USING TOPOLOGICAL ENTANGLEMENT ENTROPY TO IDENTIFY LOW ENERGY EFFECTIVE FIELD THEORIES OF FRACTIONAL CHERN INSULATORS

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Bryan Clark - Station Q

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Fractional Chern Insulators

Today's goal:

* Write down prototypical wave-functions for fractional chern insulator.

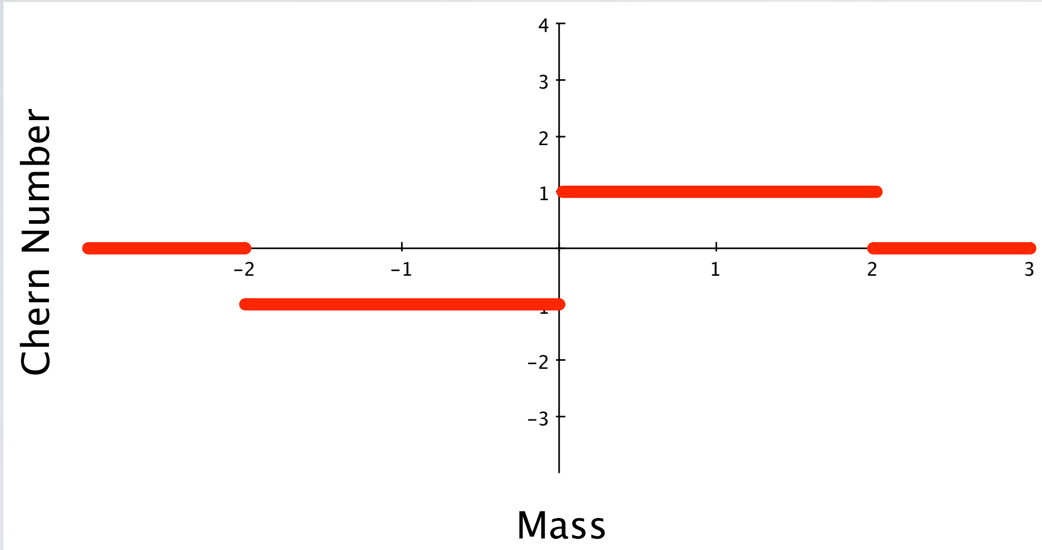
* Establish the low energy effective field theory from topological entanglement entropy.

* Look at some aspects of entanglement (finite size, etc.)

Related work to Zhang, Vishwanath http://arxiv.org/abs/1209.2424

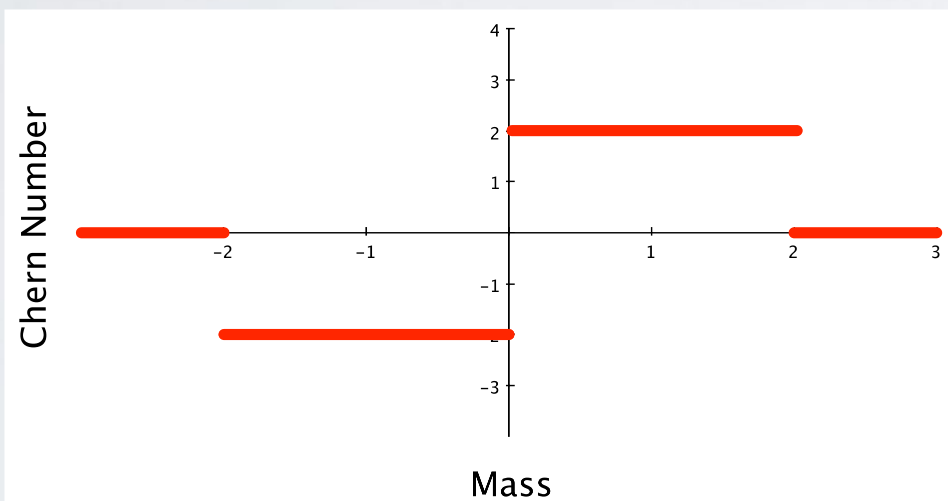
Non-interacting Chern insulators

$$h(k) = \sin k_x \sigma_x + \sin k_y \sigma_y + (M - \cos k_x - \cos k_y)\sigma_z$$



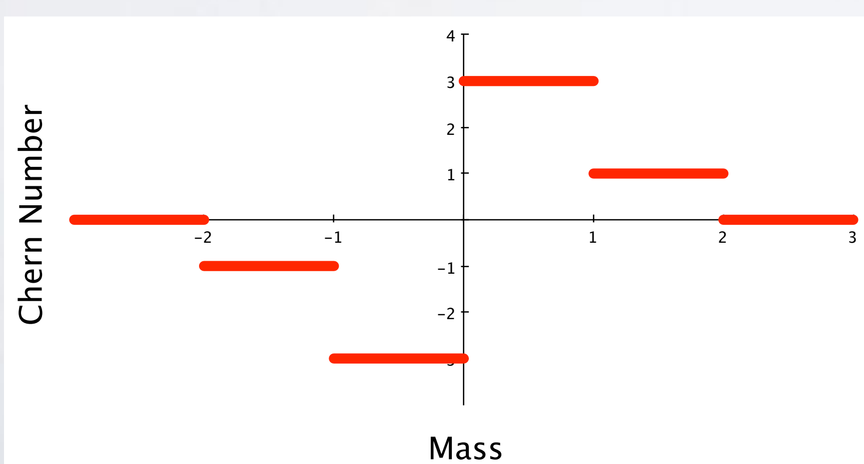
C=1

$$h(k) = \sin(k_x + k_y)\sigma_x + \sin(k_x - k_y)\sigma_y + (M - \cos(k_x + k_y) - \cos(k_x - k_y))\sigma_z$$



C=2

$$h(k) = \sin 2k_x \sigma_x + \sin 2k_y \sigma_y + (M - \cos k_x - \cos k_y)\sigma_z$$



C=1,3

Interactions....

integer quantum hall $\longrightarrow$ fractional quantum hall

chern insulator $\longrightarrow$ fractional chern insulator

$\Psi_{\mathrm{TB}} \longrightarrow$ Ground state of h(k)

$\Psi_{\mathrm{int}} \longrightarrow \langle R|\Psi_{\mathrm{int}}\rangle \equiv \langle R|\Psi_{TB}\rangle^2$

Want: Low energy effective field theories.

