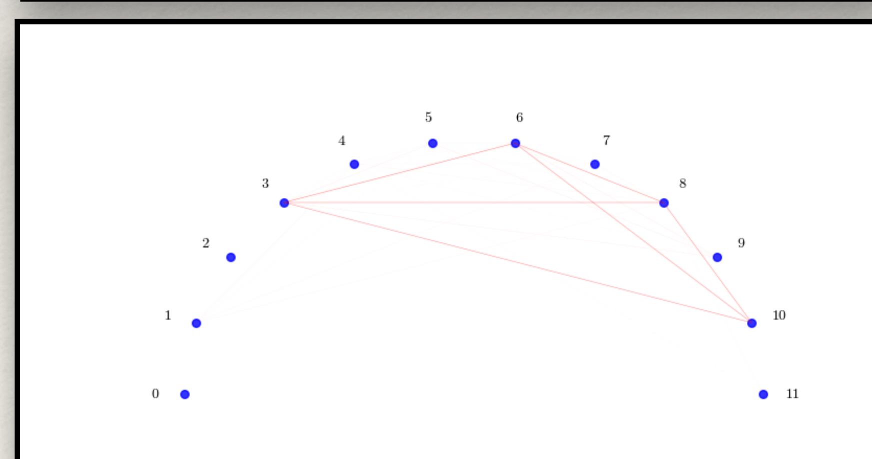
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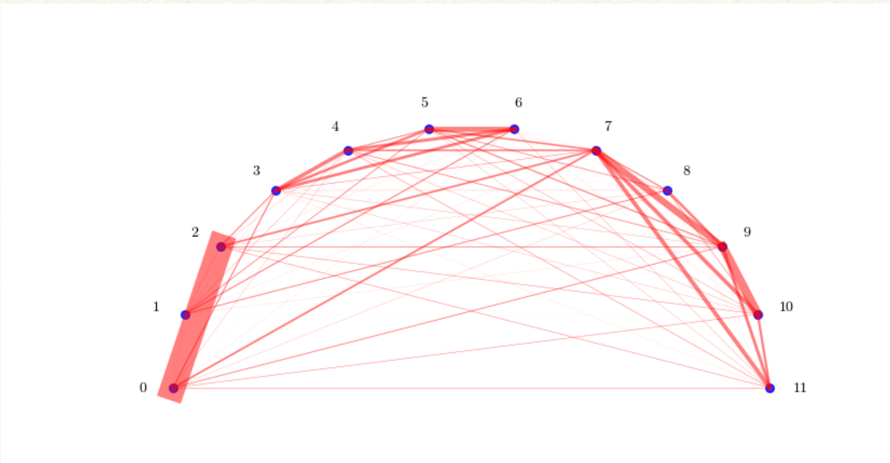
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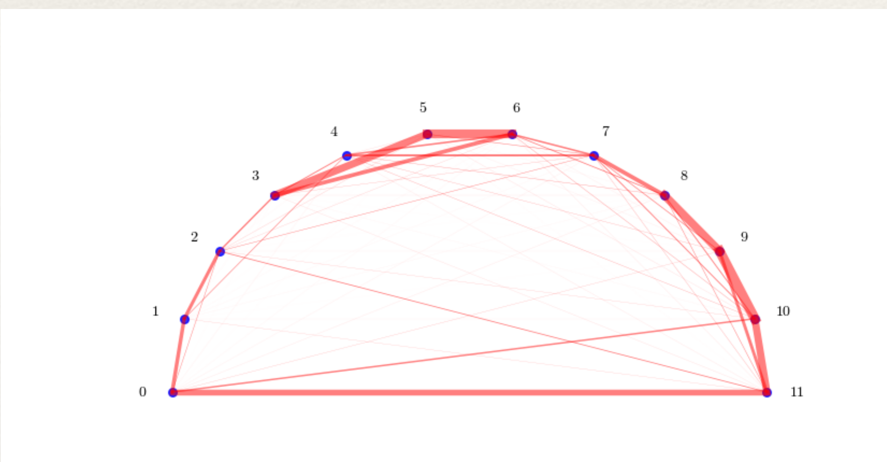
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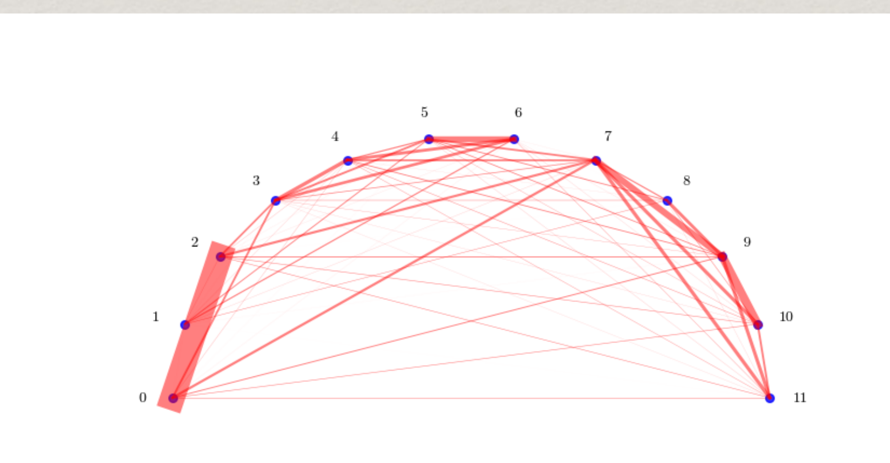
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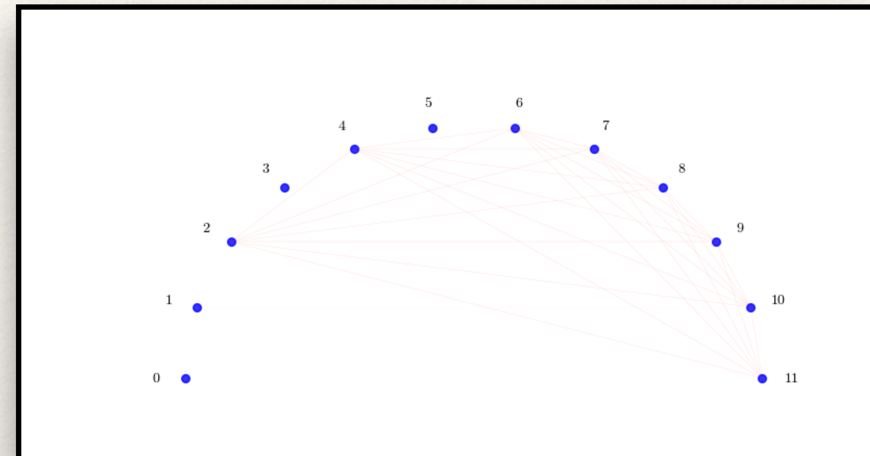
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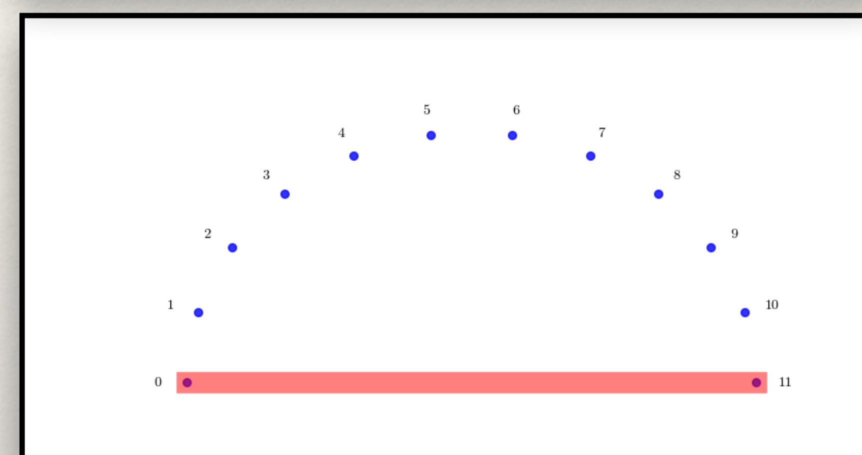
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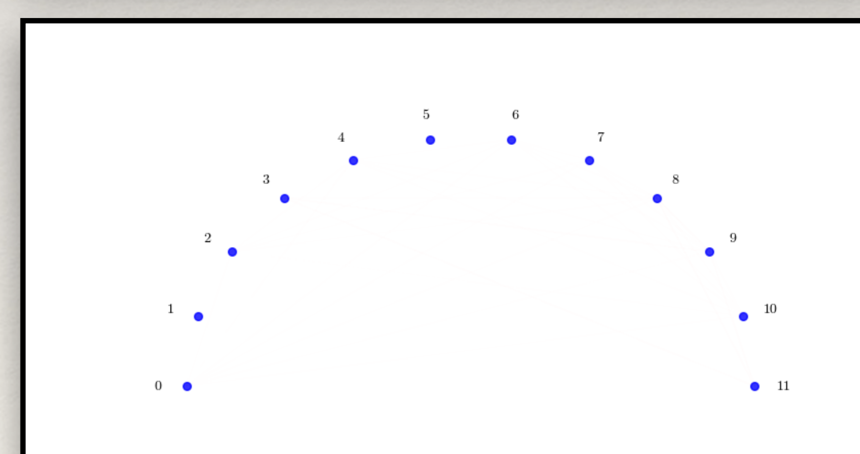
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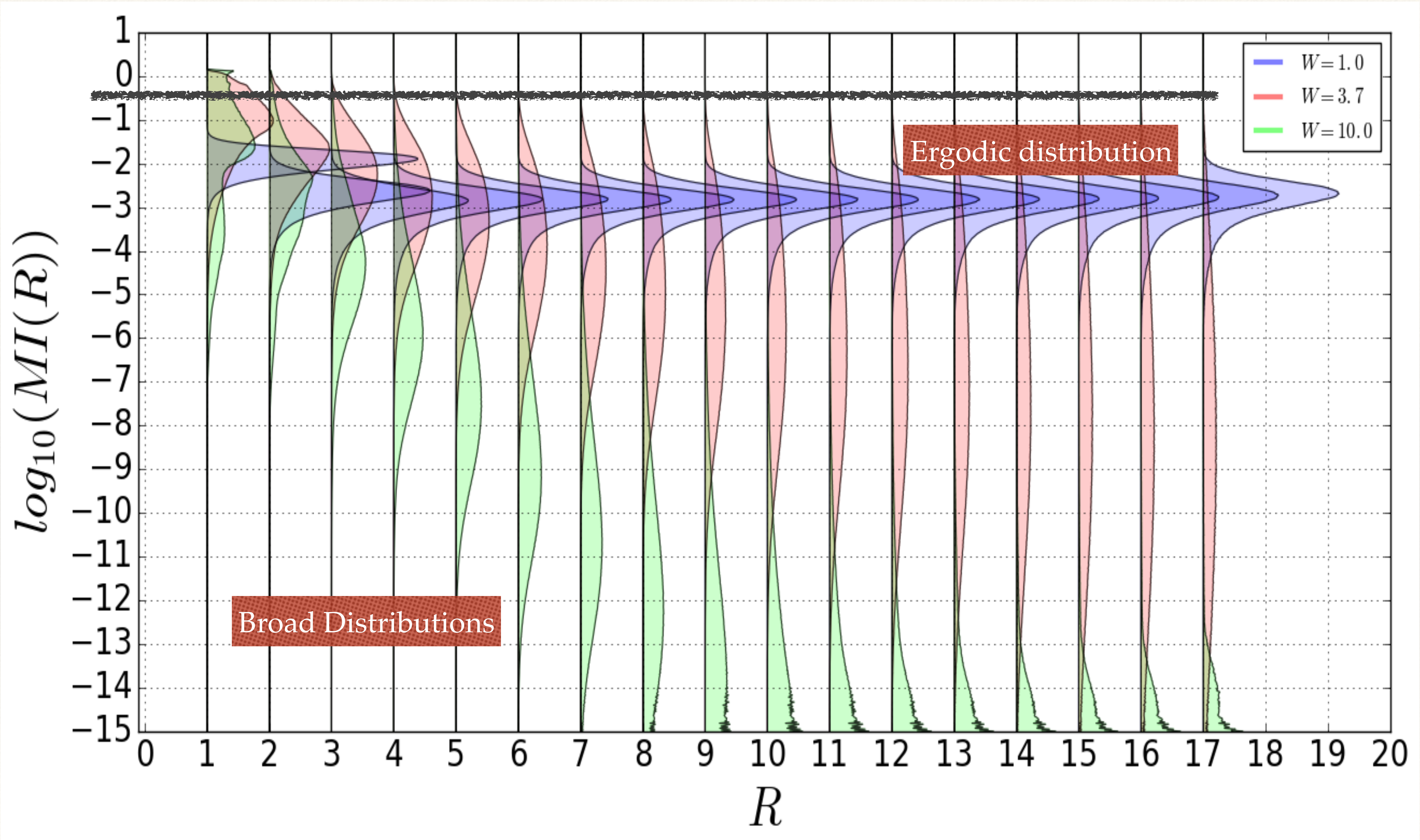
Distributions of Mutual Information in MBL



