A WELL-SPRING FOR SPIN LIQUIDS ON KAGOME AND HYPER-KAGOME LATTICES

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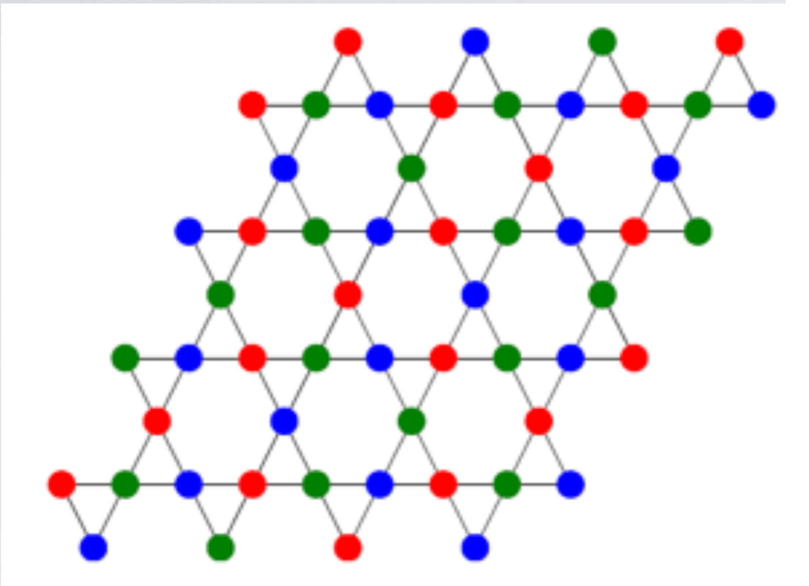
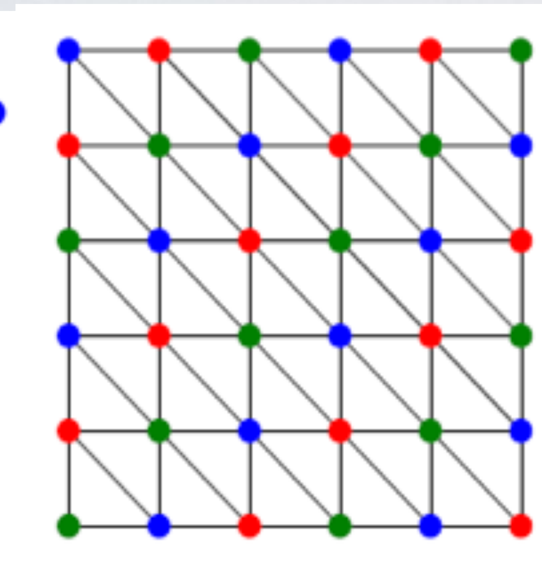
Iowa State University - December 1, 2016

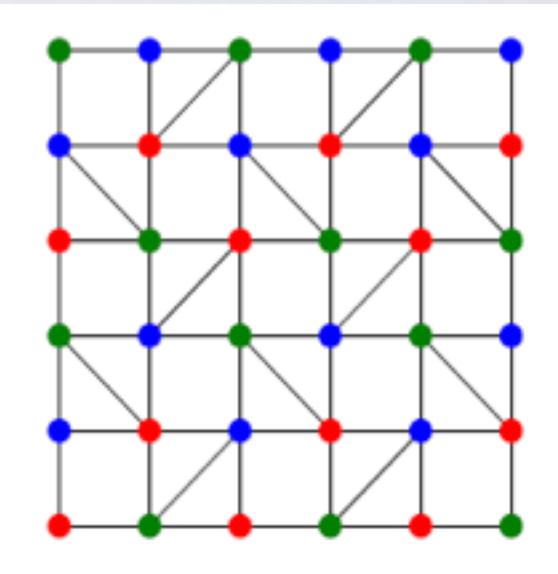
The story of frustrated magnetism is really the story of insulating materials with spin degrees of freedom which live on a non-bipartite lattice.



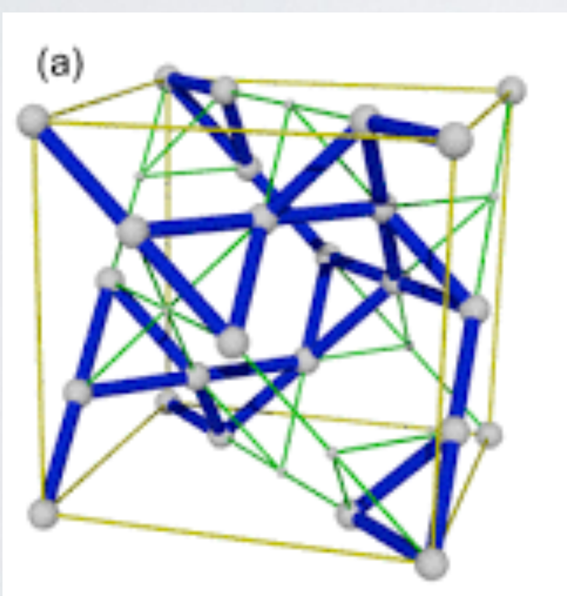
Kagome



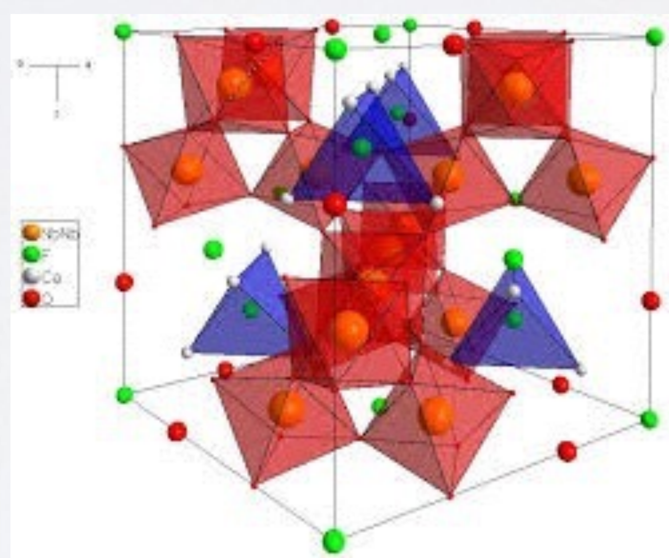
Triangular



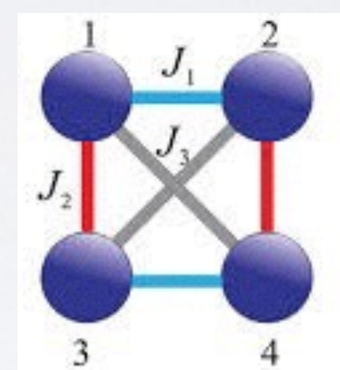
Shastry-Sutherland



Hyperkagome

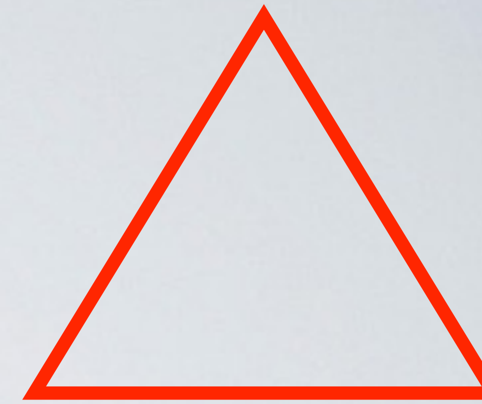


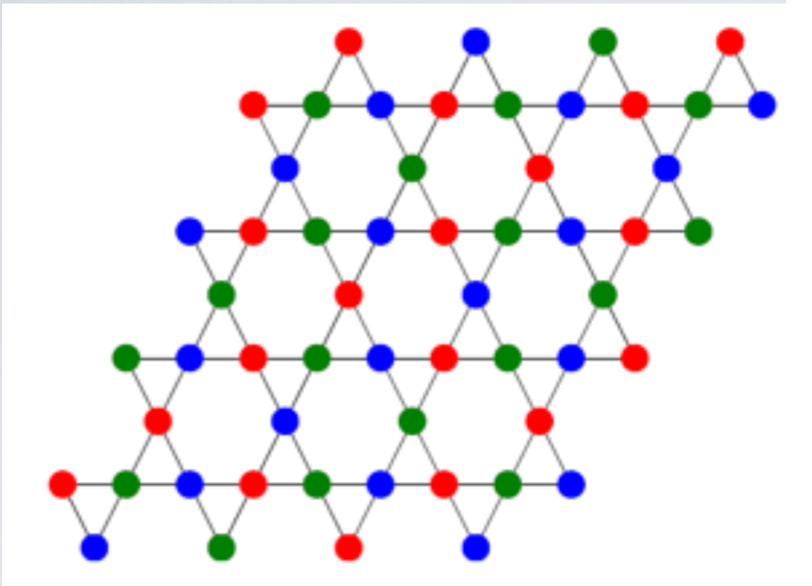
Pyrochlore



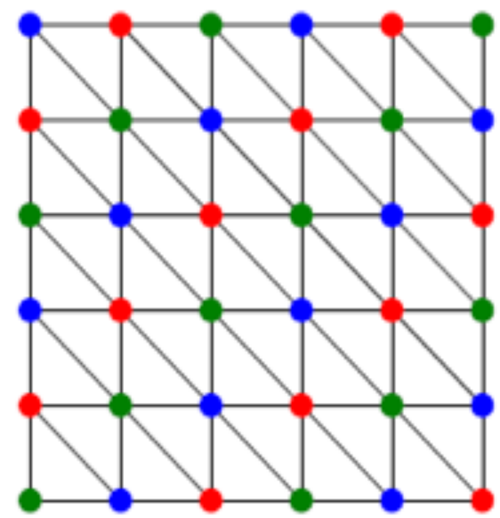
J1-J2 square

The story of frustrated magnetism is really the story of triangles.

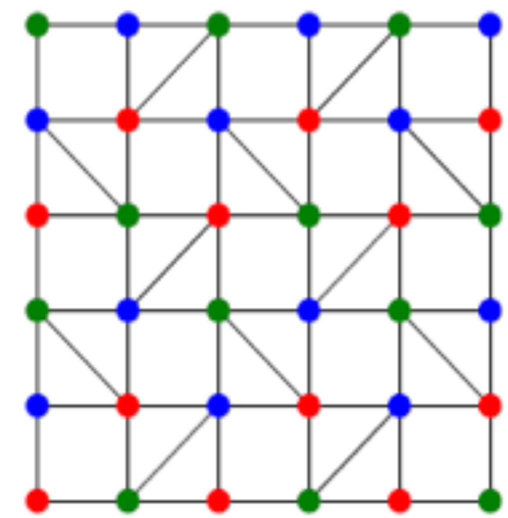




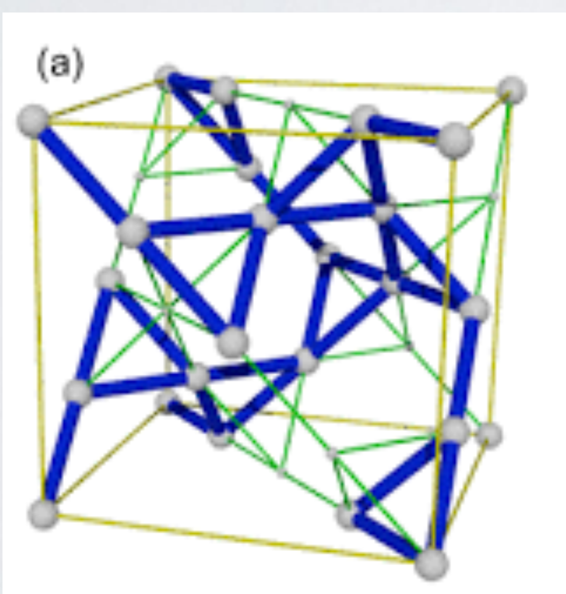
Kagome



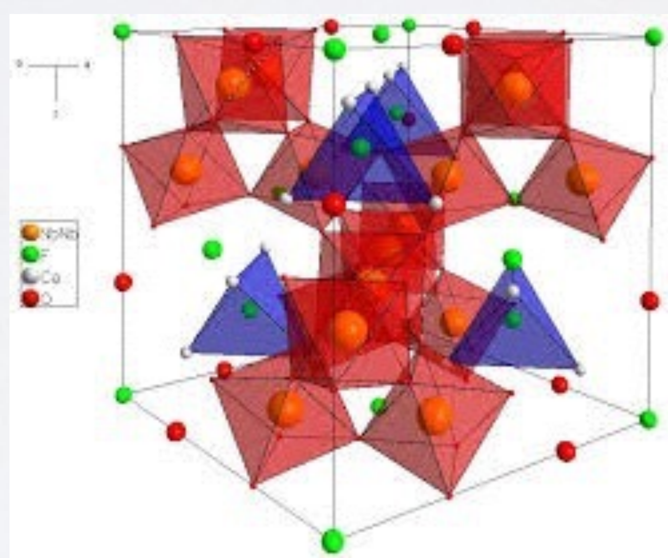
Triangular



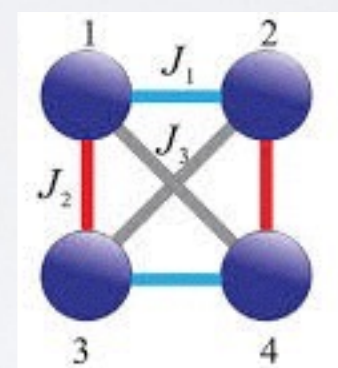
Shastry-Sutherland



Hyperkagome



Pyrochlore



J1-J2 square

The history of frustrated magnetism started in 1973

when Phil Anderson suggested that the n.n. Heisenberg model on the triangular lattice wasn't a neel state (frustration!)

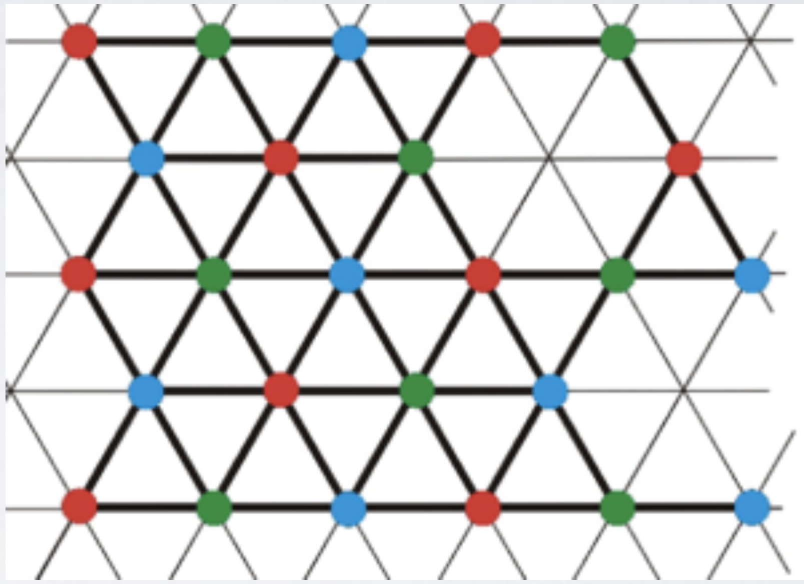
Spin 1/2 quantum Hamiltonian's

$$H_{xy} = \sum_{\langle i,j \rangle} S_i^x S_j^x + S_i^y S_j^y$$

$$H_{xxz} = H_{xy} + J_z \sum_{ij} S_i^z S_j^z$$

$$J_z = 1$$

Ground State



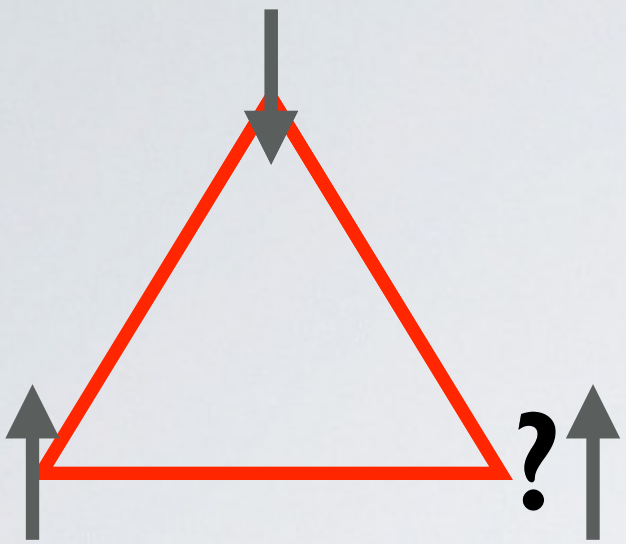
RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR?*

P. W. Anderson
Bell Laboratories, Murray Hill, New Jersey 07974
and
Cavendish Laboratory, Cambridge, England

(Received December 5, 1972; Invited**)

1973: Anderson predicts the Heisenberg model on the triangle lattice is a spin liquid.

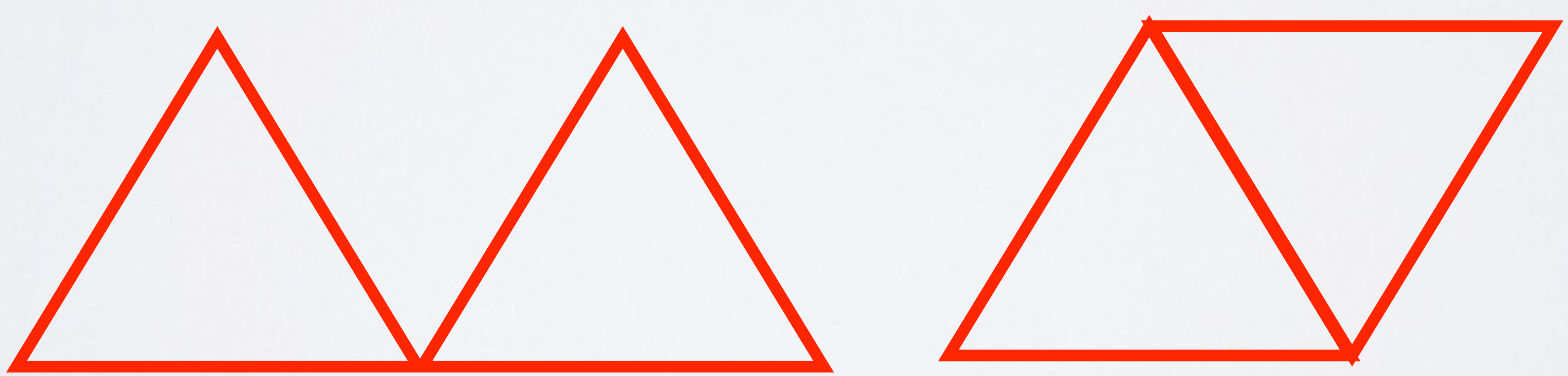
frustration!



$$H_{xxz} = H_{xy} + J_z \sum_{ij} S_i^z S_j^z \quad \text{(ising limit } J_z \to \infty\text{)}$$

$$H_{\text{ising}} = \sum_{ij} S_i^z S_j^z$$

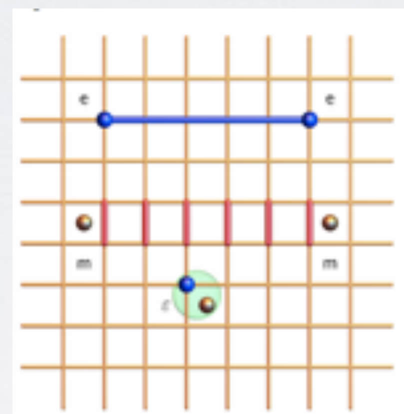
When you paste together many triangles, there are many degenerate states



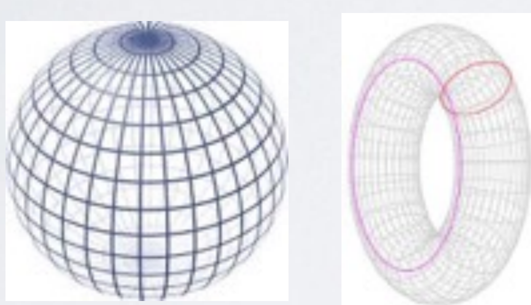
Instead he suggested it was a spin-liquid.