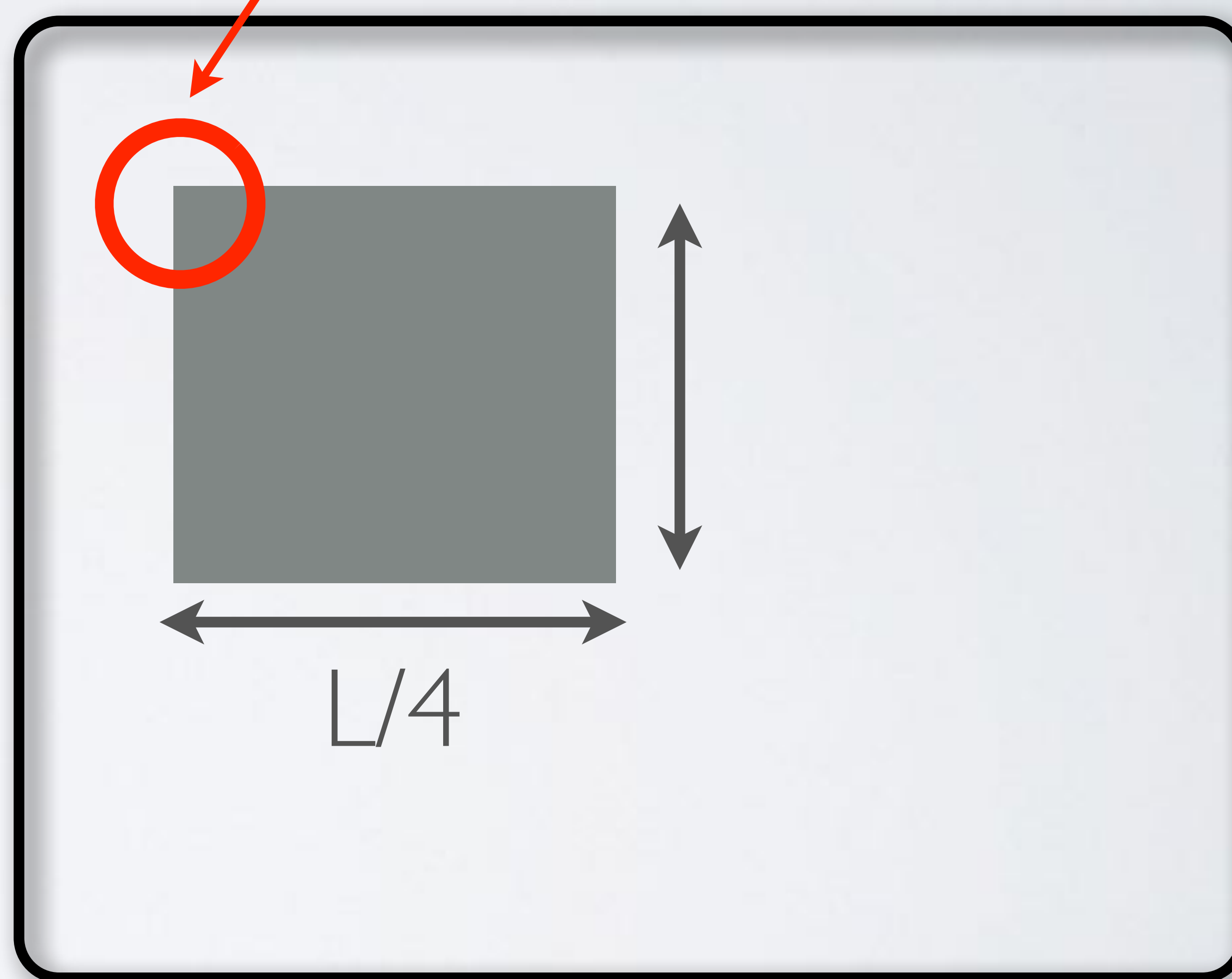
Topological Entanglement Entropy

Area Law: $S(L) = \alpha L - \gamma_0 - n_c c_1 + \ldots$

topological entanglement entropy

corner term

non-universal constant

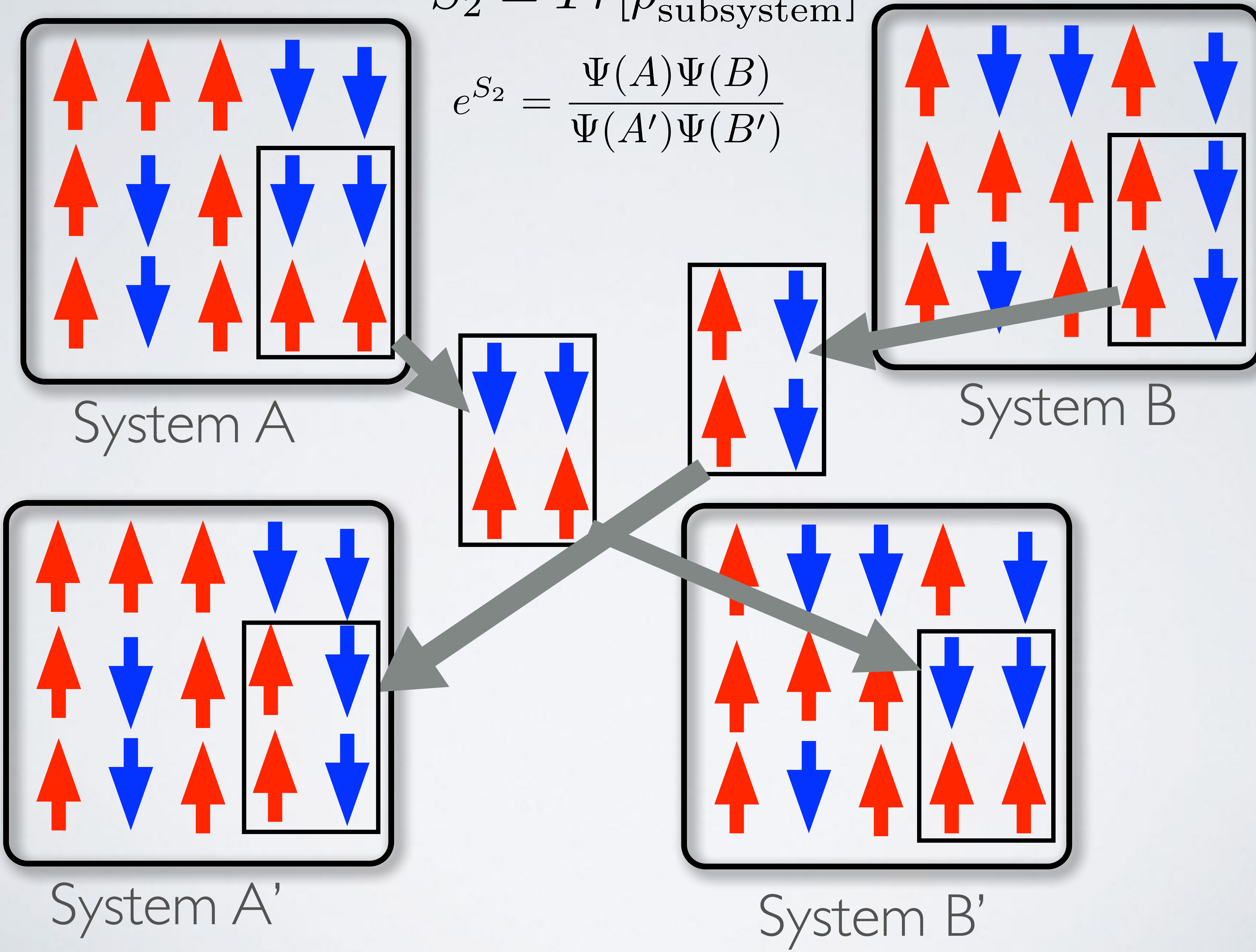


Want to compute topological entanglement entropy.

Variational Monte Carlo

$$S_2 = Tr[\rho^2_{\text{subsystem}}]$$

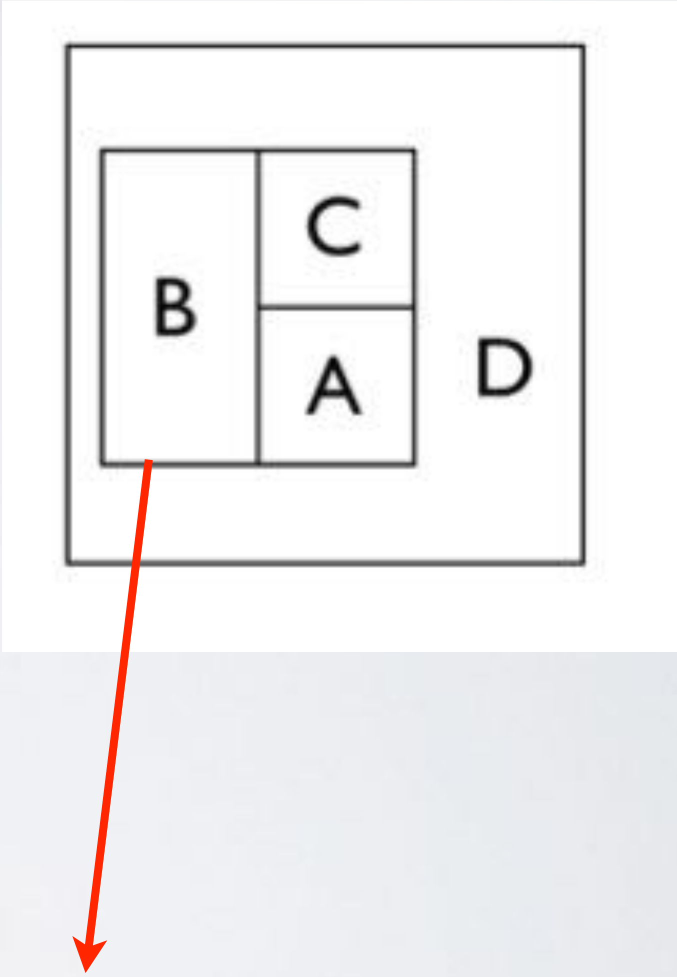
