THE MOTHER OF ALL STATES ON THE KAGOME QUANTUM ANTIFERROMAGNET

University of Illinois at Urbana Champaign
with Hitesh Changlani, Dmitrii Kochkov, Krishna Kumar, Eduardo Fradkin

ICM

Other talks I didn't give that I'm happy to chat about.

An inverse approach to strongly correlated systems

Many-Body Localization and Holography

Beyond many-body localization: eigenstates with logarithmic entanglement.

Finite Temperature Variational Monte Carlo

Wave-functions from Deep Neural Nets (without the hype)

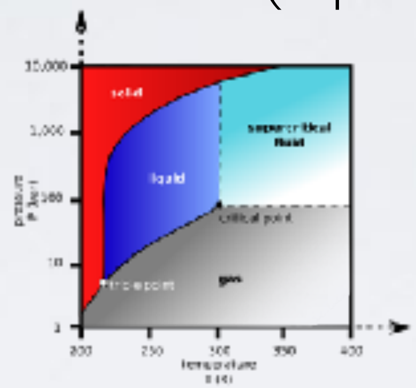
What we found?

Hamiltonian (i.e. matrix) with exponentially many ground states.

Why it's interesting?

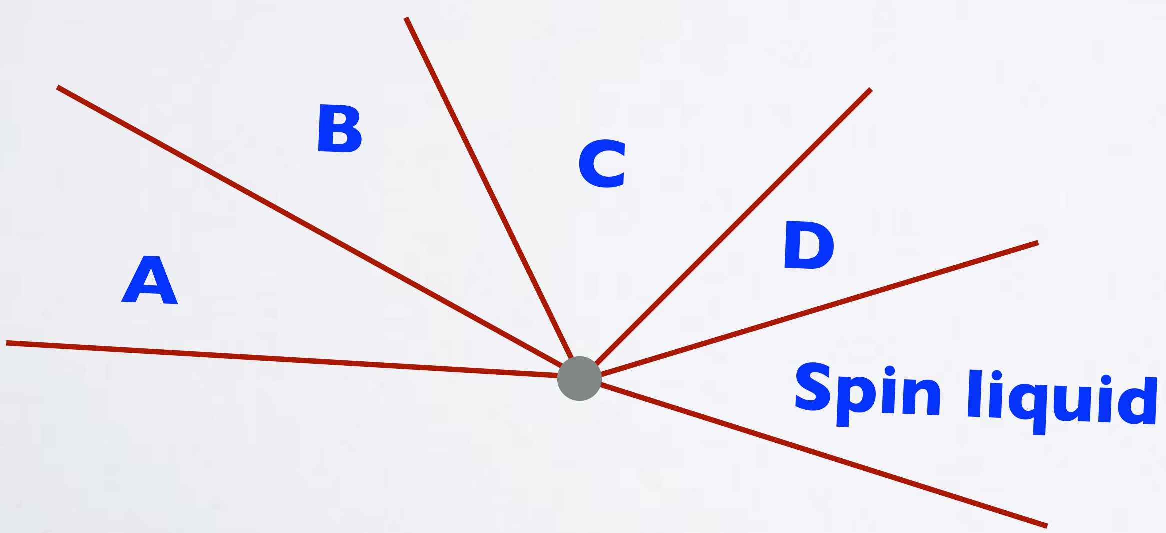
Each ground state represents a phase of matter (liquid, solid, gas, anti-ferromagnet, etc)

Physicists like to make phase diagrams



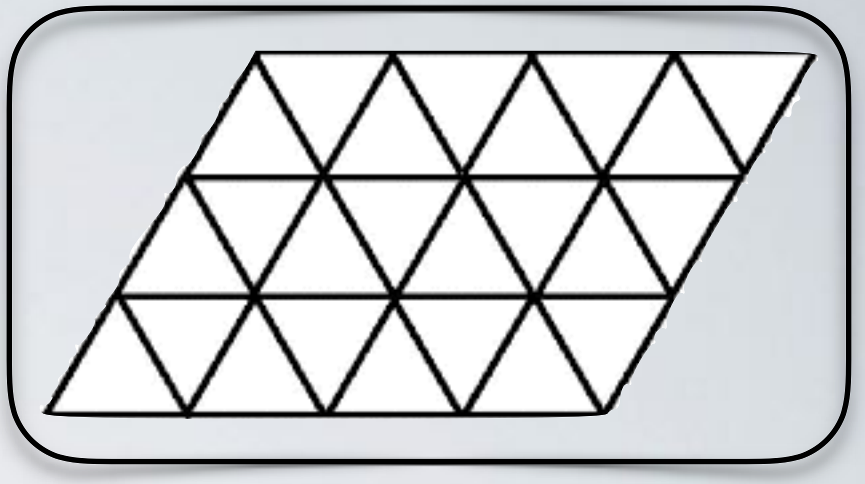
Exponential ground state means some parameters where an exponential number of phases meet.

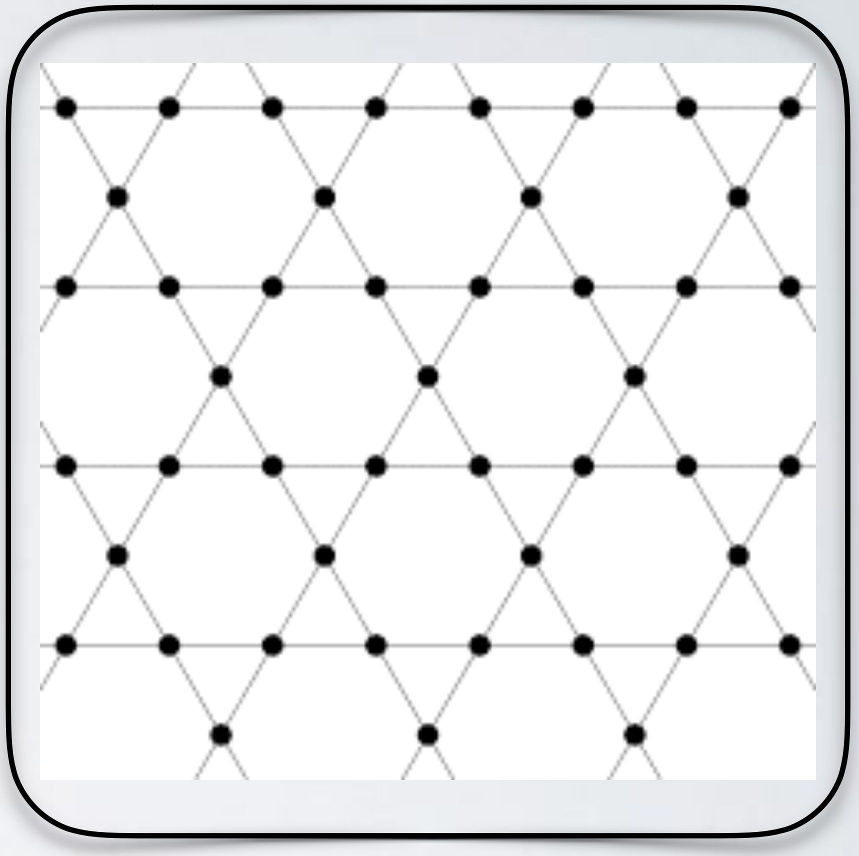
This means that it sources all interesting phases on a class of materials.



Including a particularly interesting (and useful for quantum computing) phase: a *spin-liquid.*

Frustrated Quantum Magnets.....





Insulators with spin 1/2 on lattices of pasted triangles.

Kagome quantum materials.

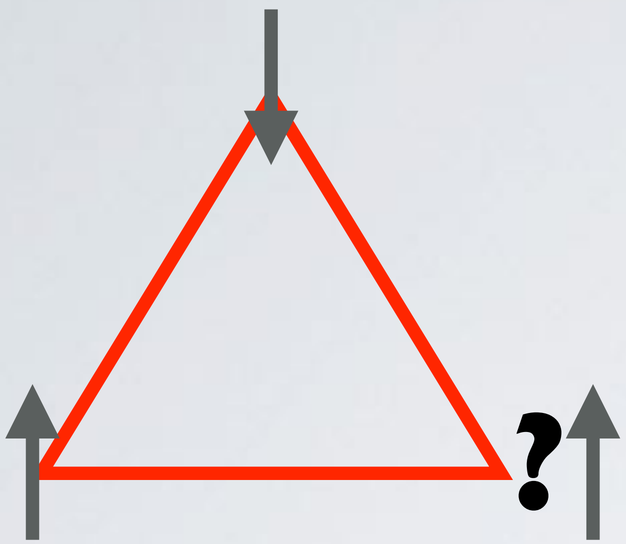
Herbertsmithite

Vesigniette

Kapellasite

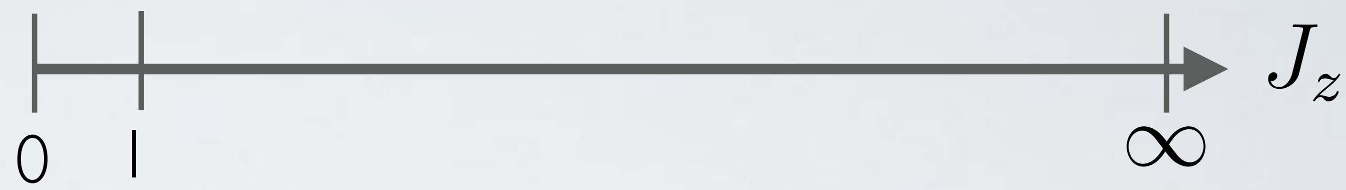
Volborthite

The story of frustration.

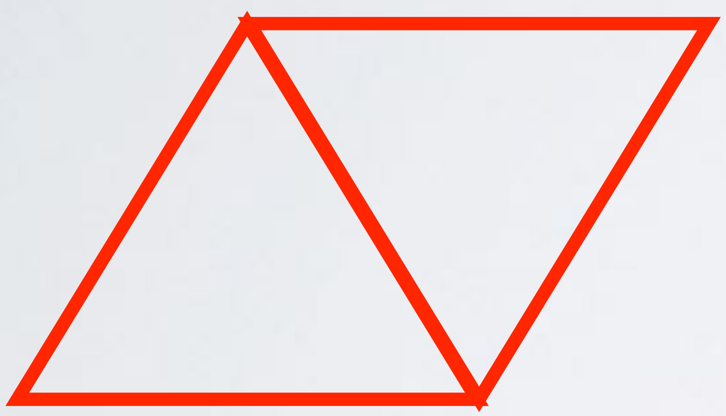


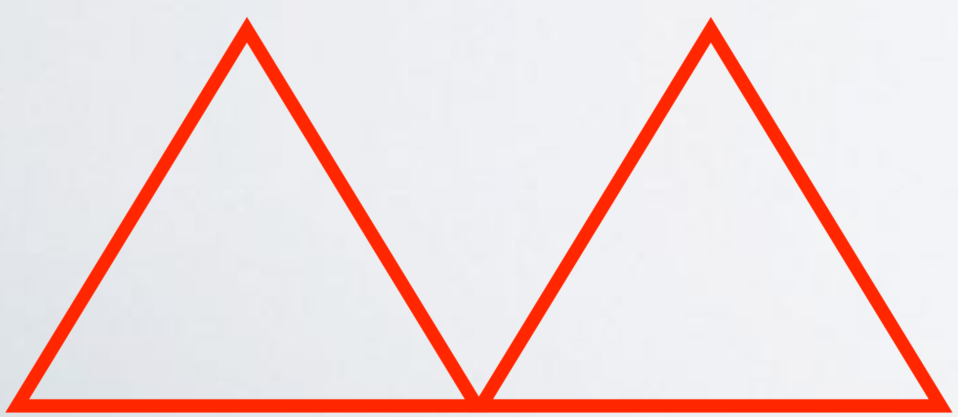
$$H_{xxz} = H_{xy} + J_z \sum_{ij} S_i^z S_j^z \quad \text{(ising limit } J_z \to \infty\text{)}$$

$$H_{\text{ising}} = \sum_{ij} S_i^z S_j^z$$



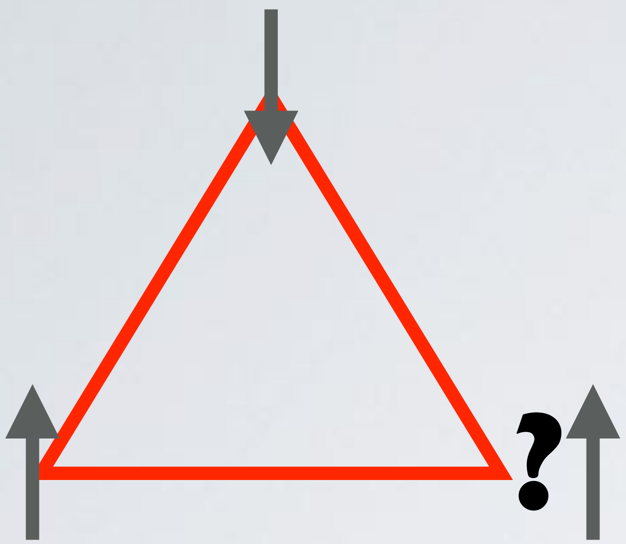
In this Ising limit, however you paste together triangles, there are many degenerate states





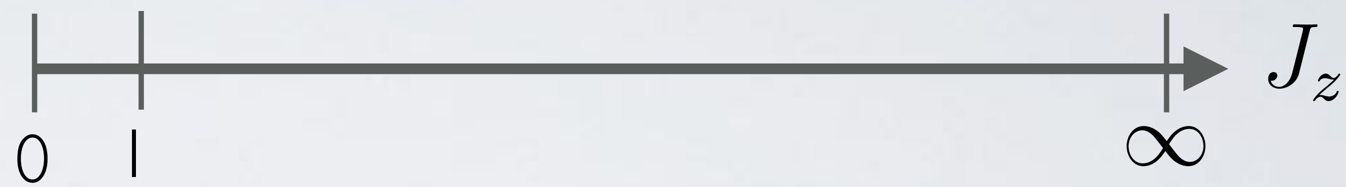
Frustrated magnets!

The *old* story of frustration.

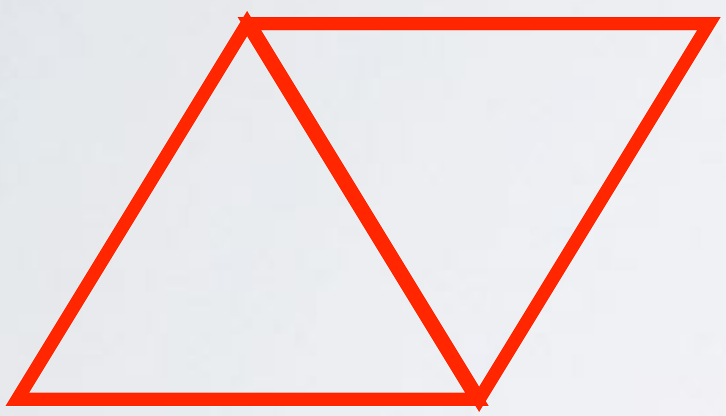


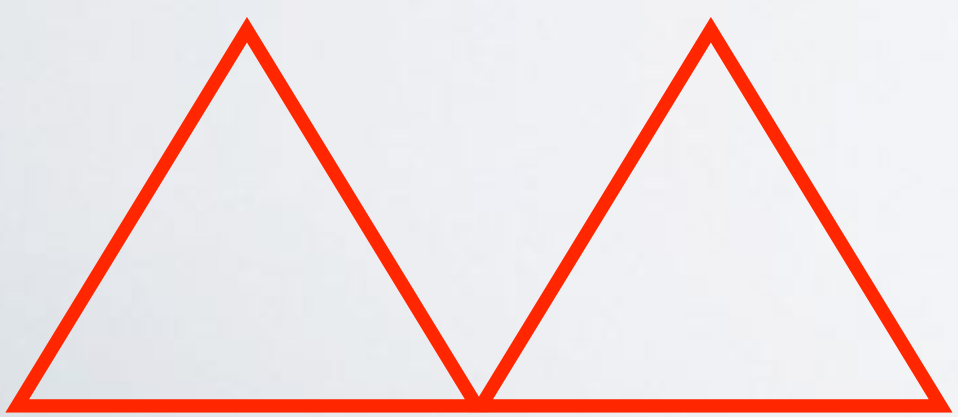
$$H_{xxz} = H_{xy} + J_z \sum_{ij} S_i^z S_j^z \quad \text{(ising limit } J_z \to \infty\text{)}$$

$$H_{\text{ising}} = \sum_{ij} S_i^z S_j^z$$



In this Ising limit, however you paste together triangles, there are many degenerate states





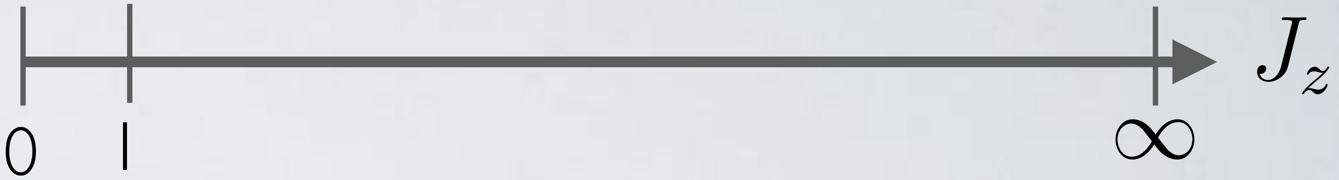
Frustrated magnets!