Q: What does the

- Standard Deviation
- Skewness
- Mean

of the distribution do?

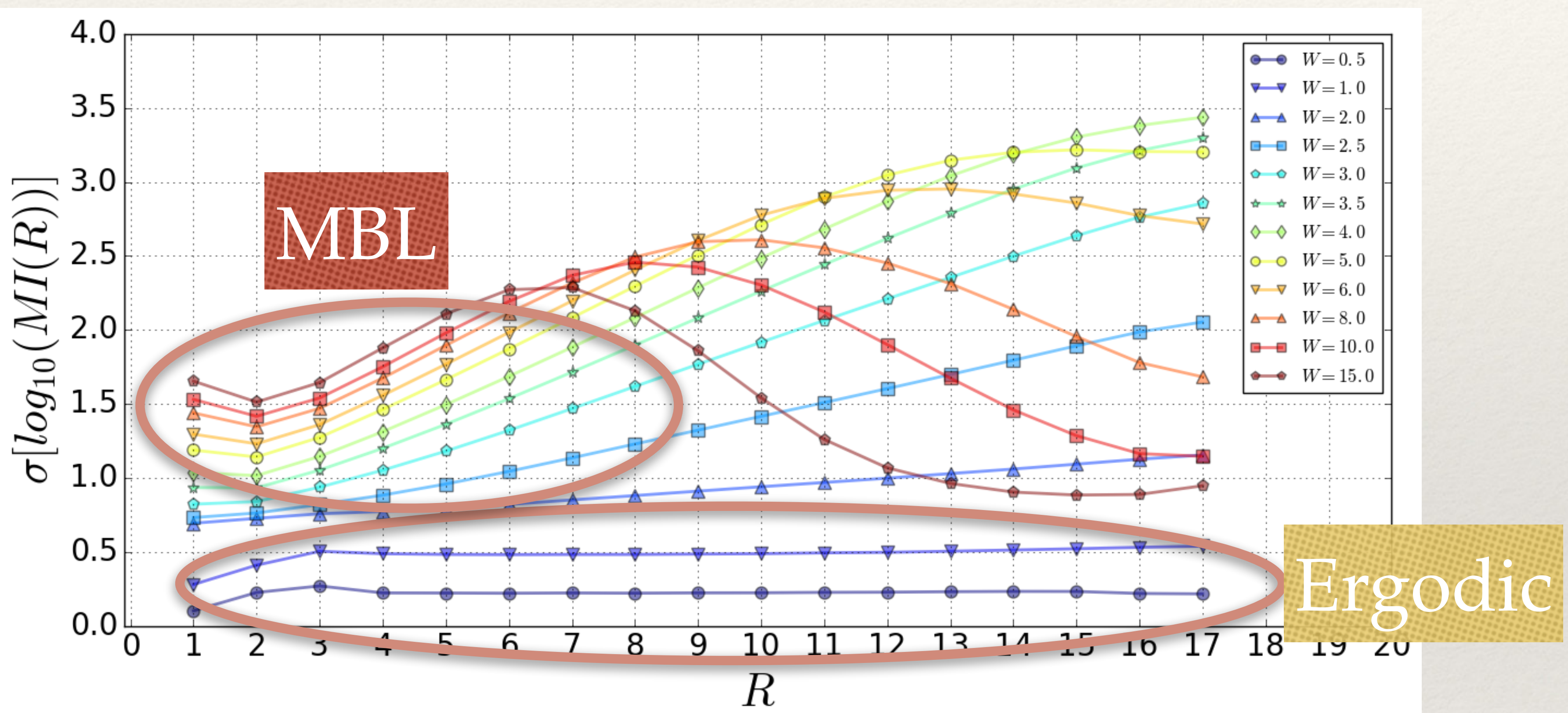
Q: What does the

Standard Deviation

- Skewness
- Mean

of the distribution do?