$$e^{S_2} = \frac{\Psi(A)\Psi(B)}{\Psi(A')\Psi(B')}$$



Kitaev ABC Construction

Area Law: $S(L) = \alpha L - \gamma_0 - n_c c_1 + \ldots$

TEE: $-\gamma_0 = 2S_A - 2S_{AB} + S_{ABC}$



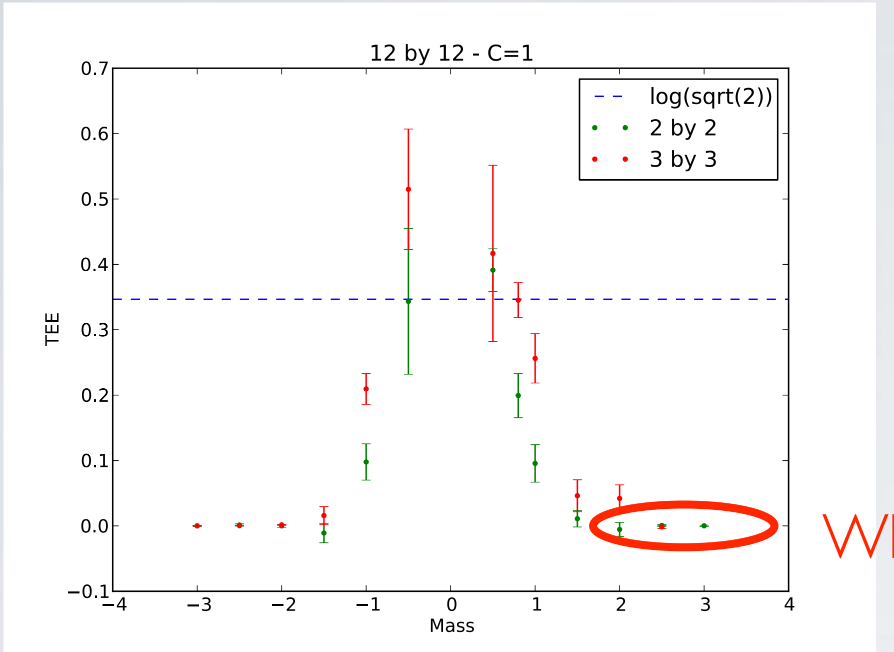
Cancels out *linear* and *corner* terms (in thermodynamic limit)

Topological phases: Degenerate ground states which can't be distinguished locally.

All boxes are local. Doesn't matter which ground state we live in.