1. No order at T=0

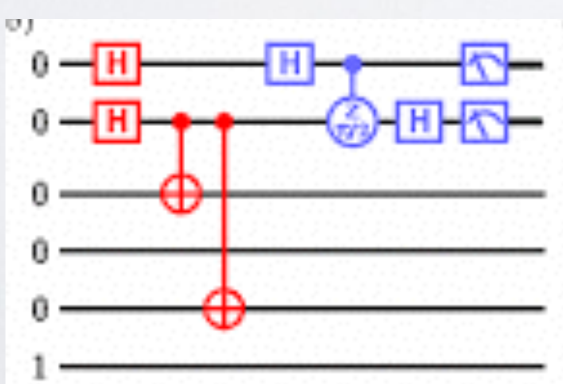
2. Non-locally created local excitations



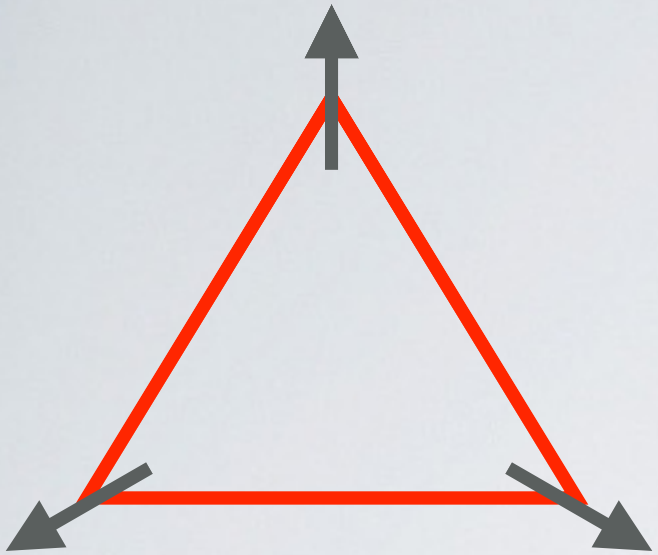
3. Topological Degeneracy



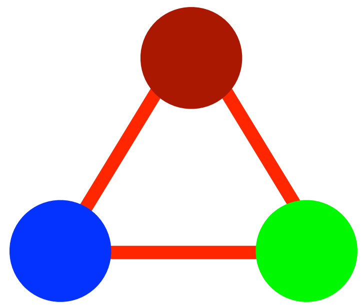
4. Long-range Entanglement



But it wasn't….instead it was a 120 degree ordered state



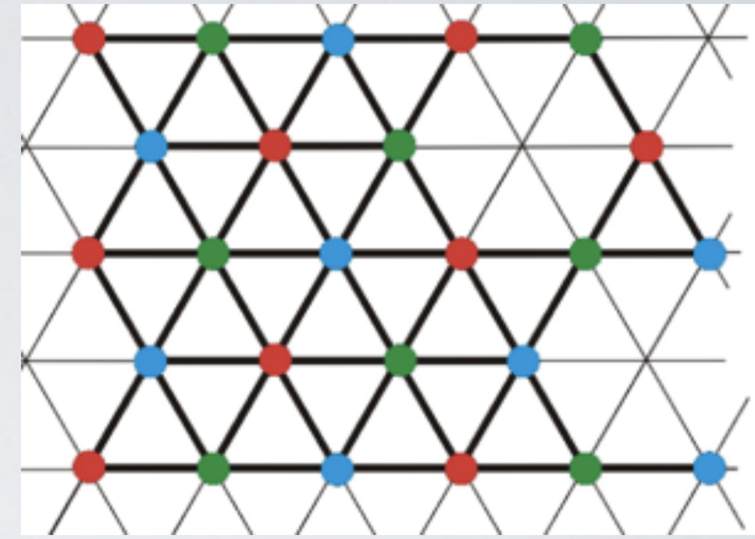
Define
3 "colors"



$$|a\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

$$|b\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + \omega|\downarrow\rangle\right)$$

$$|c\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + \omega^2|\downarrow\rangle\right)$$

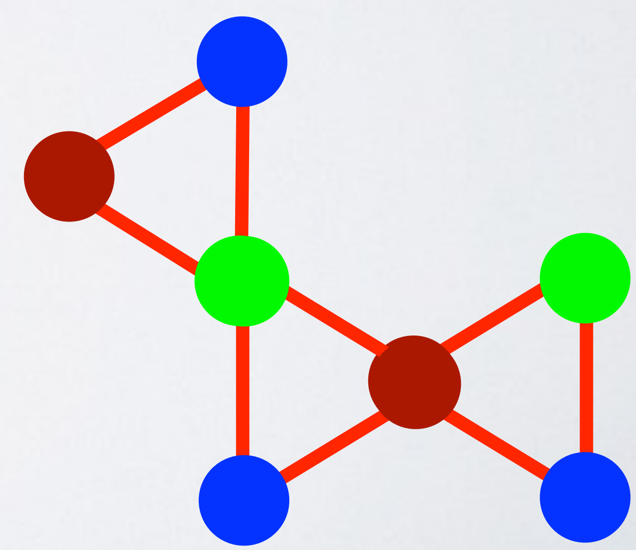


"Morally" this state but not exactly this state.

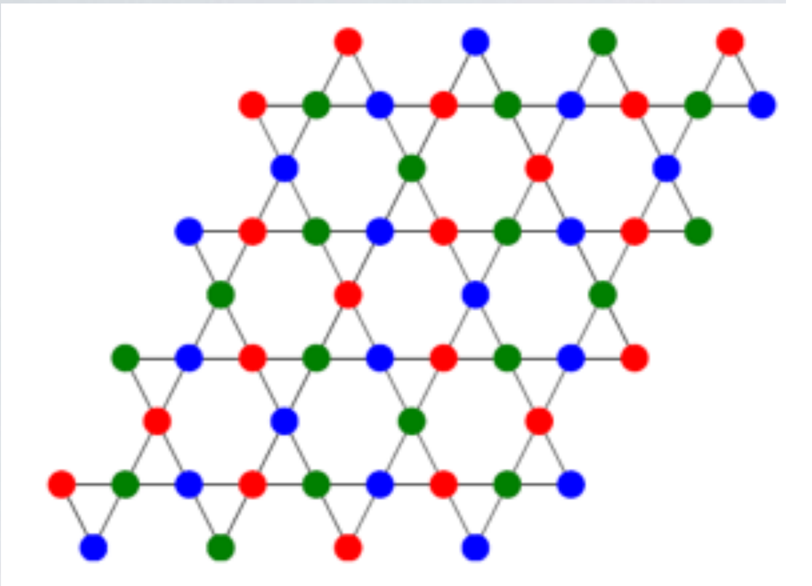
$$(|0\rangle + |1\rangle) \otimes (|0\rangle + \omega|1\rangle) \otimes (|0\rangle + \omega^2|1\rangle)$$

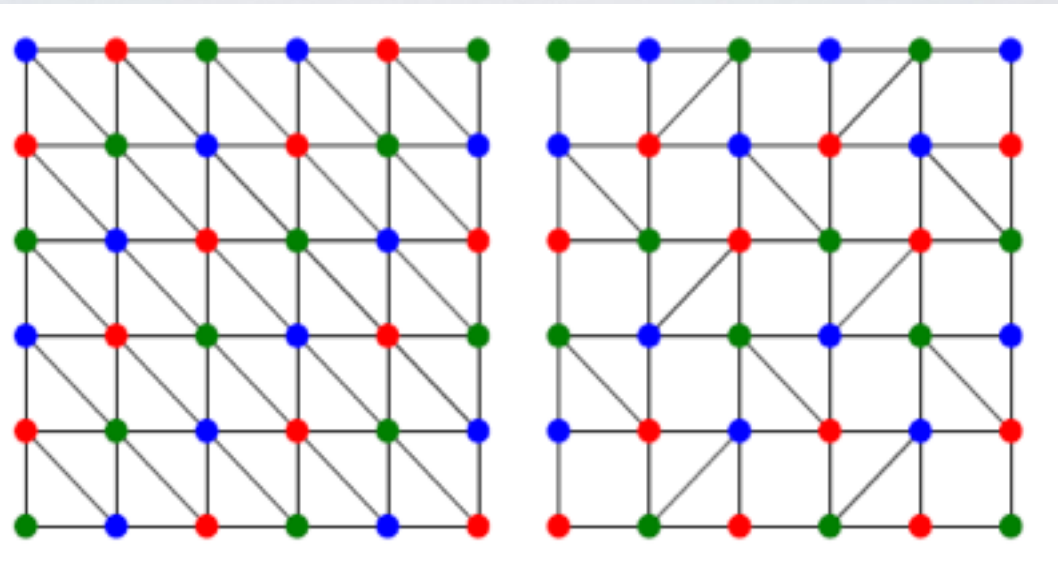
By projection ~~$|000\rangle + |111\rangle$~~ $+ |100\rangle + \omega|010\rangle + \omega^2|001\rangle + \ldots$

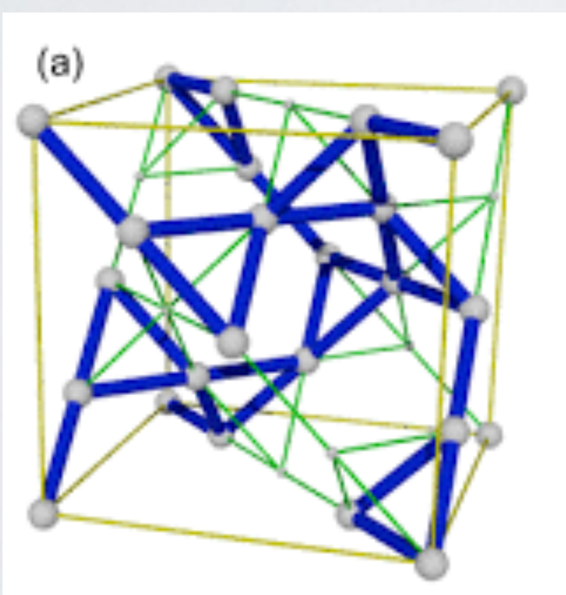
This is a high-energy eigenstate
but projection removed it for us



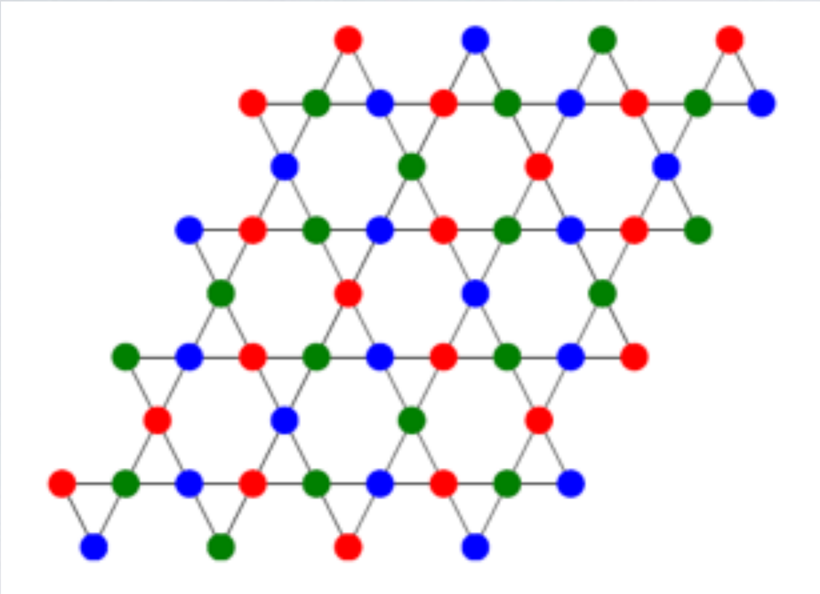
But there are other lattices of pasted-together triangles (shastry-sutherland, kagome, hyperkagome) (also all frustrated!)







Among these, kagome stands out both experimentally and theoretically



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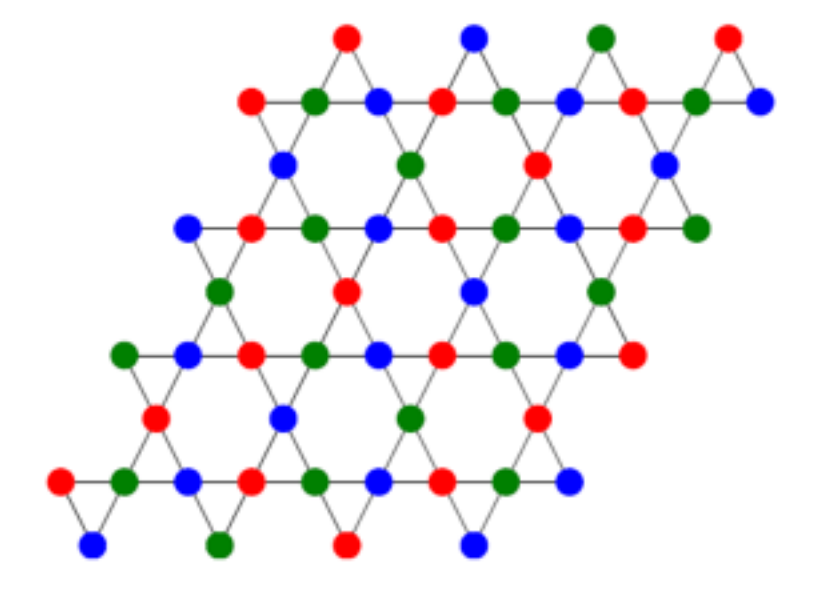

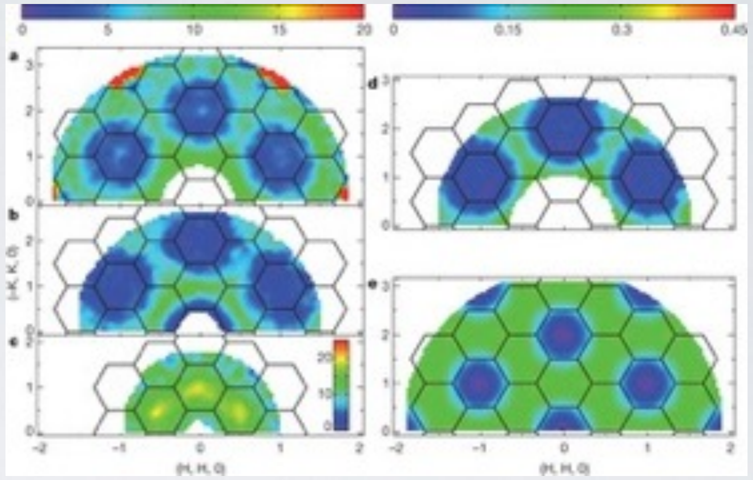

Nature 492, 406–41

Herbertsmithite

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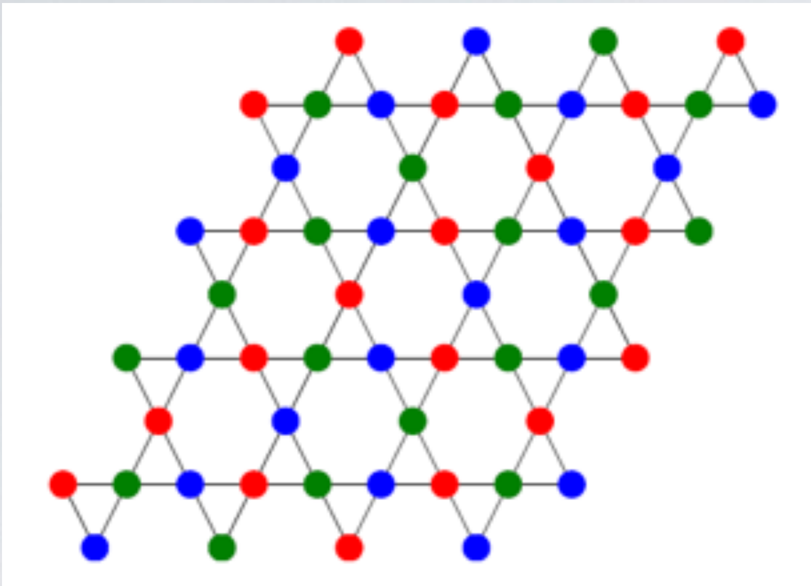
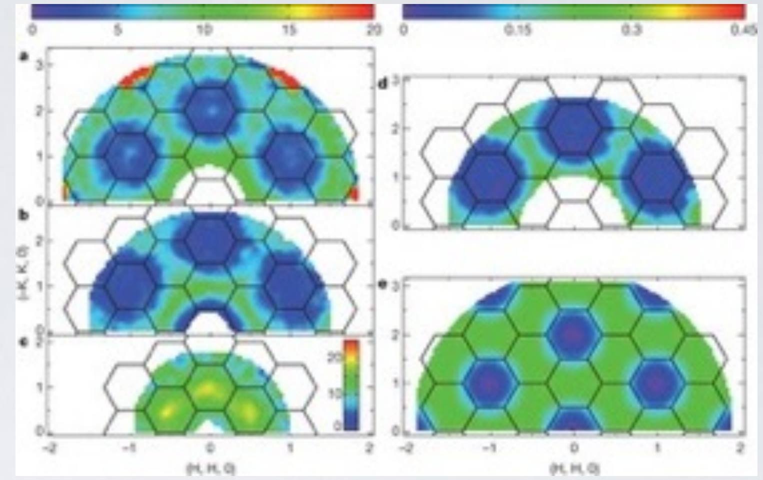