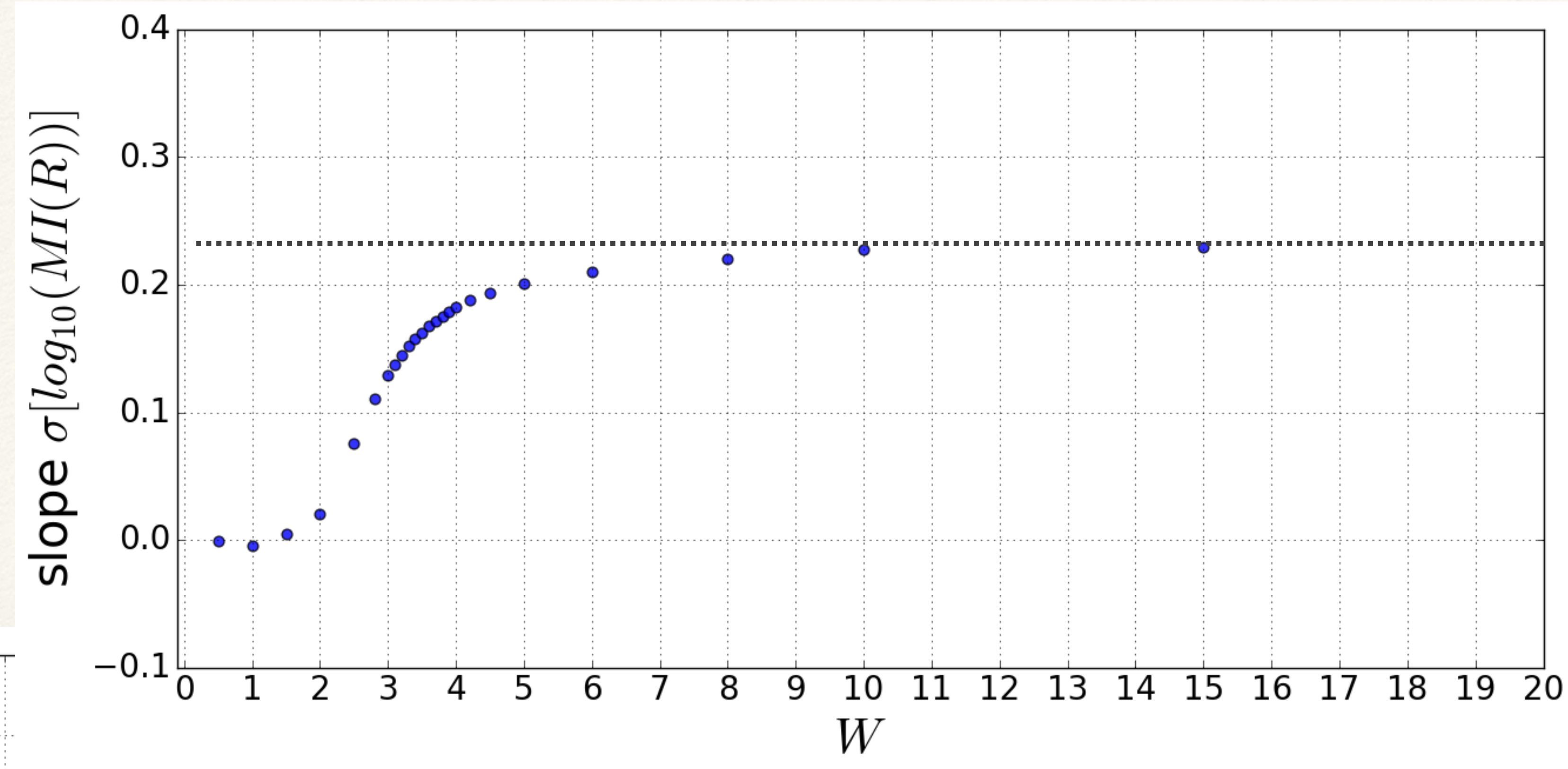
Machine Precision

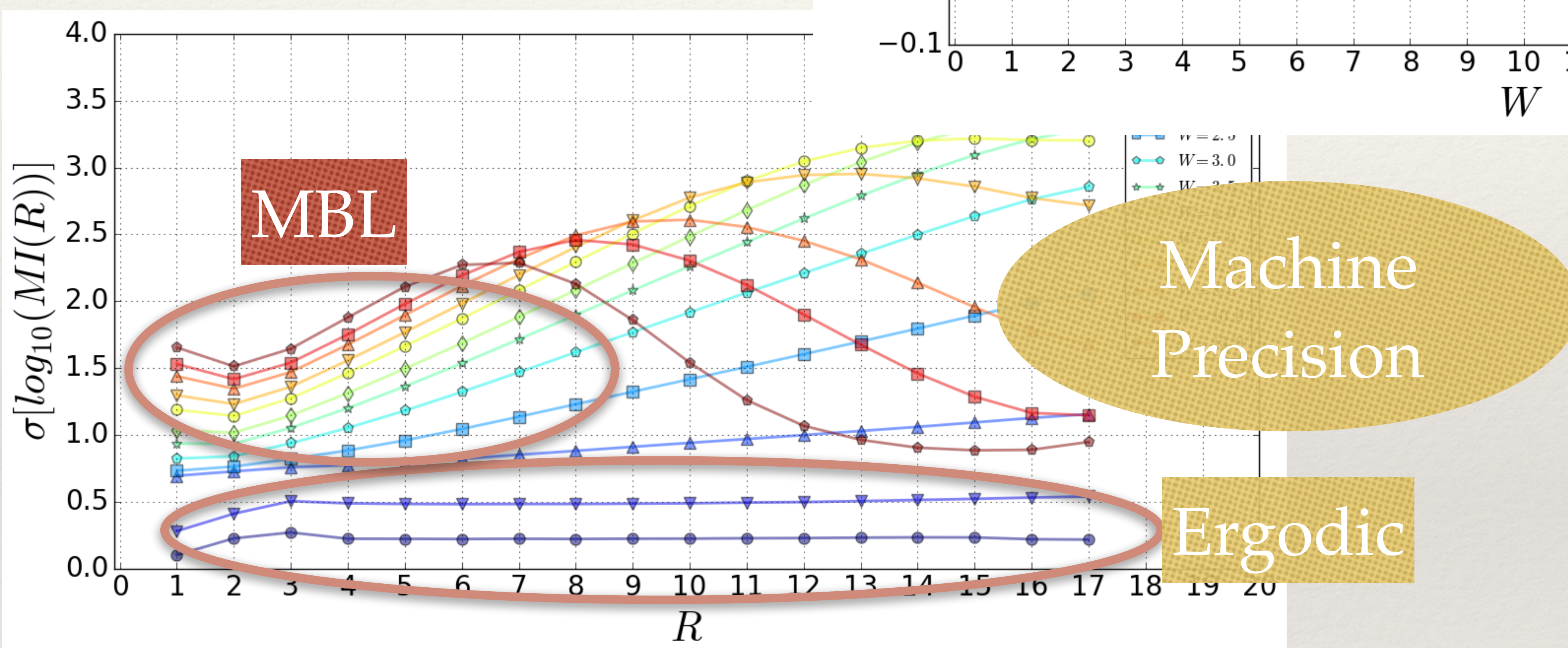


Q: What does the

- Standard Deviation
- Skewness
- Mean

of the distribution do?





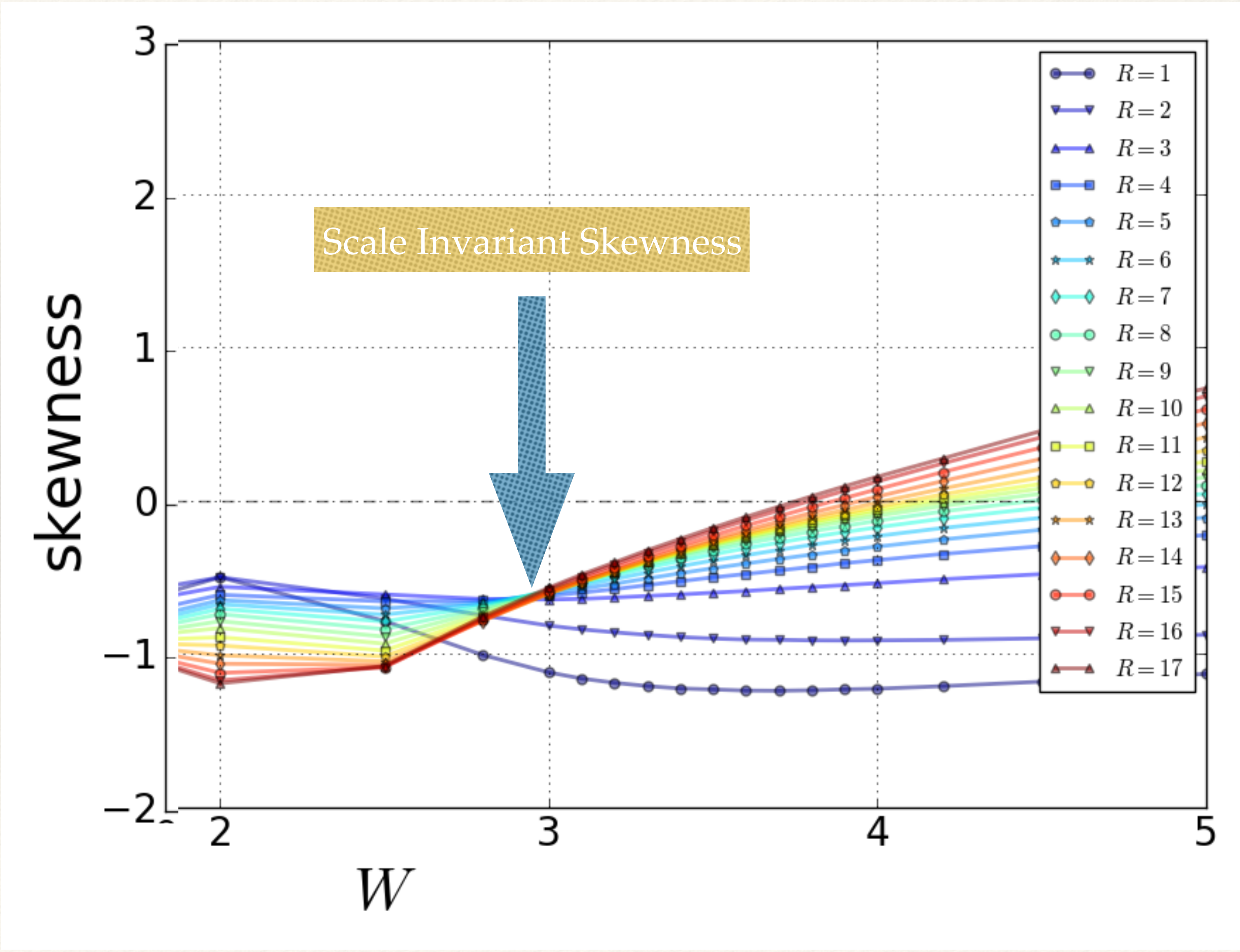
Q: What does the

- Standard Deviation

Skewness

- Mean

of the distribution do?

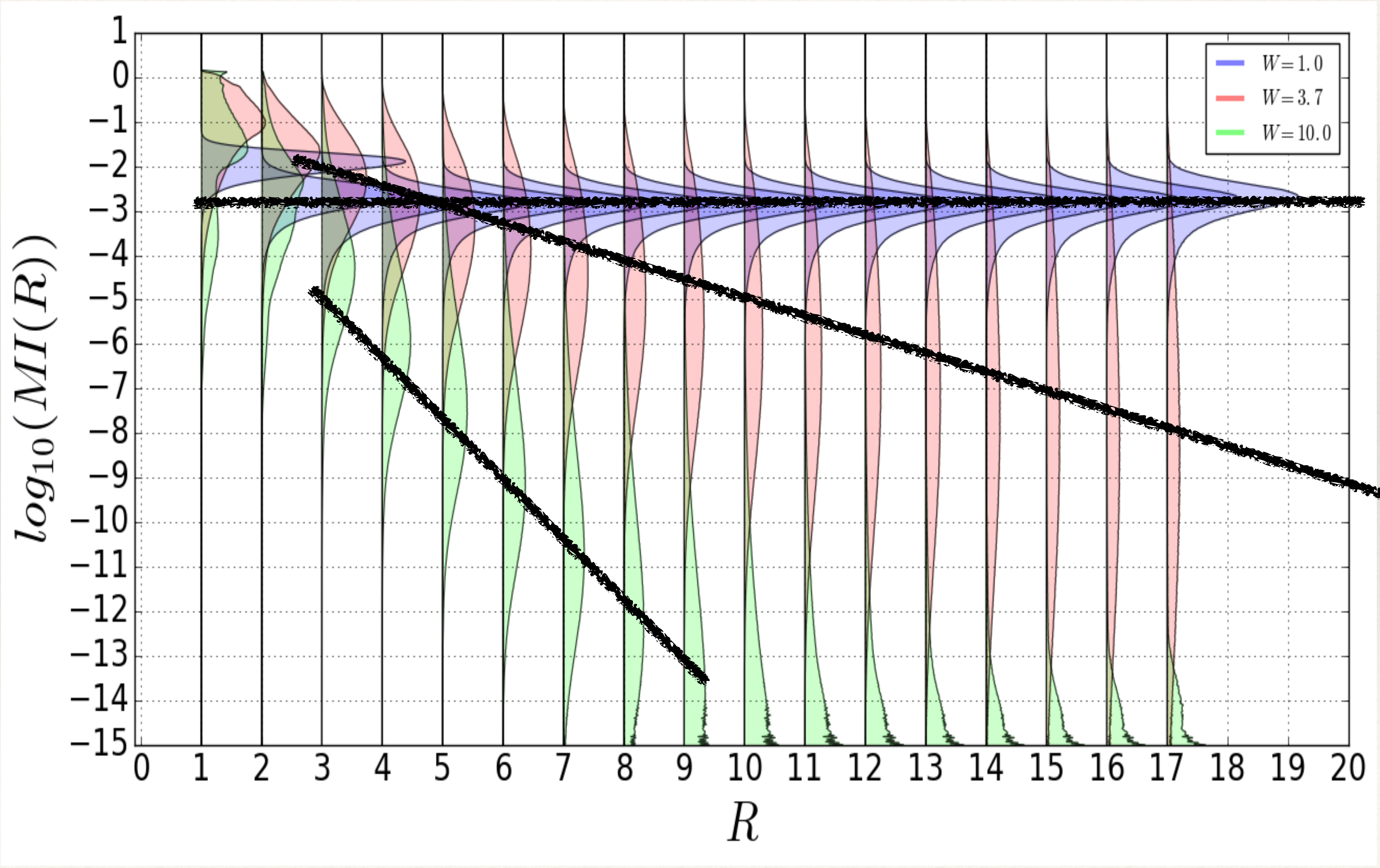


Q: What does the

- Standard Deviation
- Skewness

Mean

of the distribution do?