Fractional Chern Insulators

$$SU(2)_C \quad \ln\left[\sqrt{2/(C+2)}\,\sin(\pi/(C+2))\right]$$

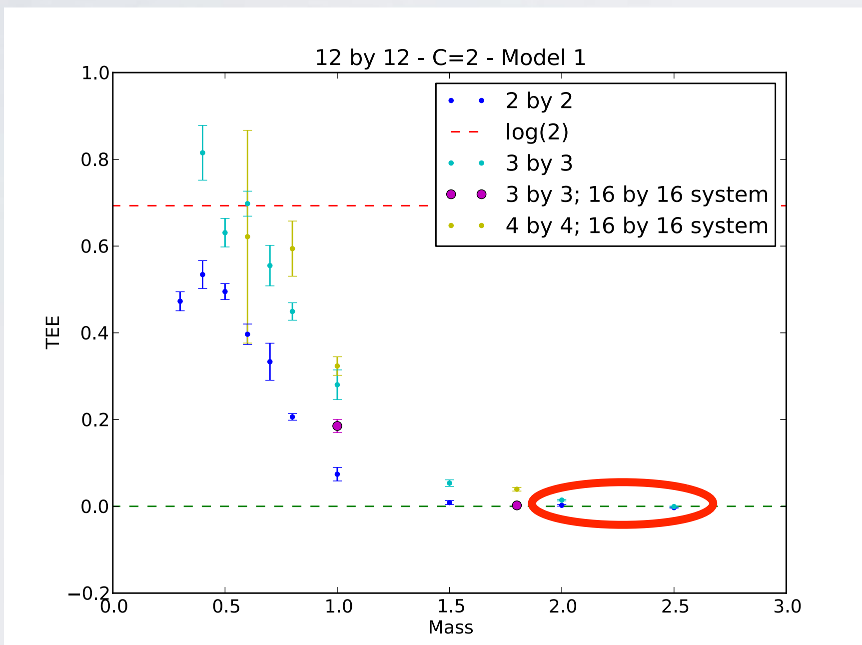


When C=0, TEE=0

0.365 +/- 0.02

$\ln\sqrt{2} \approx 0.347$

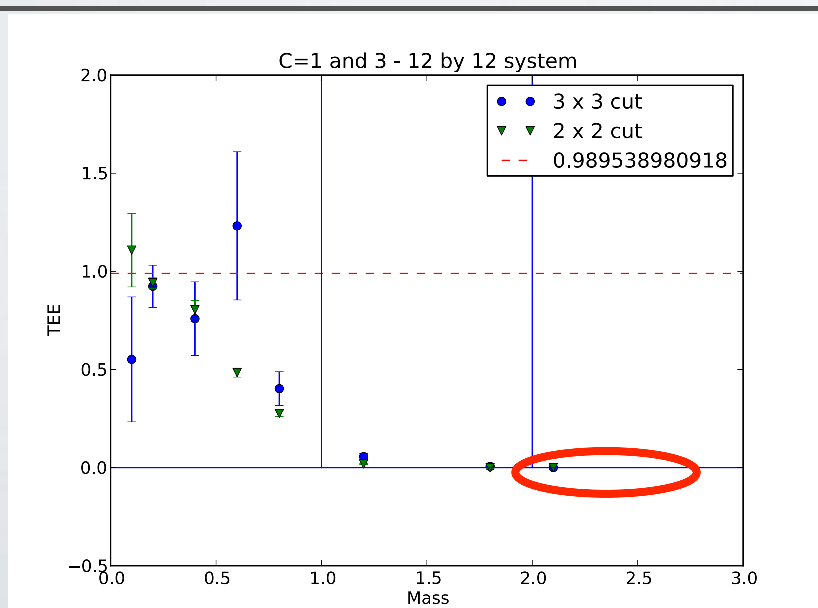
0.9 standard deviations



0.675 +/- 0.019