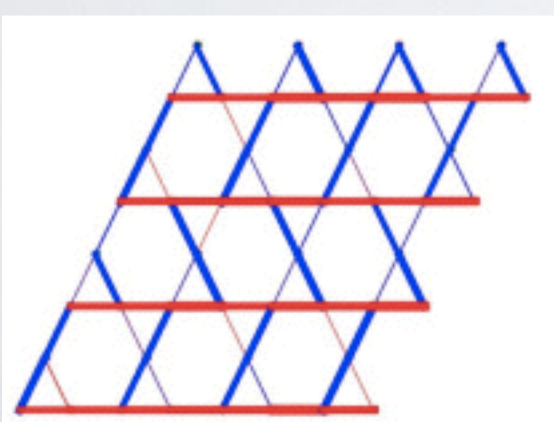
Nature 492, 406–41

Herbertsmithite

Volborthite

Kapellasite

Vesigniette



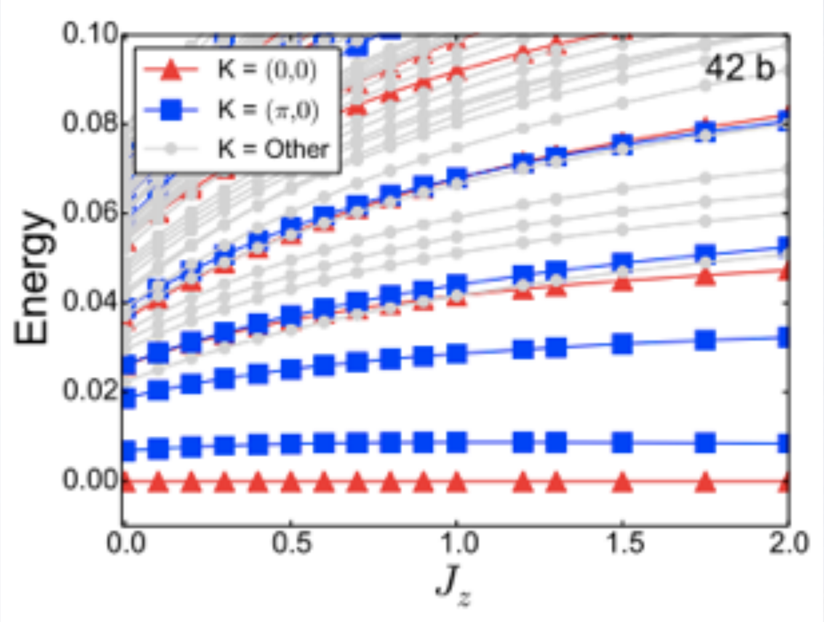
Phys. Rev. Lett. 111, 187205

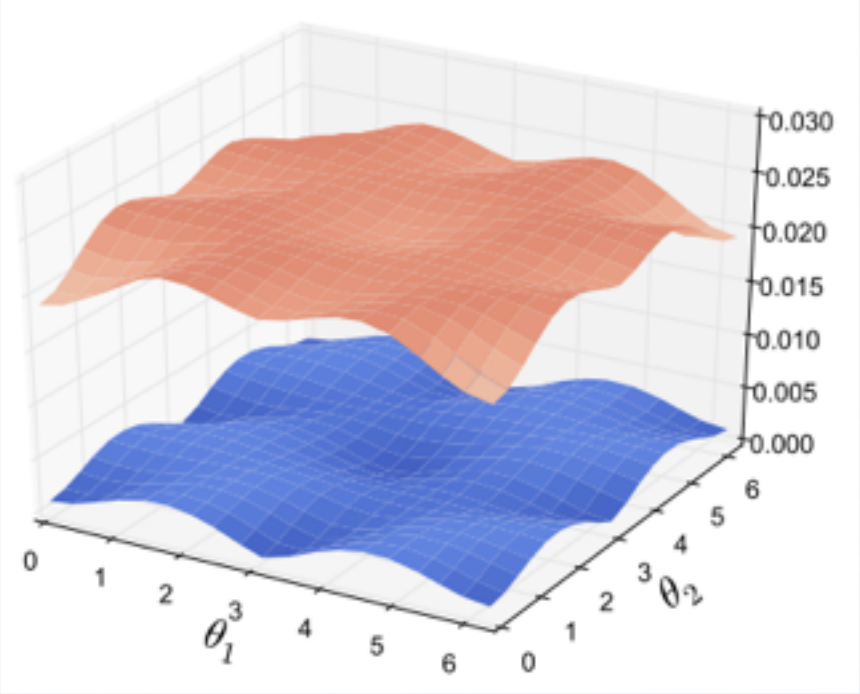
For example, we've found a spontaneously broken chiral spin-liquid (how?)

$$\nu = 1/2$$

$$\frac{m_\uparrow - m_\downarrow}{m_\uparrow + m_\downarrow} = 2/3$$

$$H_{xy} = \sum_{\langle i,j \rangle} S_i^x S_j^x + S_i^y S_j^y$$





2 Degenerate Ground State (for all twists)

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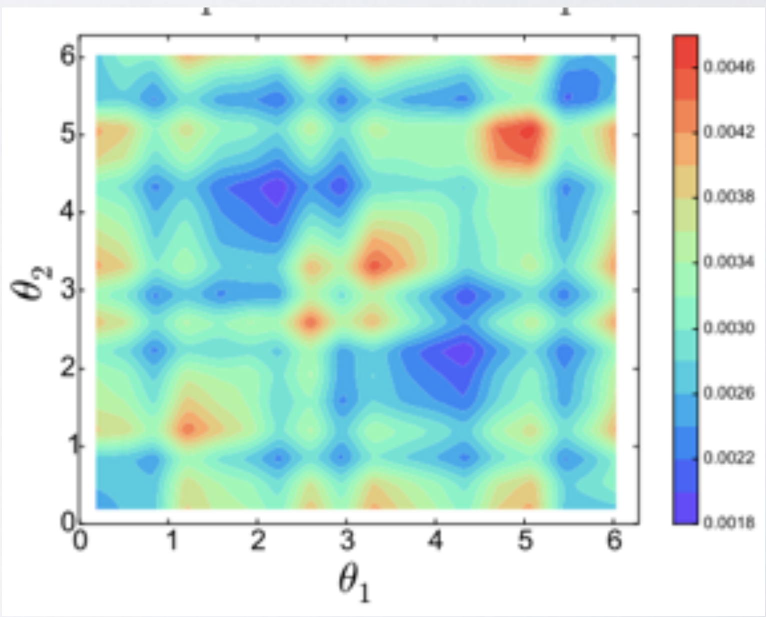
$$H_{xy} = \sum_{\langle i,j \rangle} S_i^x S_j^x + S_i^y S_j^y$$

$$C = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{2\pi} B(\theta_1, \theta_2) d\theta_1 d\theta_2$$

$$\begin{aligned} B(\theta_1,\theta_2) = \mathrm{Log}\{ & \langle \psi(\theta_1,\theta_2) | \psi(\theta_1+\delta\theta_1,\theta_2)\rangle \\ & \times \langle \psi(\theta_1+\delta\theta_1,\theta_2) | \psi(\theta_1+\delta\theta_1,\theta_2+\delta\theta_2)\rangle \\ & \times \langle \psi(\theta_1+\delta\theta_1,\theta_2+\delta\theta_2) | \psi(\theta_1,\theta_2+\delta\theta_2)\rangle \\ & \times \langle \psi(\theta_1,\theta_2+\delta\theta_2) | \psi(\theta_1,\theta_2)\rangle \} \end{aligned} \tag{4.3}$$



Chern Number: 1/2

For example, we've found a spontaneously broken chiral spin-liquid (how?)

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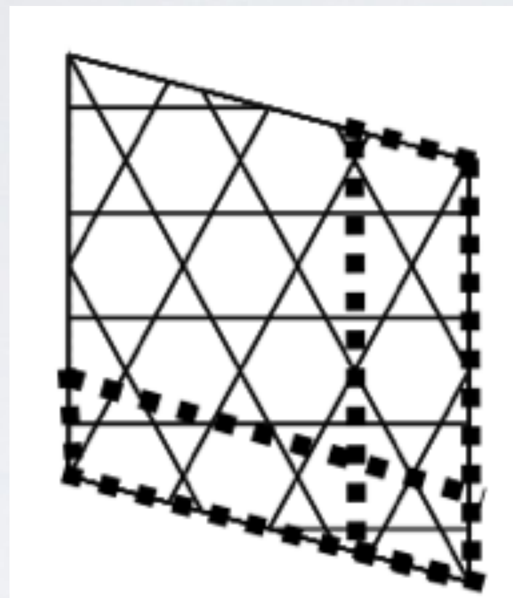
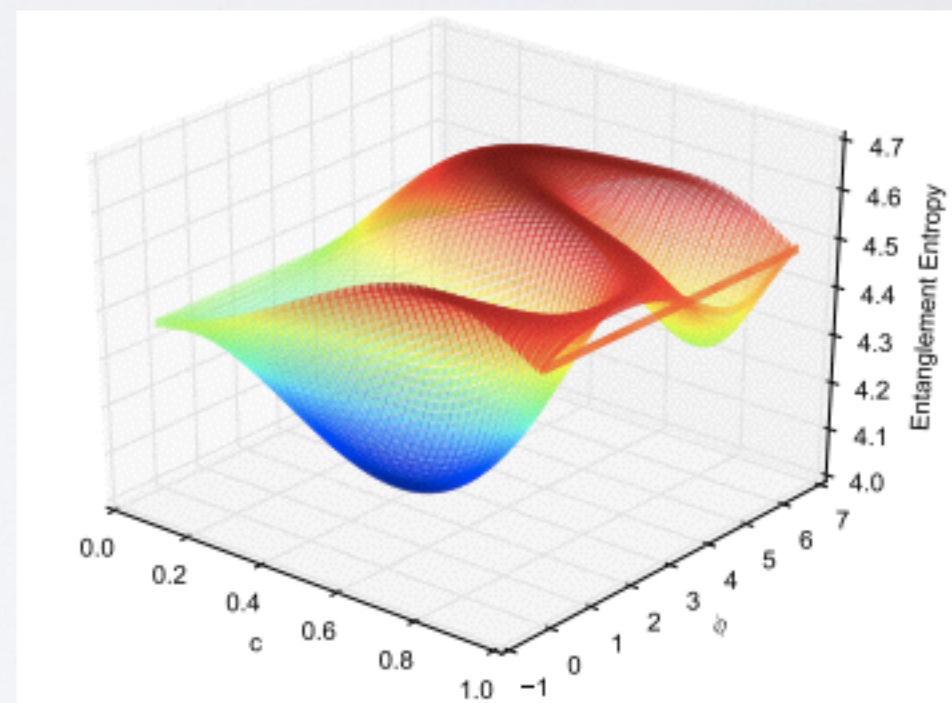


$$S = \rho_A \ln \rho_A$$

$$|GS_1> + ce^{i\theta}|GS_2>$$

Local Minimally Entangled States



Modular Matrix

$$\begin{pmatrix} 0.705 & 0.694 \\ 0.694 & -0.736e^{-i0.088} \end{pmatrix}$$

Among these, kagome stands out both experimentally and theoretically

Z2 spin liquid heisenberg (White/Huse)

Chiral spin liquid: 2/3 plateau (this work)

1/3 plateau (Donna Sheng)

Sz=0 chiral (Bela Bauer, Andreas Ludwig)

Sz=0 J1,J2,J3 (Donna Sheng)

In addition there is some experimental evidence for hyperkagome

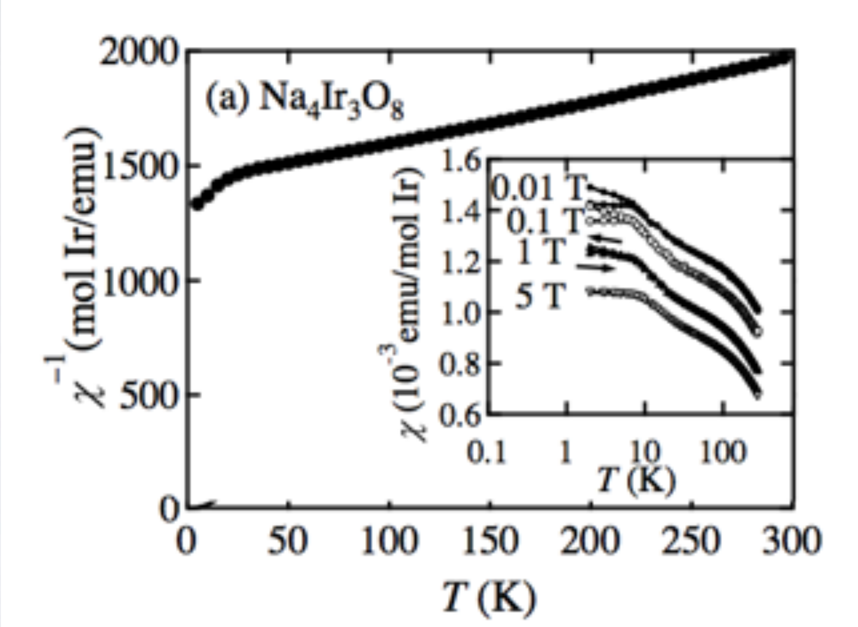
$Na_4Ir_3O_8$

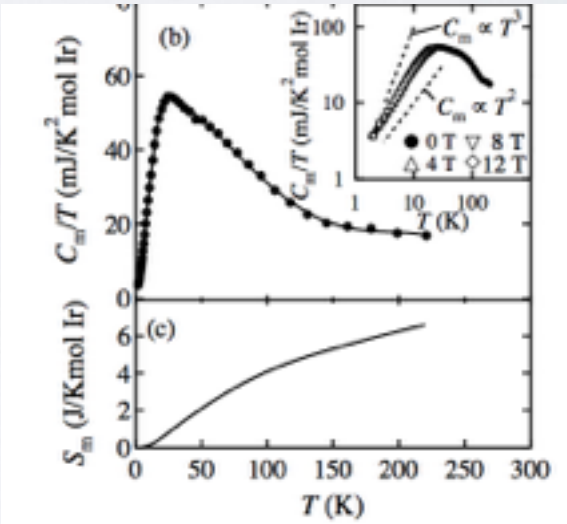
(depleted pyrochlore)

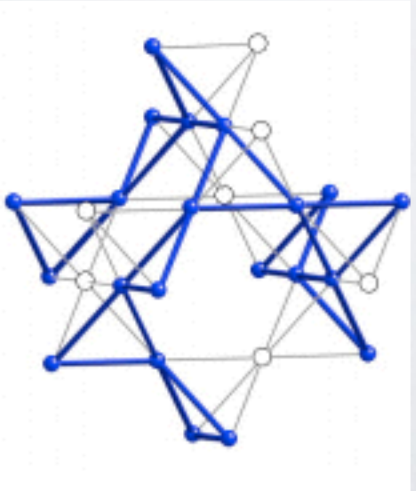
No sign of magnetic ordering down to a few Kelvin

Curie-Weiss temperature of 650K

Gapless excitations







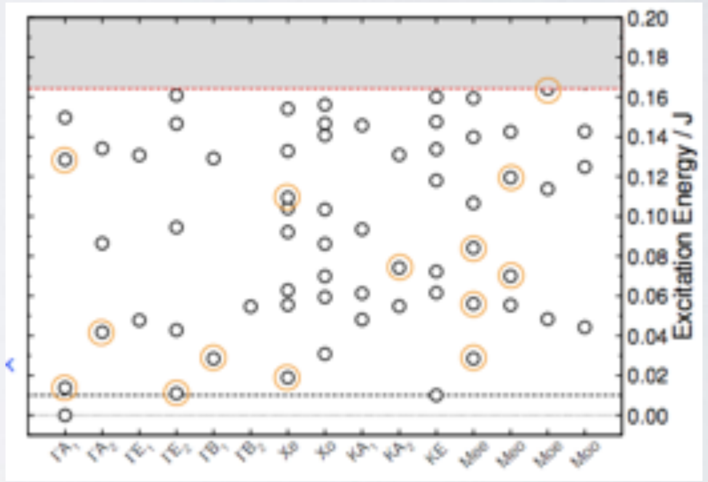
Y. Okamoto, M. Nohara, H. Aruga-Katori, and H. Takagi (2006), arXiv:0705.2821

The ising frustration doesn't seem to be a good explanation for the panalopy of spin-liquids.

(1) Why kagome and not triangular?

Both are equally frustrated in the ising limit.

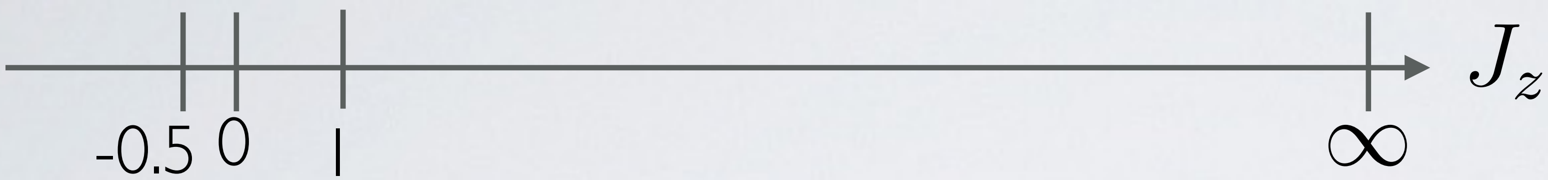
(2) Ising seems to have little to do with competing phases around the spin liquid.



(3) Mainly classical degeneracy….maybe quantum fluctuations resolve into spin-liquid but why?