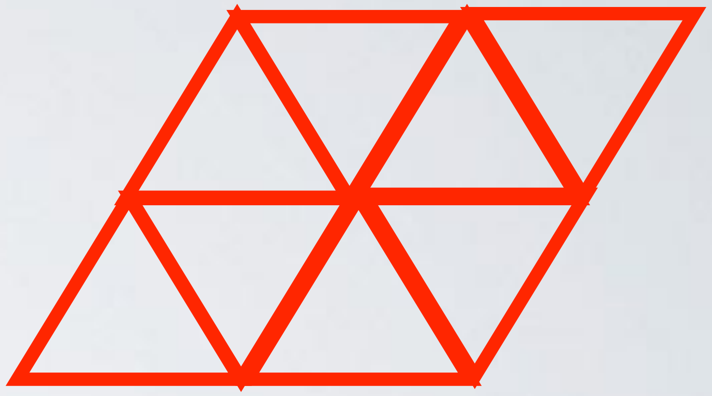
Phil Anderson suggested that the n.n. Heisenberg model on the triangular lattice wasn't a neel state (frustration!)

RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR?*

P. W. Anderson
Bell Laboratories, Murray Hill, New Jersey 07974
and
Cavendish Laboratory, Cambridge, England

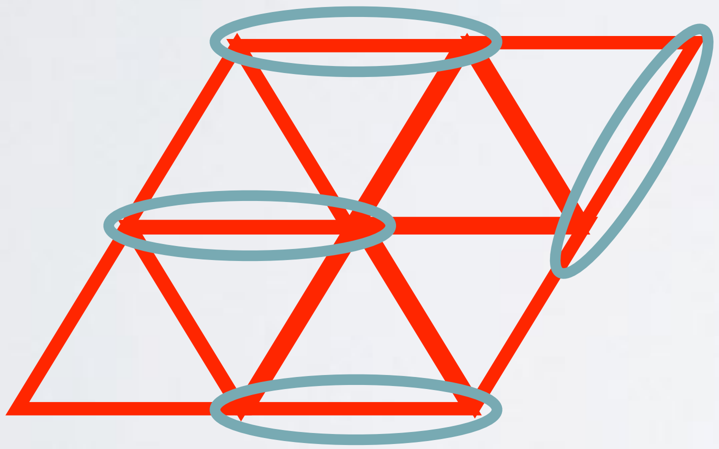
(Received December 5, 1972; Invited**)





instead, he suggested it was a RVB state.
(today we would call such a thing a spin-liquid).

$$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

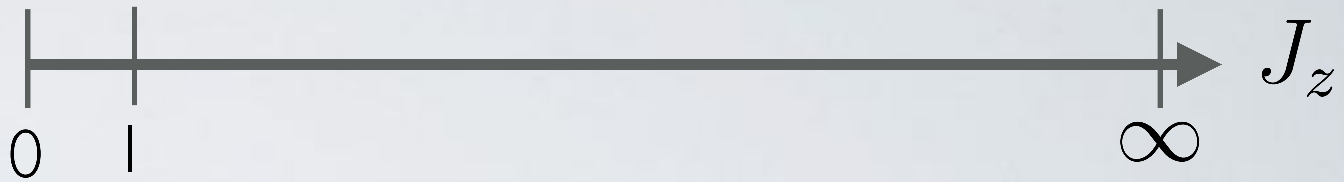


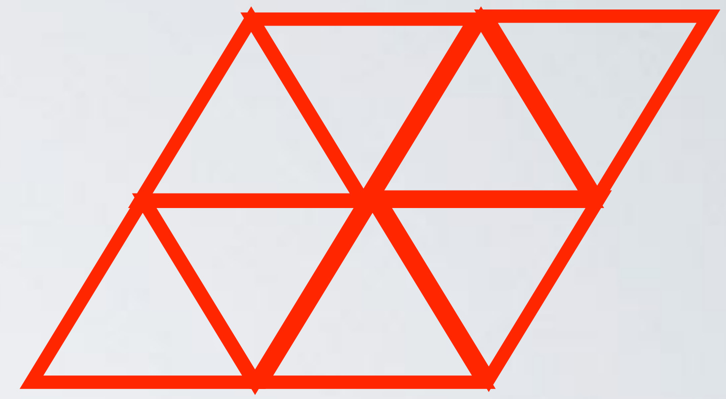
Phil Anderson suggested that the n.n. Heisenberg model on the triangular lattice wasn't a neel state (frustration!)

RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR ?*

P. W. Anderson
Bell Laboratories, Murray Hill, New Jersey 07974
and
Cavendish Laboratory, Cambridge, England

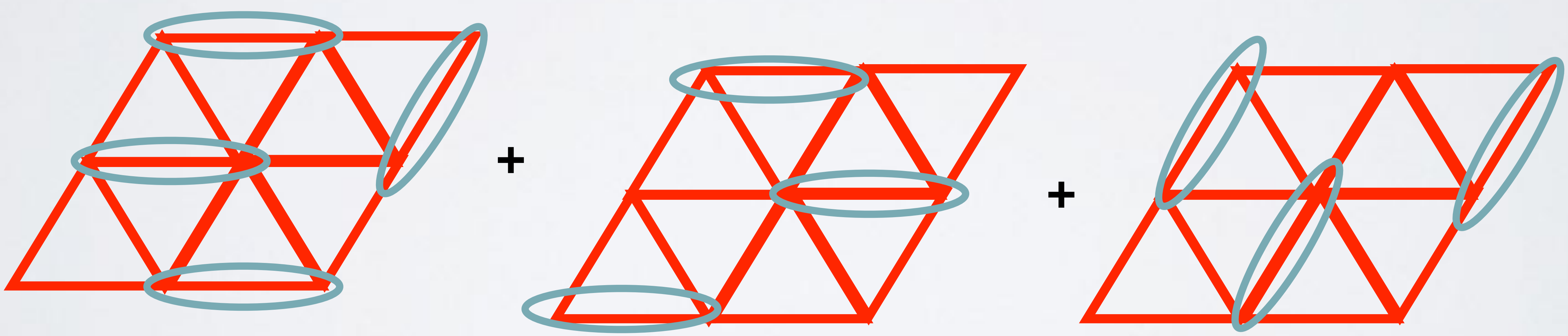
(Received December 5, 1972; Invited**)



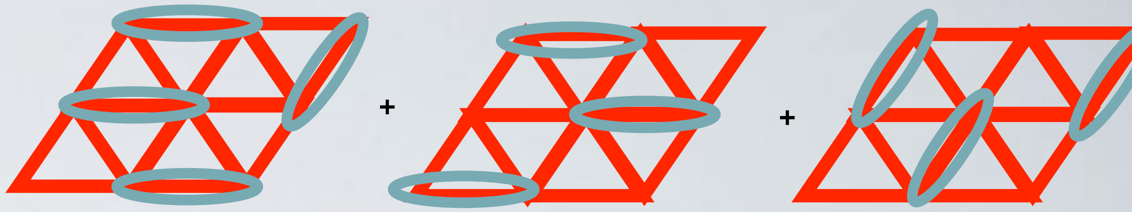


instead, he suggested it was a RVB state.
(today we would call such a thing a spin-liquid).

$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ 

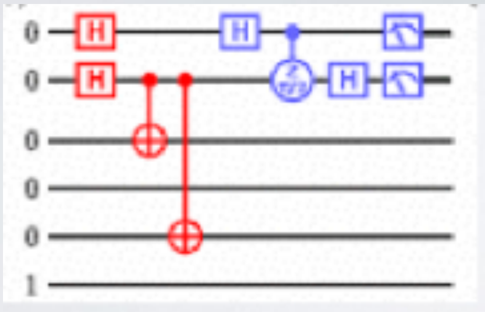


The hunt for **spin liquids** is one of the forefront areas of condensed matter research!

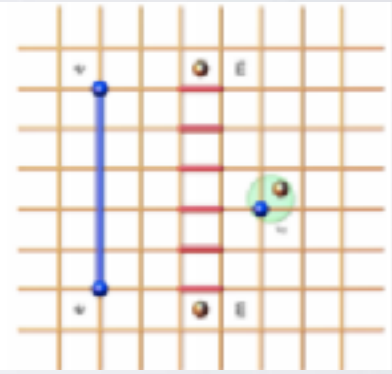


Beyond the Landau theory of phases - no broken symmetries!

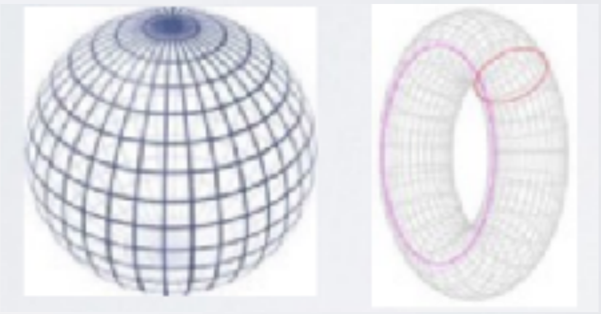
Long Range Entanglement - Can't be produced from a product state via a short quantum circuit



Fractionalized Excitations - Electron breaks into multiple emergent pieces.



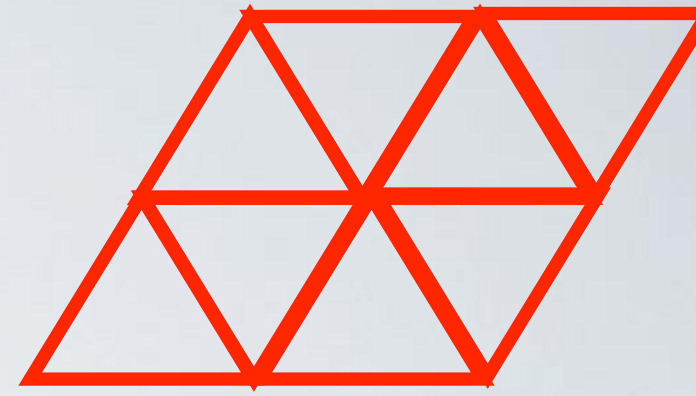
Topological Degeneracy - Manifold dependent geometry



The search for spin liquids is truly a hunt. We haven't had any good story for what sort of lattices should support spin liquids.

Was Phil Anderson right? Was the triangular lattice at the Heisenberg point a spin-liquid?

No!

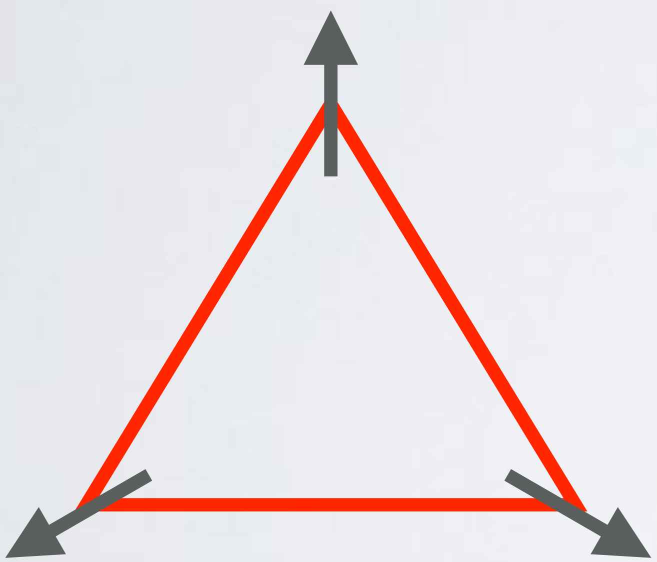


How do we know?

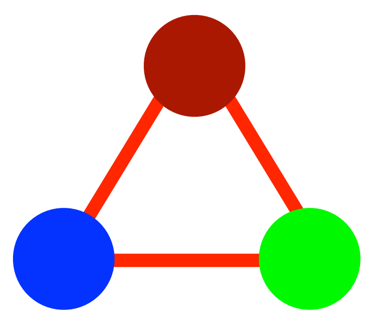
Years and years of numerics…

- Huse-Elser
- Green's Function Monte Carlo

The triangular lattice is 120 degree ordered.
No hint from the frustrated Ising limit.



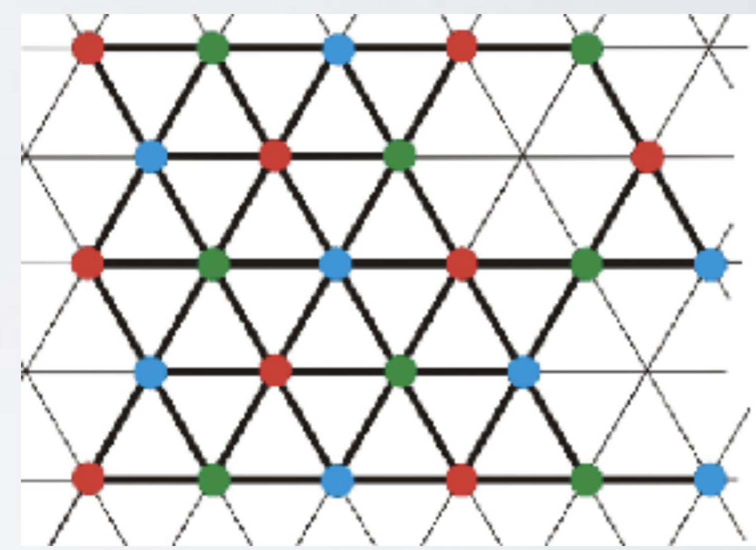
Define
3 "colors"



$$|a\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

$$|b\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + \omega|\downarrow\rangle\right)$$

$$|c\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + \omega^2|\downarrow\rangle\right)$$



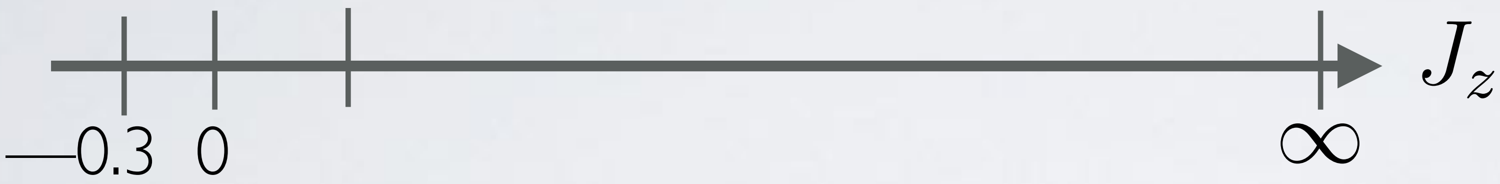
Morally, but not actually the wave-function. The actual wave-function is a highly complicated dressed version of this (that's why it took forever to verify this).

Why? Because numerics says so….

How about if we have negative Jz?

$$H_{xxz} = H_{xy} + J_z \sum_{ij} S_i^z S_j^z \quad \text{(ising limit } J_z \to \infty\text{)}$$

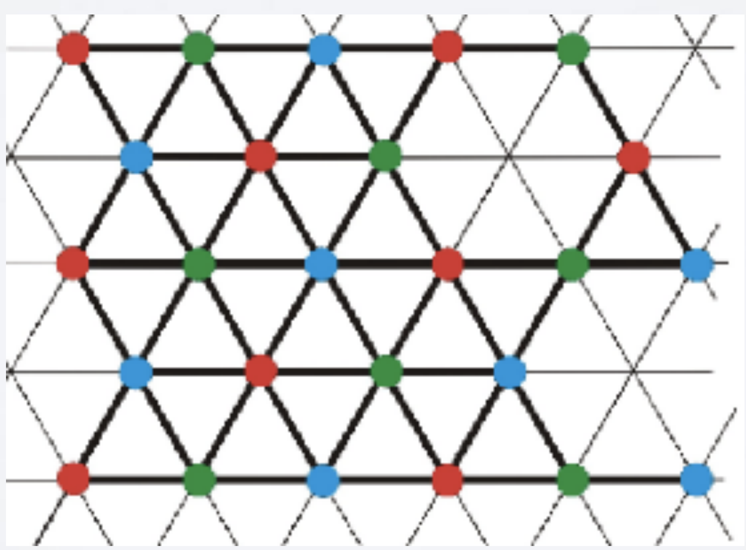
$$H_{\text{ising}} = \sum_{ij} S_i^z S_j^z$$



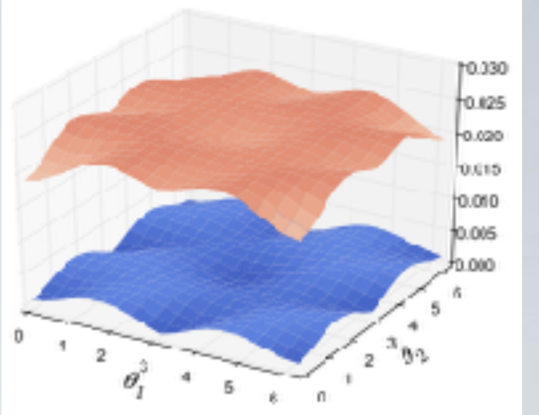
Wouldn't expect a spin-liquid.
But at least Ising-like?