$\ln 2 \approx 0.693$

0.947 standard deviations

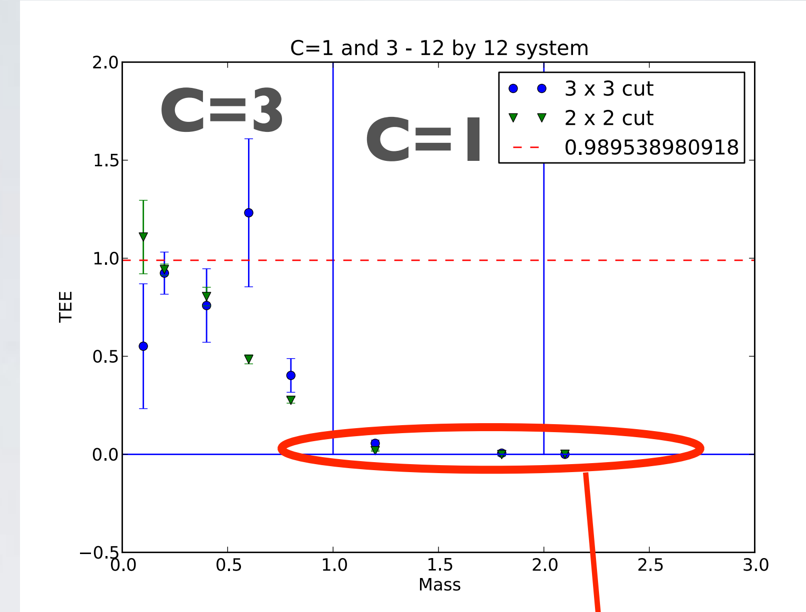


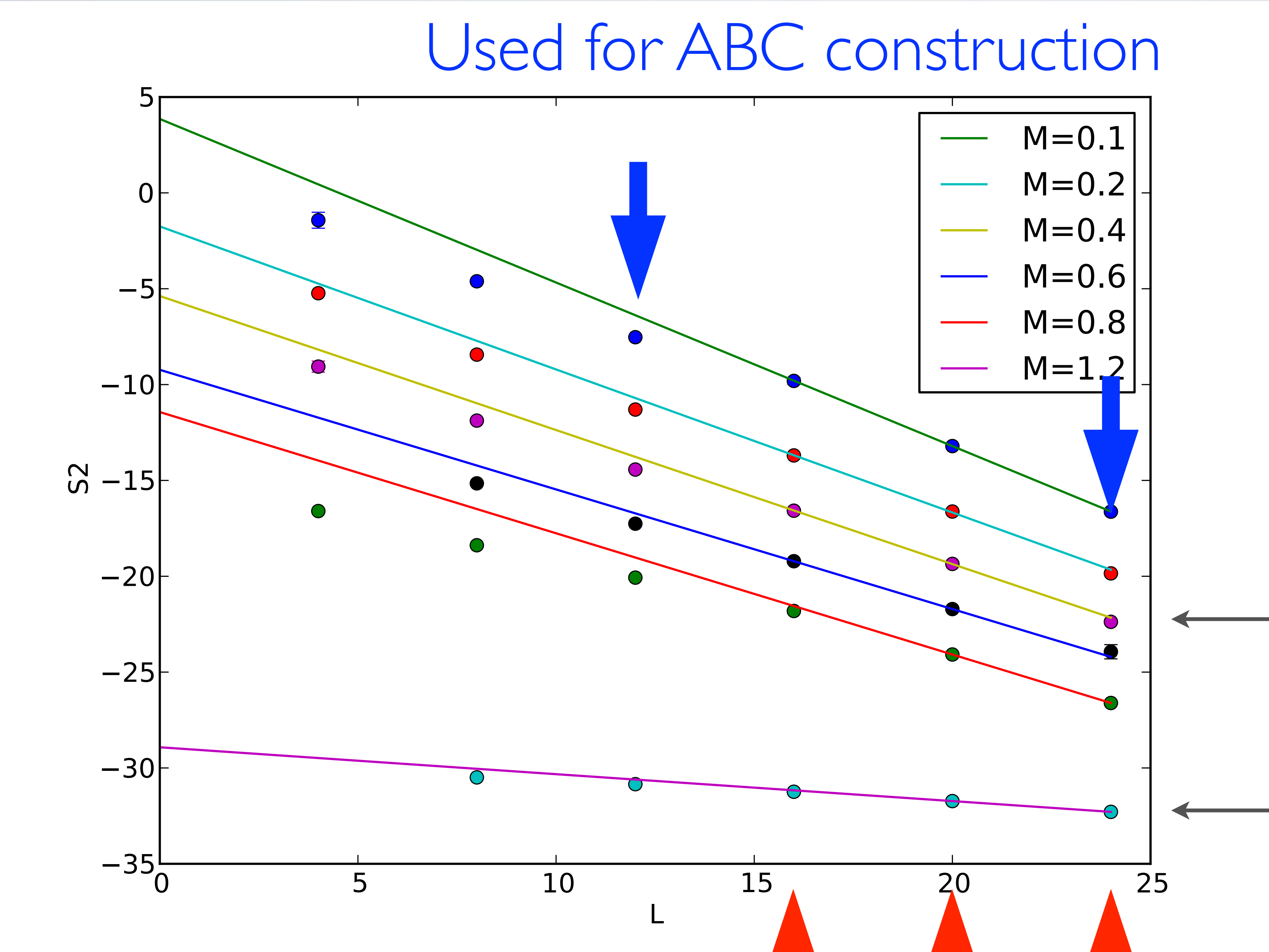
0.941 +/- 0.027

≈ 0.989

1.7 standard deviations

Let's focus on one model.





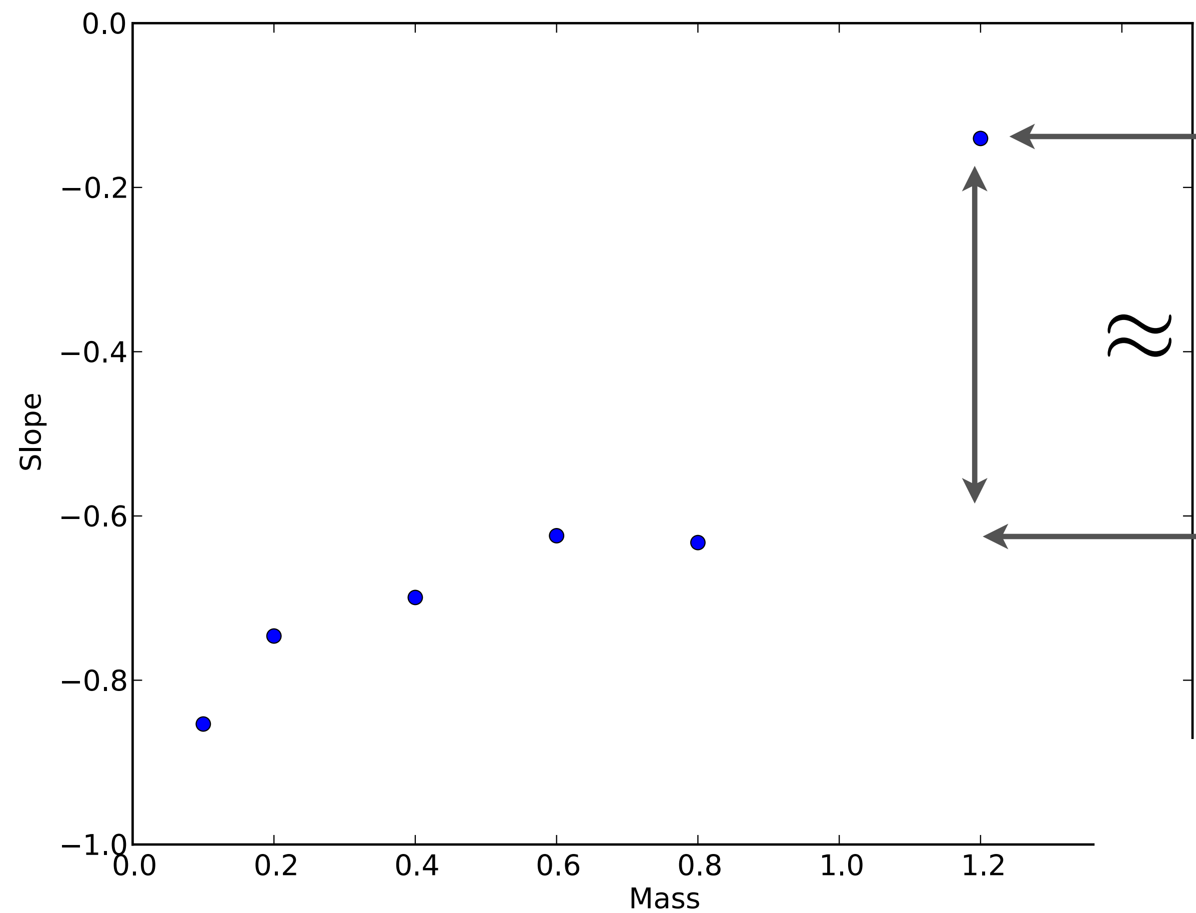
C=3

C=I

Why?