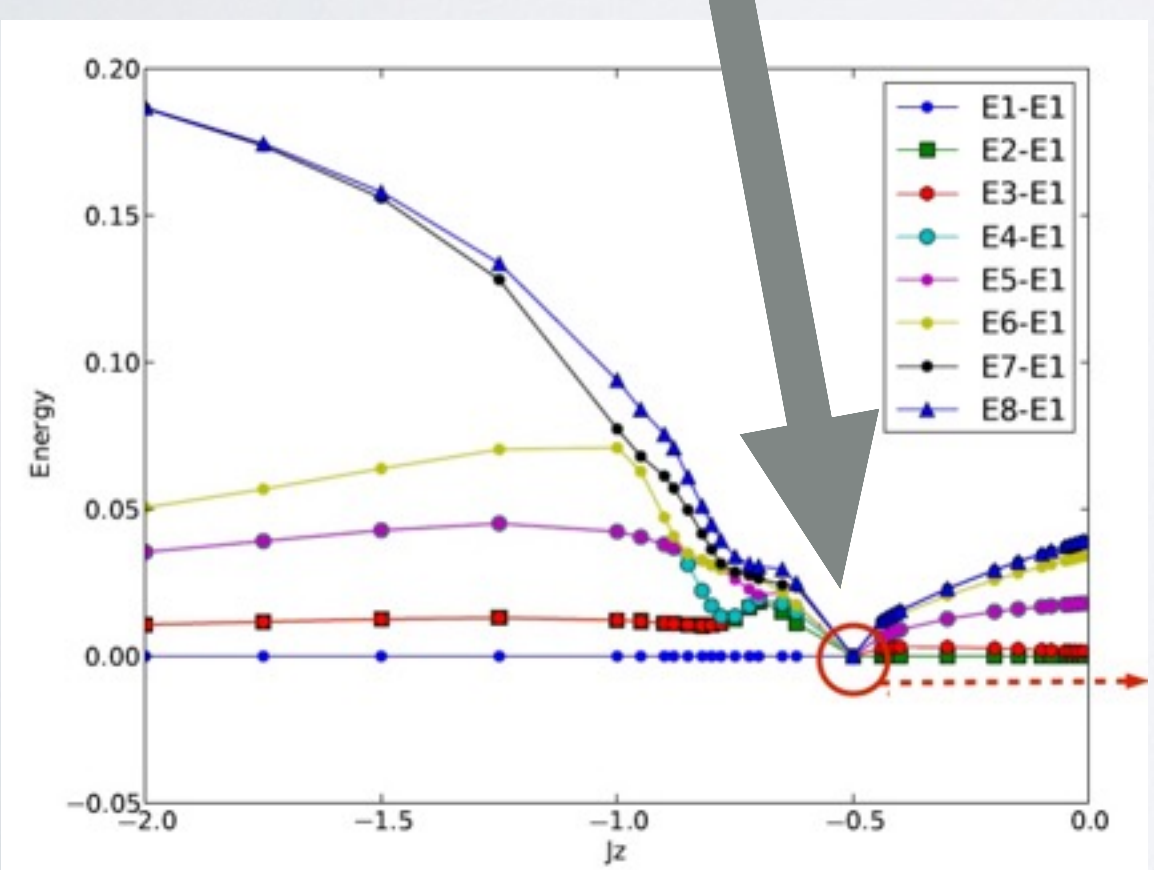
A new answer (amazing it hasn't been known for 30 years)

$$H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$



massive exact degeneracy in the XXZ model!

exactly -J/4



Who ordered that?

$$H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$

Who ordered that?

$$H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$

$$|1\rangle \equiv |\uparrow\uparrow\uparrow\rangle$$

$$|2\rangle \equiv \frac{1}{\sqrt{3}}\Big(|\uparrow\uparrow\downarrow\rangle + \omega|\uparrow\downarrow\uparrow\rangle + \omega^2|\downarrow\uparrow\uparrow\rangle\Big)$$

$$|3\rangle \equiv \frac{1}{\sqrt{3}}\Big(|\uparrow\uparrow\downarrow\rangle + \omega^2|\uparrow\downarrow\uparrow\rangle + \omega|\downarrow\uparrow\uparrow\rangle\Big)$$

$$|4\rangle \equiv \frac{1}{\sqrt{3}}\Big(|\downarrow\downarrow\uparrow\rangle + \omega|\downarrow\uparrow\downarrow\rangle + \omega^2|\uparrow\downarrow\downarrow\rangle\Big)$$

$$|5\rangle \equiv \frac{1}{\sqrt{3}}\Big(|\downarrow\downarrow\uparrow\rangle + \omega^2|\downarrow\uparrow\downarrow\rangle + \omega|\uparrow\downarrow\downarrow\rangle\Big)$$

$$|6\rangle \equiv |\downarrow\downarrow\downarrow\rangle$$

Who ordered that?

$$H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$

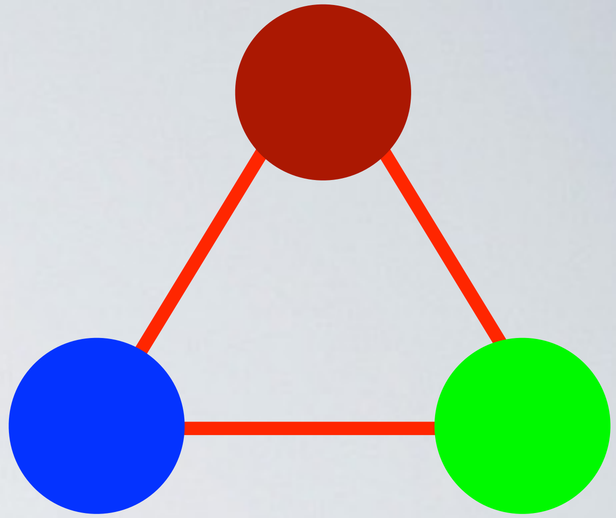
$$|+\rangle \equiv \frac{1}{\sqrt{3}}\Big(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle\Big)$$

$$|-\rangle \equiv \frac{1}{\sqrt{3}}\Big(|\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\rangle\Big)$$

$$H_{\text{tri}} = -\frac{3J}{8}\sum_{i=1}^{6} |i\rangle\langle i| + \frac{9J}{8}(|+\rangle\langle +| + |-\rangle\langle -|)$$

$$-\frac{3J}{8}(1 - |+\rangle\langle +| - |-\rangle\langle -|)$$

Who ordered that?



$$|+\rangle \equiv \frac{1}{\sqrt{3}}\left(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle\right)$$
$$|-\rangle \equiv \frac{1}{\sqrt{3}}\left(|\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\rangle\right)$$

$$-\frac{3J}{8} + \frac{3J}{2}(|+\rangle\langle+| + |-\rangle\langle-|)$$

Projectors

Constant

Positive coefficient

We want to **minimize the energy** by zeroing out the projectors

Frustration Free!

Many Triangles

$$H = \sum_{\text{tri}} H_{\text{tri}} = \frac{3}{2} \sum_{\text{tri}} P_{\text{tri}} - \frac{3}{8} N_{\text{tri}}$$

$$P_{\text{tri}} \equiv |+\rangle\langle+| + |-\rangle\langle-|$$

NOTE: projectors on triangles **DO NOT** commute with each other!!!

So an arbitrary eigenstate on one triangle need not be COMPATIBLE with other triangles

$$H = \sum_{\text{tri}} H_{\text{tri}} = \frac{3}{2} \sum_{\text{tri}} P_{\text{tri}} - \frac{3}{8} N_{\text{tri}}$$

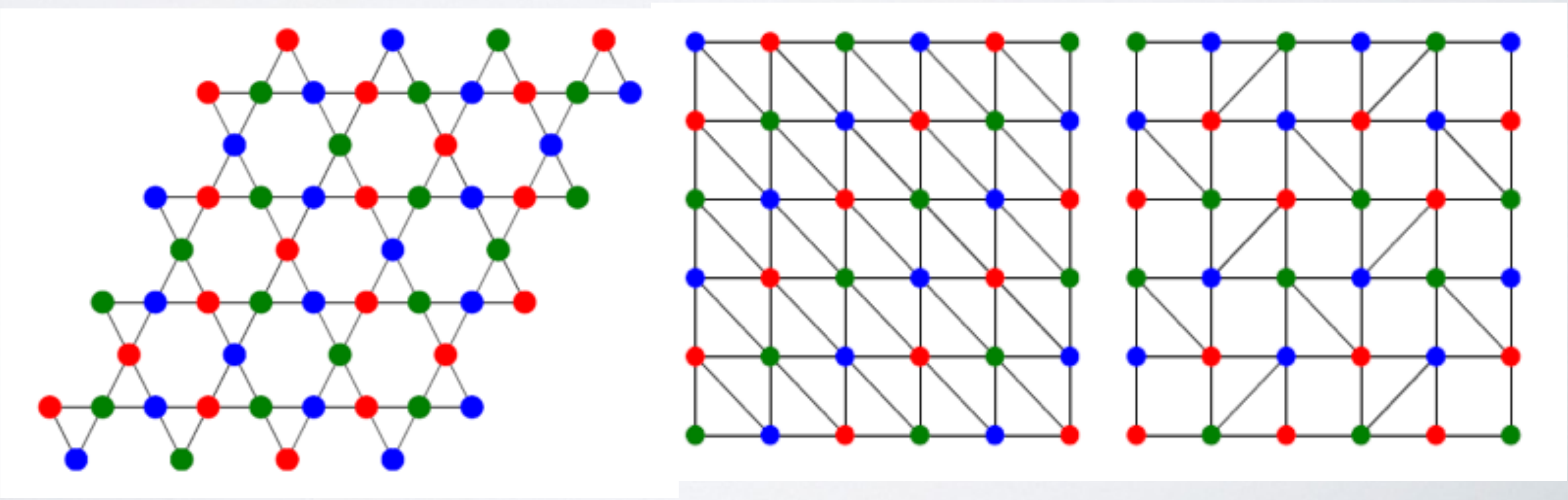
$$|a\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle + |\downarrow\rangle\Big)$$
$$|b\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle + \omega|\downarrow\rangle\Big)$$
$$|c\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle + \omega^2|\downarrow\rangle\Big)$$

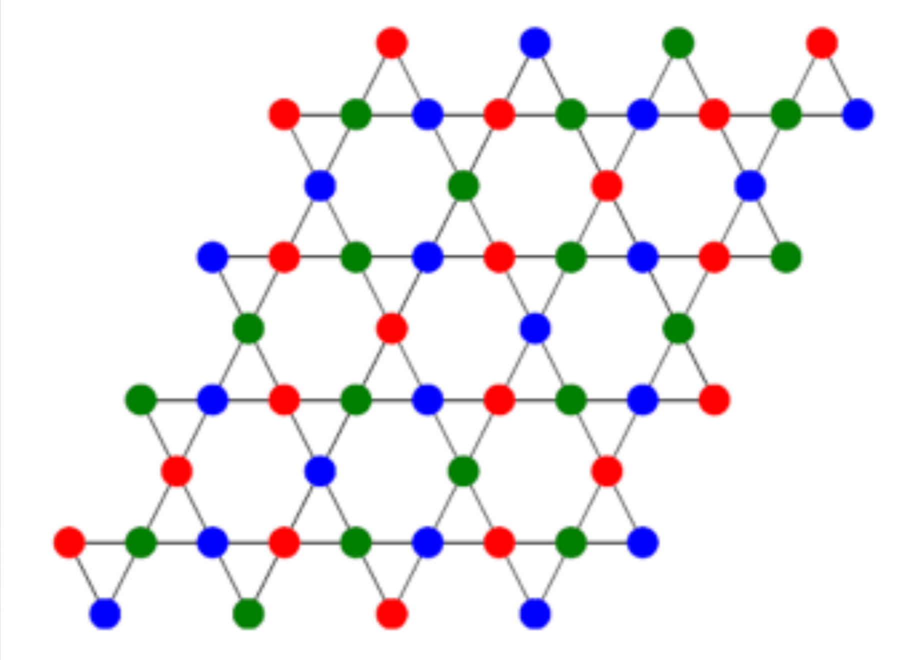
$$P_{\text{tri}} \equiv |+\rangle\langle+| + |-\rangle\langle-|$$

We want projector to annihilate our proposed solution

$$|\psi\rangle \equiv \prod_s \otimes |C_s\rangle_s$$

As long as we have ONLY one color per triangle the tensor product of all colors is an EXACT eigenstate





$$|a\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle + |\downarrow\rangle\Big)$$

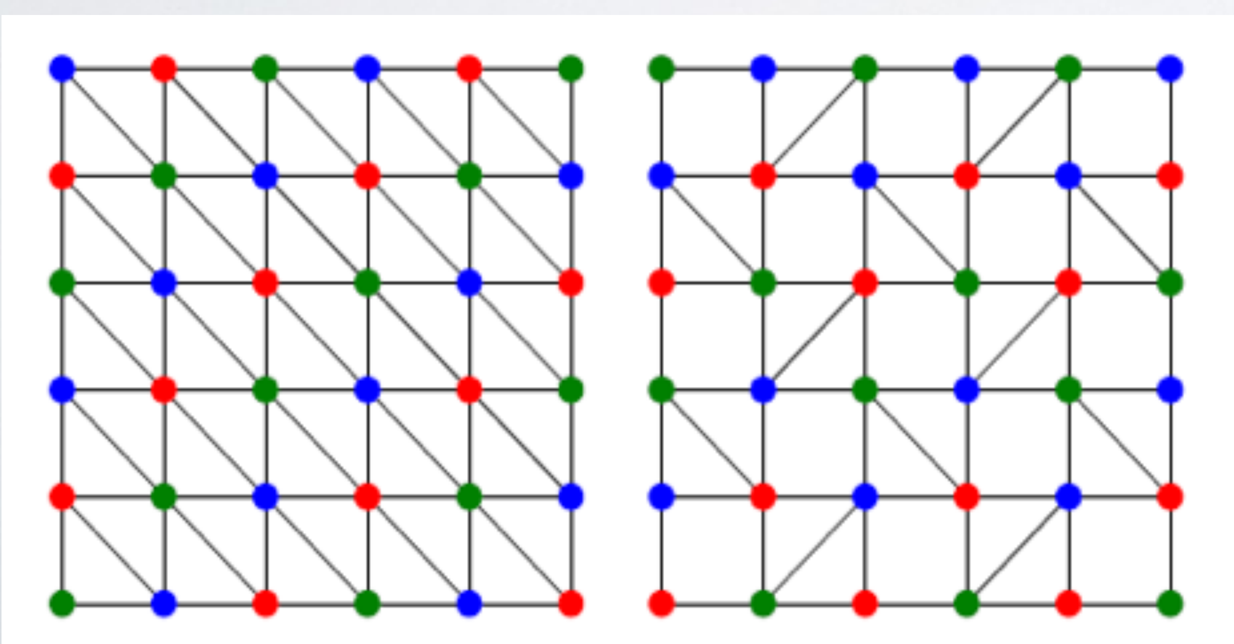
$$|b\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle + \omega|\downarrow\rangle\Big)$$

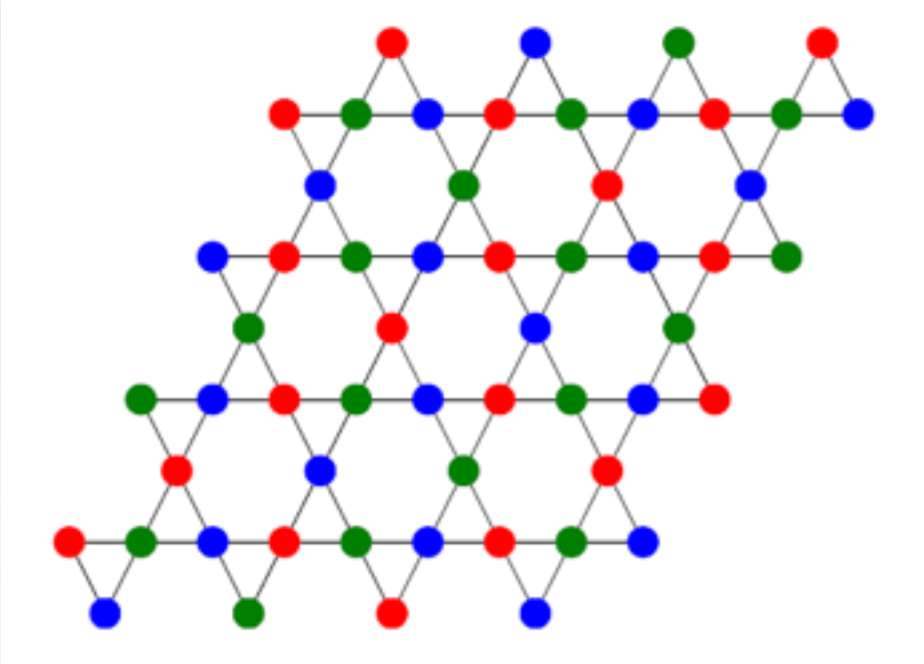
$$|c\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle + \omega^2|\downarrow\rangle\Big)$$

An exponential number of colorings! 1.208^N (from Baxter)

$$|\psi\rangle \equiv \prod_s \otimes |C_s\rangle_s$$

Only one (or two) colorings.





$$|a\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle + |\downarrow\rangle\Big)$$

$$|b\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle + \omega|\downarrow\rangle\Big)$$

$$|c\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle + \omega^2|\downarrow\rangle\Big)$$

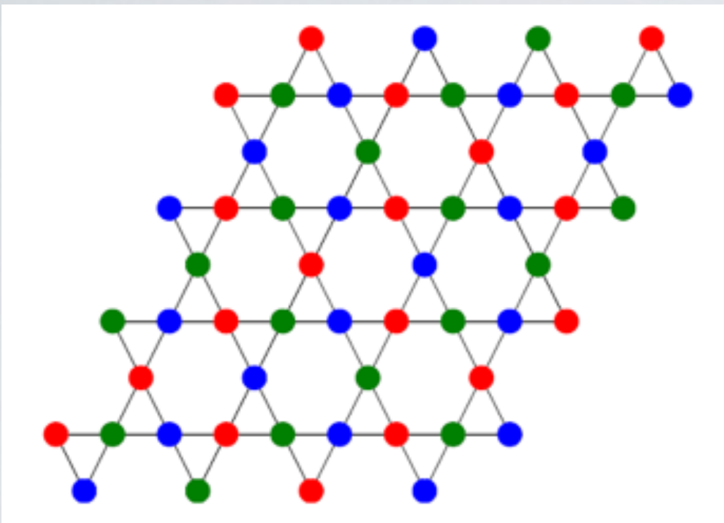
An exponential number of colorings!

$$|\psi\rangle \equiv \prod_s \otimes |C_s\rangle_s$$

But this mixes Sz sectors, (particle number in boson language)

Eigenstates in a fixed Sz sector

Lattice	Ising configs	Colorings
2x2x3	924	8
3x2x3	48620	16
4x2x3	2.7 million	32



$$|\psi\rangle \equiv \prod_s \otimes |C_s\rangle_s$$

But this mixes Sz sectors, (particle number in boson language)

⇩ Project to definite sector

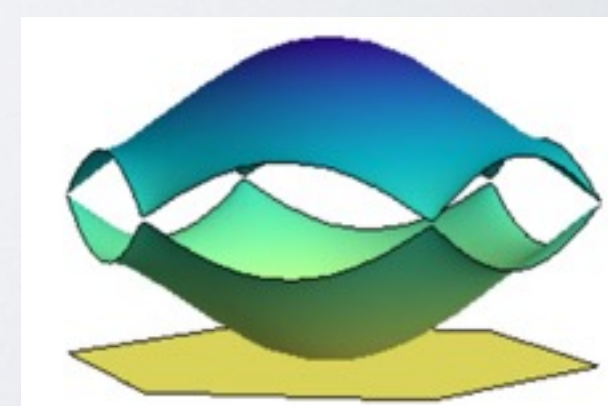
$$|\psi^C\rangle \equiv P_{S_z}\left(\prod_{\text{valid}} \otimes |C_s\rangle\right)$$

These are all still eigenstates!

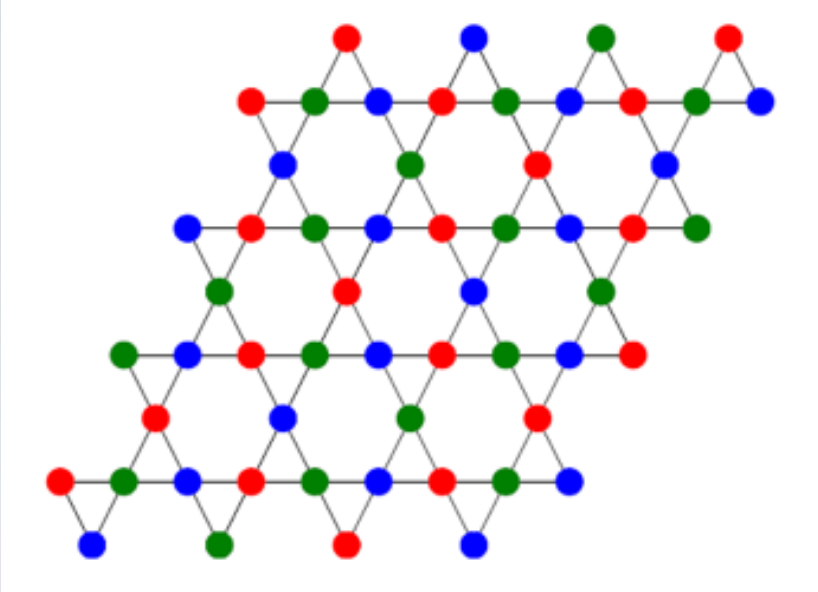
Modes may be linearly dependent. Their rank may be less then the number of colorings.