Nope...

coplanar



Kagome spin liquids everywhere....



Z2 (or Dirac) Spin Liquid — Heisenberg (White/Huse)

Chiral Spin Liquid — 2/3 Plateau (this work)

1/3 Plateau (Donna Sheng)

Chiral Term (Bela Bauer, Andreas Ludwig)

J1, J2, J3 (Donna Sheng)

Why? Because numerics says so….

Experimental evidence for spin-liquids in hyperkagome

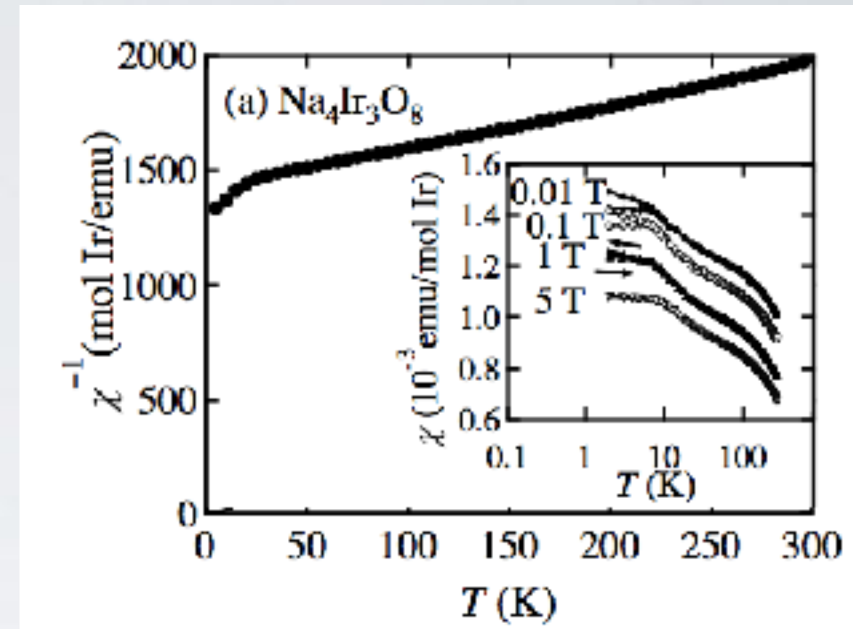
$Na_4Ir_3O_8$

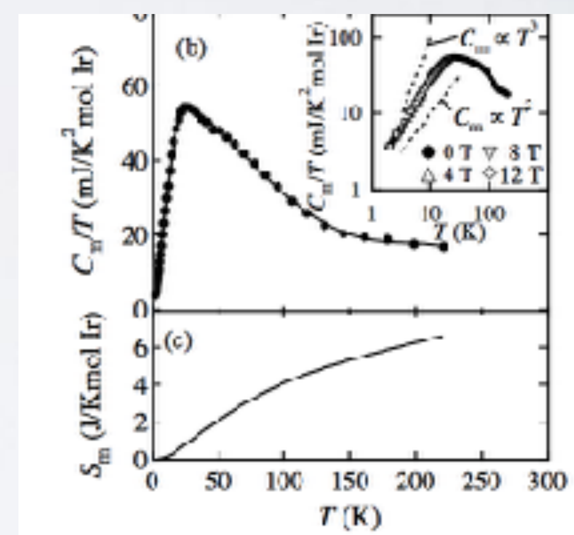
(depleted pyrochlore)

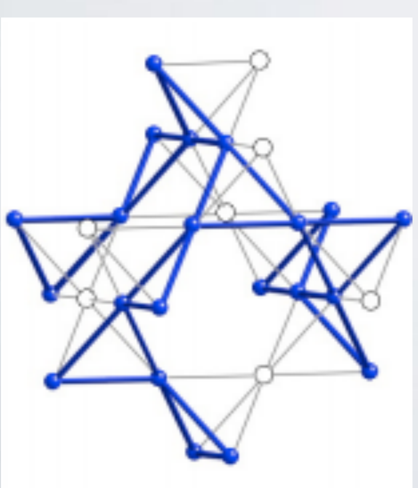
No sign of magnetic ordering down to a few Kelvin

Curie-Weiss temperature of 650K

Gapless excitations







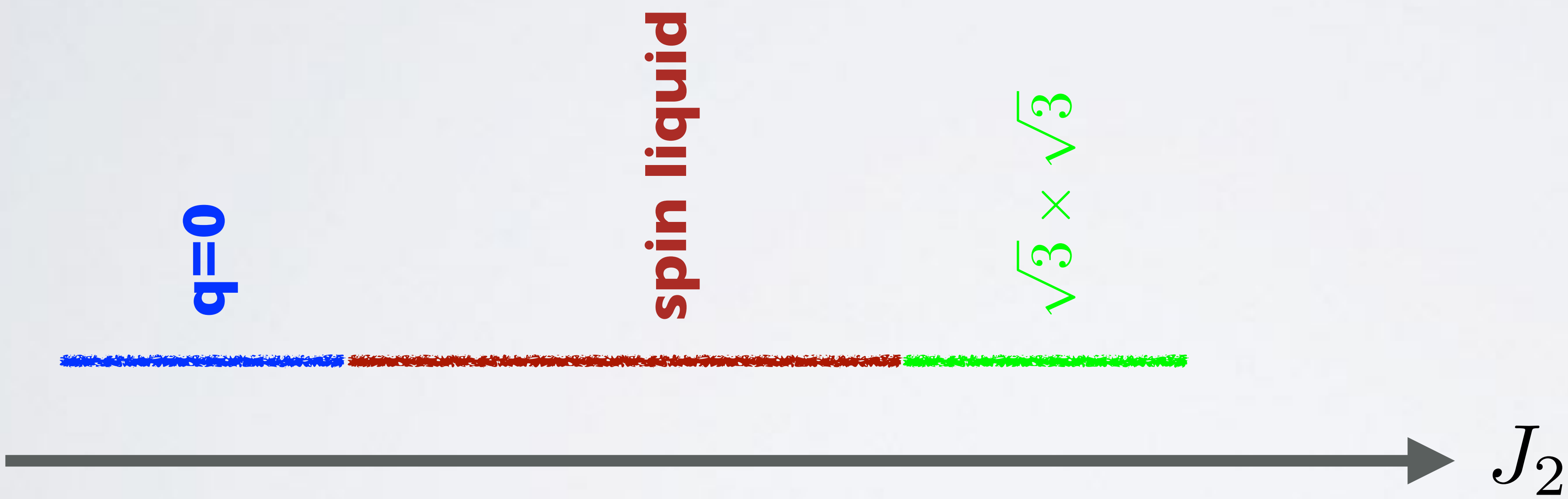
Y. Okamoto, M. Nohara, H. Aruga-Katori, and H. Takagi (2006), arXiv:0705.2821

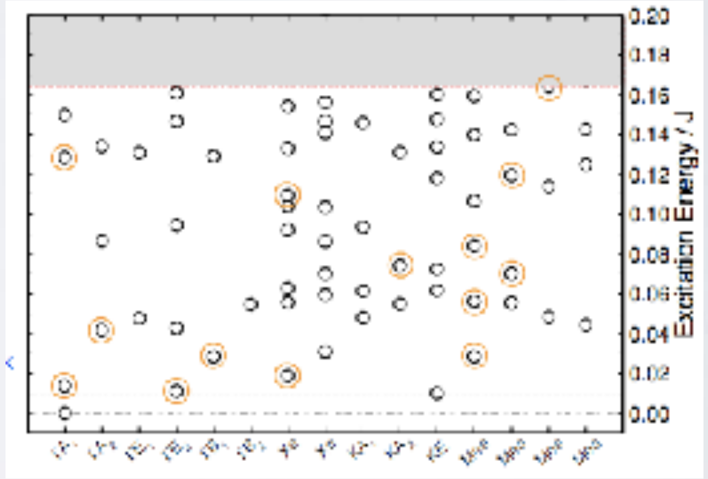
The Ising frustration doesn't seem to be a good explanation for anything.

(1) Why hyperkagome and kagome and not triangular?

Both are equally frustrated in the ising limit.

(2) Ising seems to have little to do with the coplanar phases

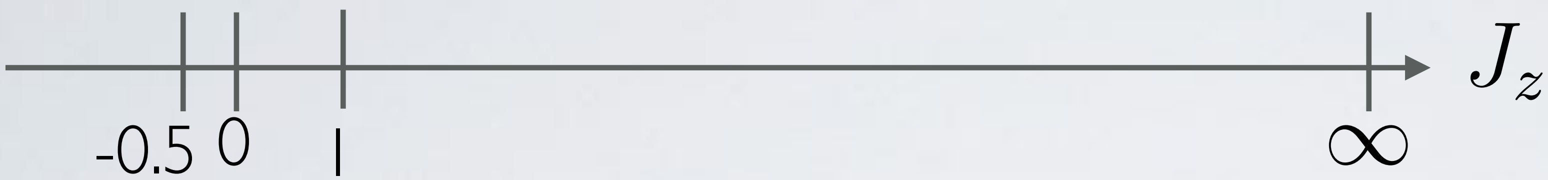




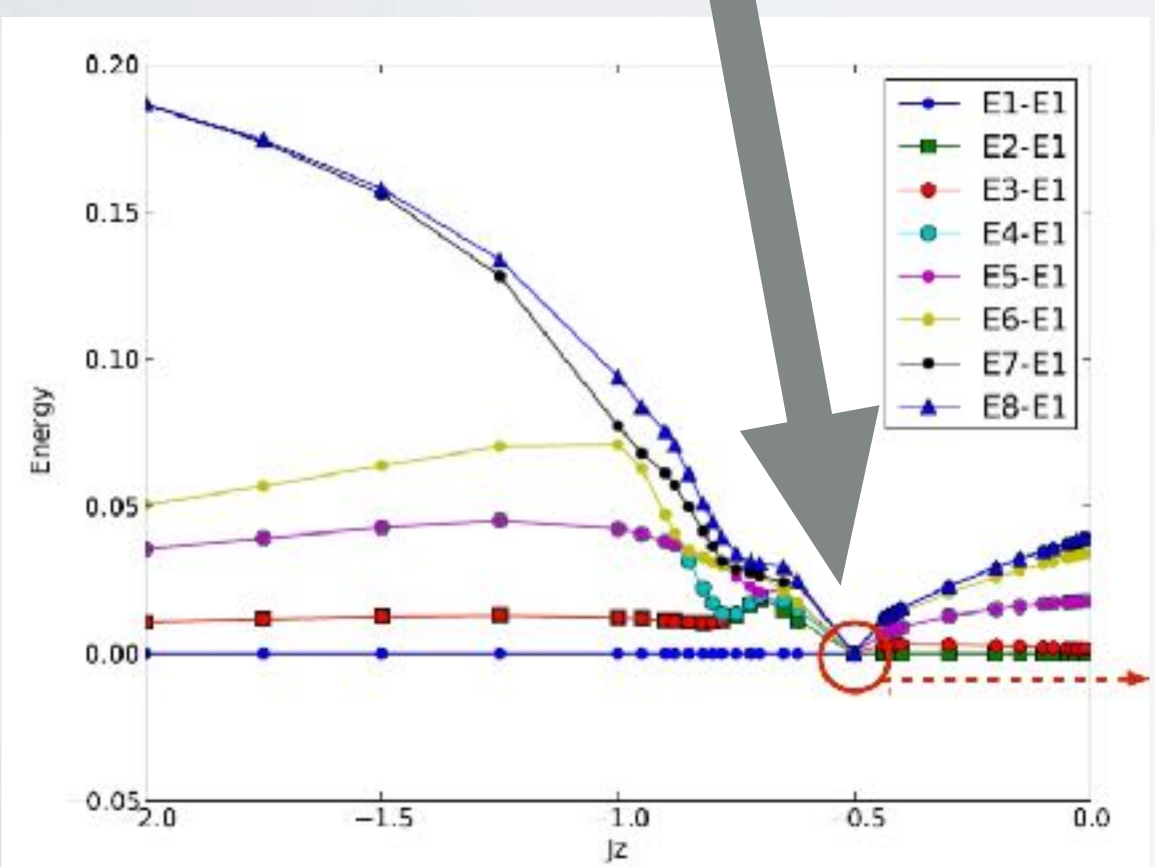
(3) Why does kagome have so many real phases and so many competing low-energy states?

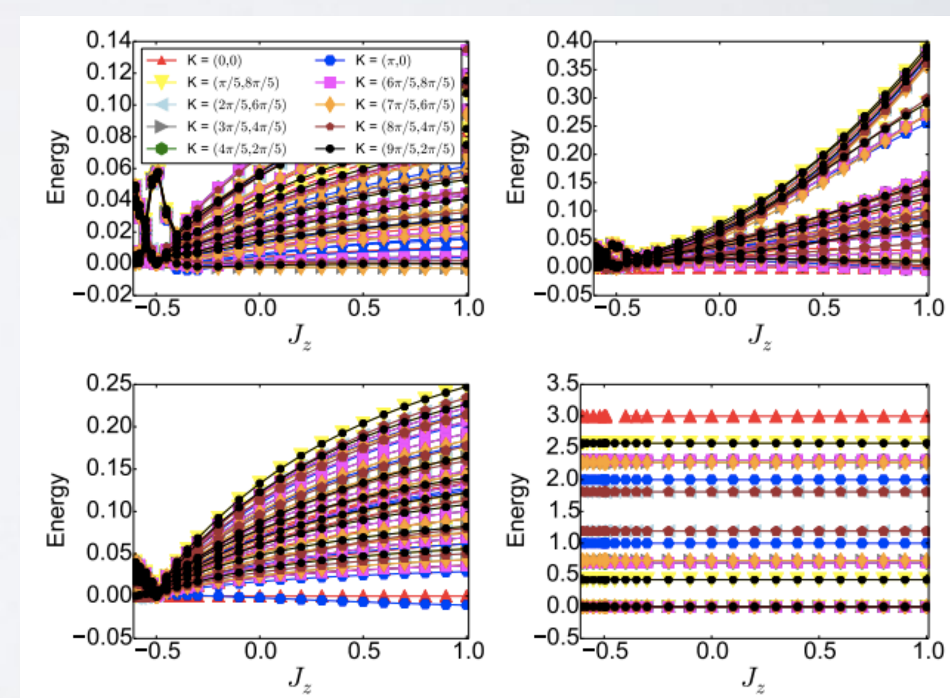
An interesting discovery…. (amazing it hasn't been known for 30 years)

$$H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$



On the kagome:
massive exact degeneracy in the XXZ model!
exactly -J/4

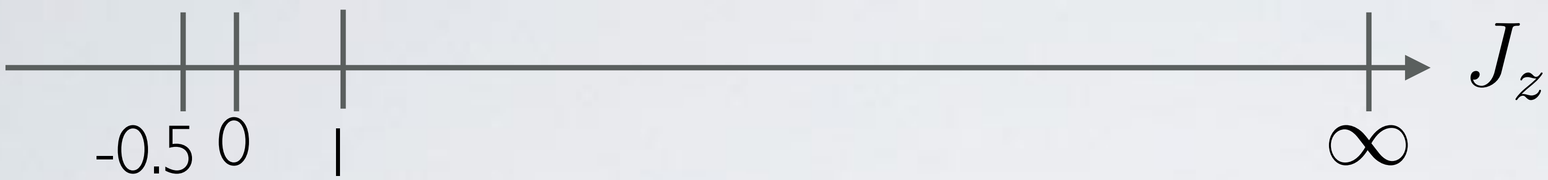




What's going on? Who ordered this?

An interesting discovery.... (amazing it hasn't been known for 30 years)

$$H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$

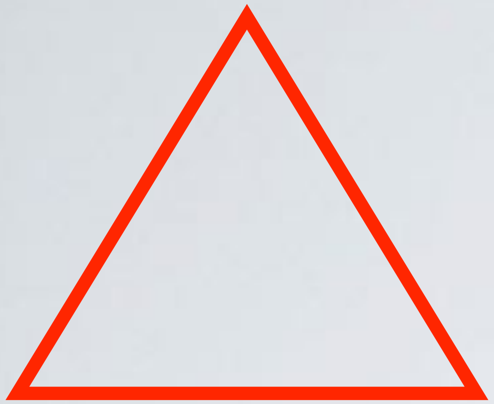


On the triangle:
no massive exact degeneracy.
exactly -J/2

What's going on? Who ordered this?