Area Law

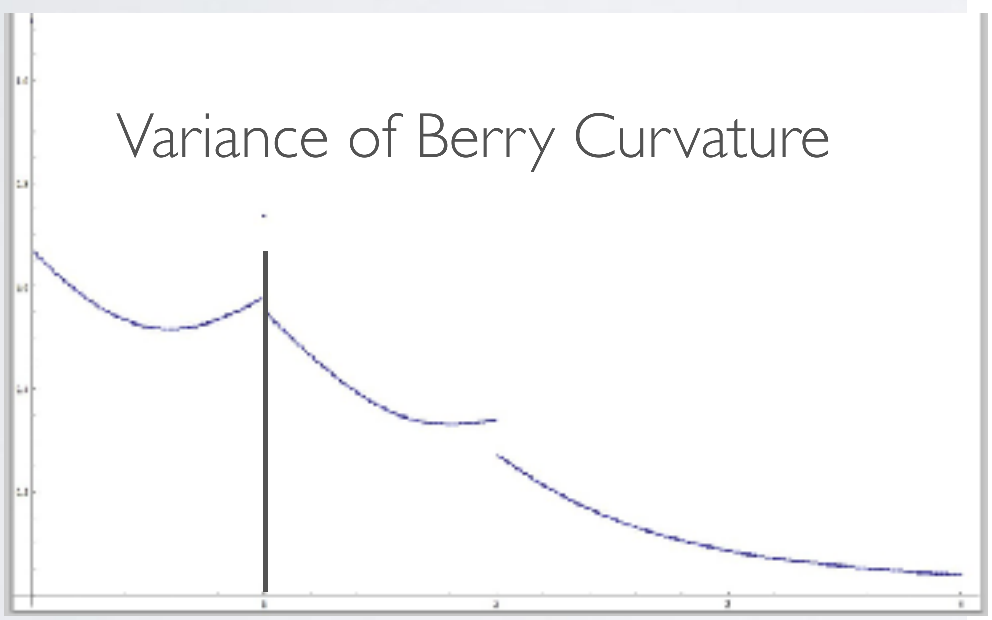
ABC construction must be canceling out higher order in L terms.



C=1

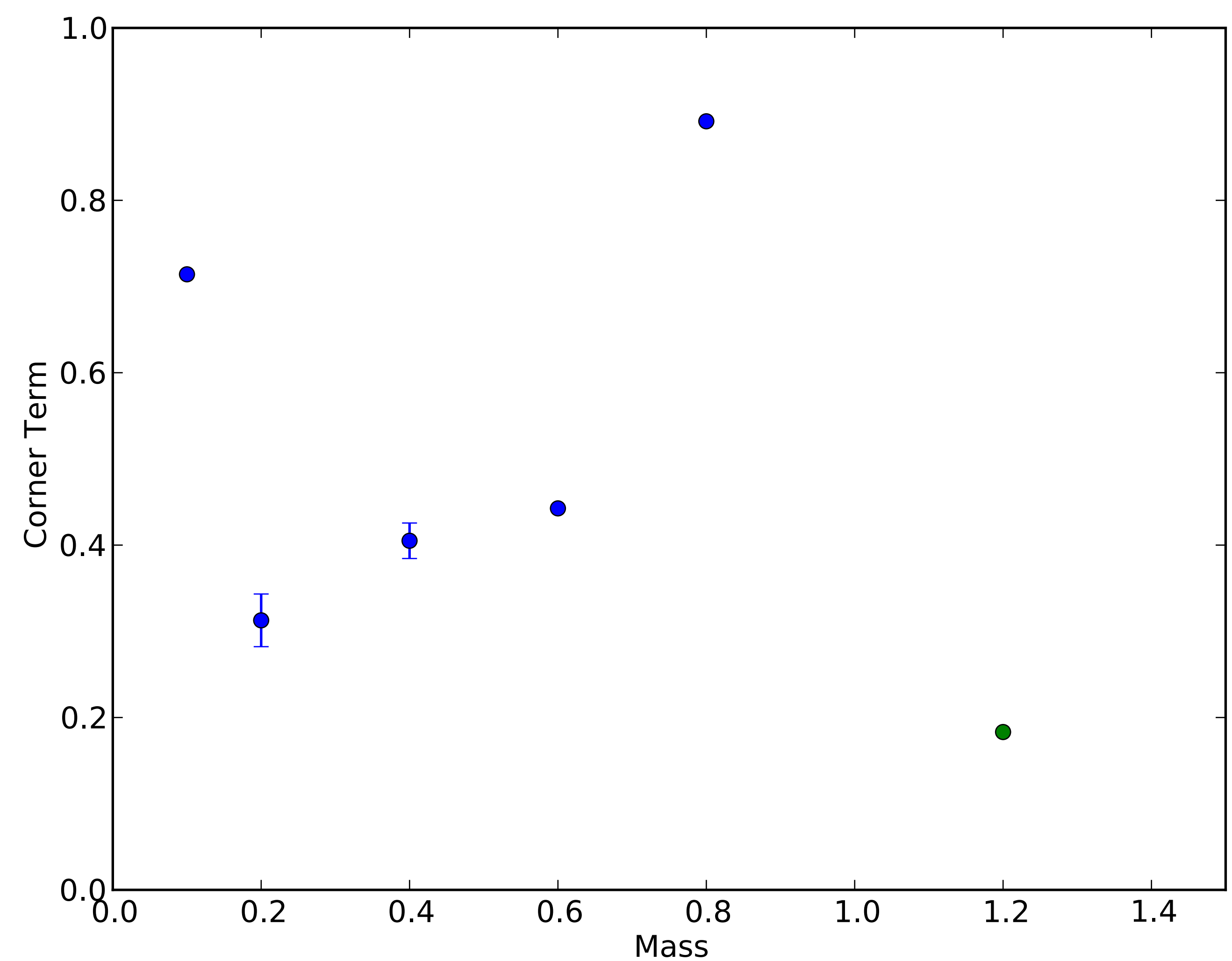
$\approx$ factor of 3 for non-universal α