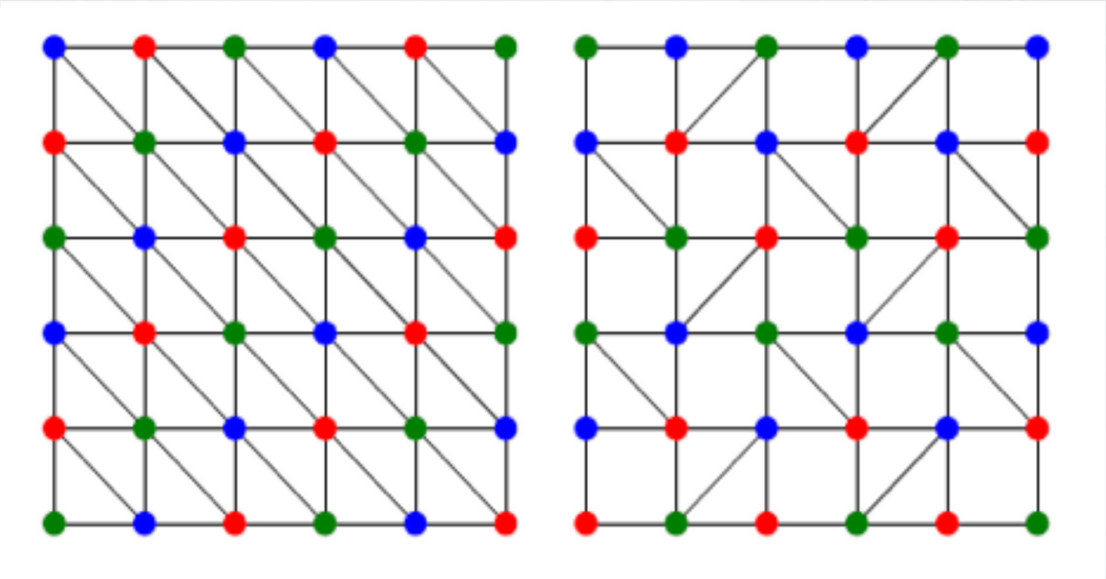
Additional Subtelty: These are not always all the eigenstates.

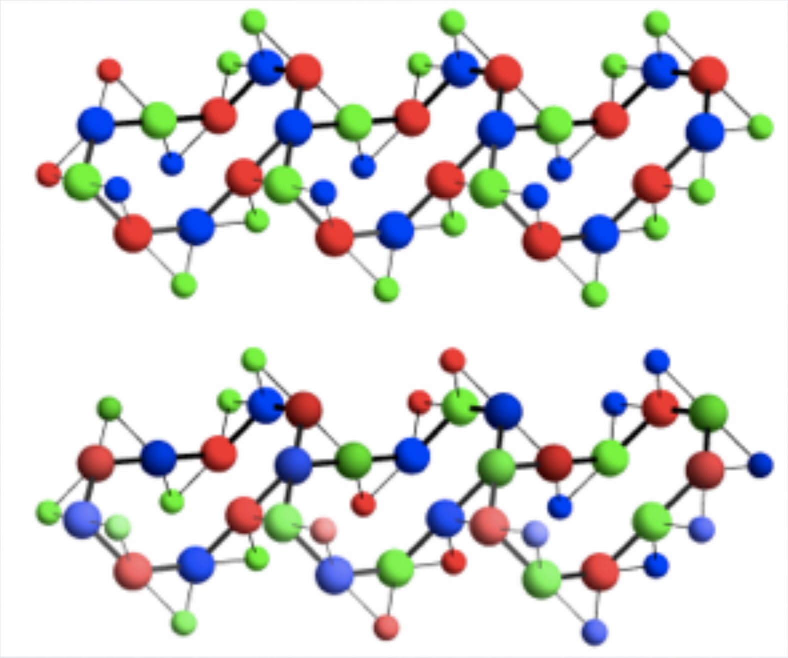
The “one-boson” particle number sector reproduces the known flat band.



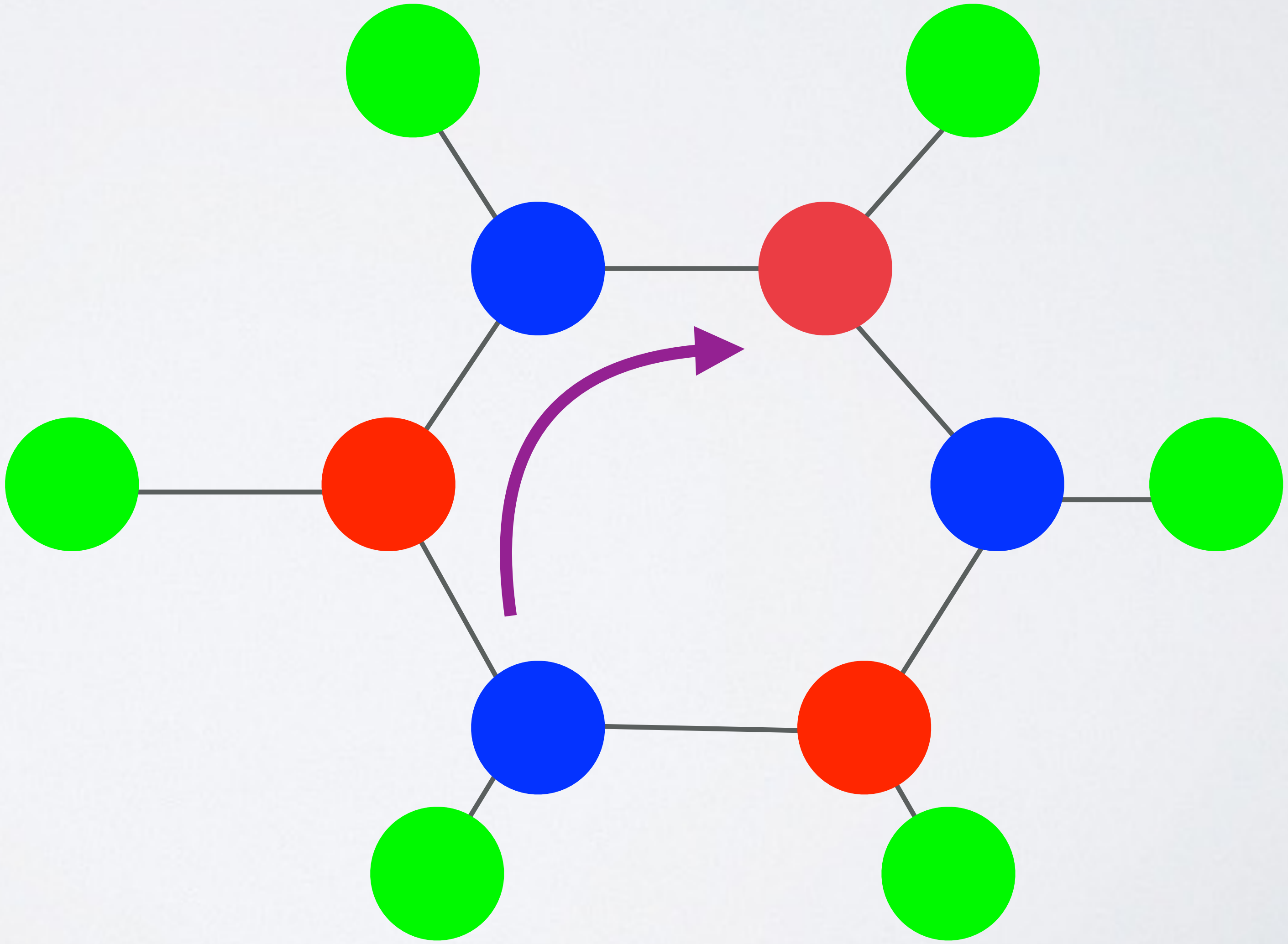
Quantum coloring



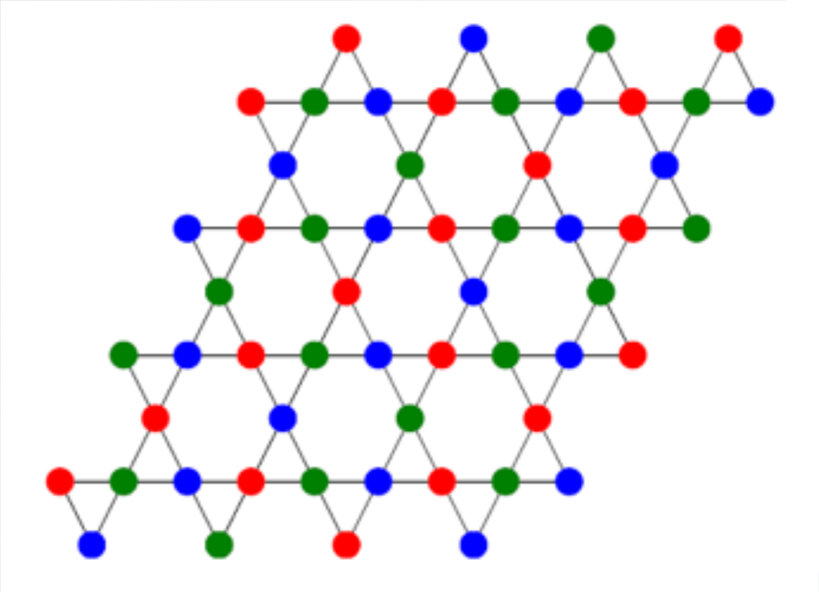


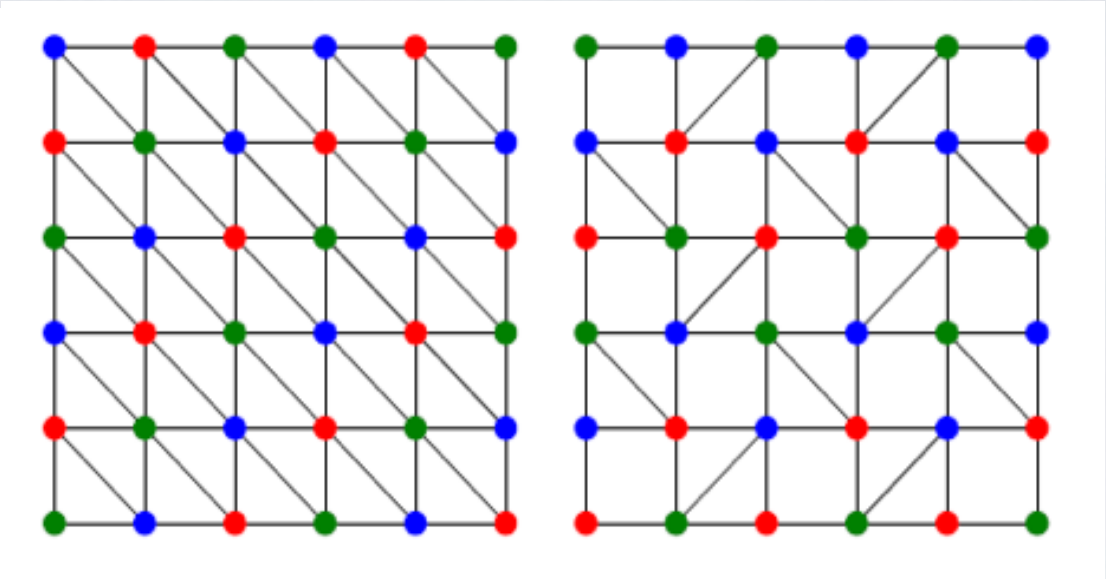


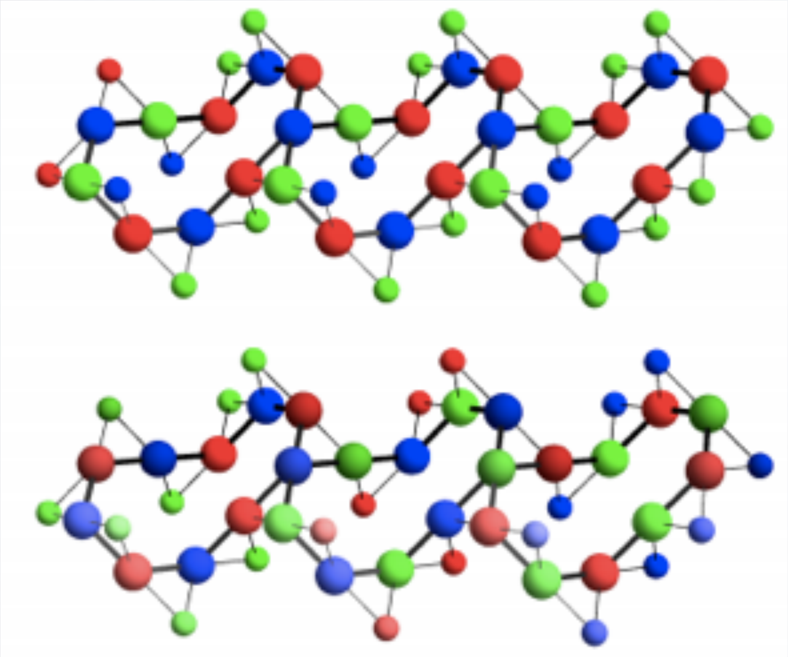
Consider kagome…



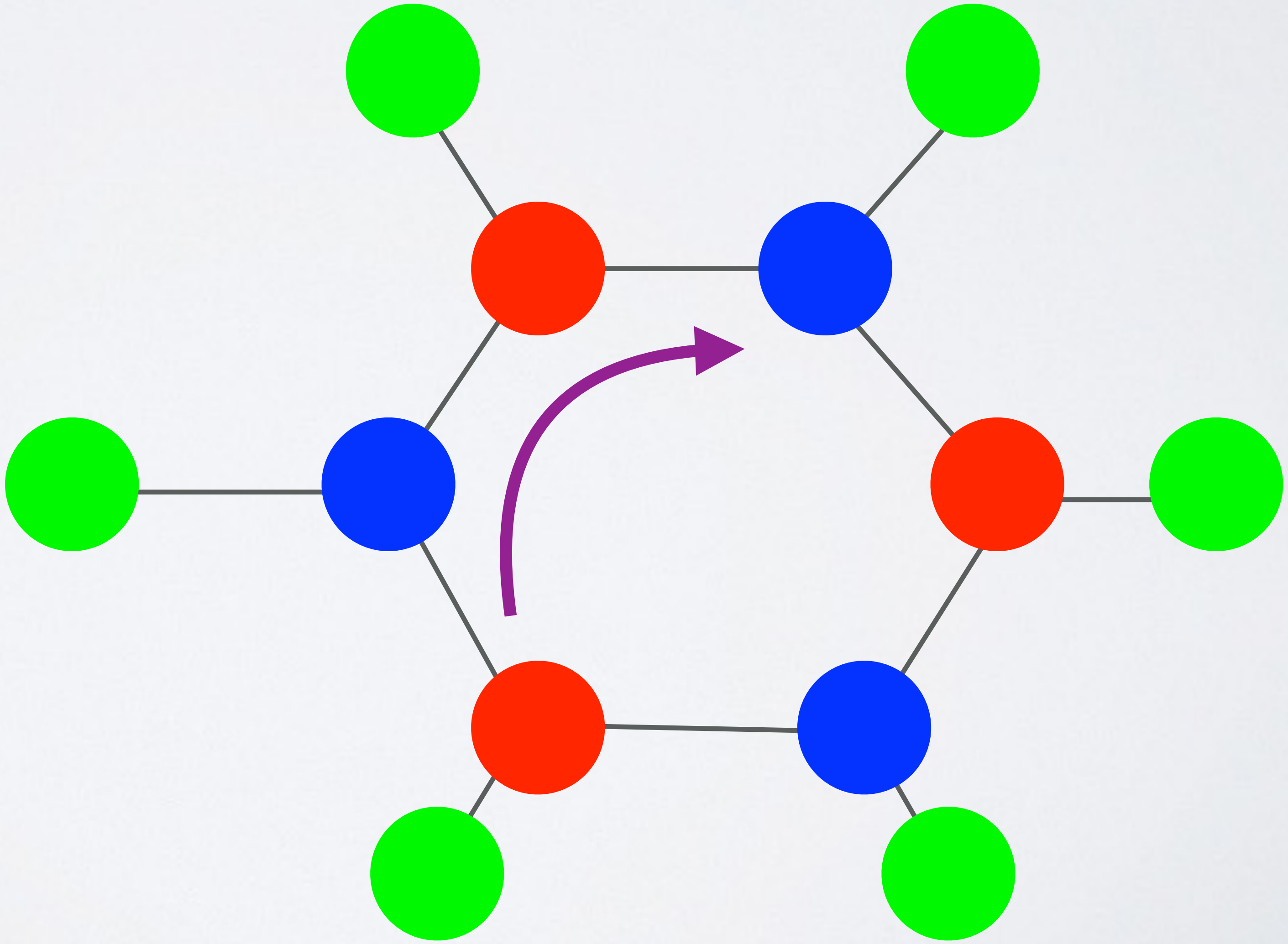
Quantum coloring







Consider kagome…



Kagome has flat bands for one particle.