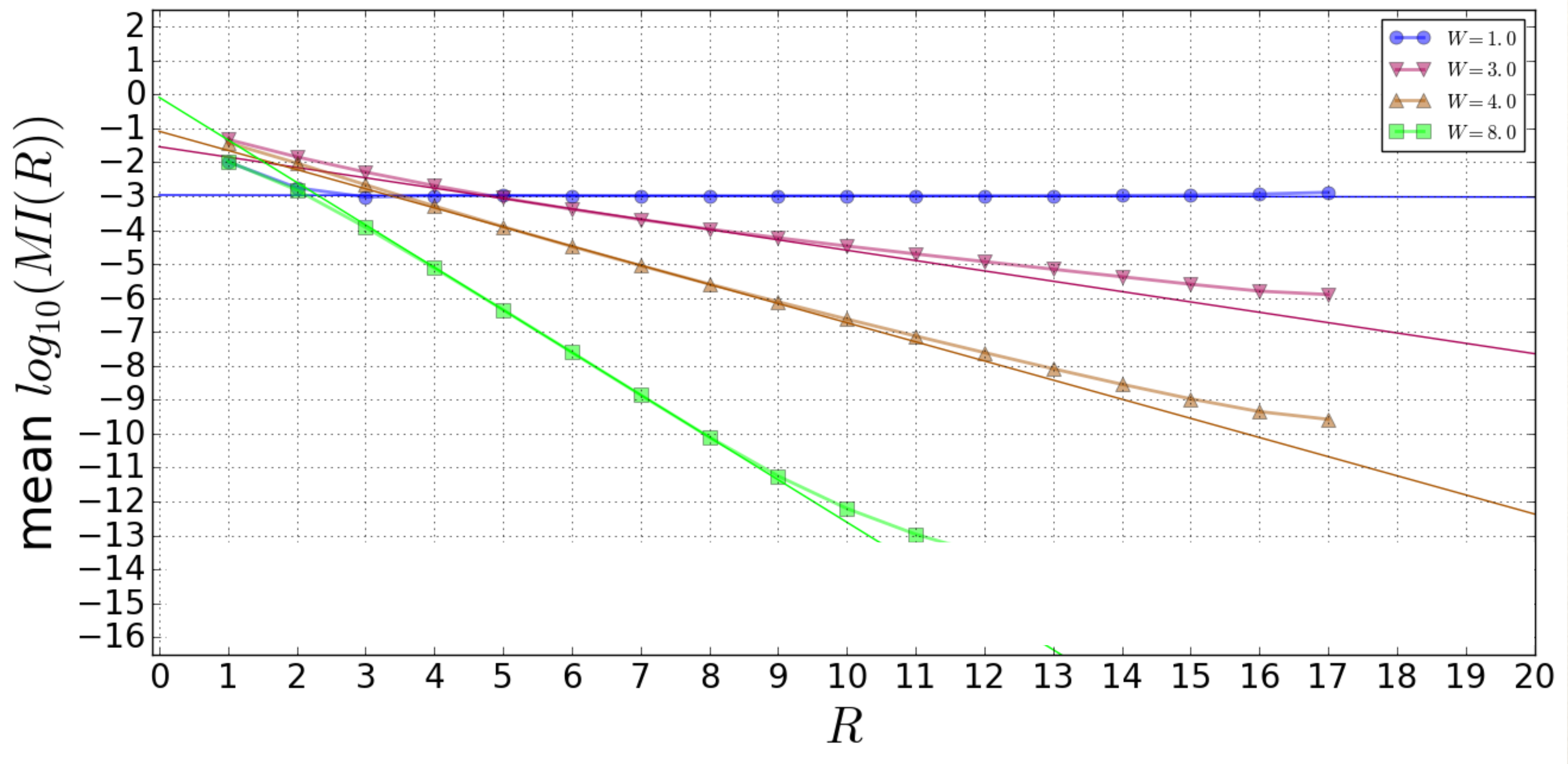


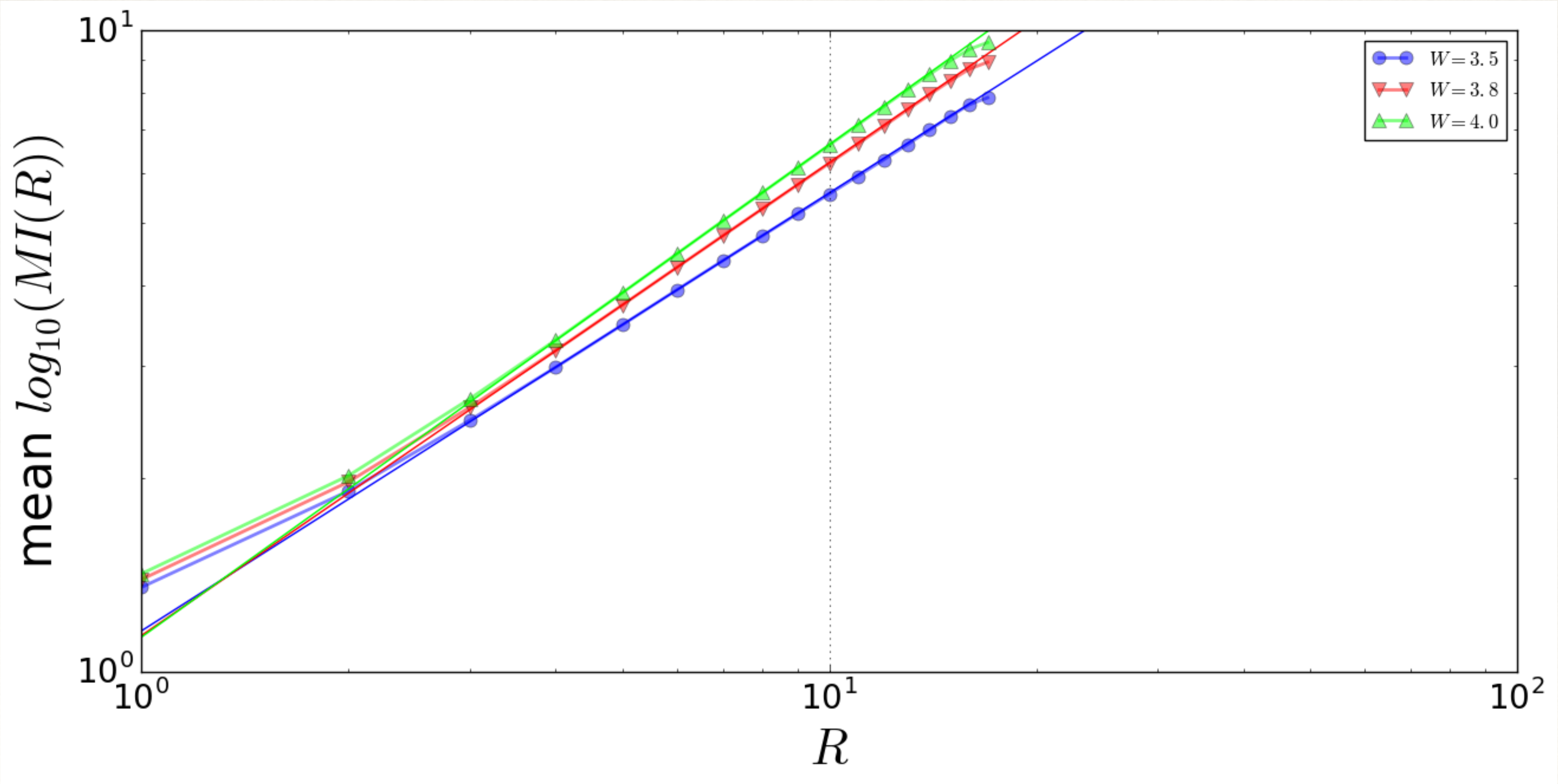
Q: What does the

- Standard Deviation
- Skewness

Mean

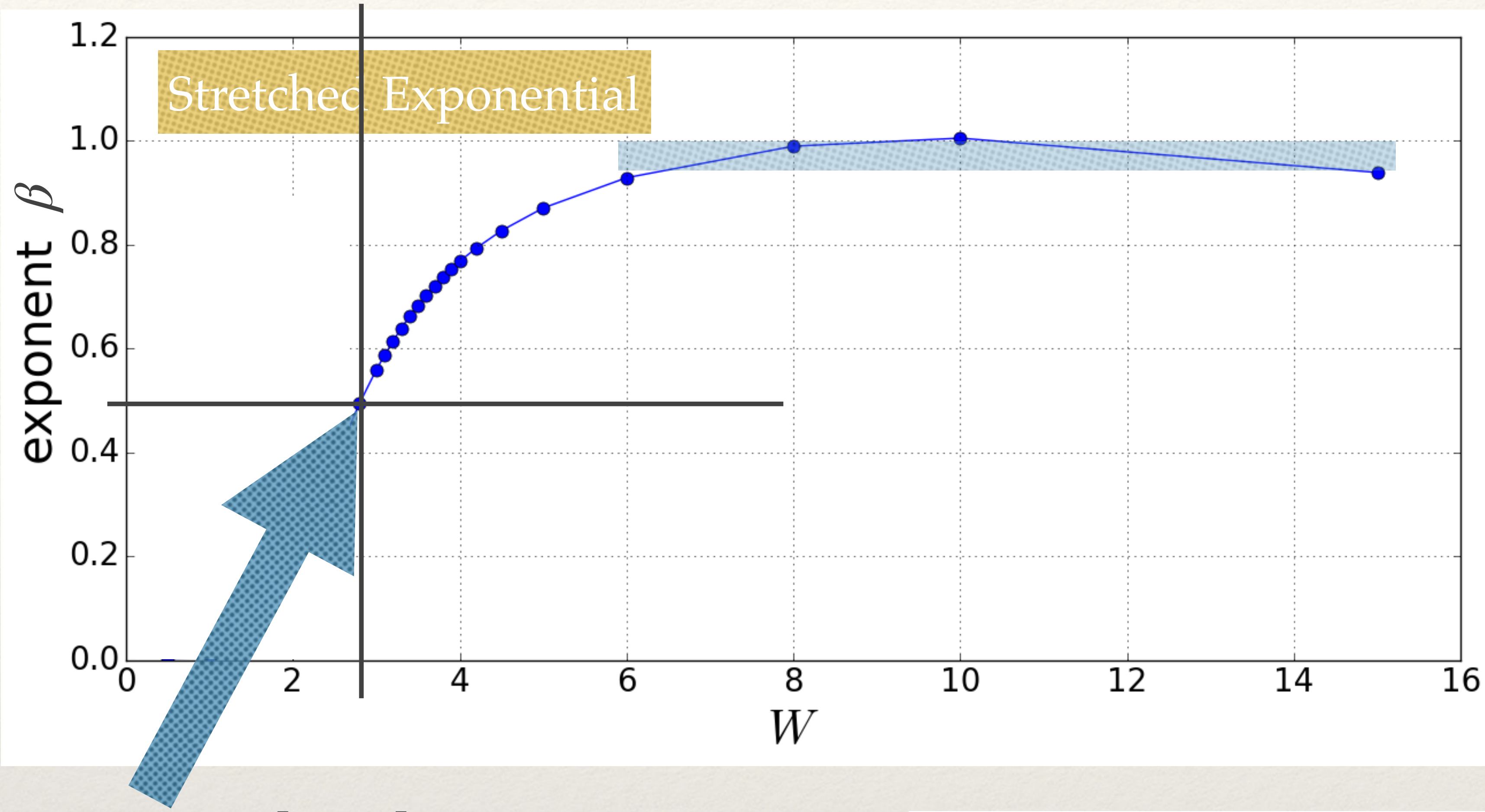
of the distribution do?