Who ordered that?

$$H_{XXZ0} = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$

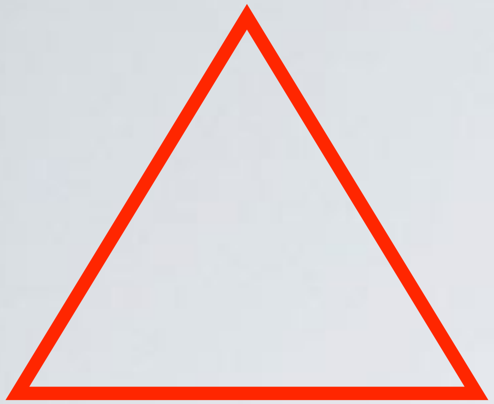


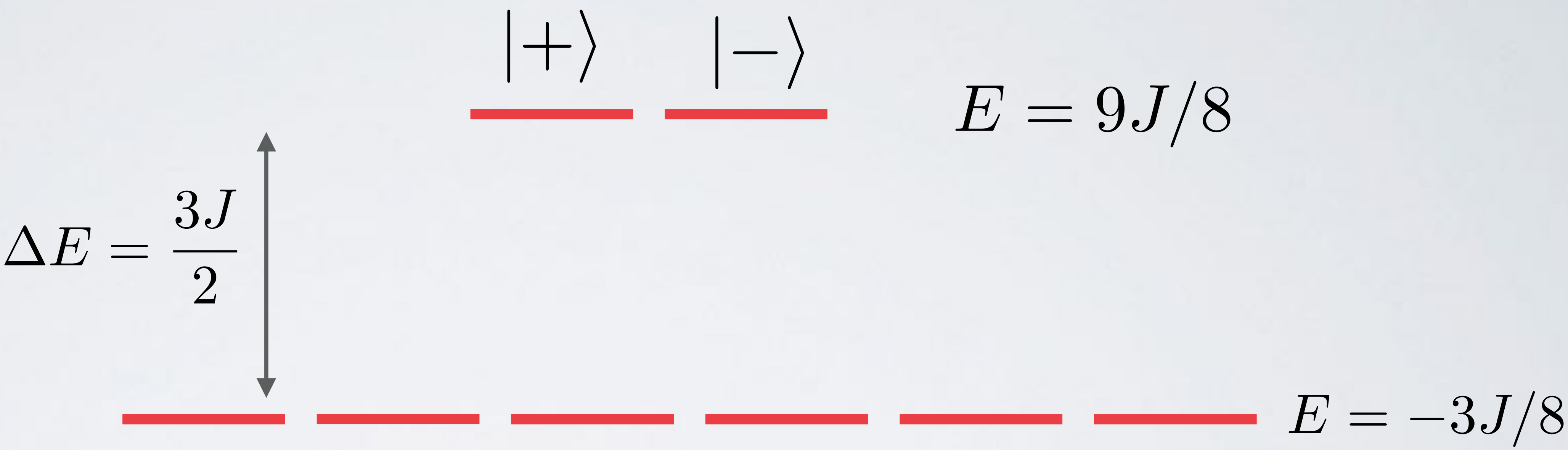
$$E = 9J/8$$

$$E = -3J/8$$

Who ordered that?

$$H_{XXZ0} = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$





$$-\frac{3J}{8} + \frac{3J}{2}(|+\rangle\langle+| + |-\rangle\langle-|)$$

Projectors

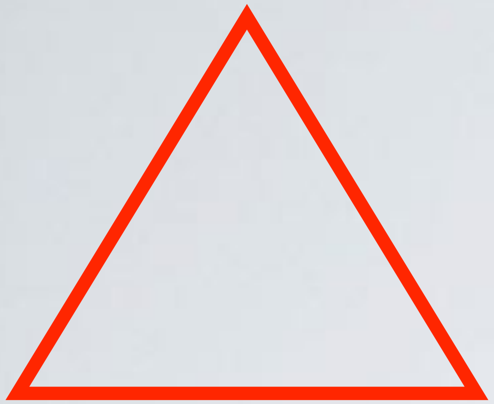
Constant

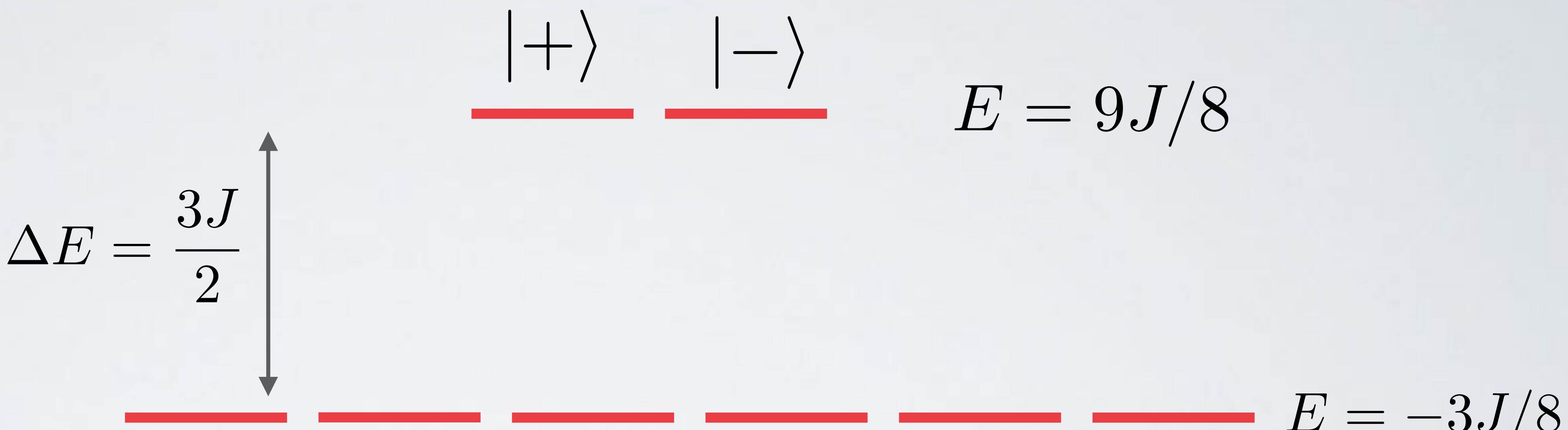
Positive coefficient

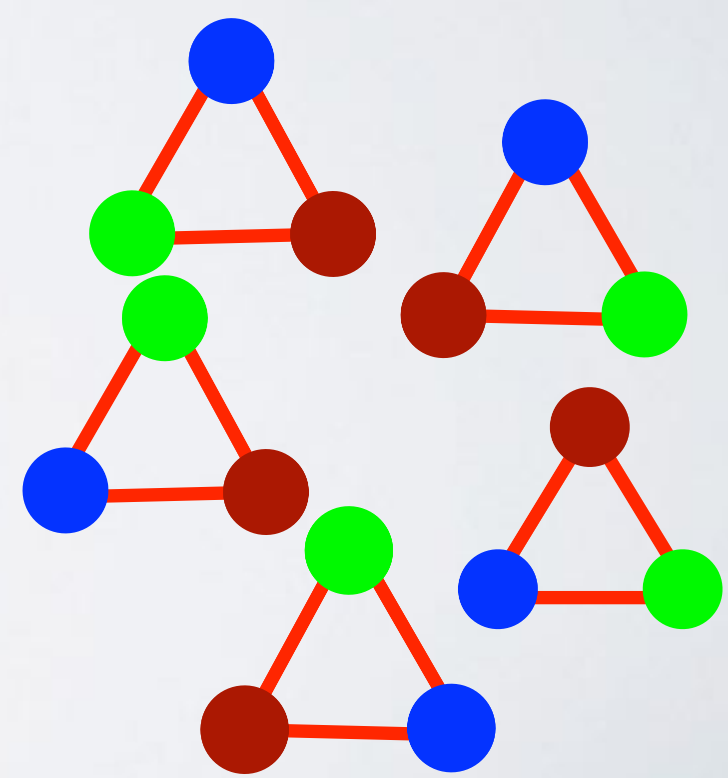
We want to **minimize the energy** by zeroing out the projectors

Who ordered that?

$$H_{XXZ0} = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$







$$-\frac{3J}{8} + \frac{3J}{2}\left(|+\rangle\langle+| + |-\rangle\langle-|\right)$$

Projectors

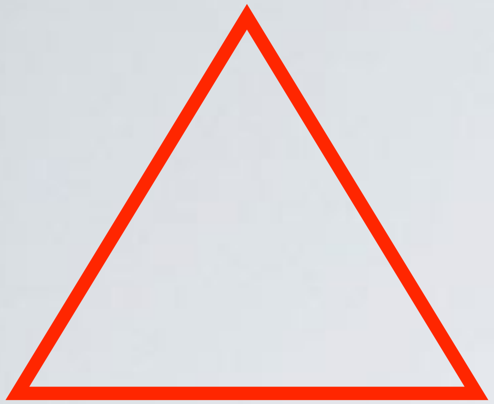
Constant

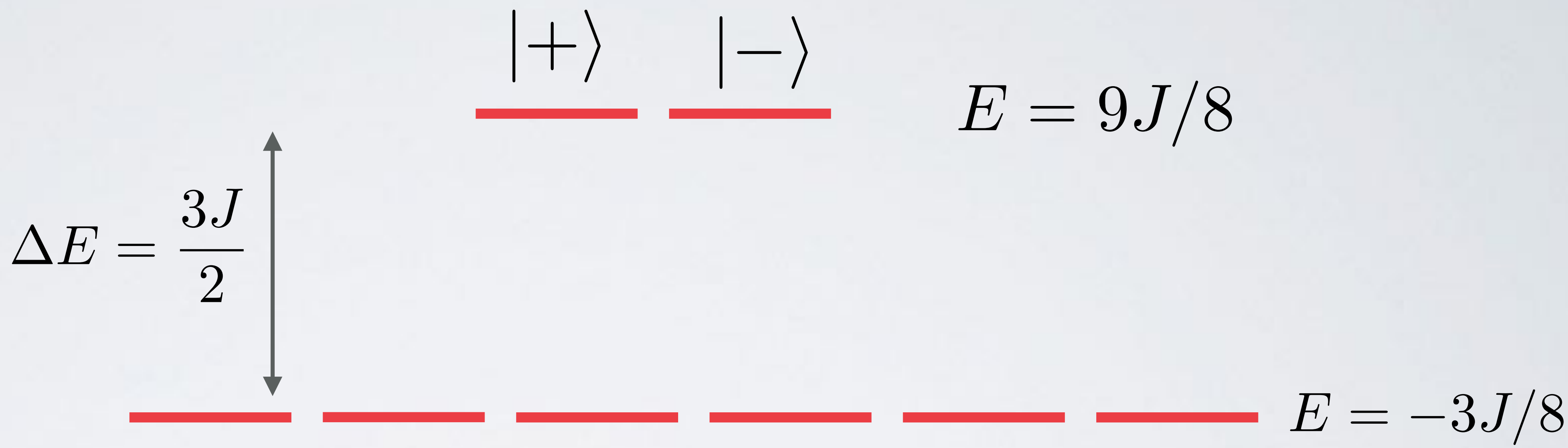
Positive coefficient

We want to **minimize the energy** by zeroing out the projectors

Who ordered that?

$$H_{XXZ0} = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$





$$-\frac{3J}{8} + \frac{3J}{2}\left(|+\rangle\langle+| + |-\rangle\langle-|\right)$$

Projectors

Constant

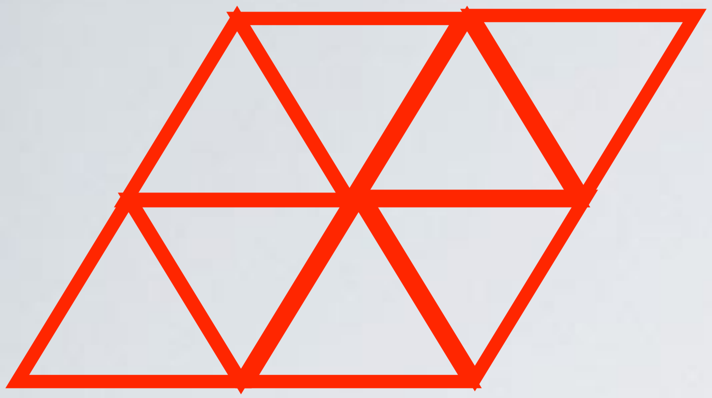
Positive coefficient

We want to **minimize the energy** by zeroing out the projectors

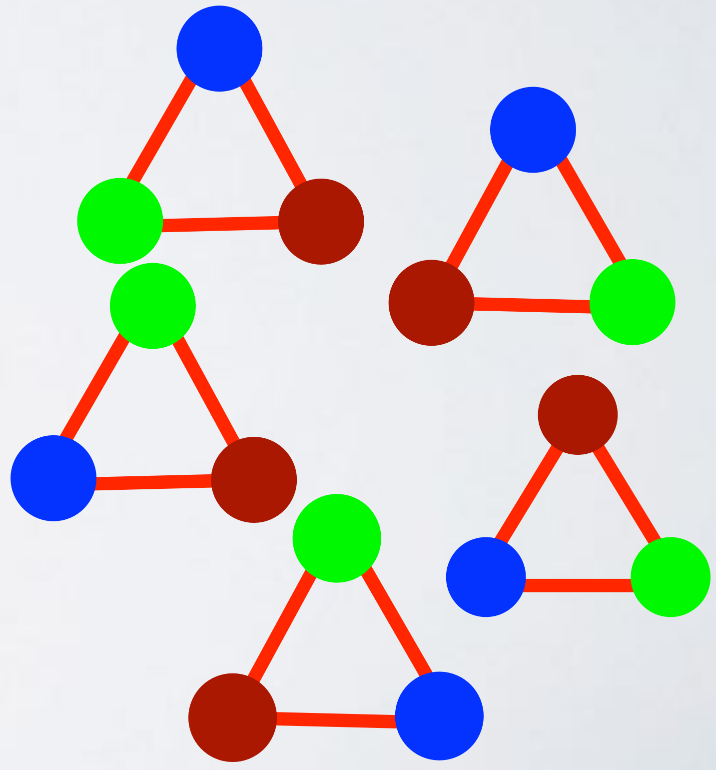
$$|1\rangle \equiv |\uparrow\uparrow\uparrow\rangle$$
$$|2\rangle \equiv \frac{1}{\sqrt{3}}\left(|\uparrow\uparrow\downarrow\rangle + \omega|\uparrow\downarrow\uparrow\rangle + \omega^2|\downarrow\uparrow\uparrow\rangle\right)$$
$$|3\rangle \equiv \frac{1}{\sqrt{3}}\left(|\uparrow\uparrow\downarrow\rangle + \omega^2|\uparrow\downarrow\uparrow\rangle + \omega|\downarrow\uparrow\uparrow\rangle\right)$$
$$|4\rangle \equiv \frac{1}{\sqrt{3}}\left(|\downarrow\downarrow\uparrow\rangle + \omega|\downarrow\uparrow\downarrow\rangle + \omega^2|\uparrow\downarrow\downarrow\rangle\right)$$
$$|5\rangle \equiv \frac{1}{\sqrt{3}}\left(|\downarrow\downarrow\uparrow\rangle + \omega^2|\downarrow\uparrow\downarrow\rangle + \omega|\uparrow\downarrow\downarrow\rangle\right)$$
$$|6\rangle \equiv |\downarrow\downarrow\downarrow\rangle$$

What about many triangles?

$$H = \sum_{\Delta} H_{XXZ0}$$

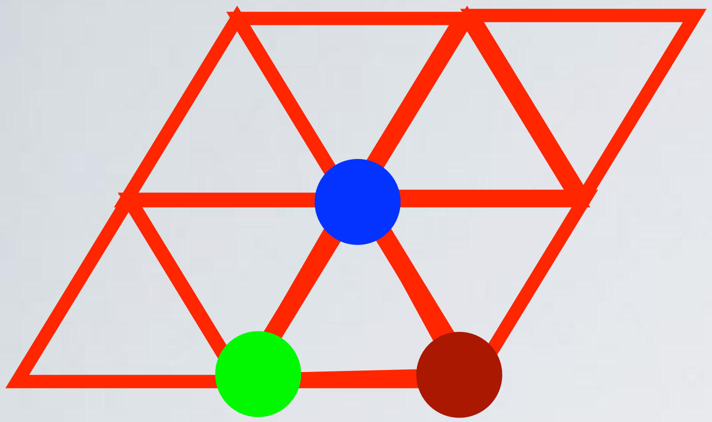


Paste together ground states over individual triangles

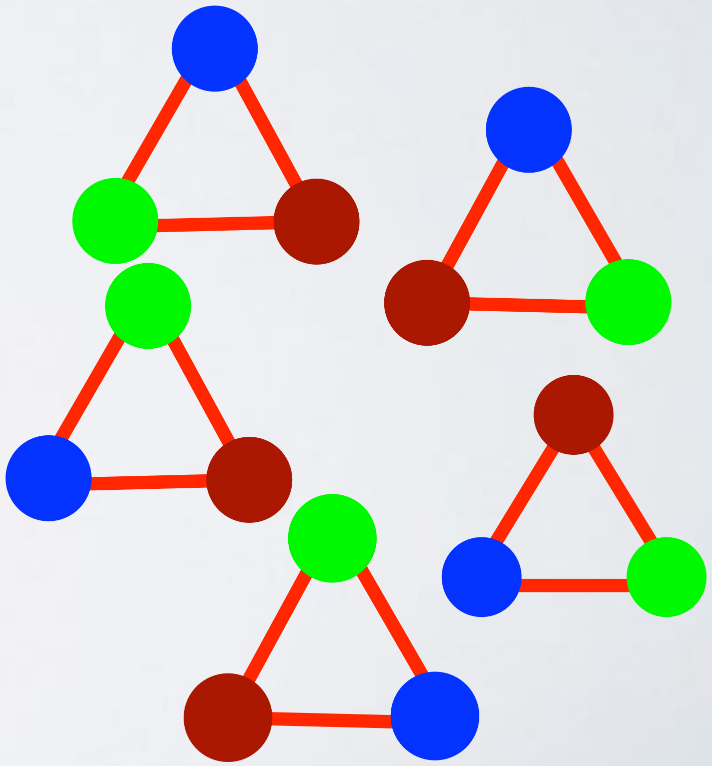


What about many triangles?