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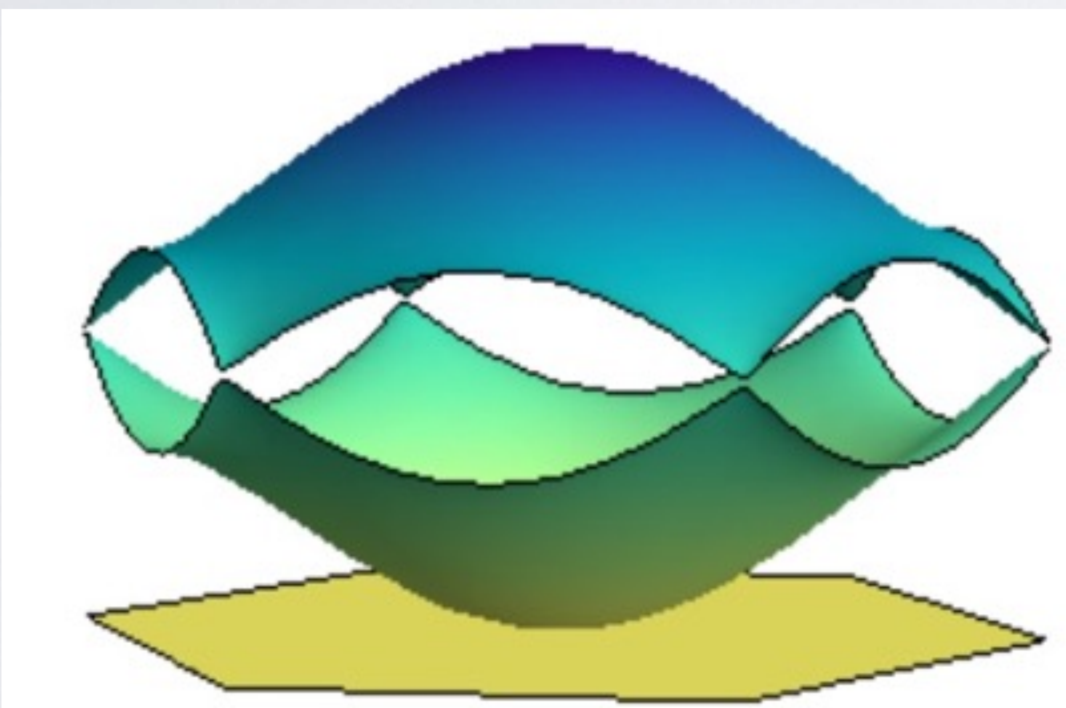
Band touching from real-space topology in frustrated hopping models

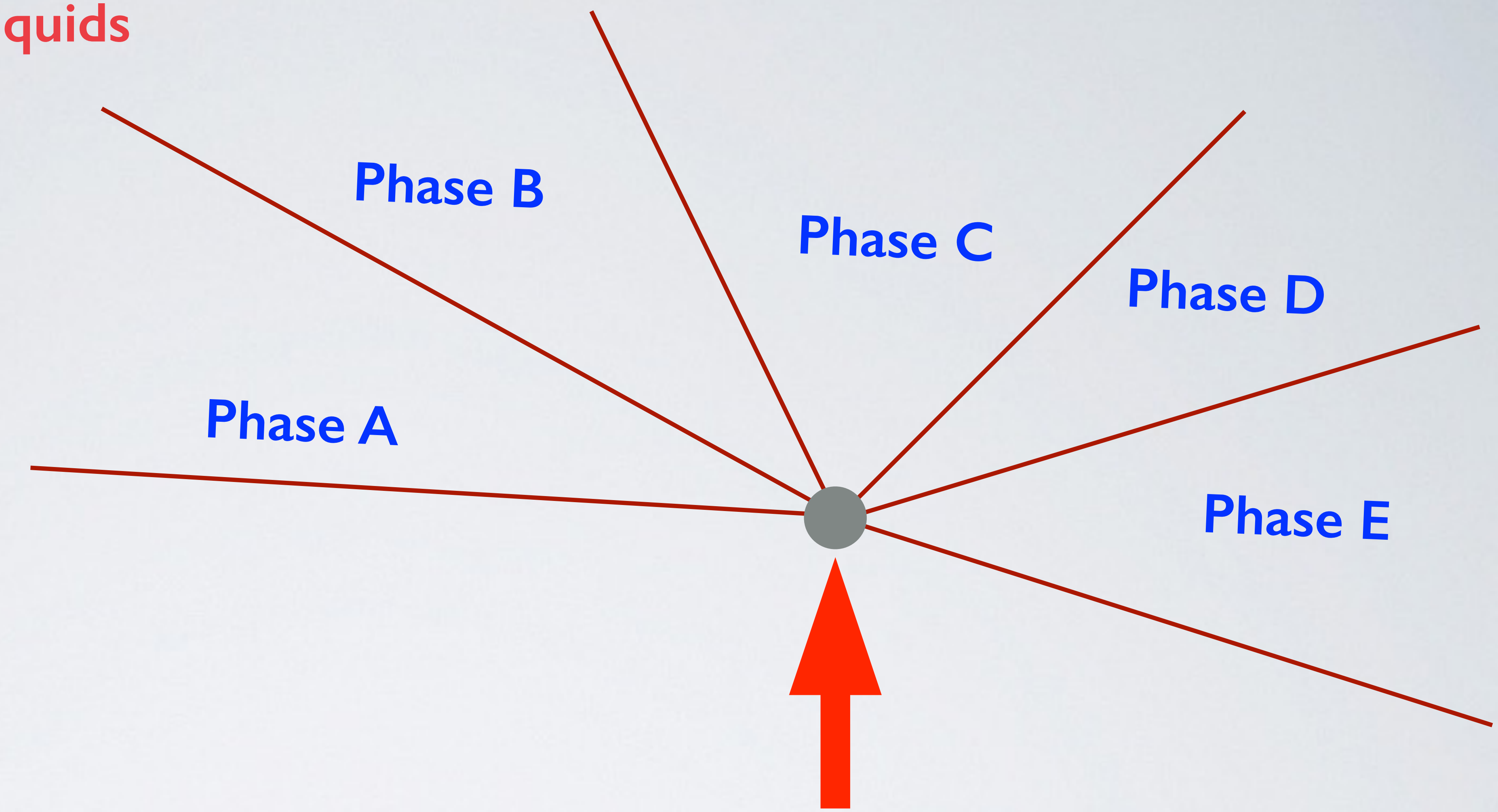
Doron L. Bergman,[1] Congjun Wu,[2] and Leon Balents[3]

$$\mathcal{H}_t = -t\sum_{\langle ij\rangle}(c_i^\dagger c_j + \text{H.c.}),$$

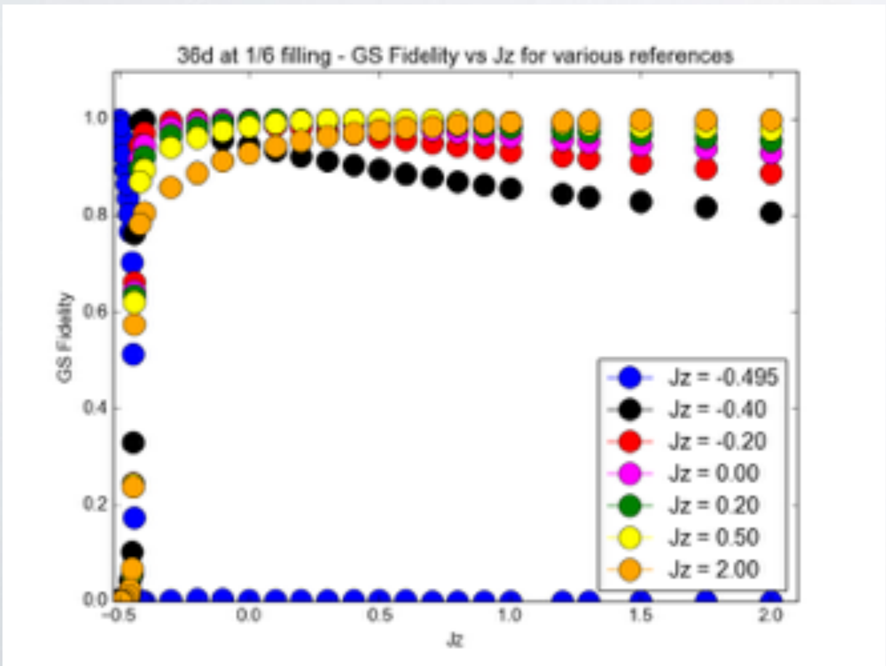


Project into one spin up

Connected to known spin liquids

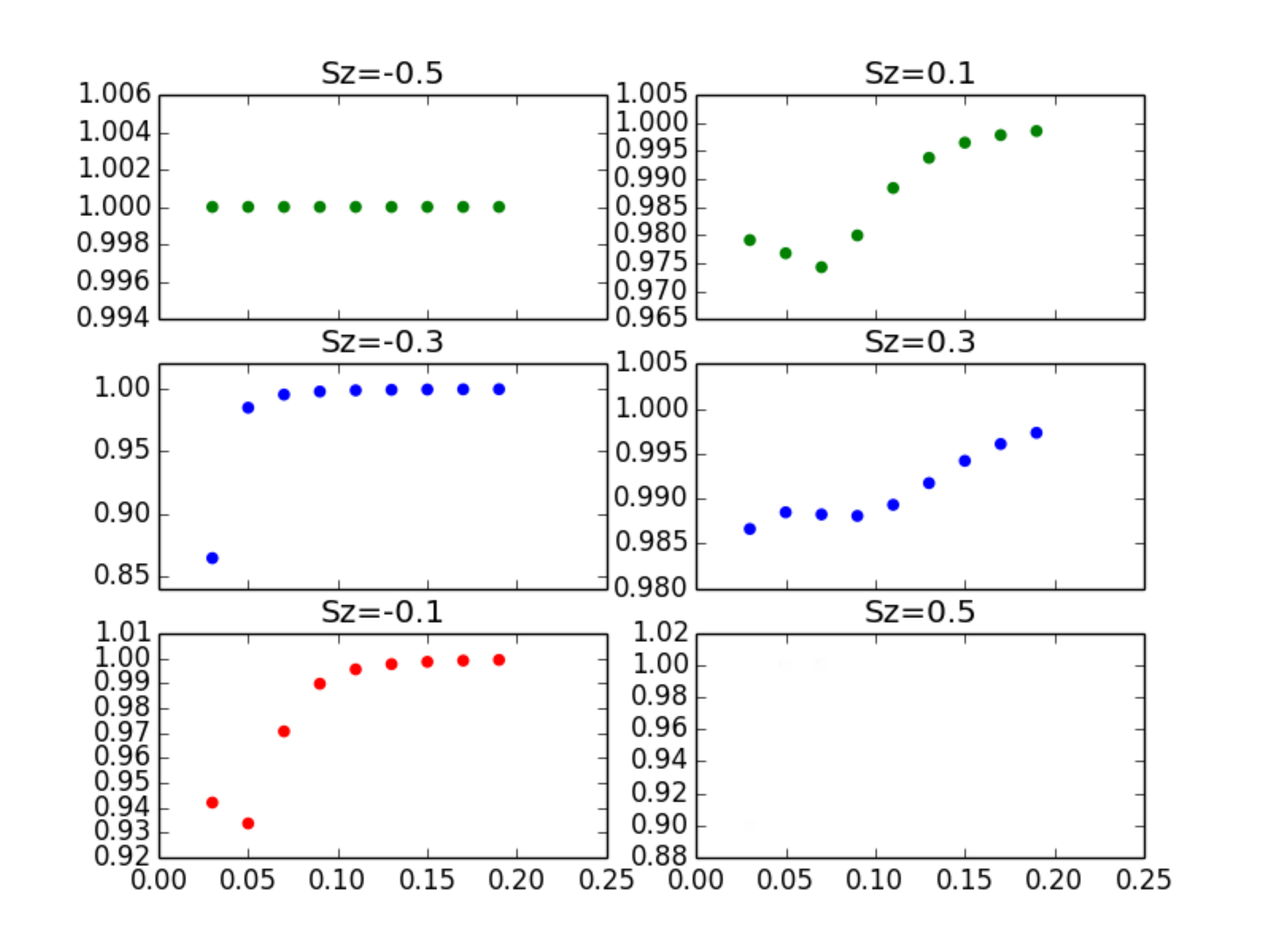


The well-spring of all phases?



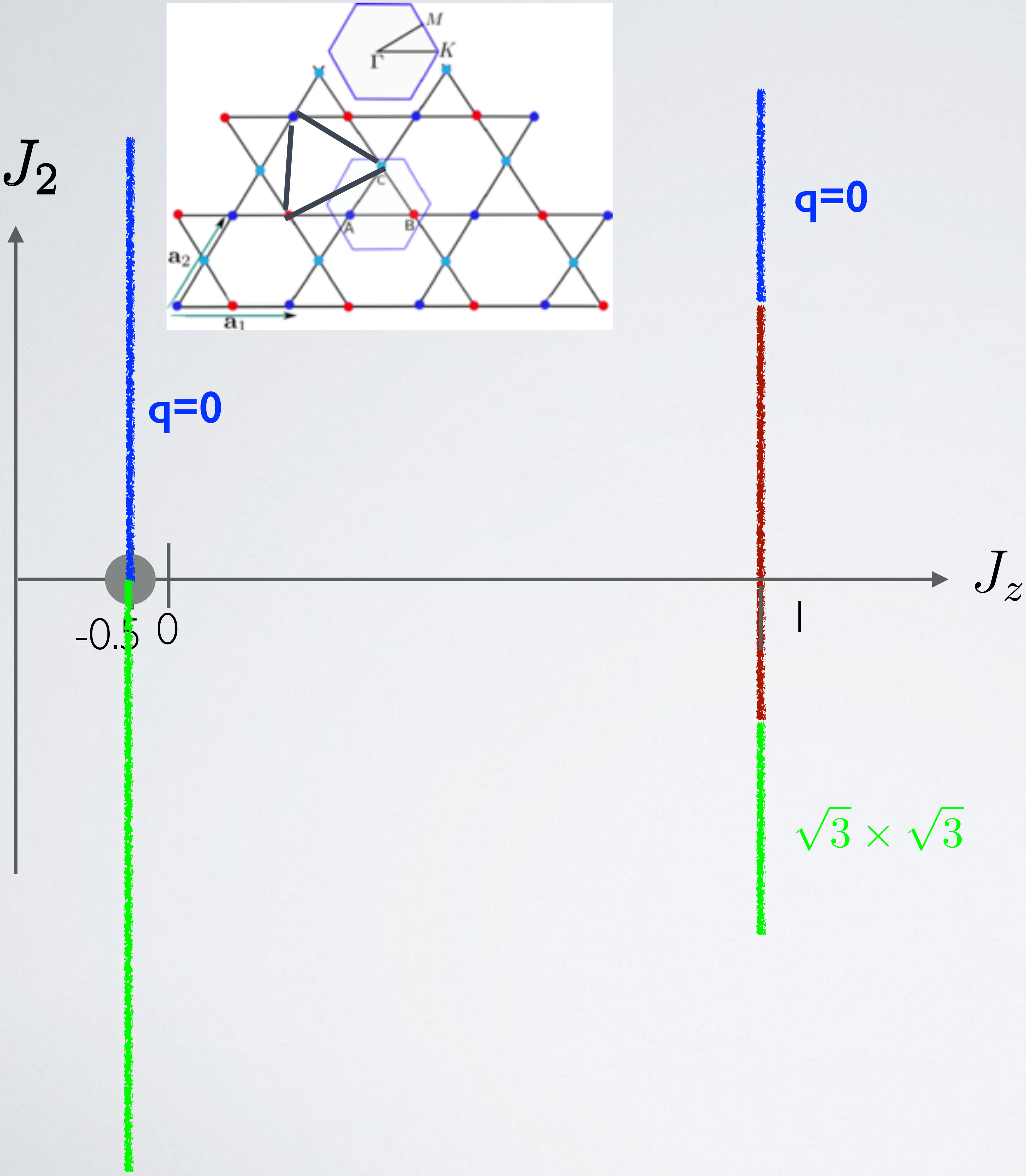
Connected to chiral spin liquid.

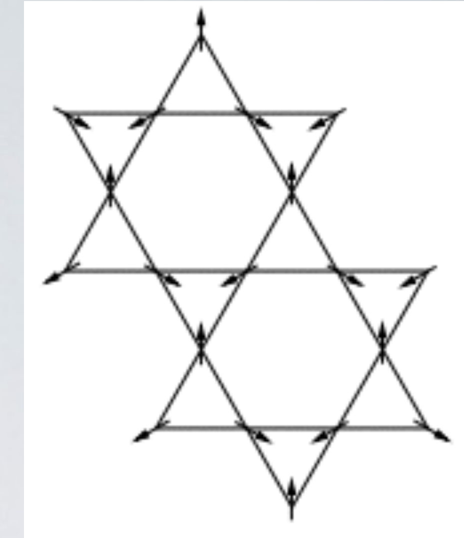
Fidelity

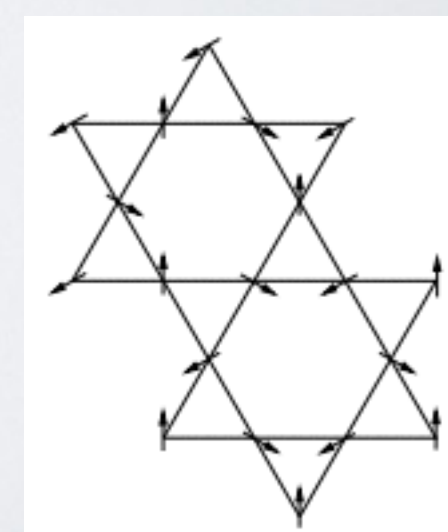


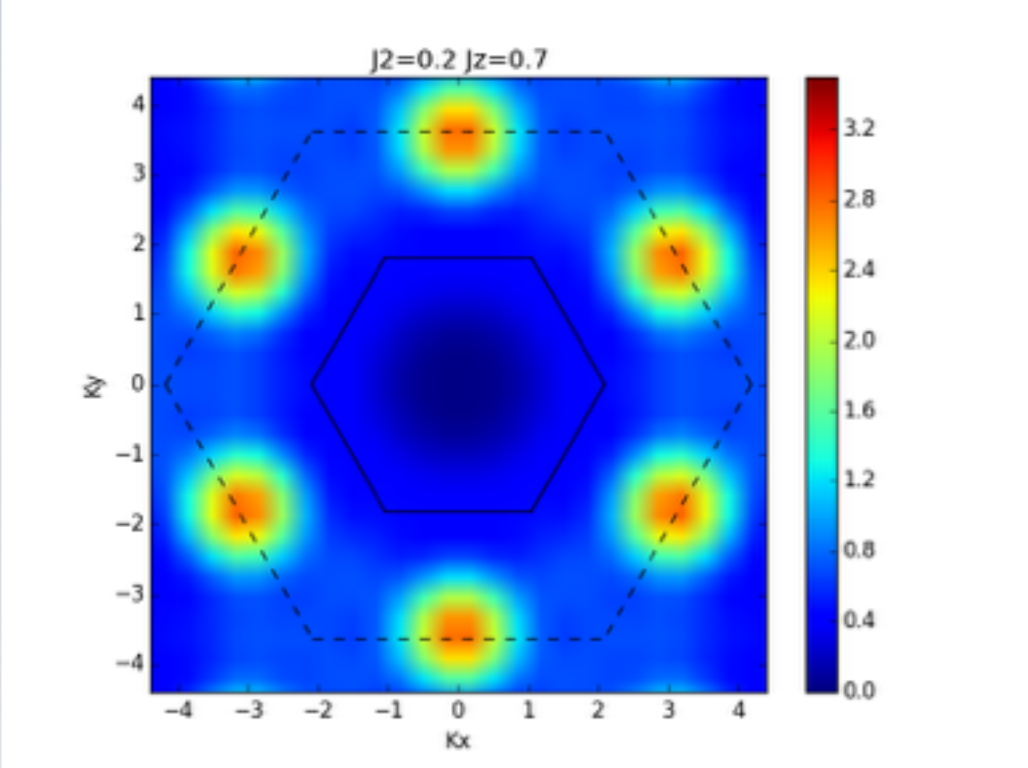
Sz=-0.5
1.006
1.004
1.002
1.000
0.998
0.996
0.994
Sz=0.1
1.005
1.000
0.995
0.990
0.985
0.980
0.975
0.970
0.965
Sz=-0.3
1.00
0.95
0.90
0.85
Sz=0.3
1.005
1.000
0.995
0.990
0.985
0.980
Sz=-0.1
1.01
1.00
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
0.00 0.05 0.10 0.15 0.20 0.25
Sz=0.5
1.02
1.00
0.98
0.96
0.94
0.92
0.90
0.88
0.00 0.05 0.10 0.15 0.20 0.25