C=3

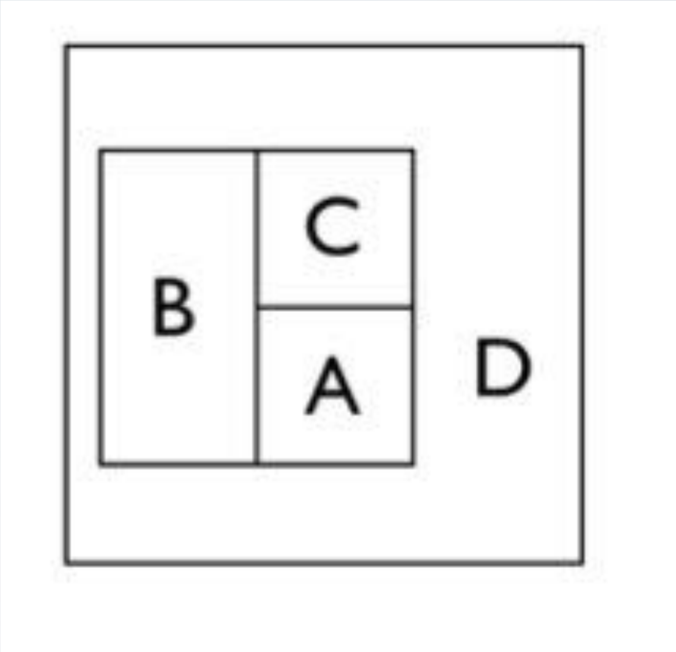


Mass

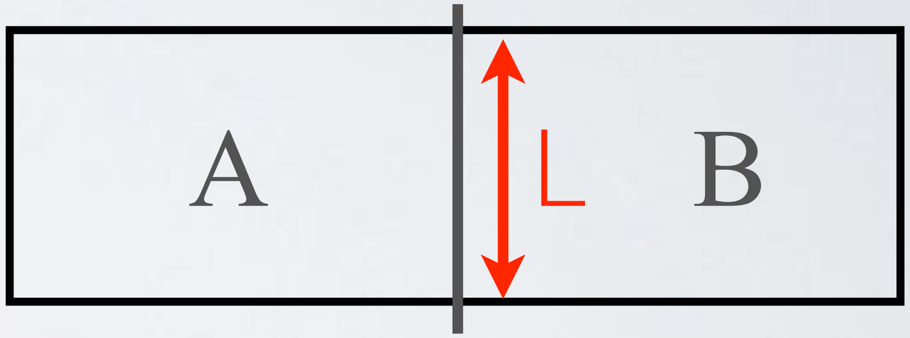
Variance of Berry curvature tracks corner term?



ABC Construction: Topologically insensitive to ground state.



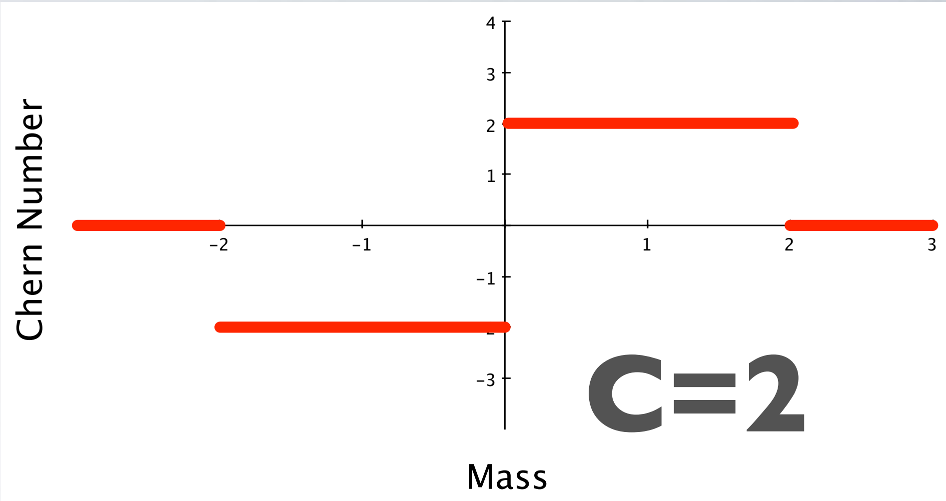
Trivial Bipartition: Sensitive to which ground state but no corner terms.