$$H = \sum_{\Delta} H_{XXZ0}$$

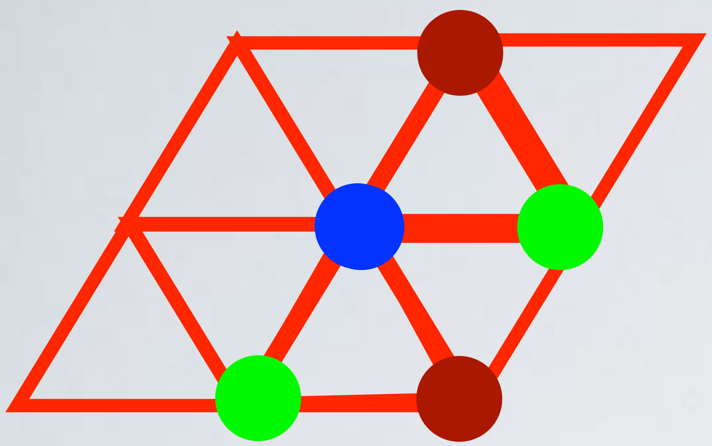


Paste together ground states over individual triangles

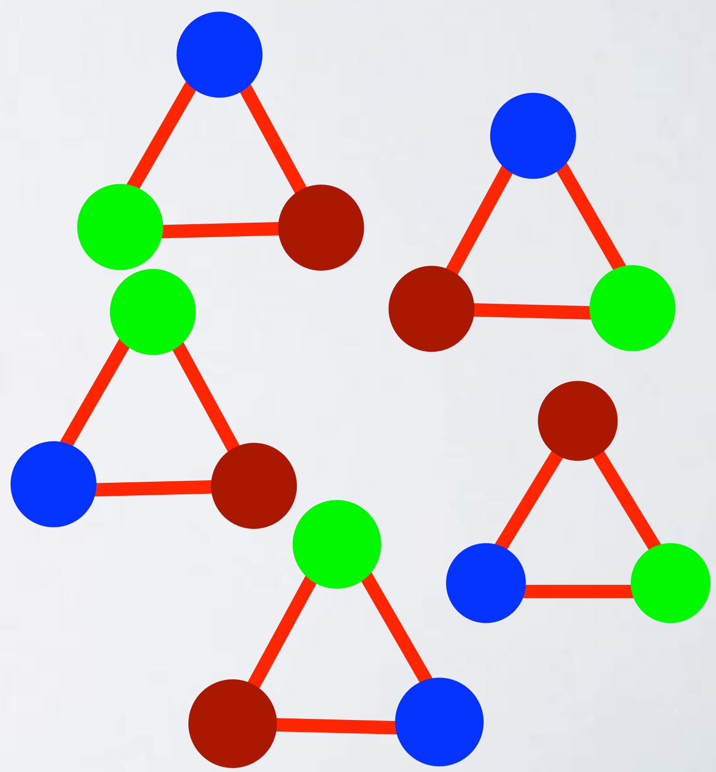


What about many triangles?

$$H = \sum_{\Delta} H_{XXZ0}$$

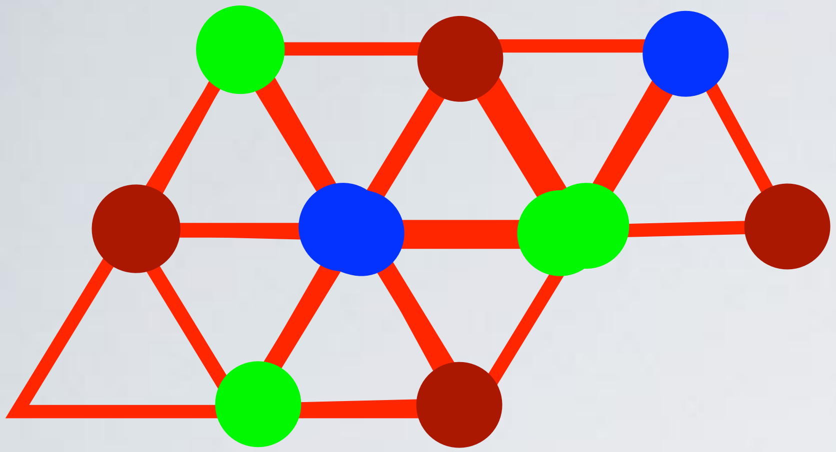


Paste together ground states over individual triangles

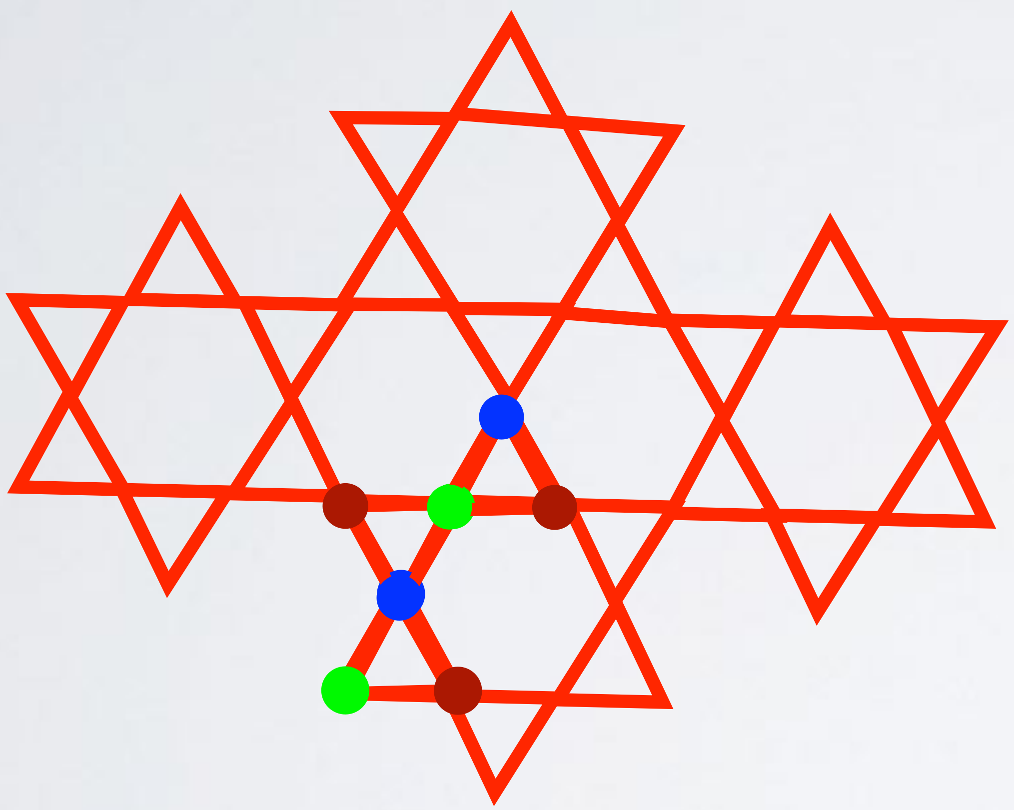


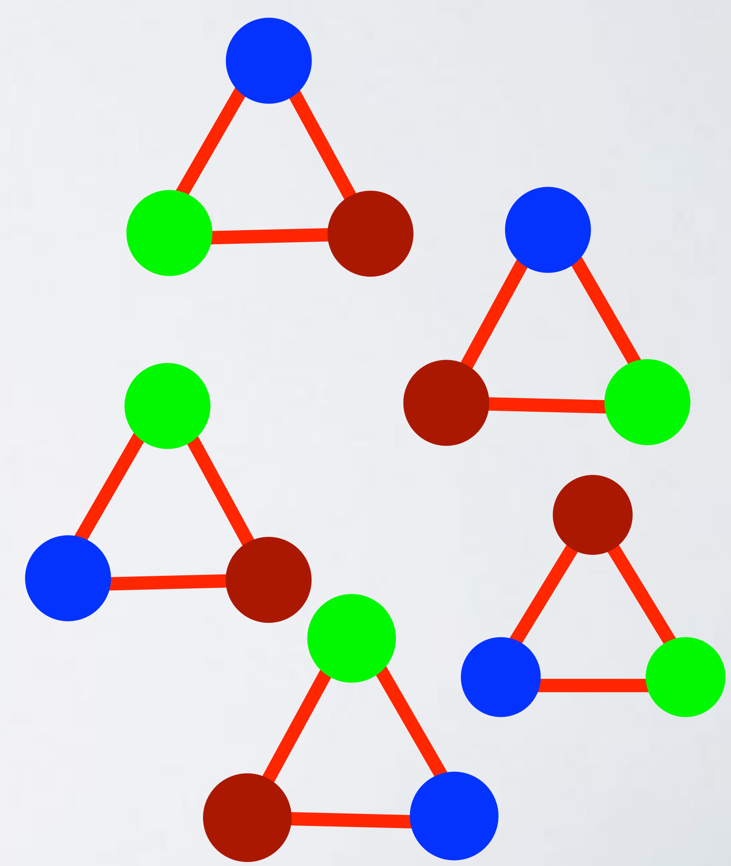
What about many triangles?

$$H = \sum_{\Delta} H_{XXZ0}$$



Paste together ground states over individual triangles

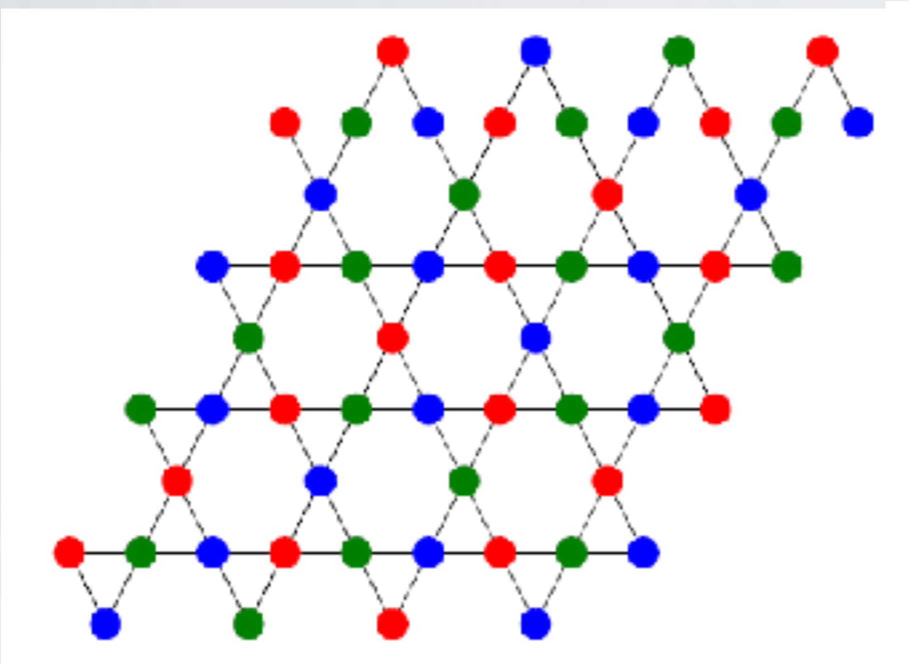


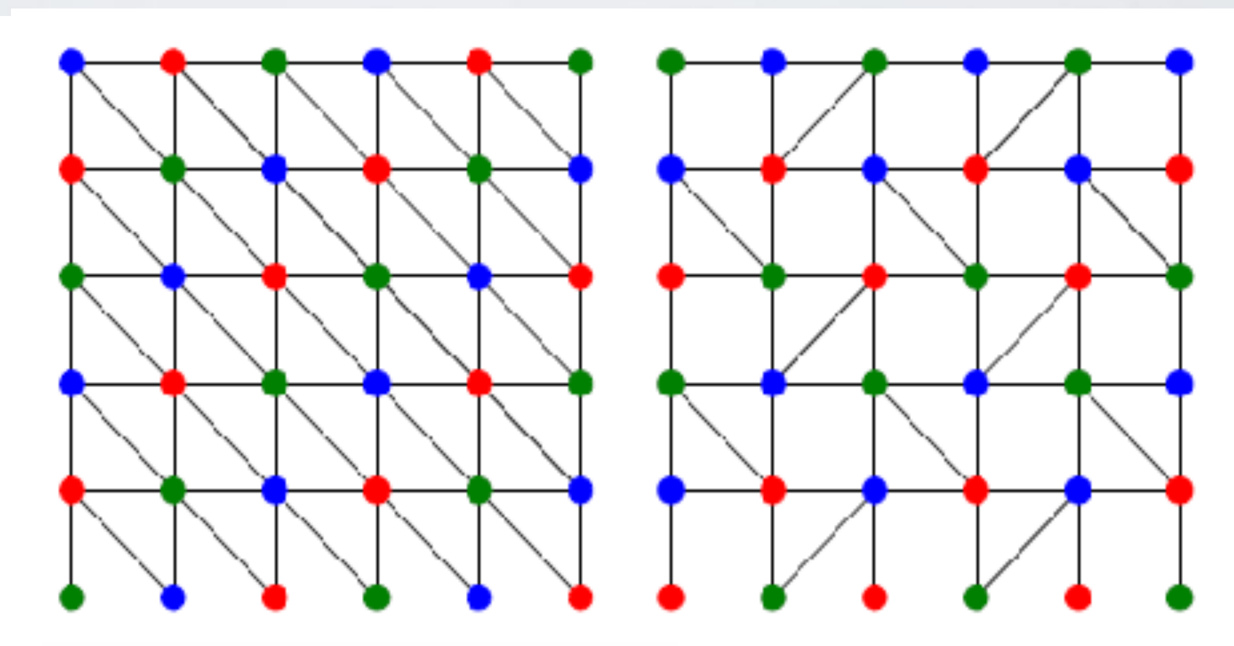


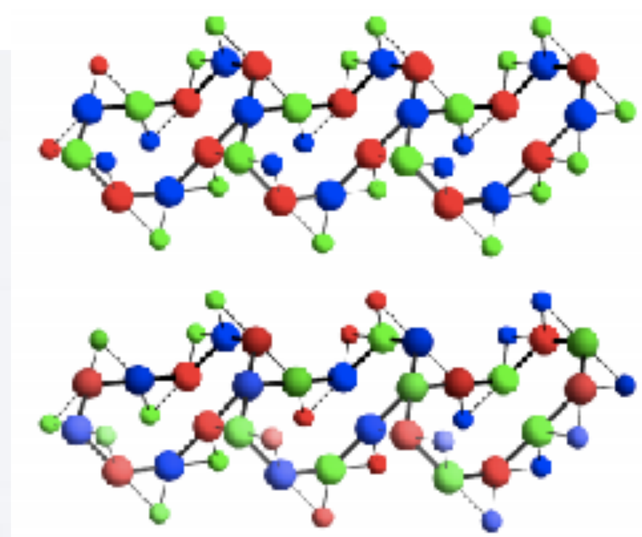
Frustration Free!
(but not commuting)

$$H = \sum_{\Delta} H_{XXZ0}$$

$$|\psi\rangle \equiv \prod_{s} \otimes |C_s\rangle_s$$







Actually there are more ground states….

$$|\psi\rangle \equiv \prod_s \otimes |C_s\rangle_s$$

This mixes Sz sectors

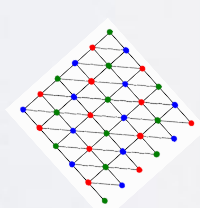
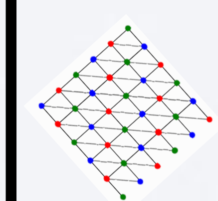
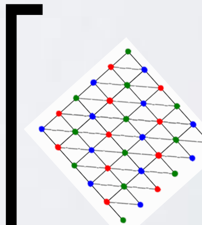
But the Hamiltonian doesn't.

$$|\psi^C\rangle \equiv P_{S_z}\left(\prod_{\text{valid}} \otimes |C_s\rangle\right)$$

So projecting to Sz sectors are ground states.