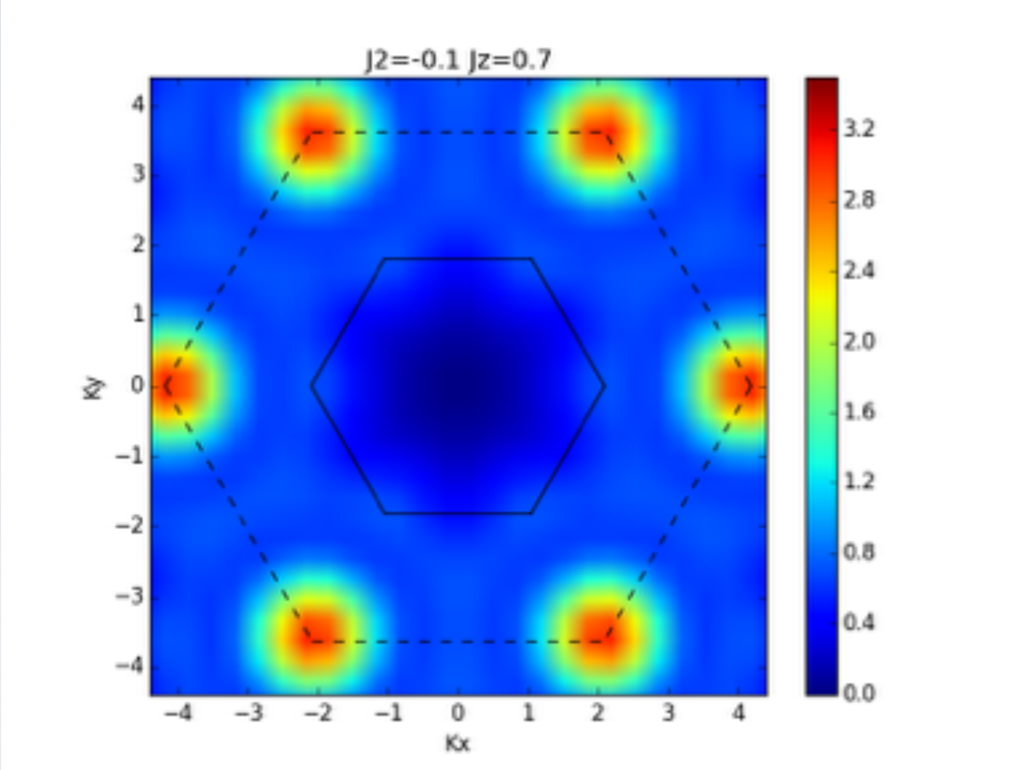
The "Herbertsmithite" spin-liquid

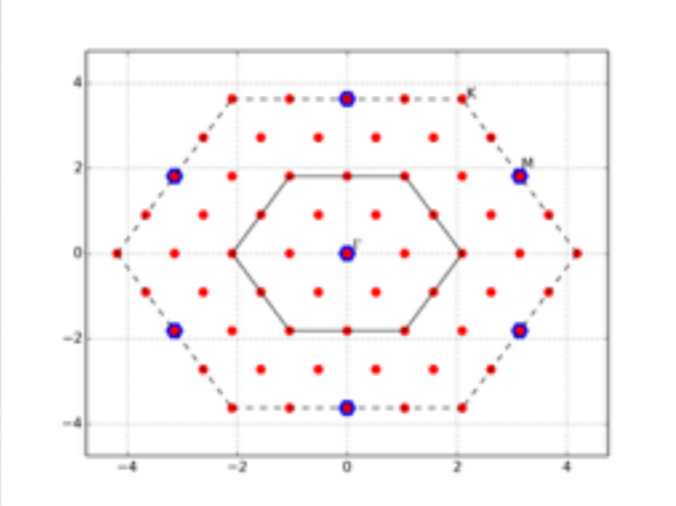




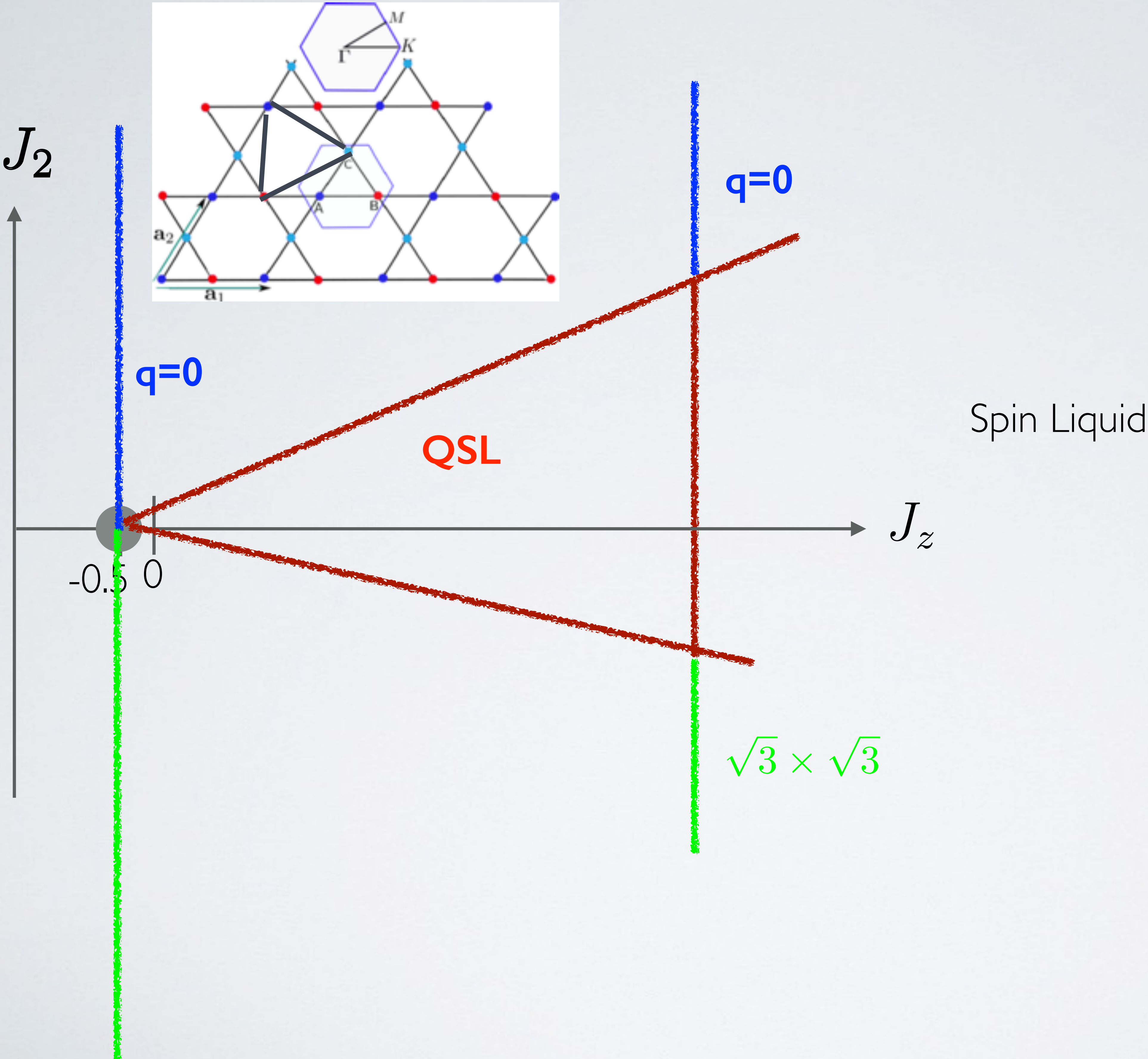



J2=0.2 Jz=0.7
Ky
Kx

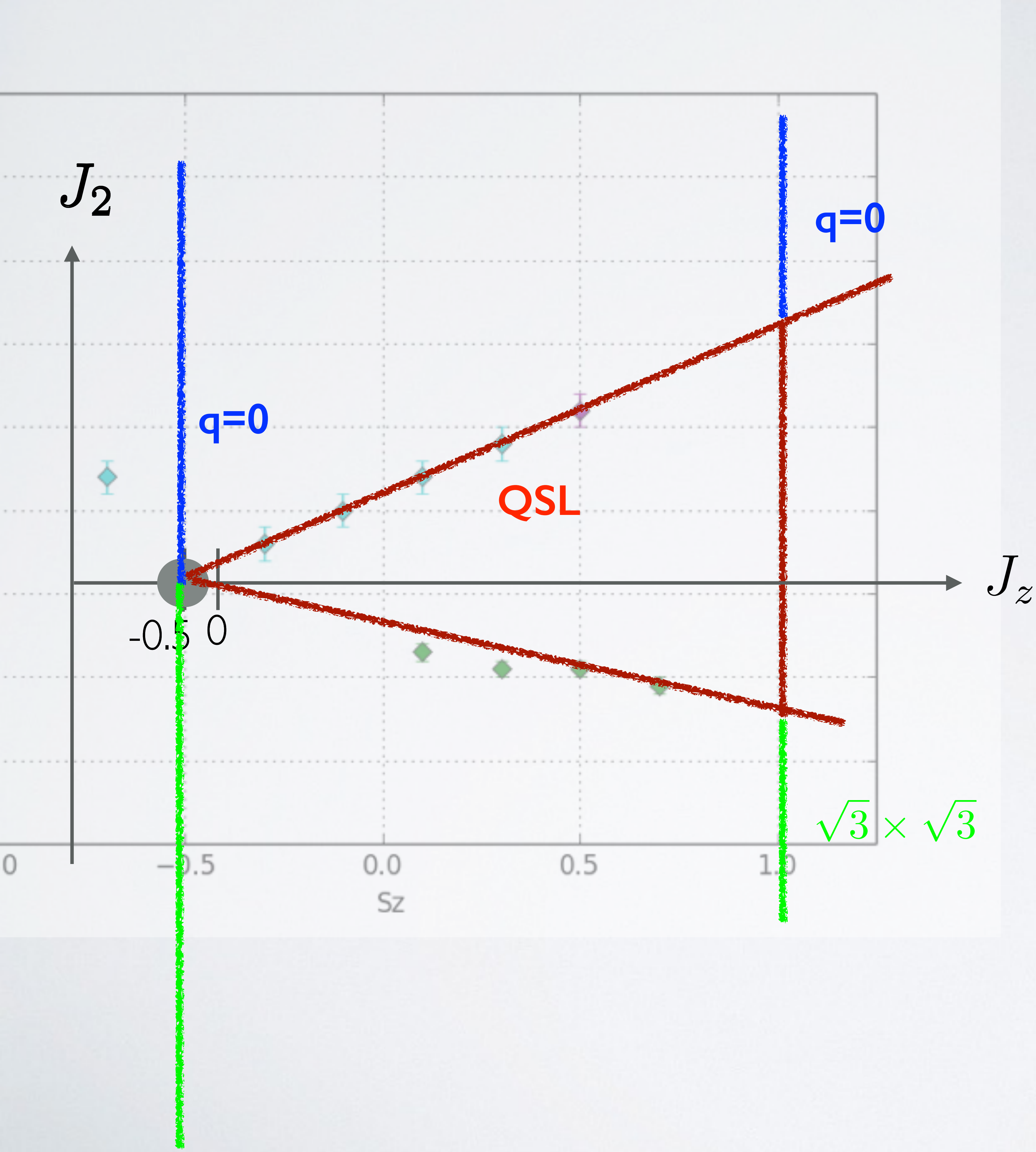

J2=-0.1 Jz=0.7
Ky
Kx



The "Herbertsmithite" spin-liquid

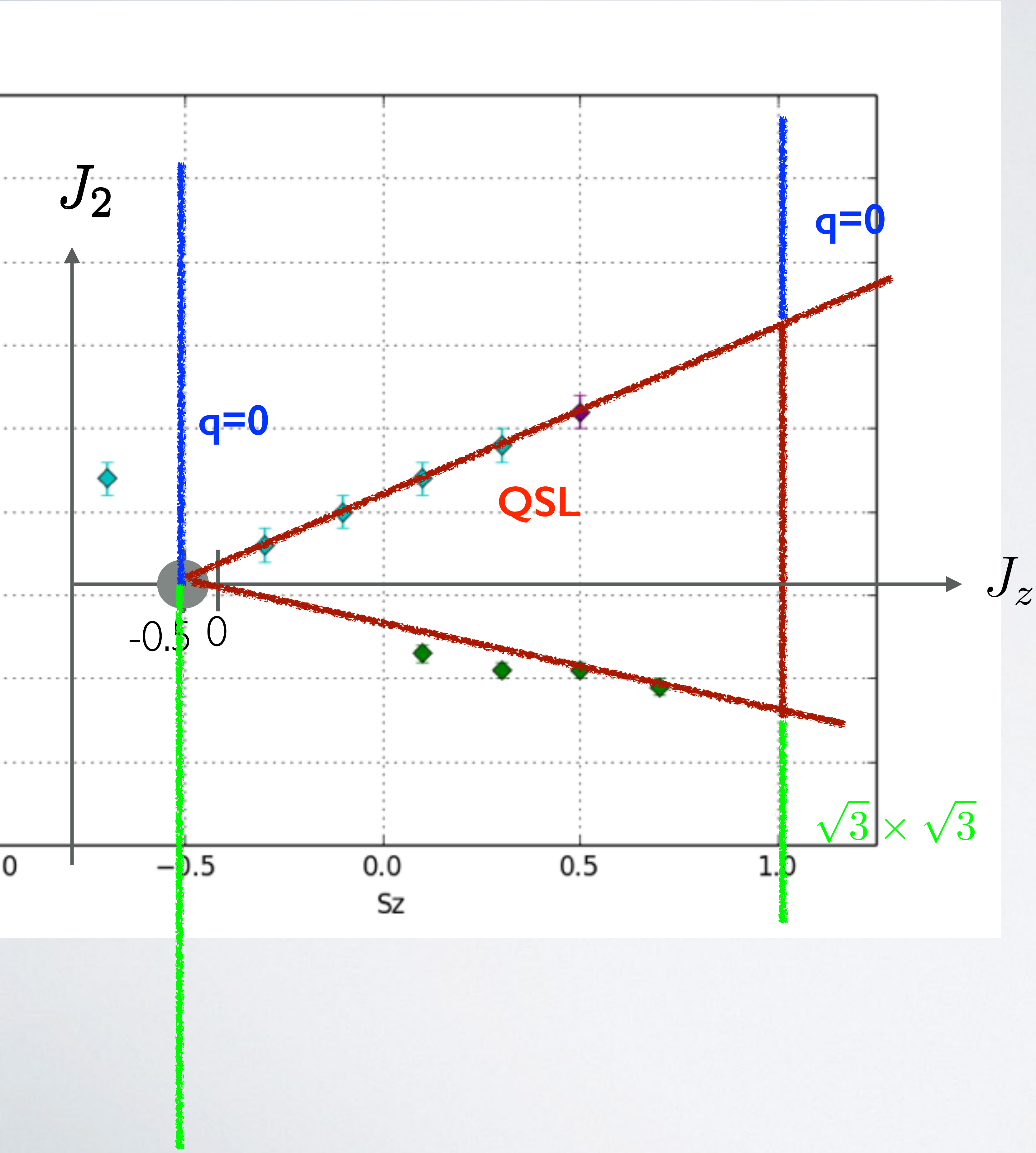


The "Herbertsmithite" spin-liquid



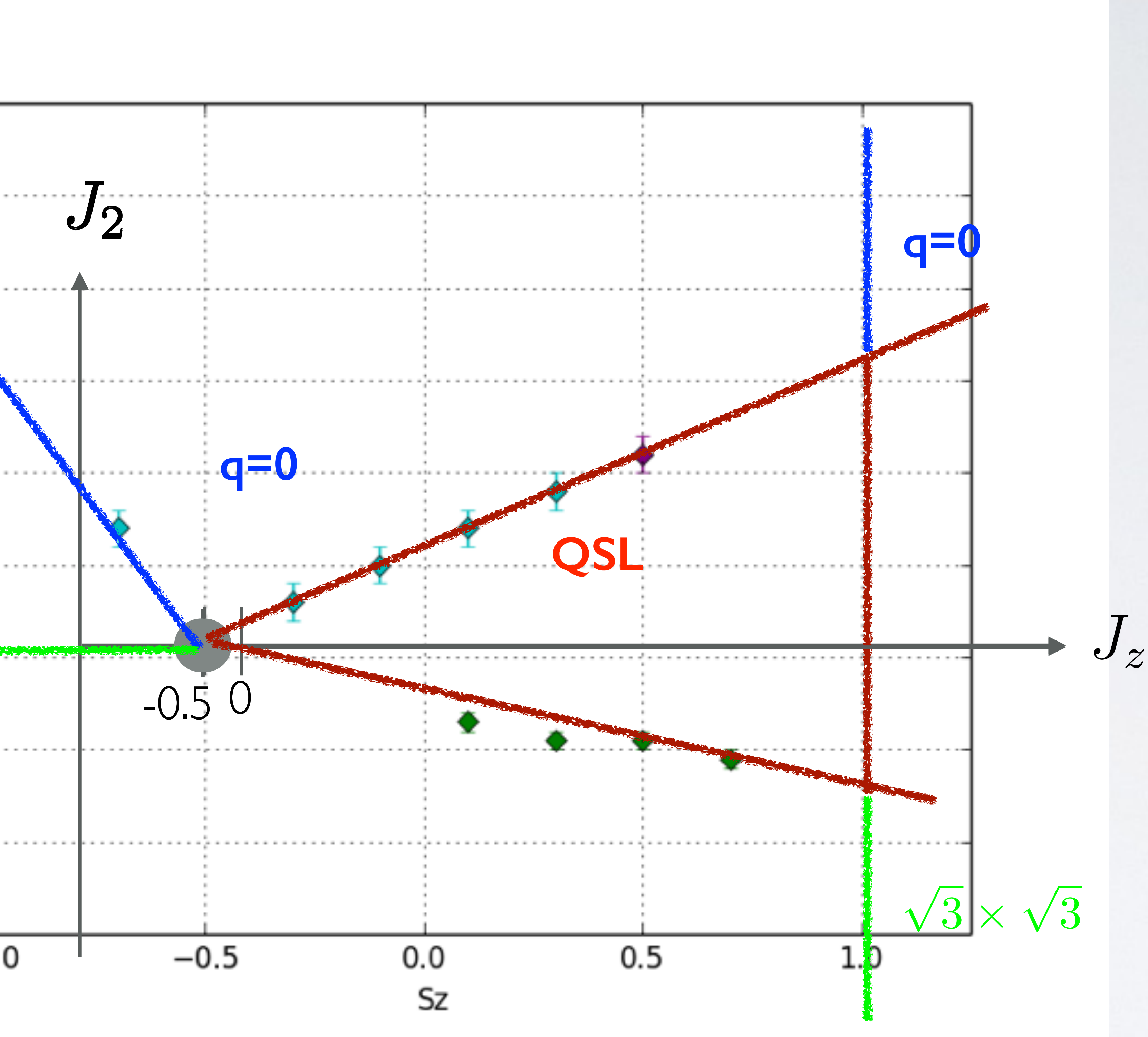
Spin Liquid