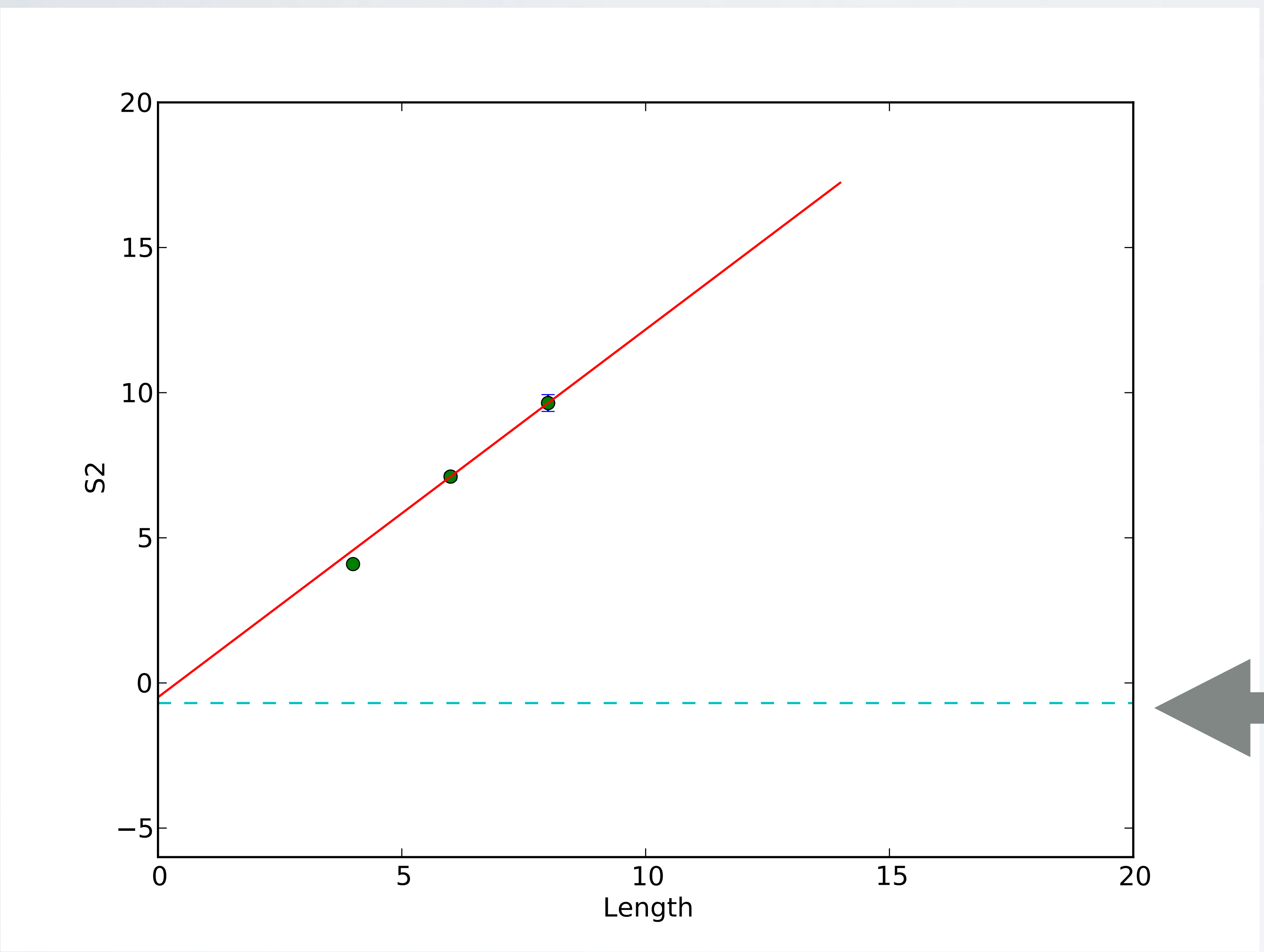
$$S(L) = \alpha L - \gamma_0 + ...$$

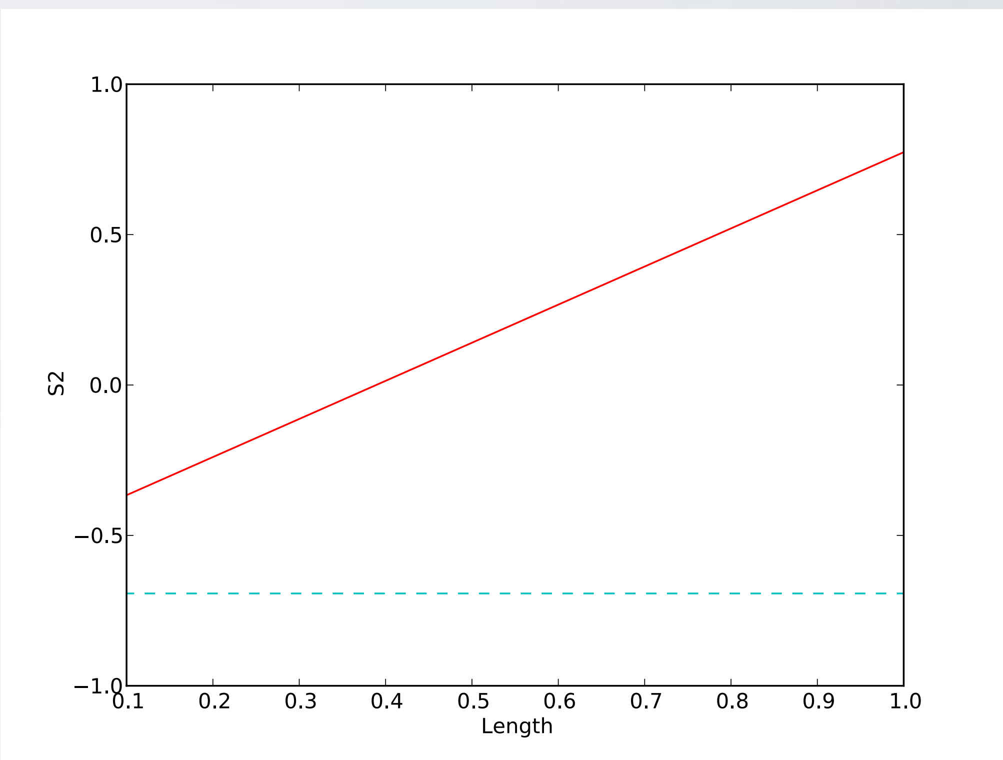
could be between 0 and TEE.

A bipartition cut



$$h(k) = \sin(k_x + k_y)\sigma_x + \sin(k_x - k_y)\sigma_y + (M - \cos(k_x + k_y) - \cos(k_x - k_y))\sigma_z$$

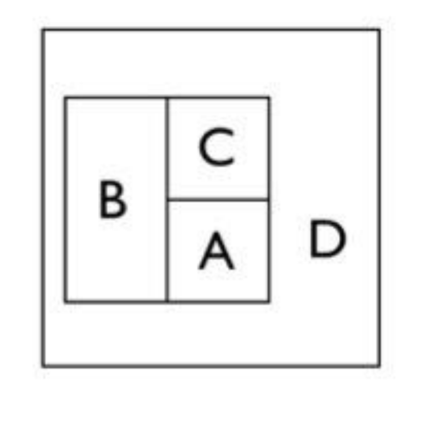


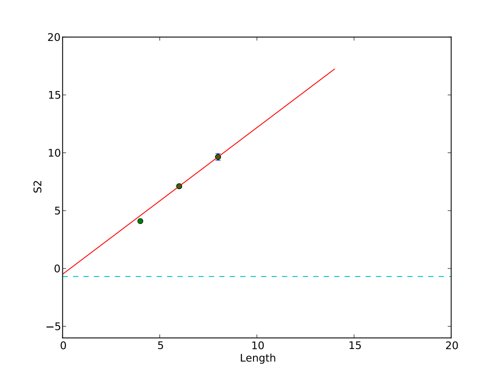


expected value

Conclusion

* Compute the TEE





* Low energy effective field theory: $SU(2)_c$

* Look at area law, corner terms, etc.