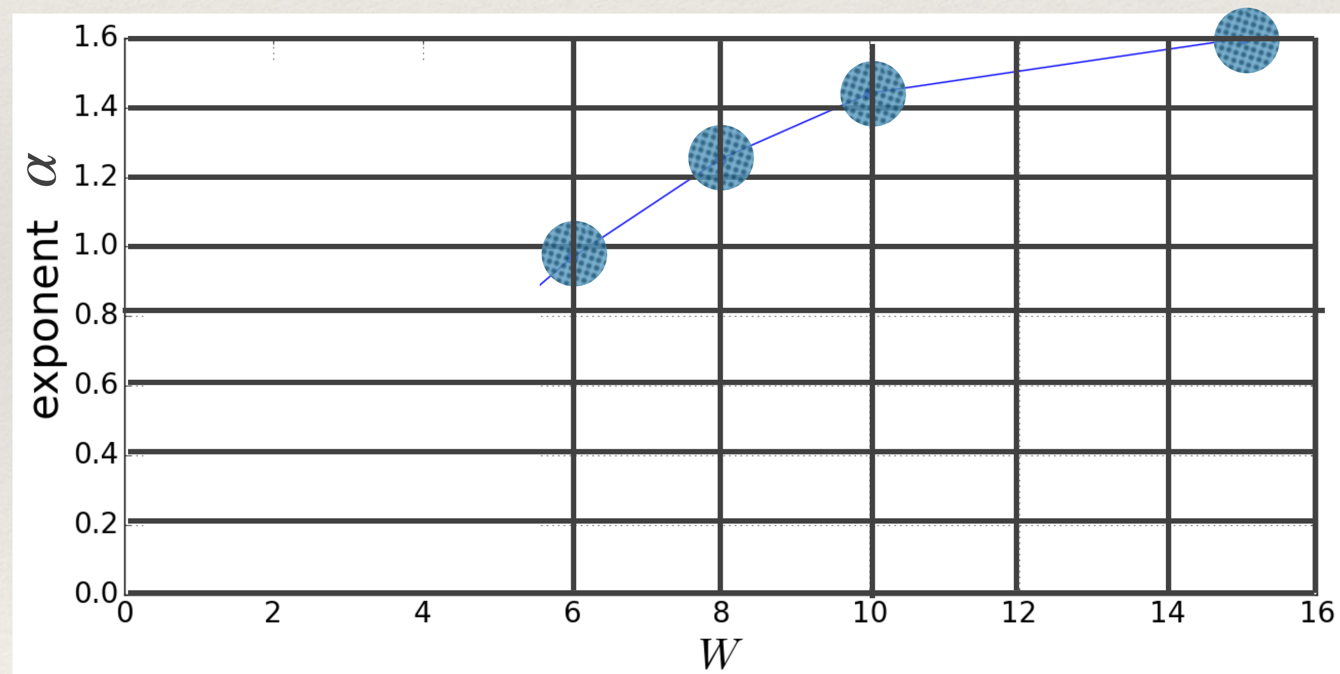


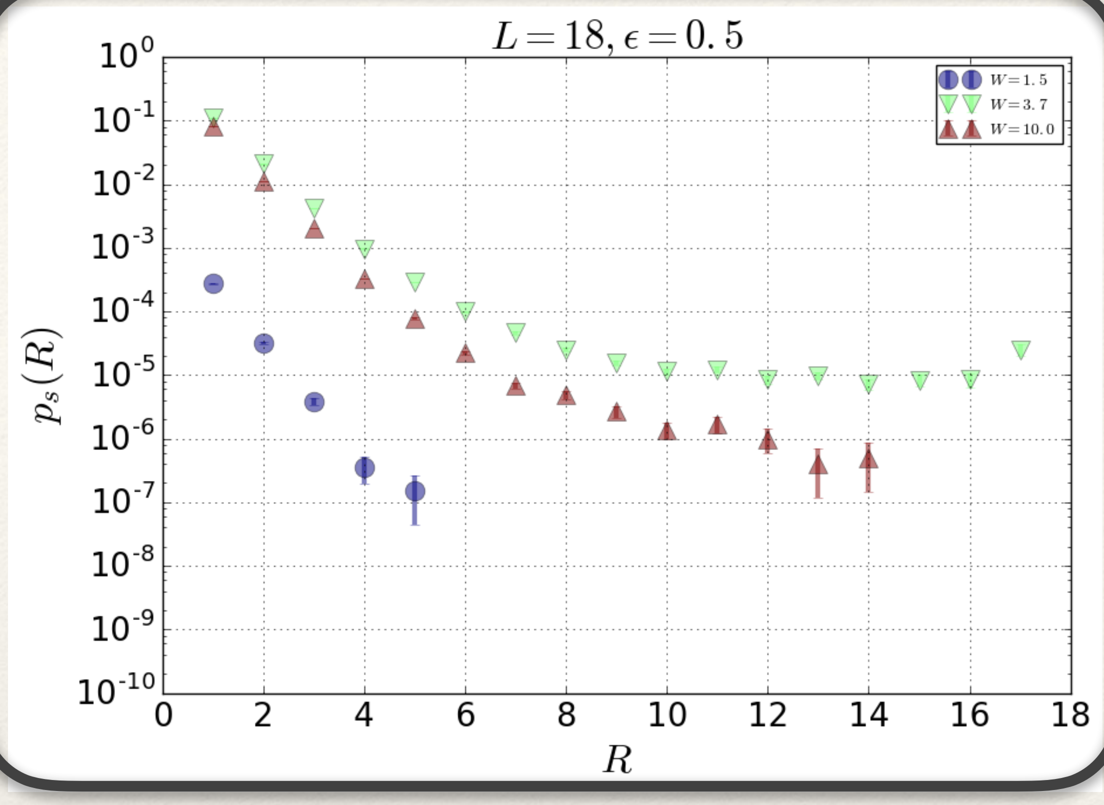
$6 > W \quad \mathrm{MI} \propto \exp[-\alpha R]$

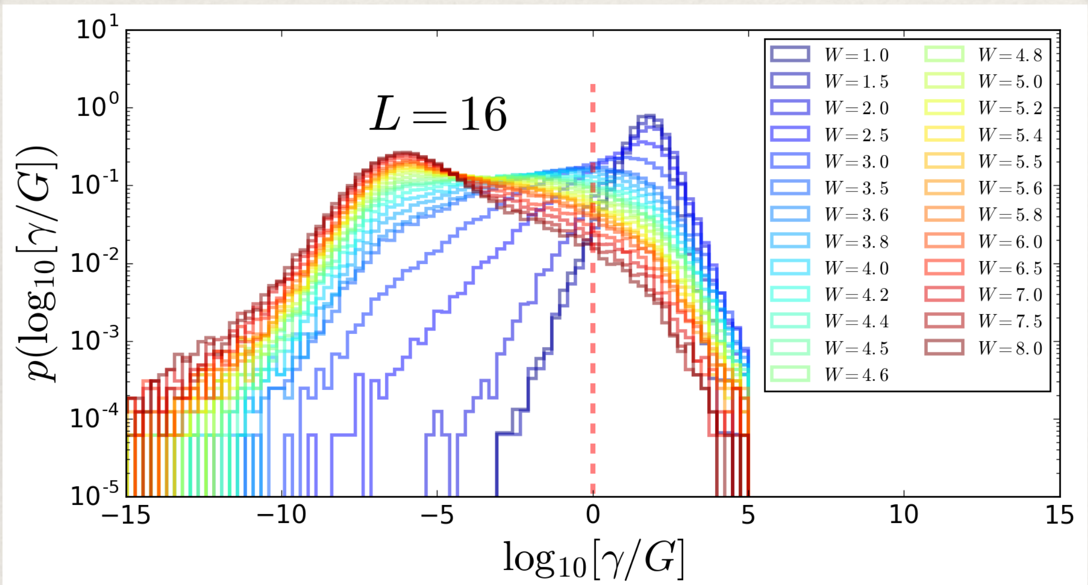
$2.7 > W > 6 \quad \mathrm{MI} \propto \exp[-A R^{\beta}]$



$\mathrm{MI} \propto \exp\left[-\sqrt{R}\right]$







$W \approx 2.8$ $\quad$ $W \approx 6.0$

$$\overline{\log[\mathrm{MI}]} \sim -AR^{1/2} \longleftrightarrow -R^{1}$$

$$\gamma_{\log(MI)} \sim -\ln(2)$$

At the transition:

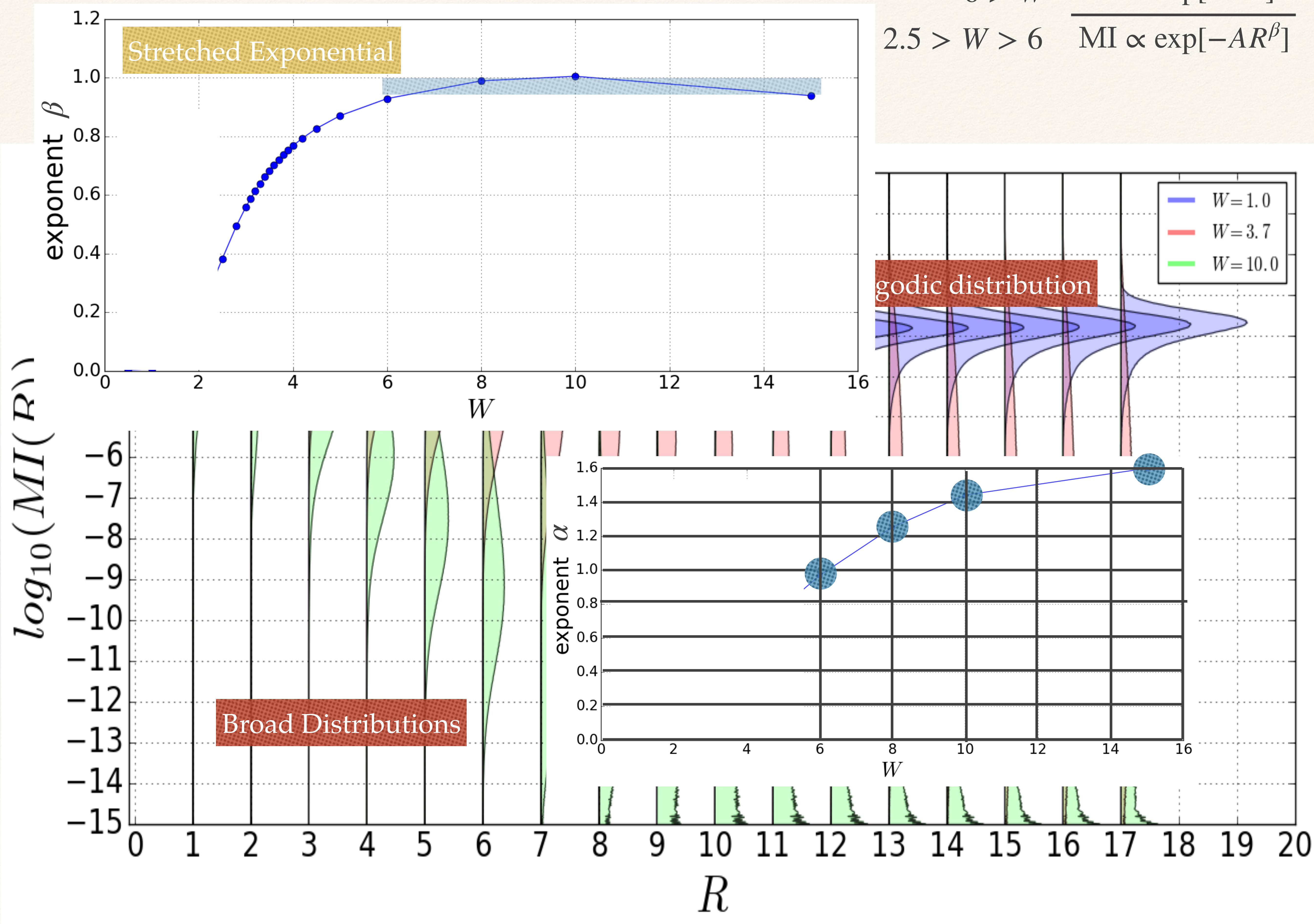
$$\overline{\log[\mathrm{MI}]} \sim -R^{1/2} \longleftrightarrow -R^{1}$$