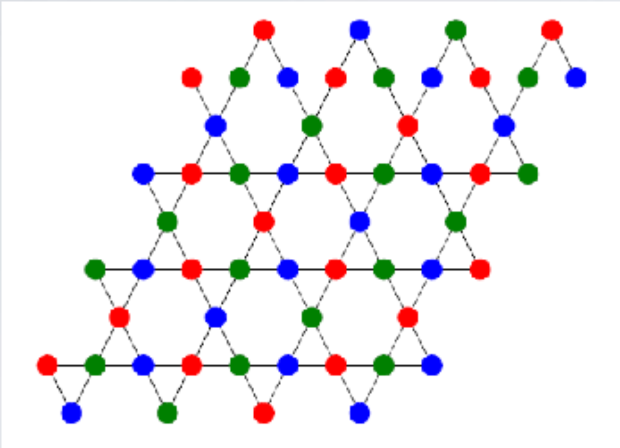
Roughly, each color gives N ground states (one per Sz sector)

(A bit of a lie because colors are non-orthogonal and may be more-so after projection)

$\Big\{ P_{S_z=0}\Big[$  $\Big]$, $P_{S_z=1}\Big[$  $\Big]$, …. , $P_{S_z=N}\Big[$  $\Big]\Big\}$

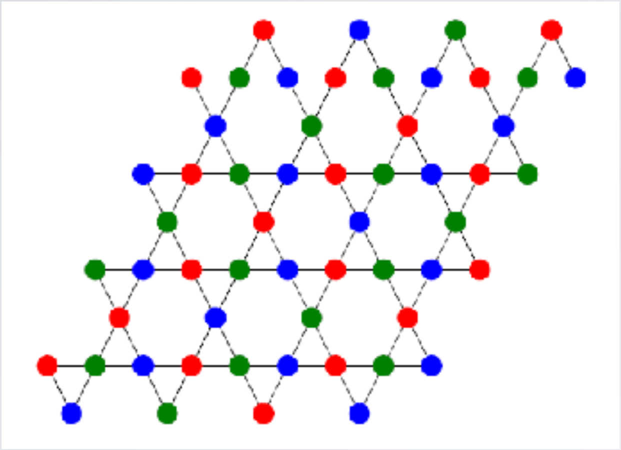
Can we use this to understand the kagome lattice?

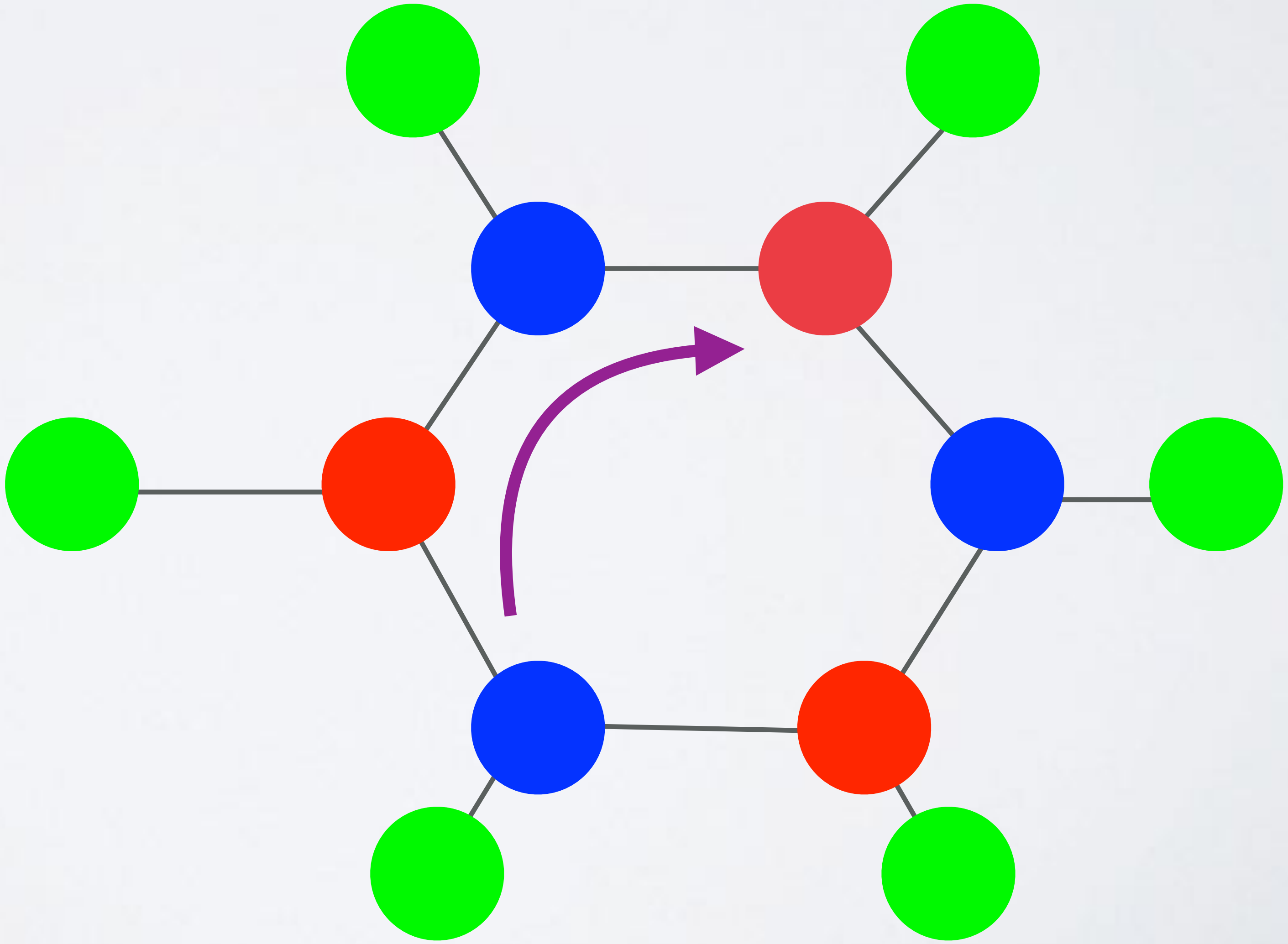
How many colorings?



Can we use this to understand the kagome lattice?

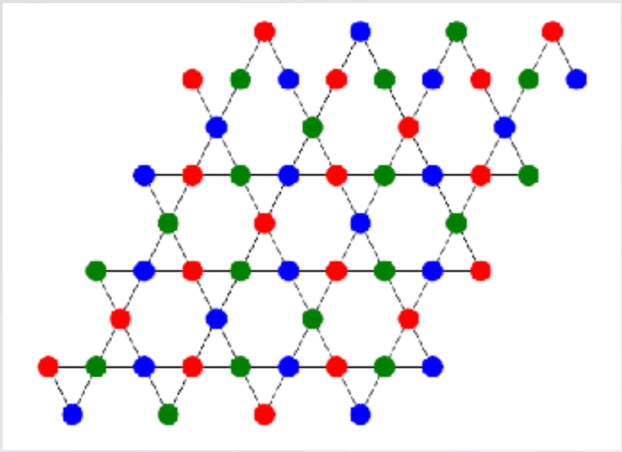
How many colorings?

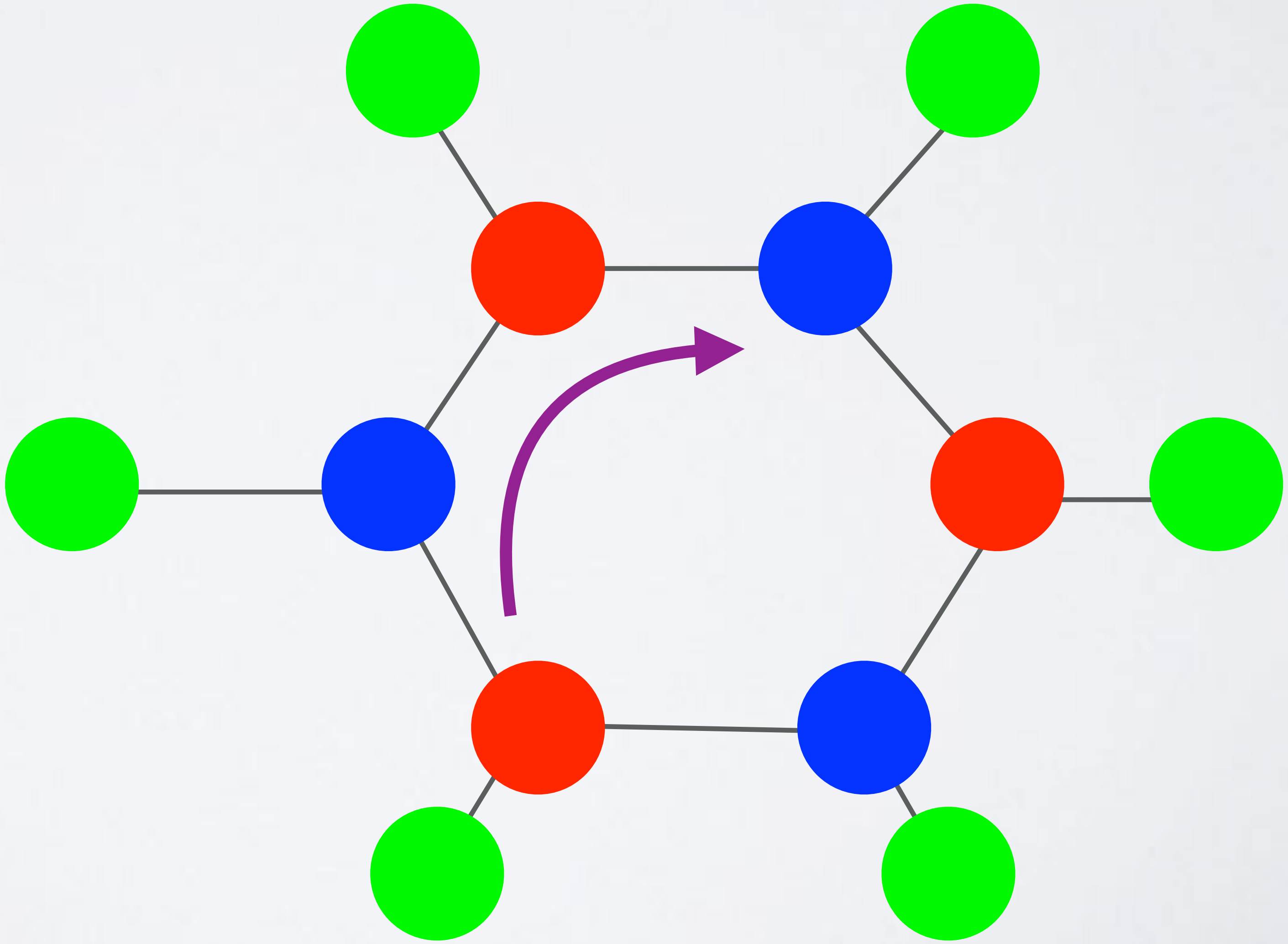




Can we use this to understand the kagome lattice?

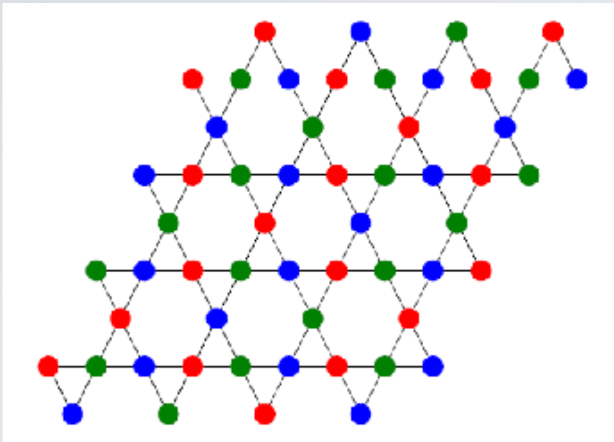
How many colorings?





Can we use this to understand the kagome lattice?

How many colorings?



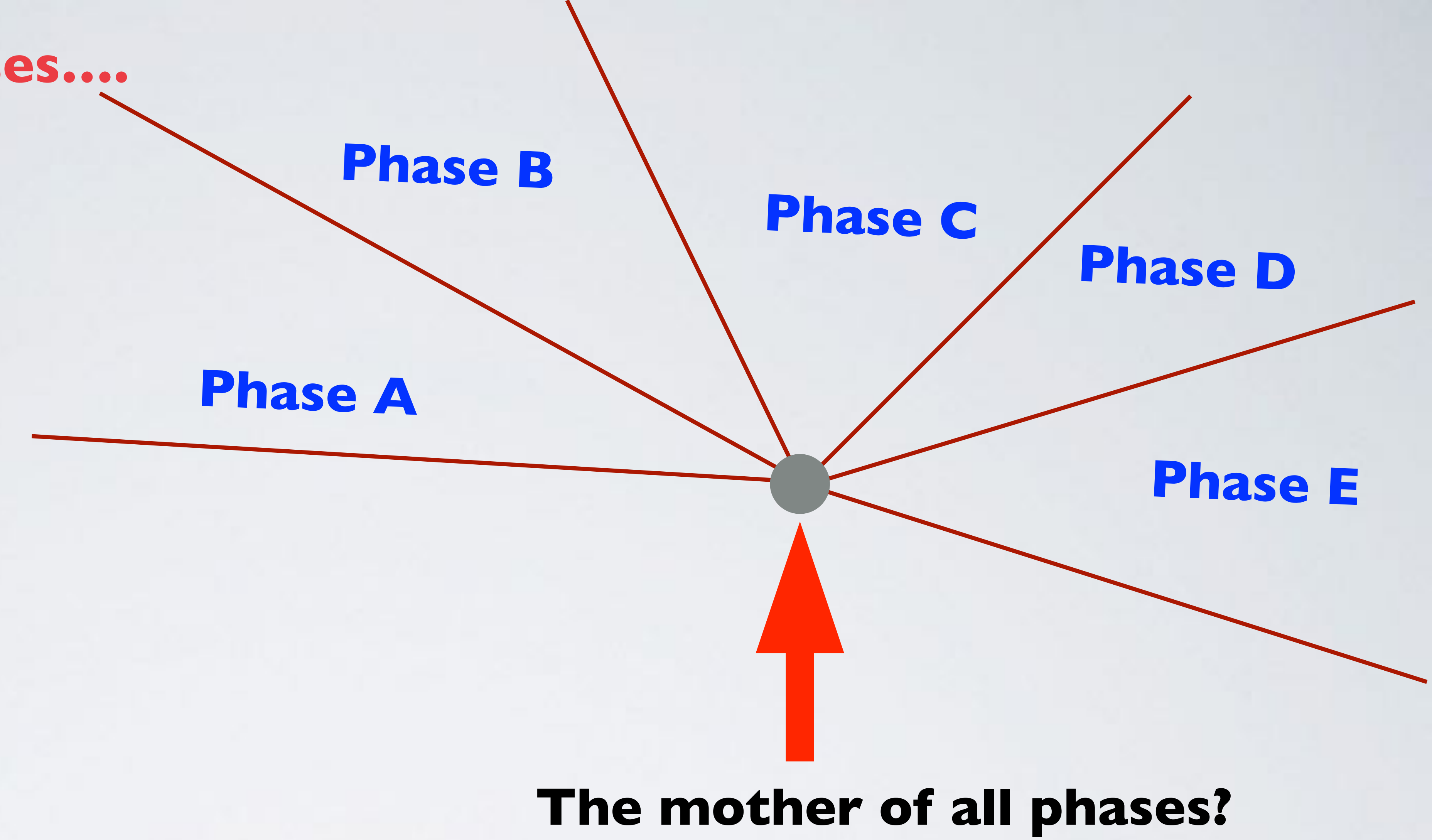
An exponential number of colorings!

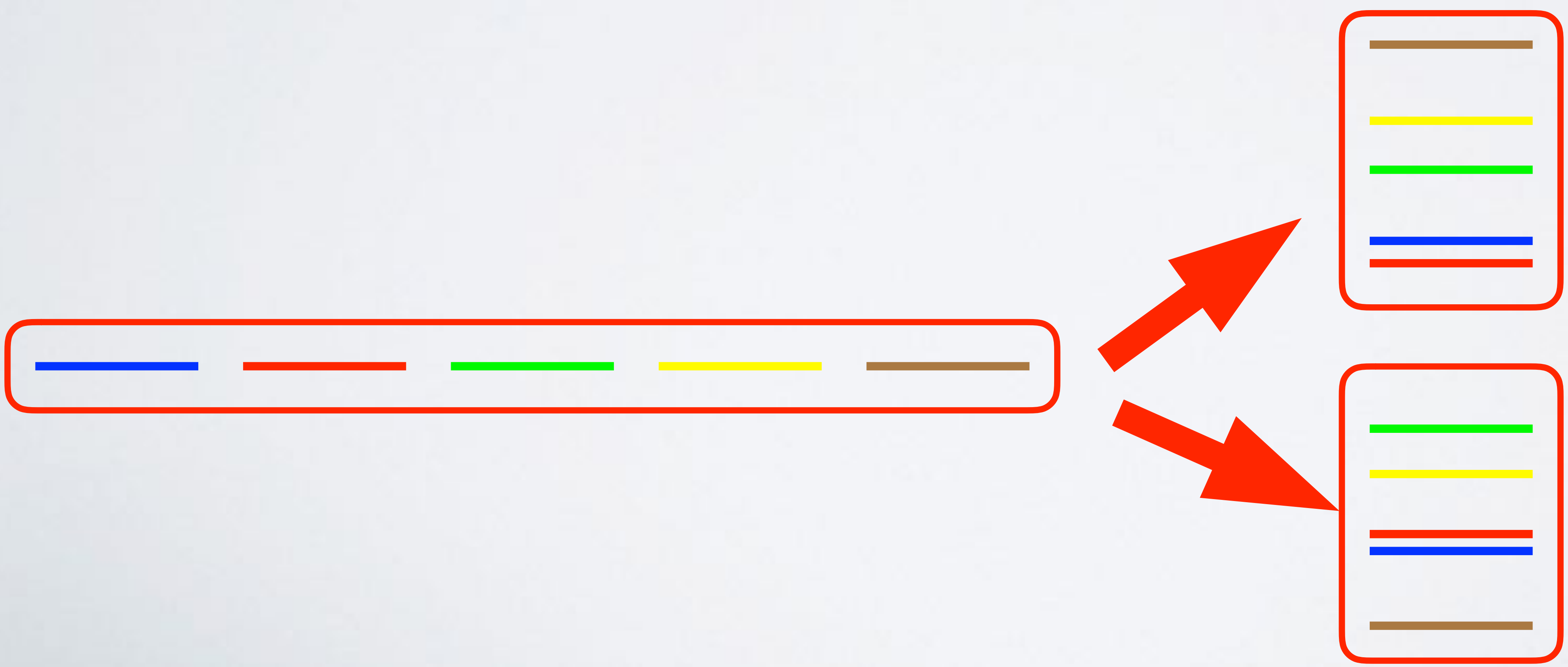
$$1.208^N$$ (from Baxter)

But much fewer then Ising configurations....

Lattice	Ising configs	Colorings
2x2x3	924	8
3x2x3	48620	16
4x2x3	2.7 million	32

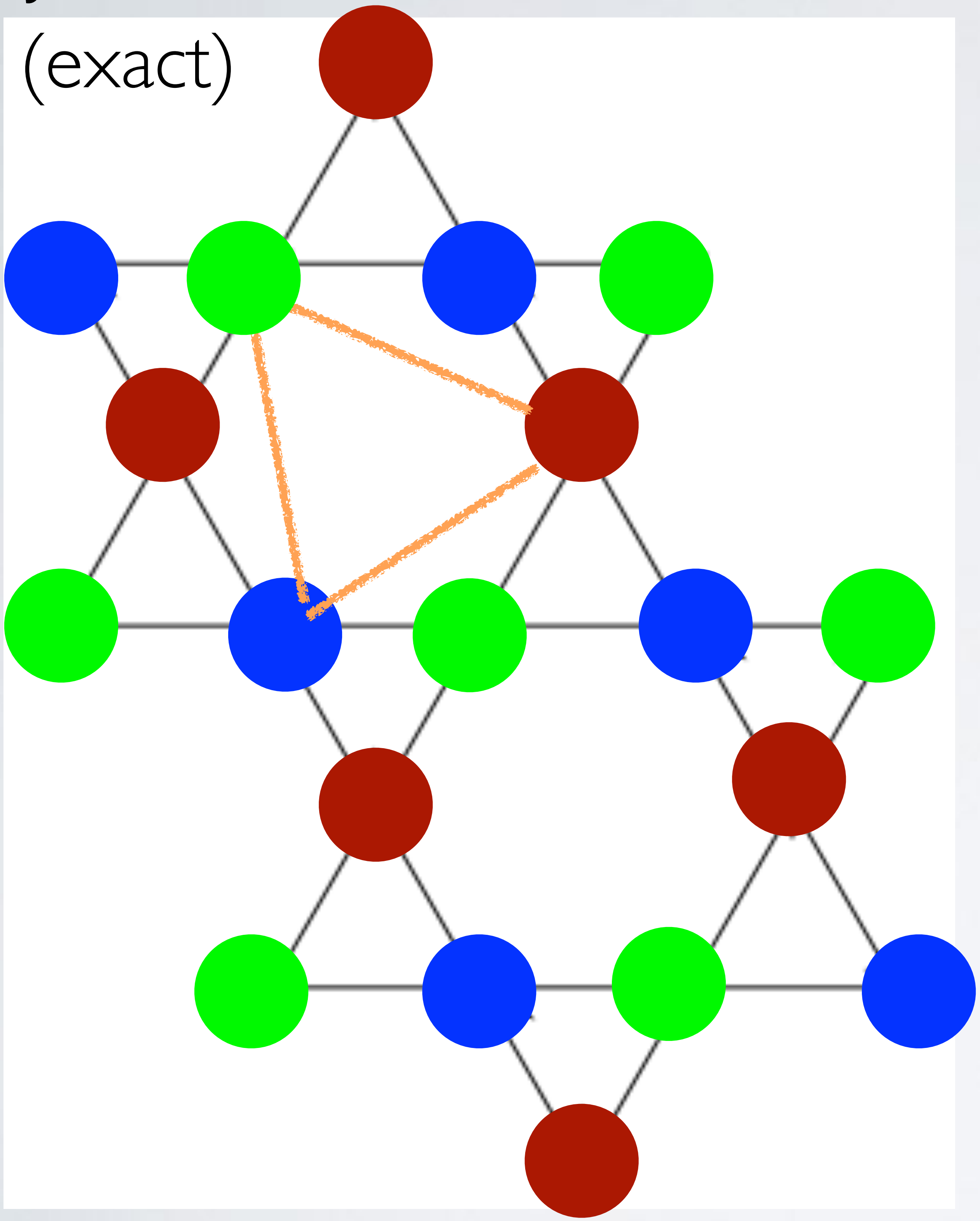
Connect to known phases....





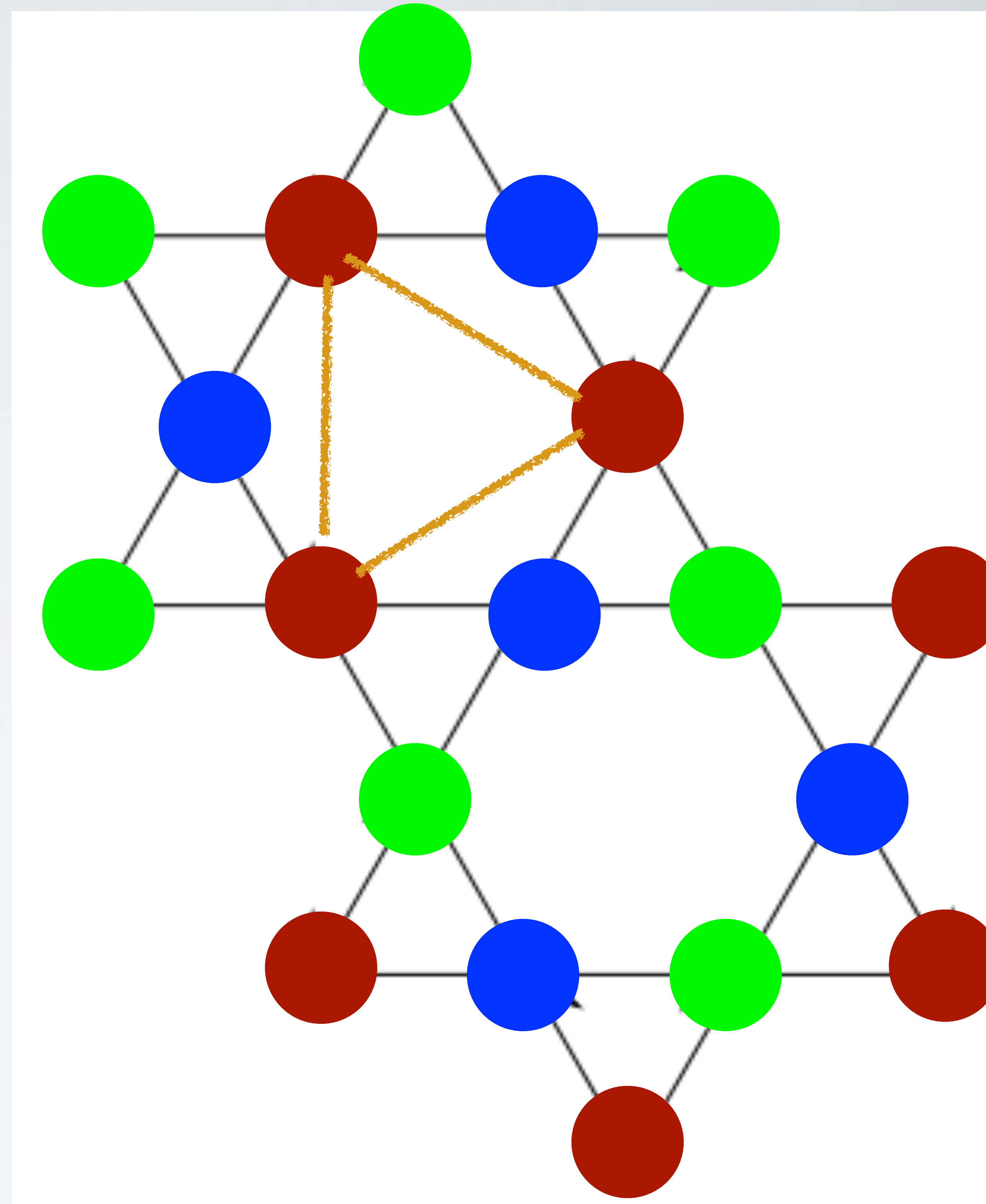
J2>0

(exact)