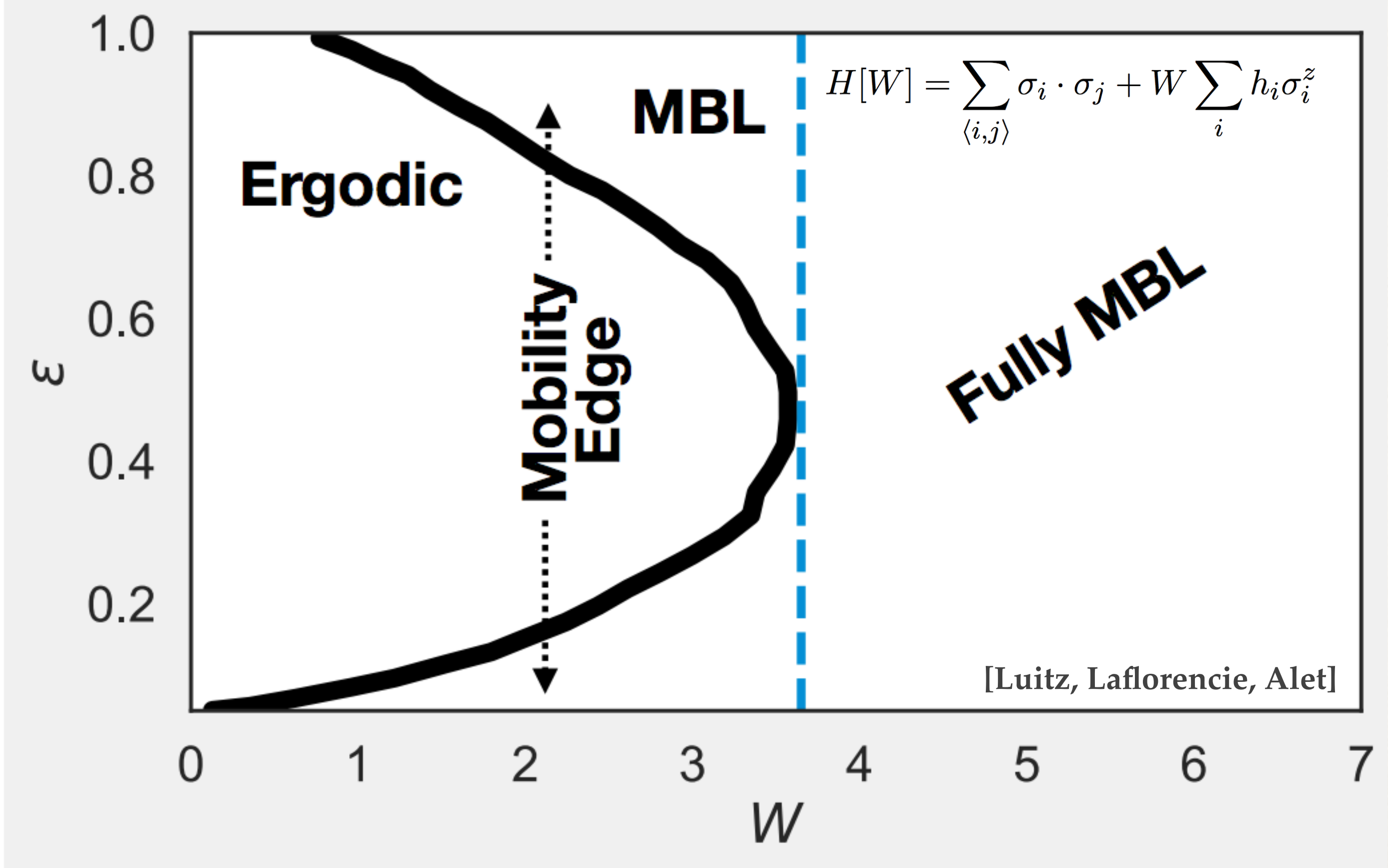
$$\sigma_{\log(MI)} \sim 0.1R \longleftrightarrow 0.2R$$

$$\gamma_{\log(MI)} \sim -\ln(2) \longleftrightarrow$$

$6 > W \quad \mathrm{MI} \propto \exp[-\alpha R]$

$2.5 > W > 6 \quad \mathrm{MI} \propto \exp[-A R^{\beta}]$





Many-Body Localization

Fully MBL

(Mobility Edge)

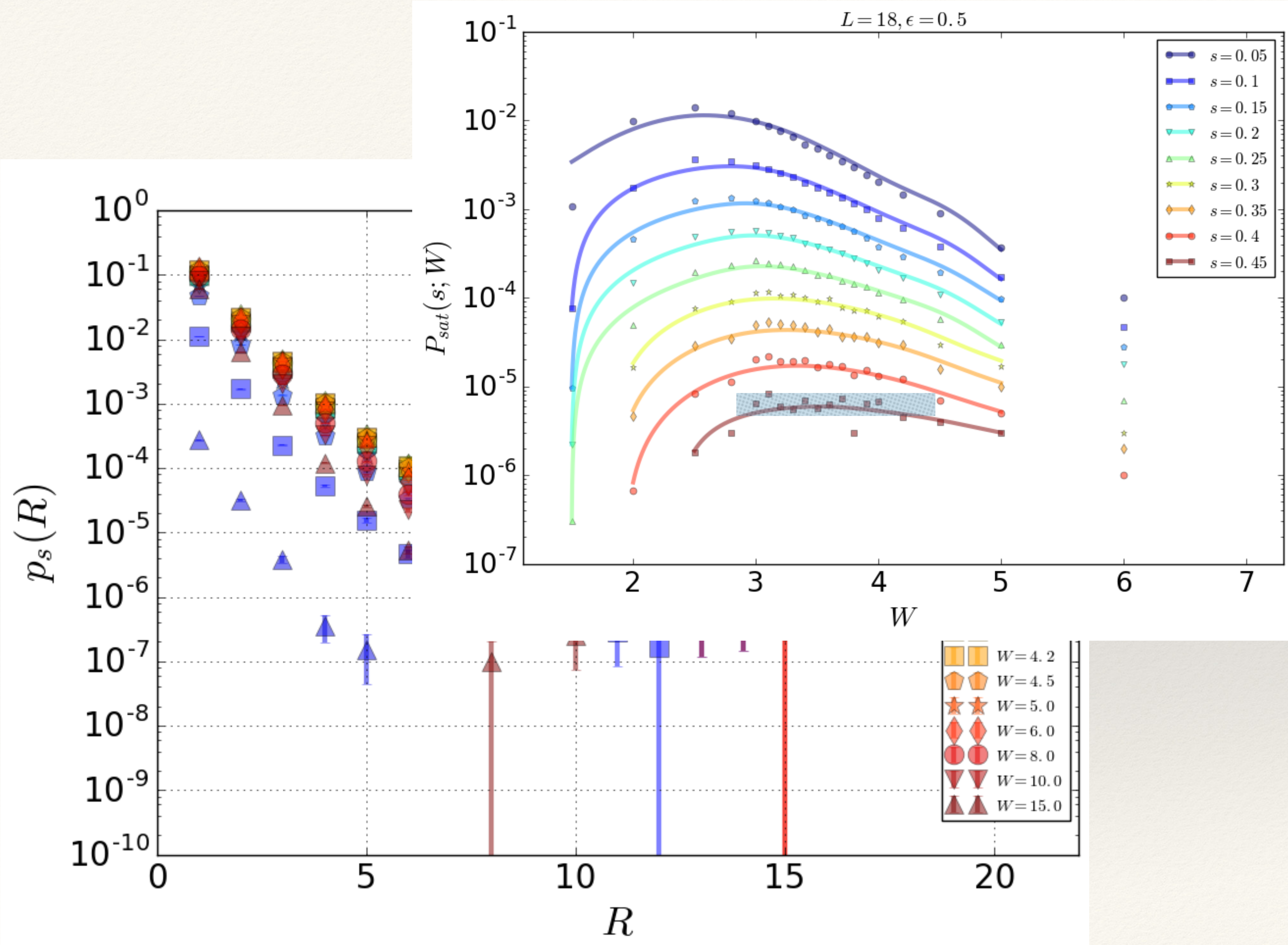
Transition

Ergodic

Florence, Italy

Resonances in MBL

Bryan Clark
with Benjamin Villalonga

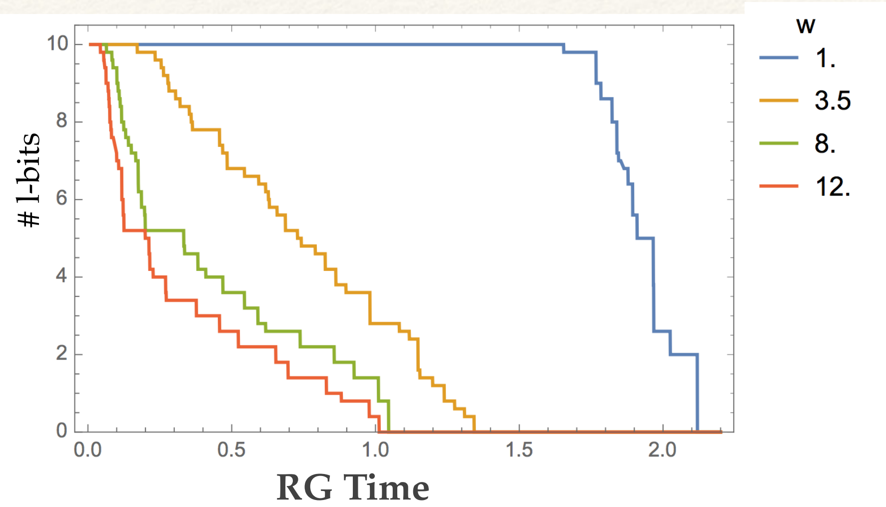


$L=18, \epsilon=0.5$
$P_{sat}(s;W)$
W
$s=0.05$
$s=0.1$
$s=0.15$
$s=0.2$
$s=0.25$
$s=0.3$
$s=0.35$
$s=0.4$
$s=0.45$
$p_s(R)$
R
$W=4.2$
$W=4.5$
$W=5.0$
$W=6.0$
$W=8.0$
$W=10.0$
$W=15.0$