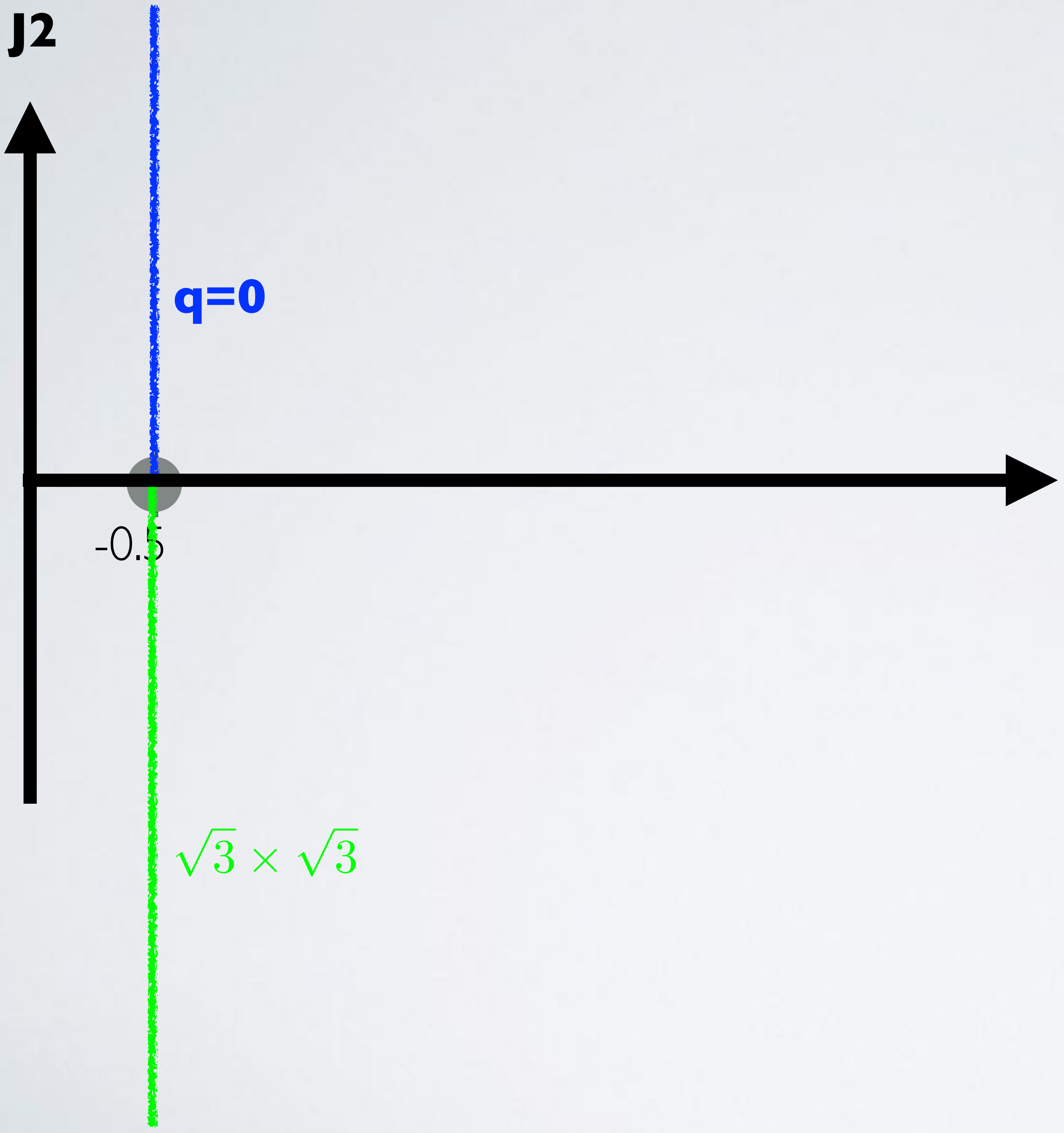


q=0

J2<0

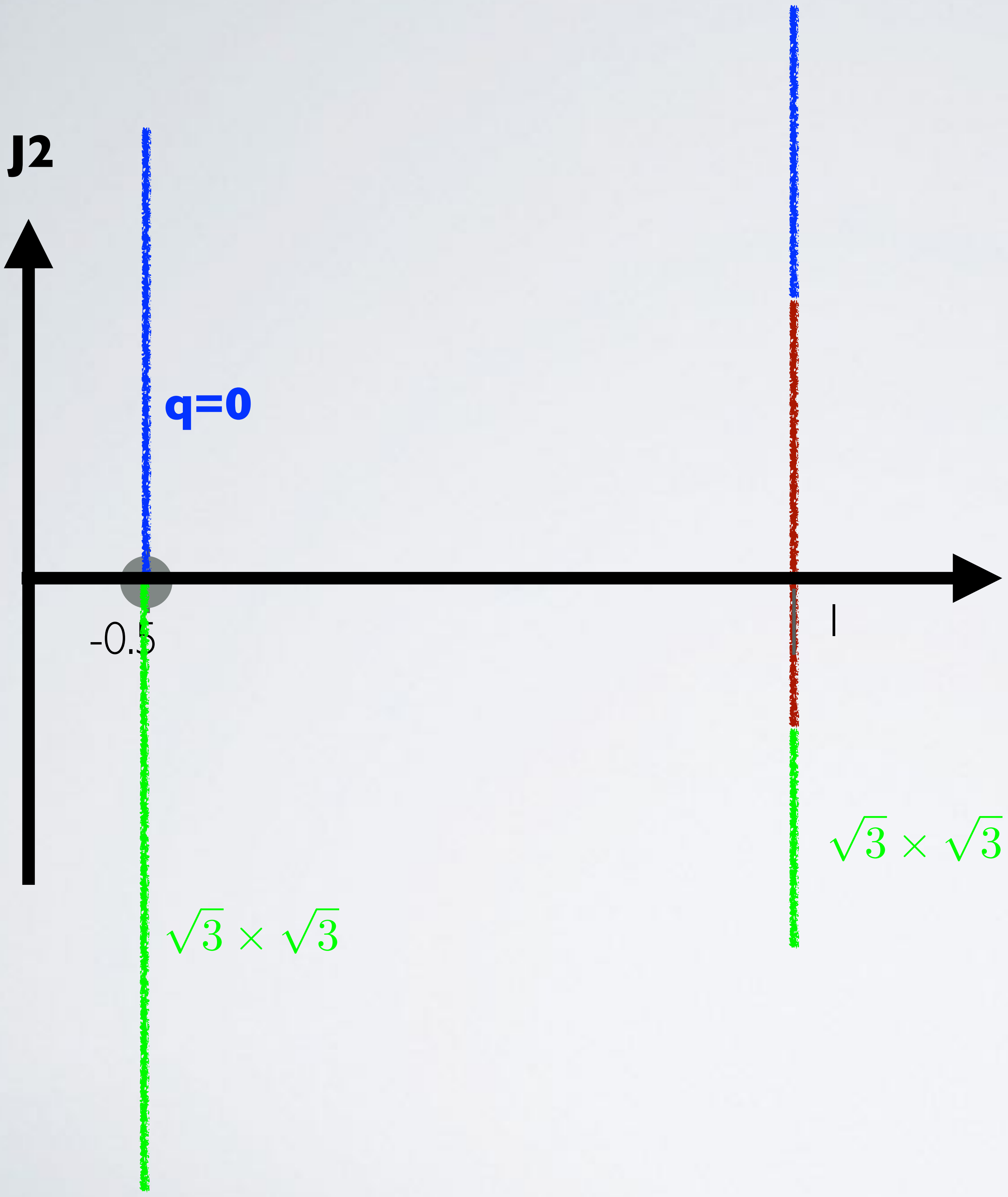


$\sqrt{3} \times \sqrt{3}$

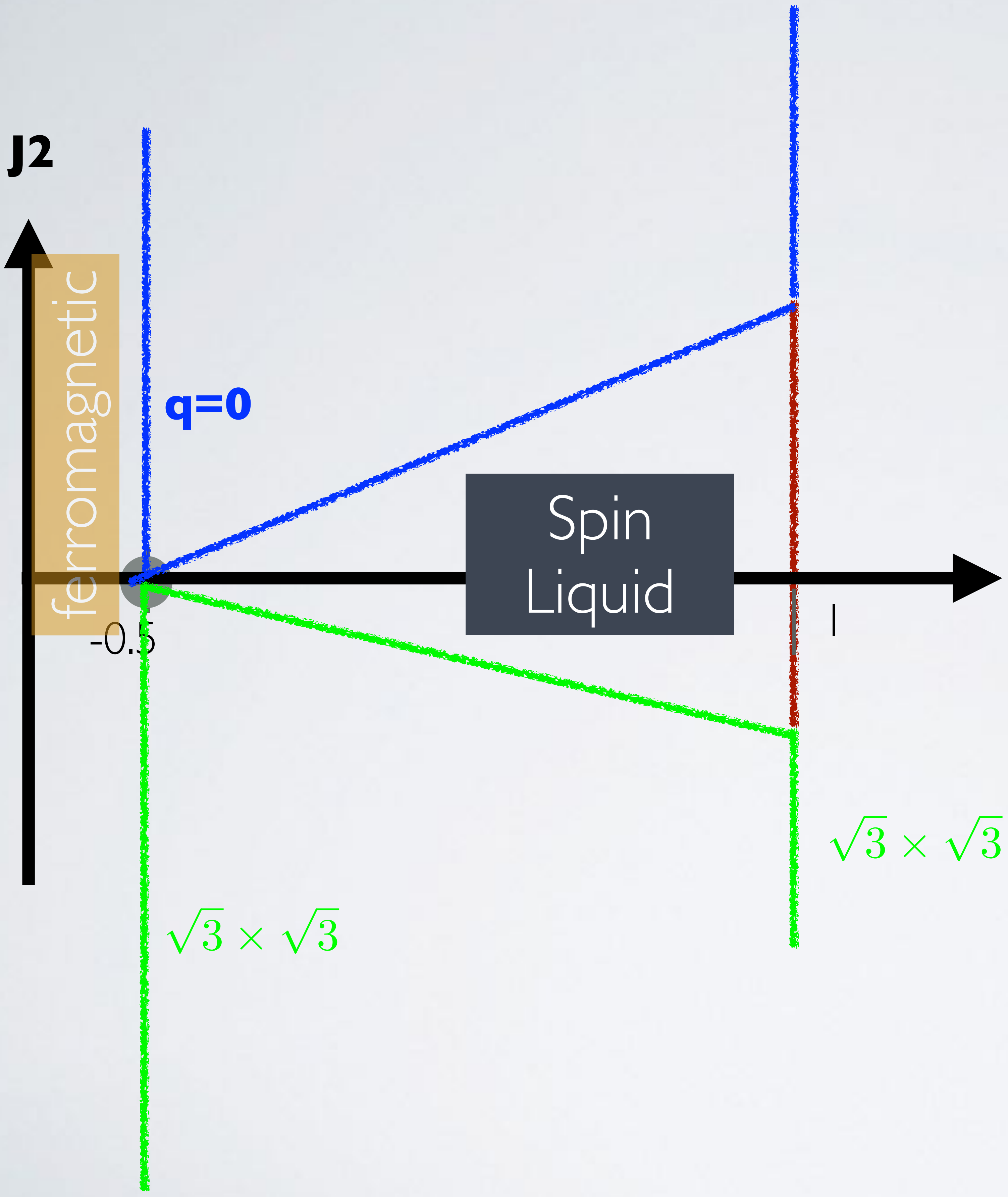


J2
q=0
-0.5
$\sqrt{3} \times \sqrt{3}$

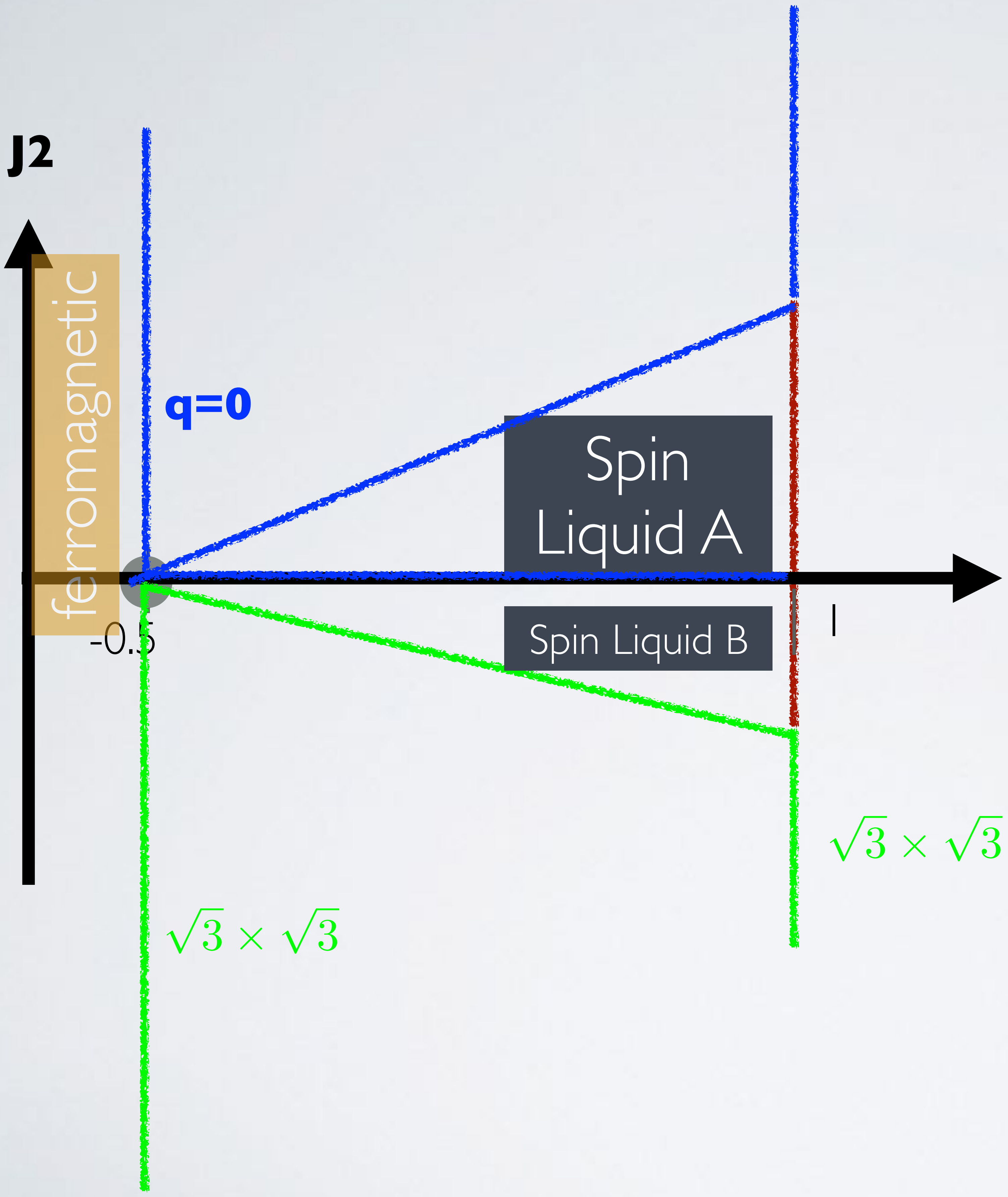
$S_z = 0$



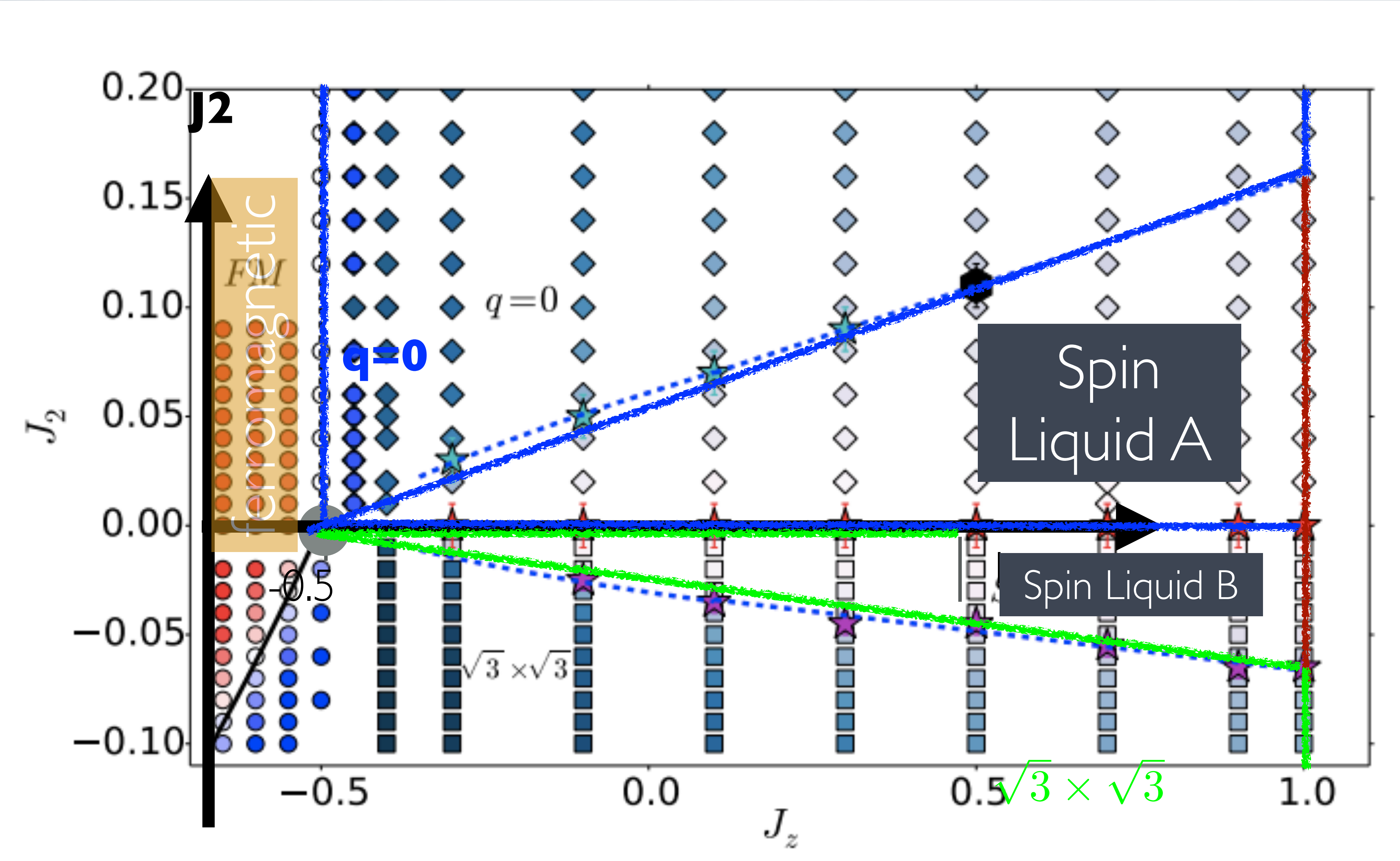
$S_z = 0$

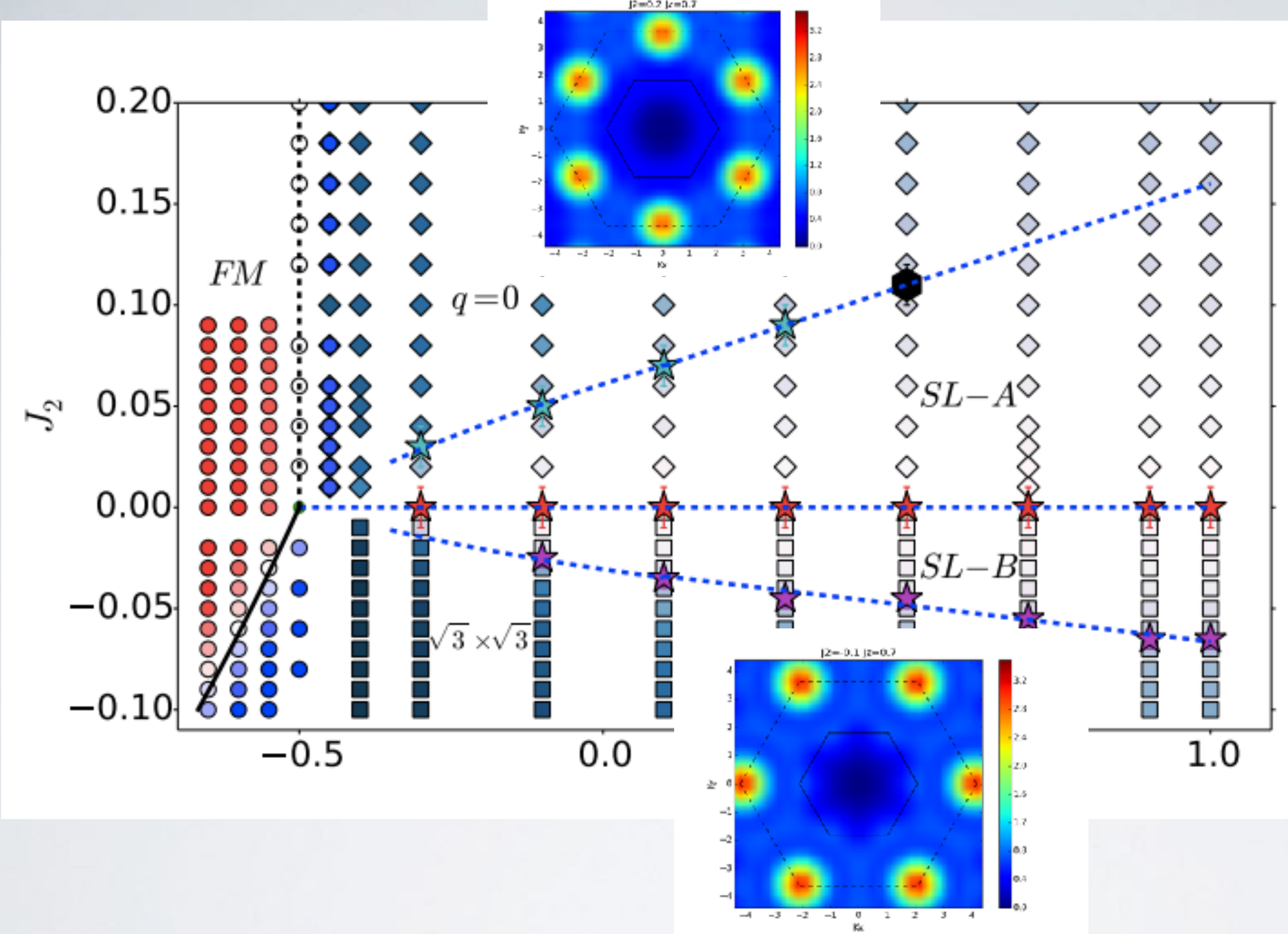

J2
ferromagnetic
q=0
Spin Liquid
-0.5
1
$\sqrt{3} \times \sqrt{3}$
$\sqrt{3} \times \sqrt{3}$

$S_z = 0$



J2
ferromagnetic
q=0
Spin Liquid A
Spin Liquid B
-0.5
1
$\sqrt{3} \times \sqrt{3}$
$\sqrt{3} \times \sqrt{3}$


J2
ferromagnetic
FM
q=0
$q=0$
Spin Liquid A
Spin Liquid B
-0.5
$\sqrt{3}\times\sqrt{3}$
$\sqrt{3}\times\sqrt{3}$
J_2
J_z
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.5
0.0
0.5
1.0



Q:Why co-planar states?

Colorings are all co-planar

Q:Why these co-planar states?

Fixed by colorings which satisfy J1-J2

Q:Why spin-liquids?

Q:Why so many competing phases?

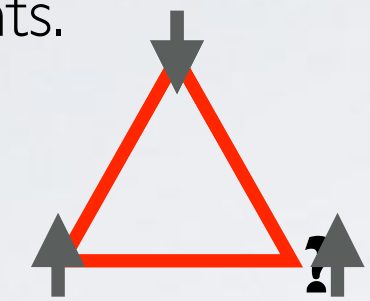
Q:Why low-energy mess?

Exponential Degeneracy

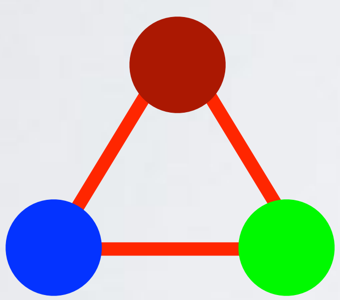
Conclusions

XXZ0 controls the physics of the Heisenberg point on lattices of pasted triangles in the way that the Ising limit doesn't.

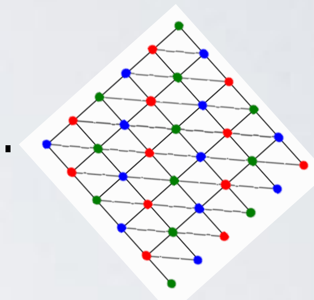
The story of frustration is not one of triangles which can't satisfy up-up-down constraints.



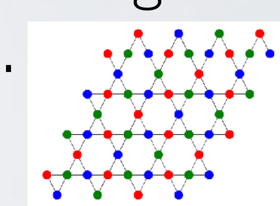
Instead, the story of frustrated magnetism is really one of coloring.



A single coloring which controls the triangular lattice.



And an exponential number of colorings which controls the kagome lattice.



From which all the known phases (and I conjecture arbitrarily many more) arise.