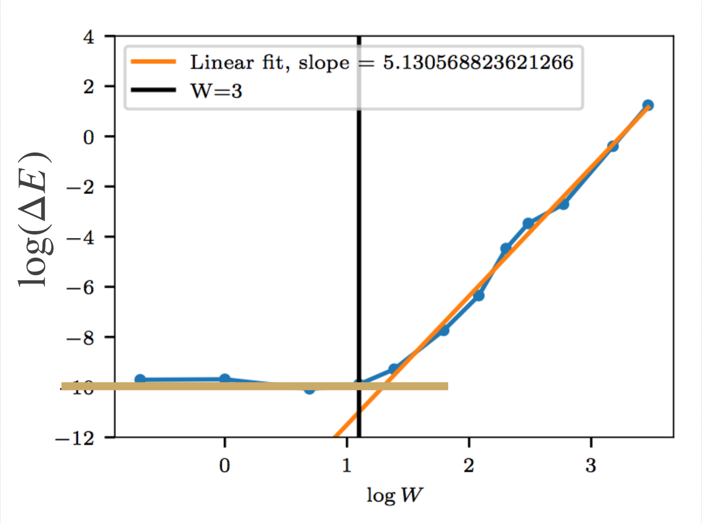
At the MBL transition…

Numerical RG:

Wegner-Wilson Flow



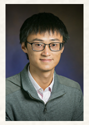
Uniform integrating out of l-bits under RG



RG integrates out final energy scale:

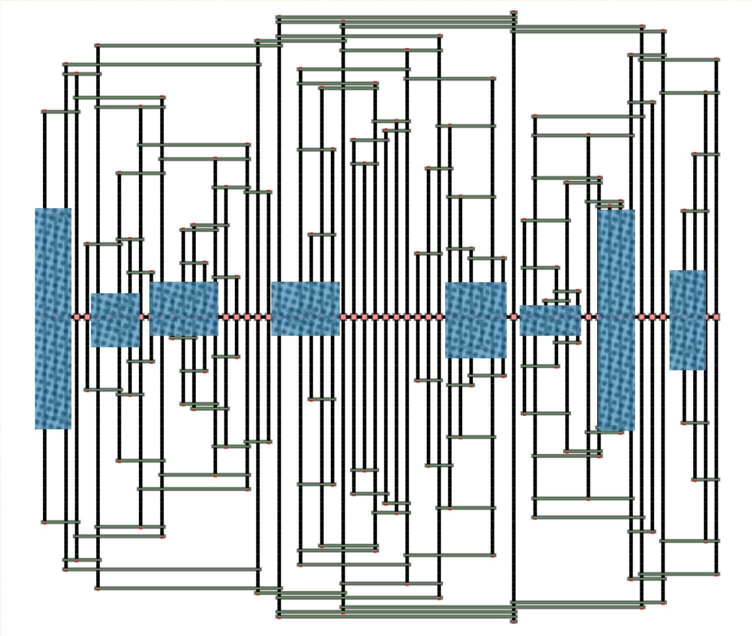
$$\Delta E \propto 2^{-L}$$

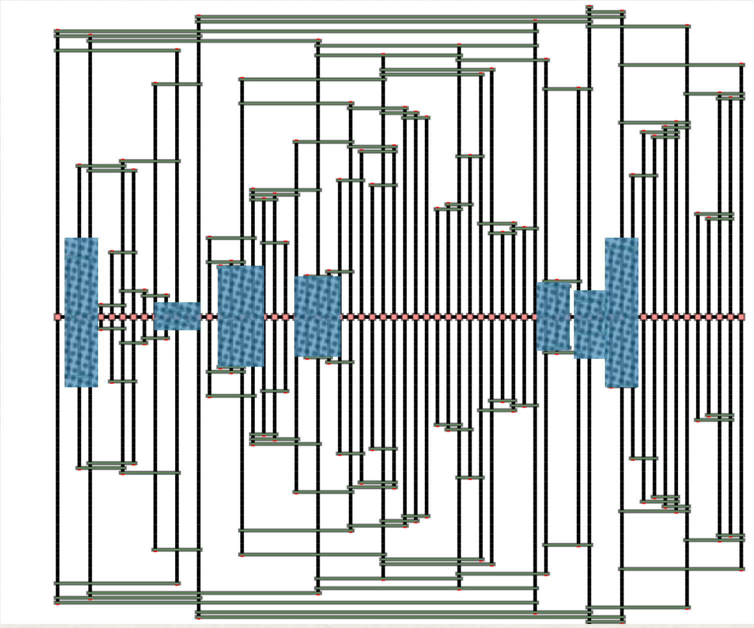
$$\Delta E \propto W^{-\alpha(L)}$$

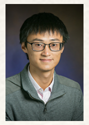


Yu-Pekker-Clark

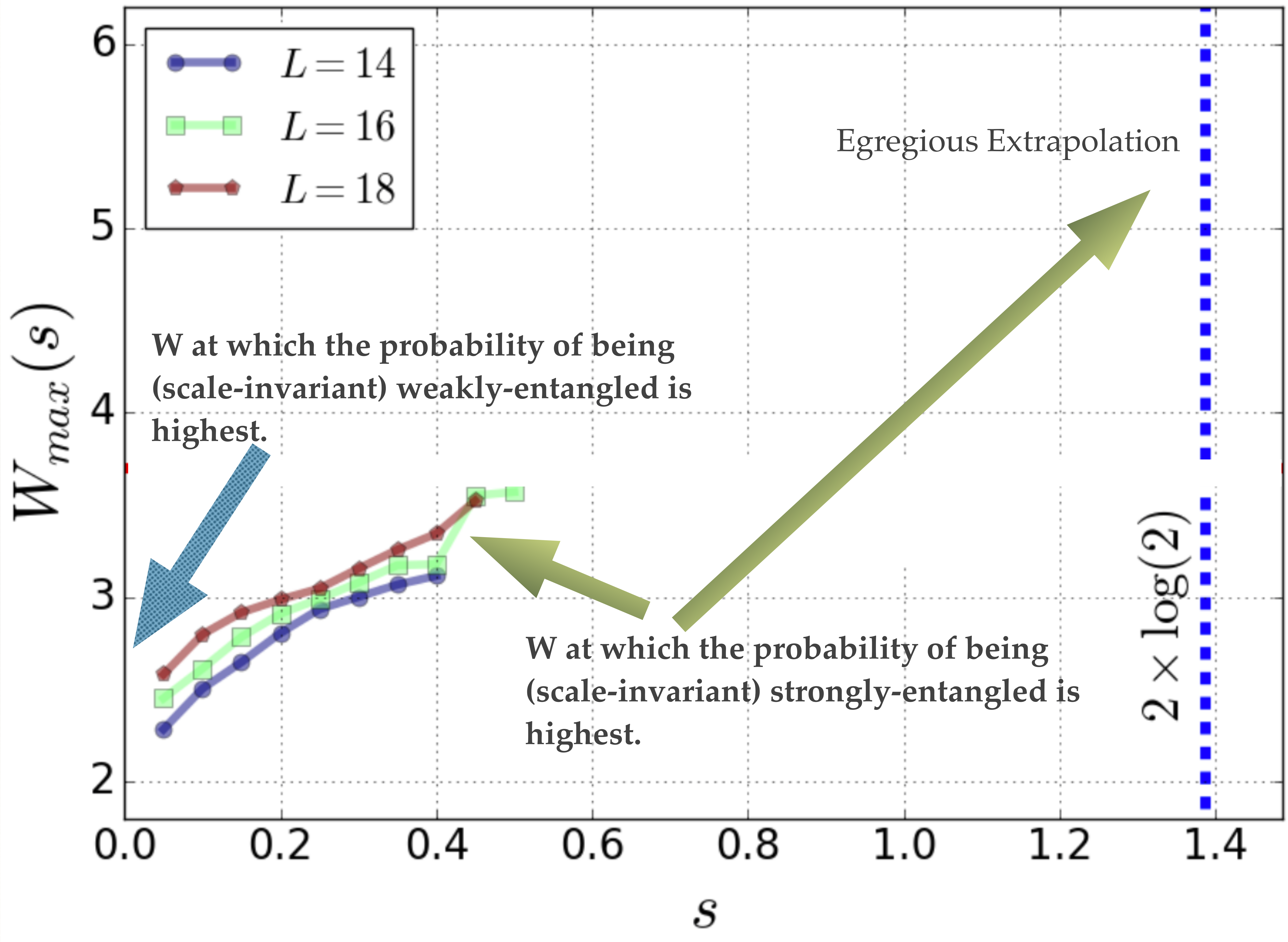
MERA



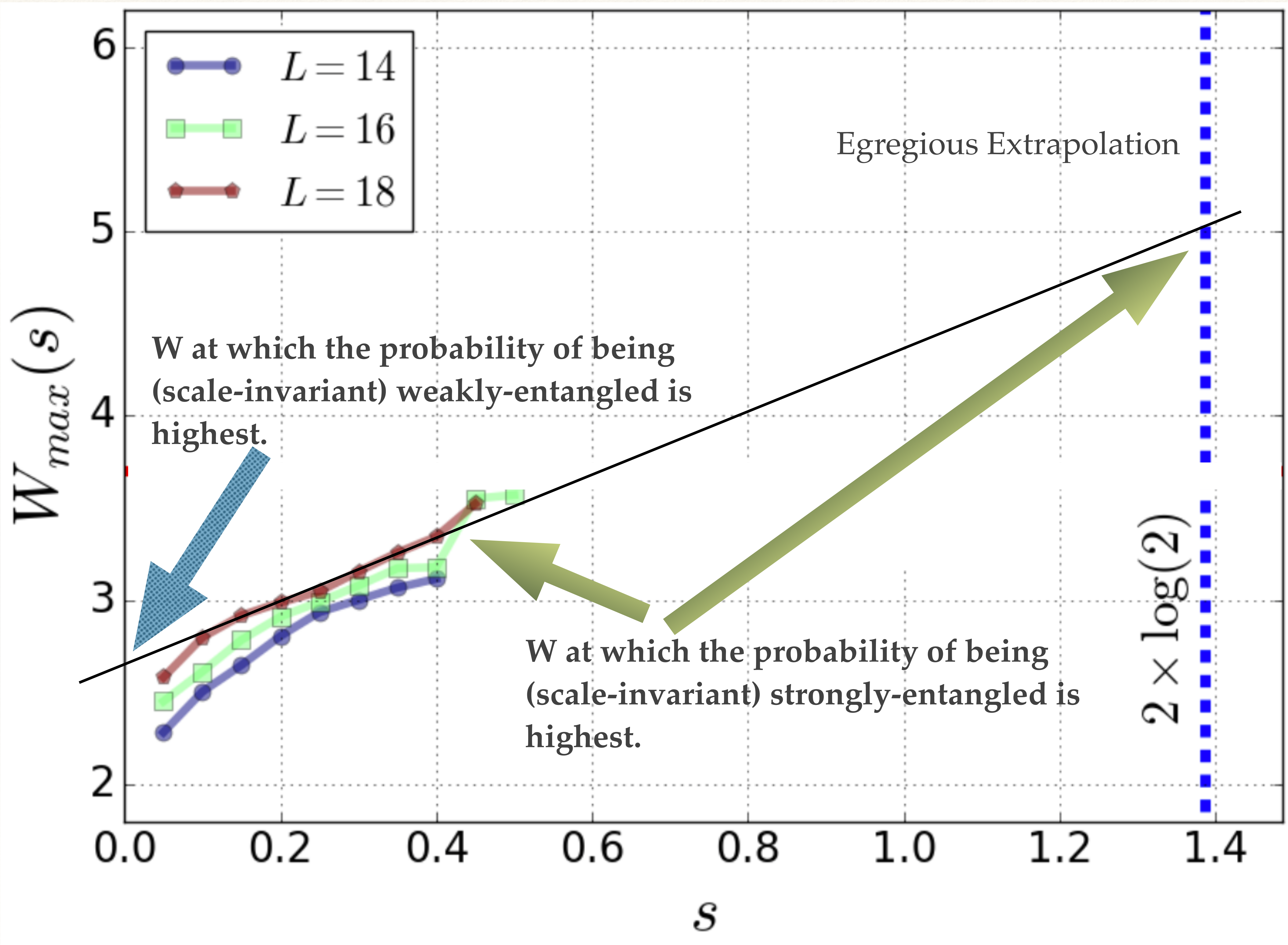




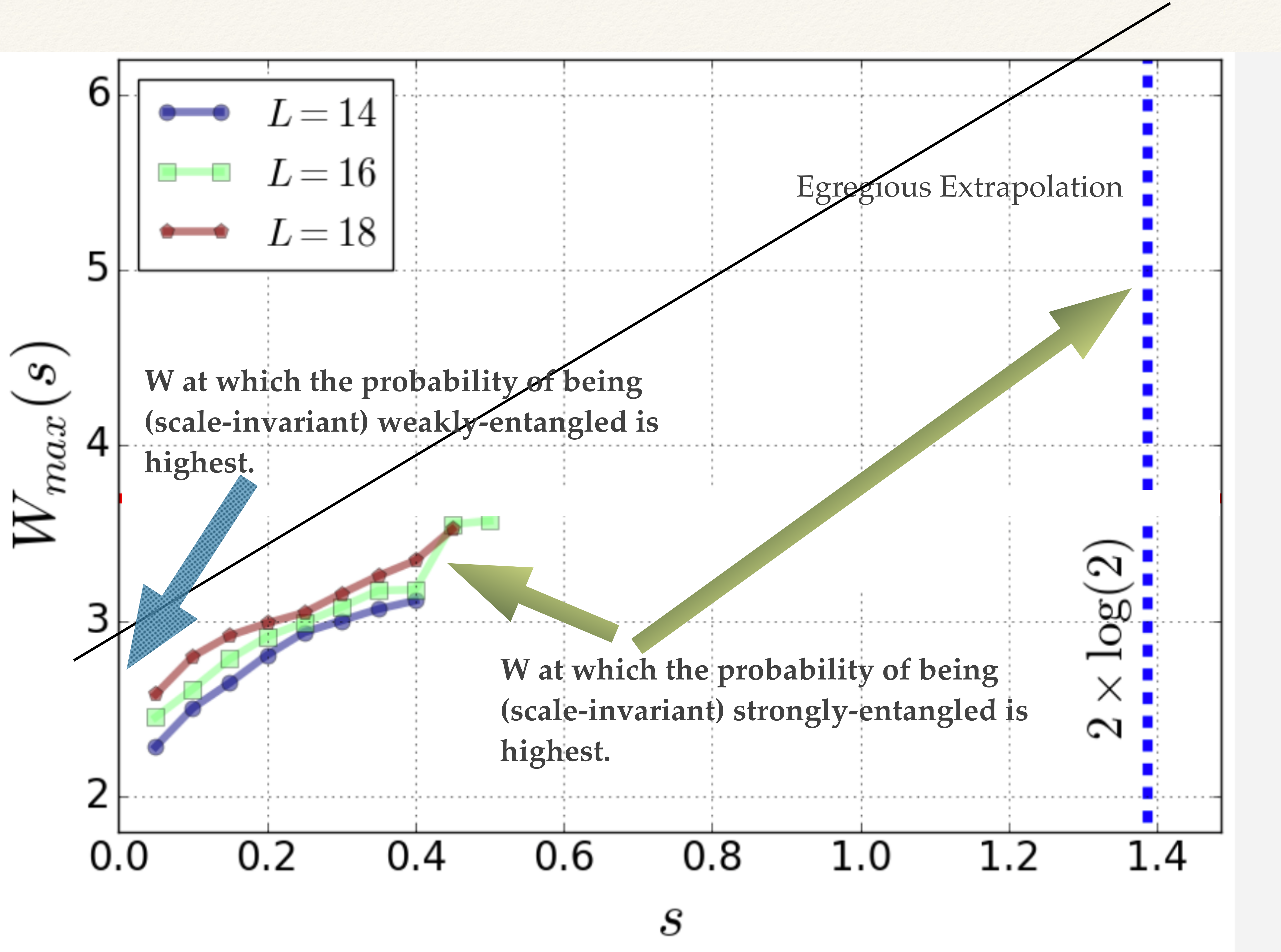
Yu-Pekker-Clark



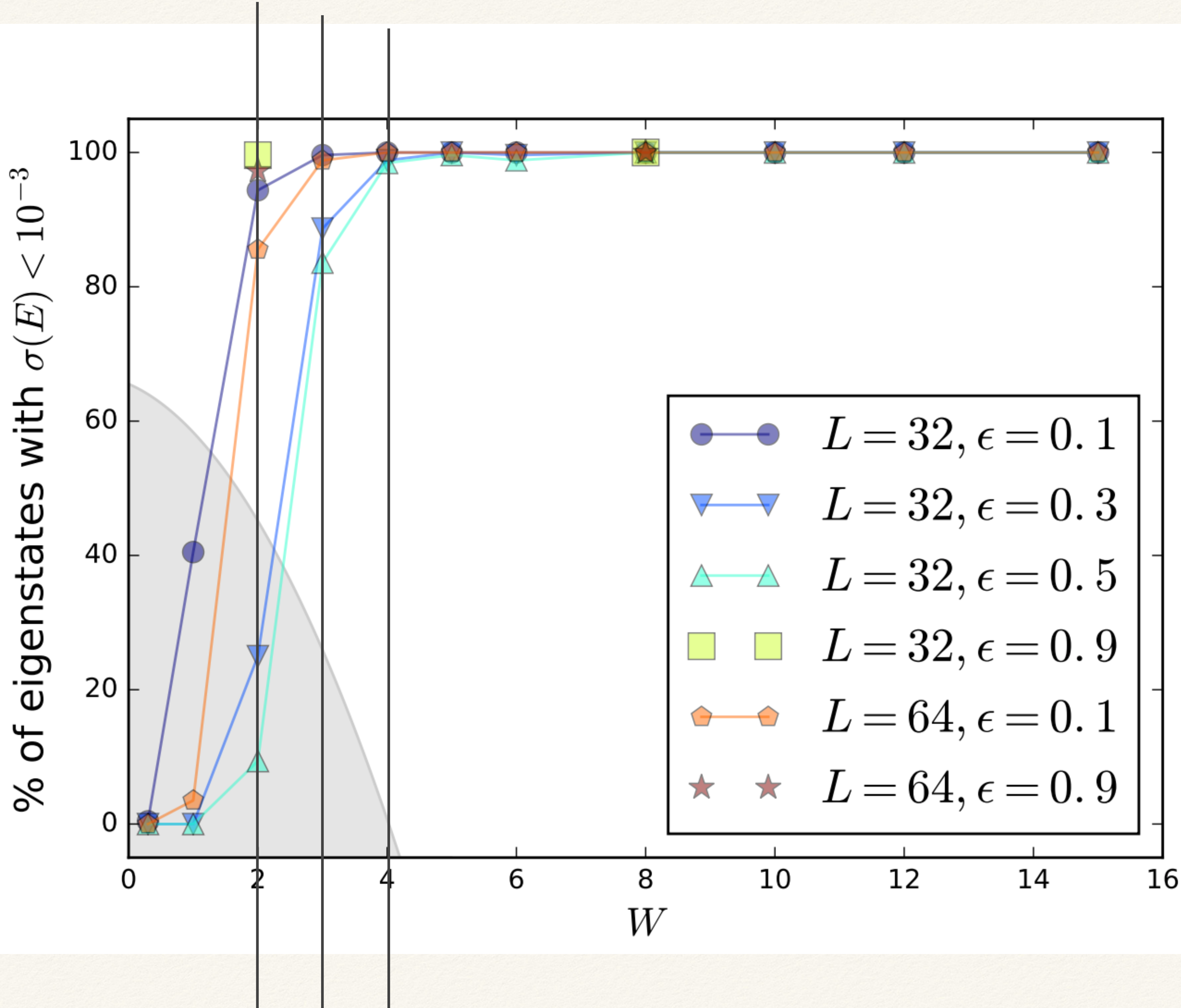
$L = 14$
$L = 16$
$L = 18$
$W_{max}(s)$
s
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
2
3
4
5
6
Egregious Extrapolation
$2 \times \log(2)$
W at which the probability of being (scale-invariant) weakly-entangled is highest.
W at which the probability of being (scale-invariant) strongly-entangled is highest.



L = 14
L = 16
L = 18
Egregious Extrapolation
W at which the probability of being (scale-invariant) weakly-entangled is highest.
W at which the probability of being (scale-invariant) strongly-entangled is highest.
2 × log(2)
$W_{max}(s)$
s
6
5
4
3
2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4



$L = 14$
$L = 16$
$L = 18$
Egregious Extrapolation
W at which the probability of being (scale-invariant) weakly-entangled is highest.
W at which the probability of being (scale-invariant) strongly-entangled is highest.
$2 \times \log(2)$
$W_{max}(s)$
s



% of eigenstates with $\sigma(E) < 10^{-3}$
100
80
60
40
20
0
0
2
4
6
8
10
12
14
16
W
$L = 32, \epsilon = 0.1$
$L = 32, \epsilon = 0.3$
$L = 32, \epsilon = 0.5$
$L = 32, \epsilon = 0.9$
$L = 64, \epsilon = 0.1$
$L = 64, \epsilon = 0.9$