The "Herbertsmithite" spin-liquid

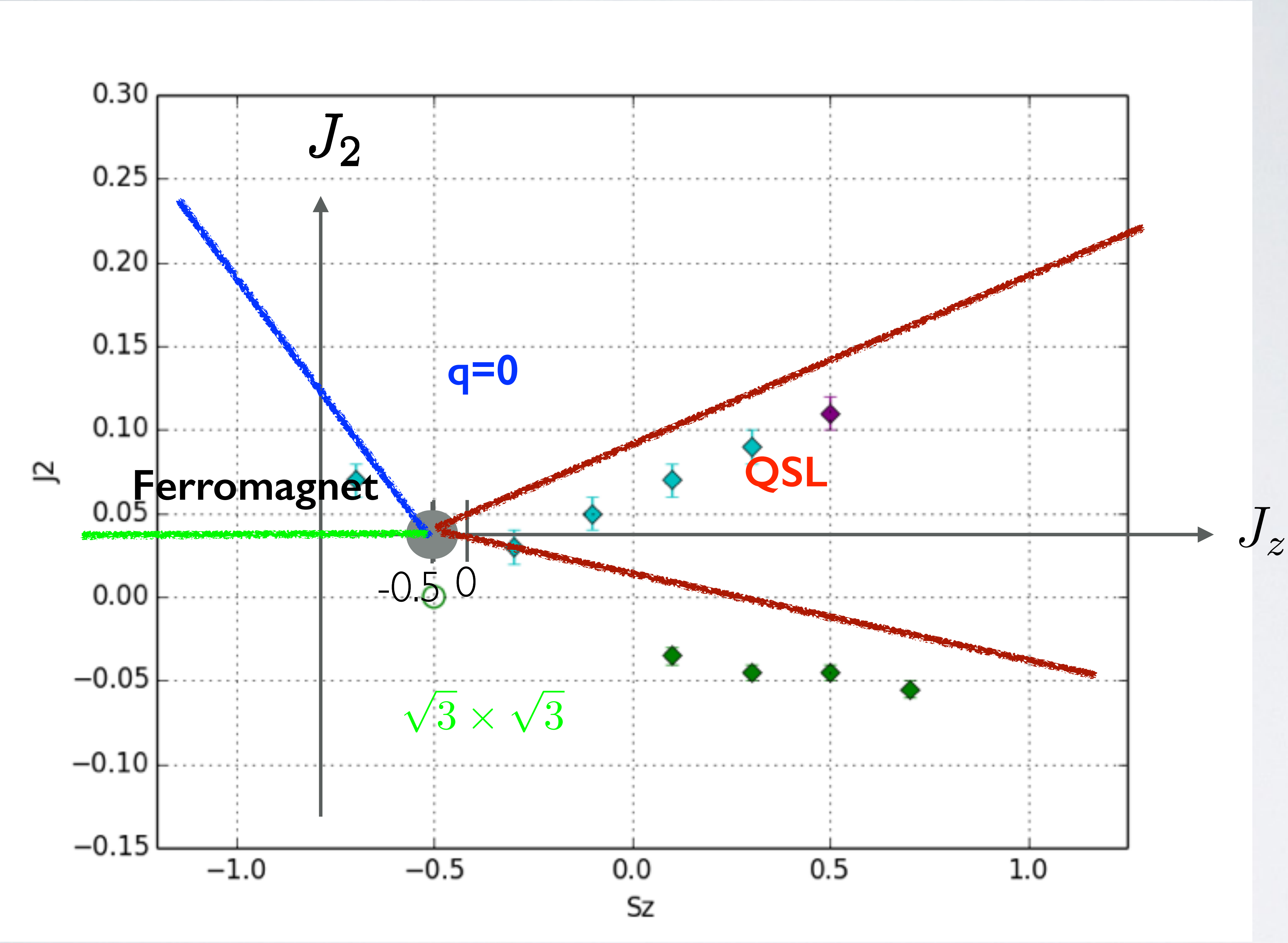


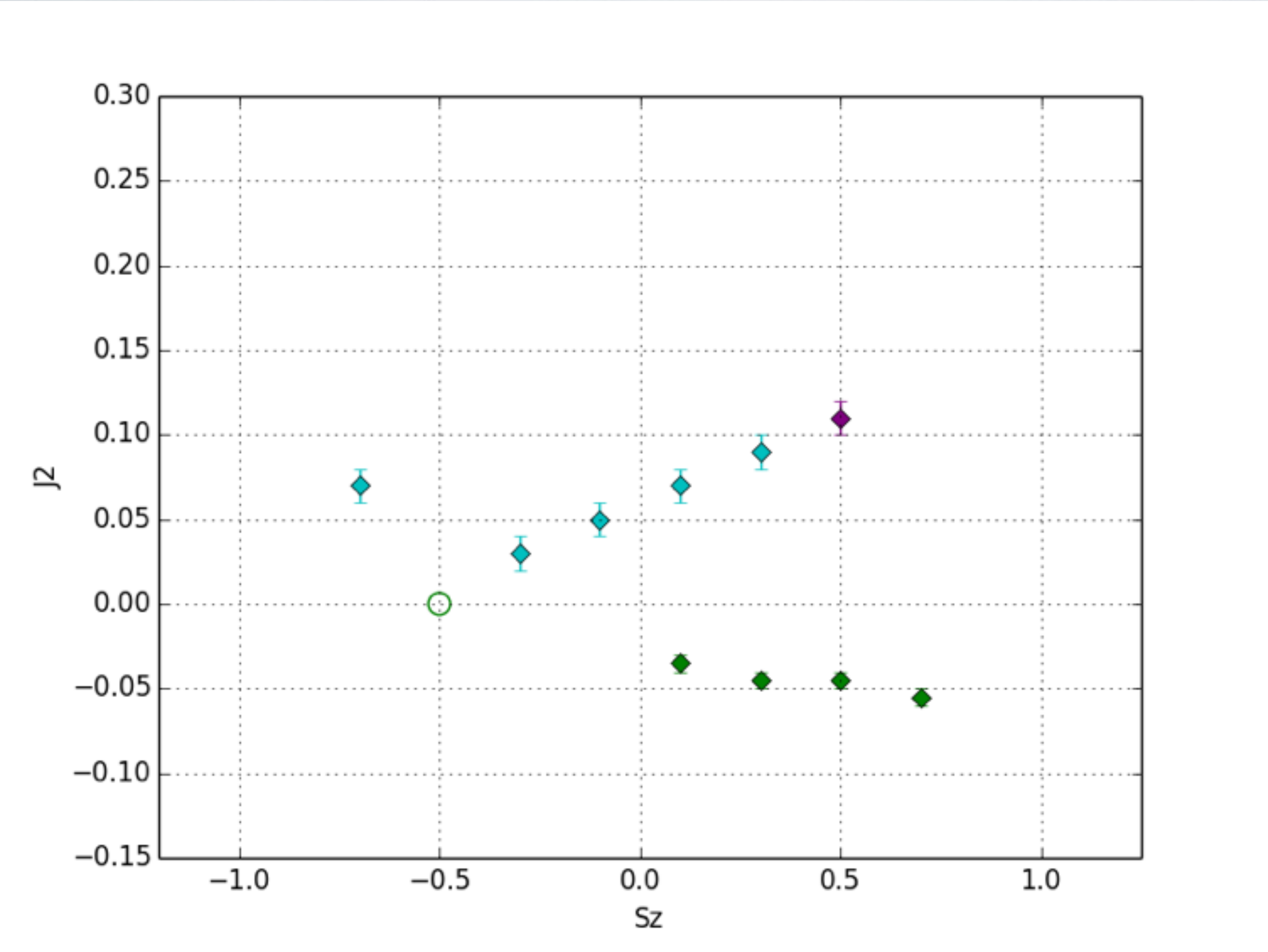
Spin Liquid

The “Herbertsmithite” spin-liquid



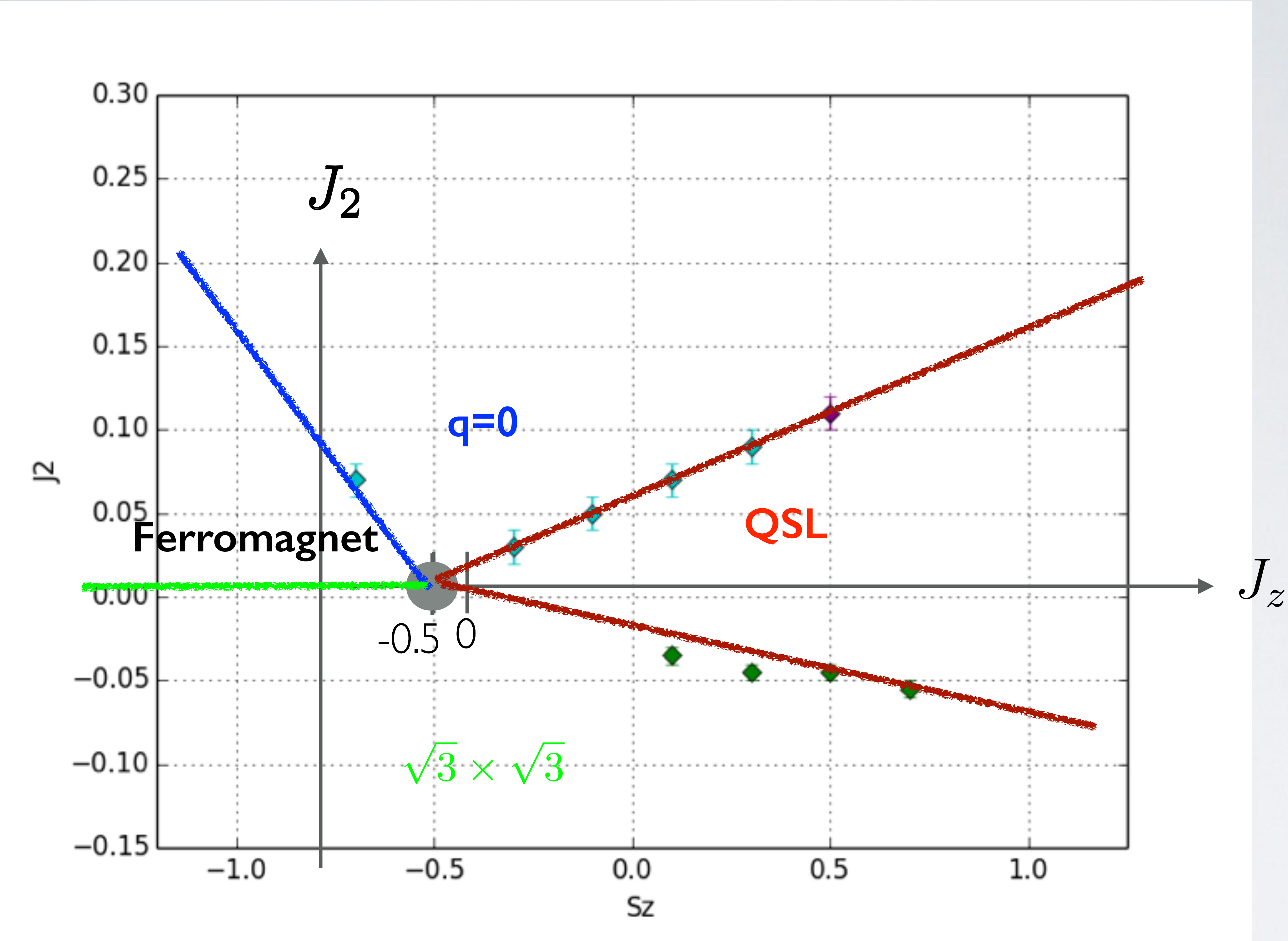
The "Herbertsmithite" spin-liquid

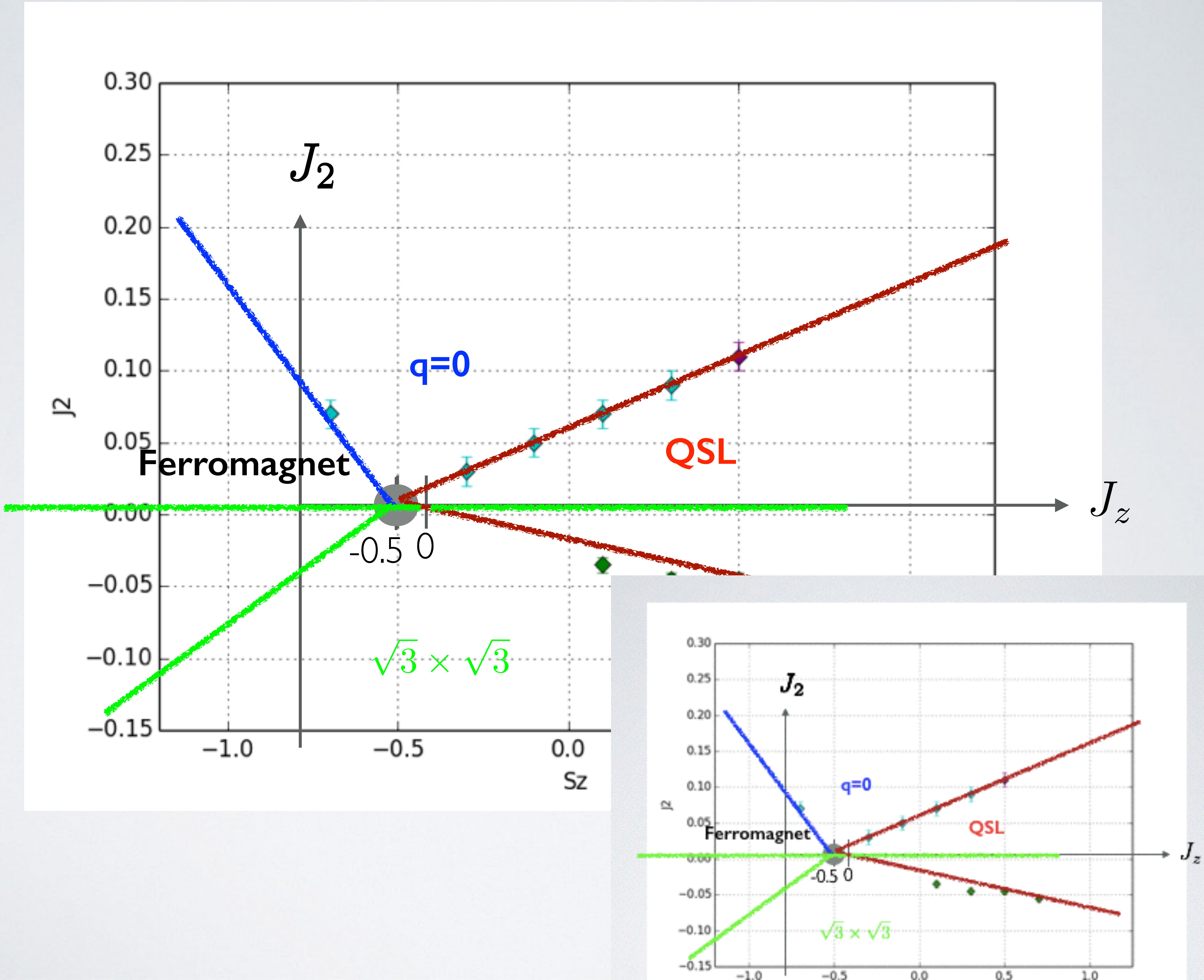




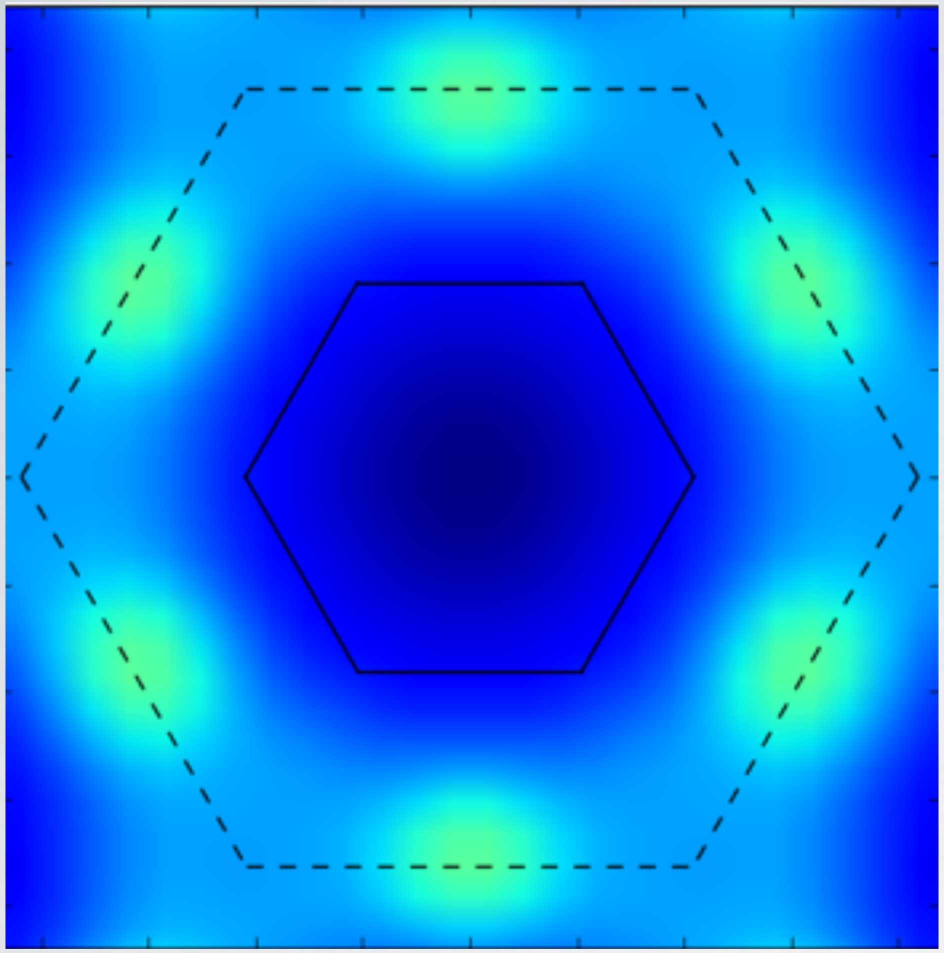
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
J2
−1.0
−0.5
0.0
0.5
1.0
Sz

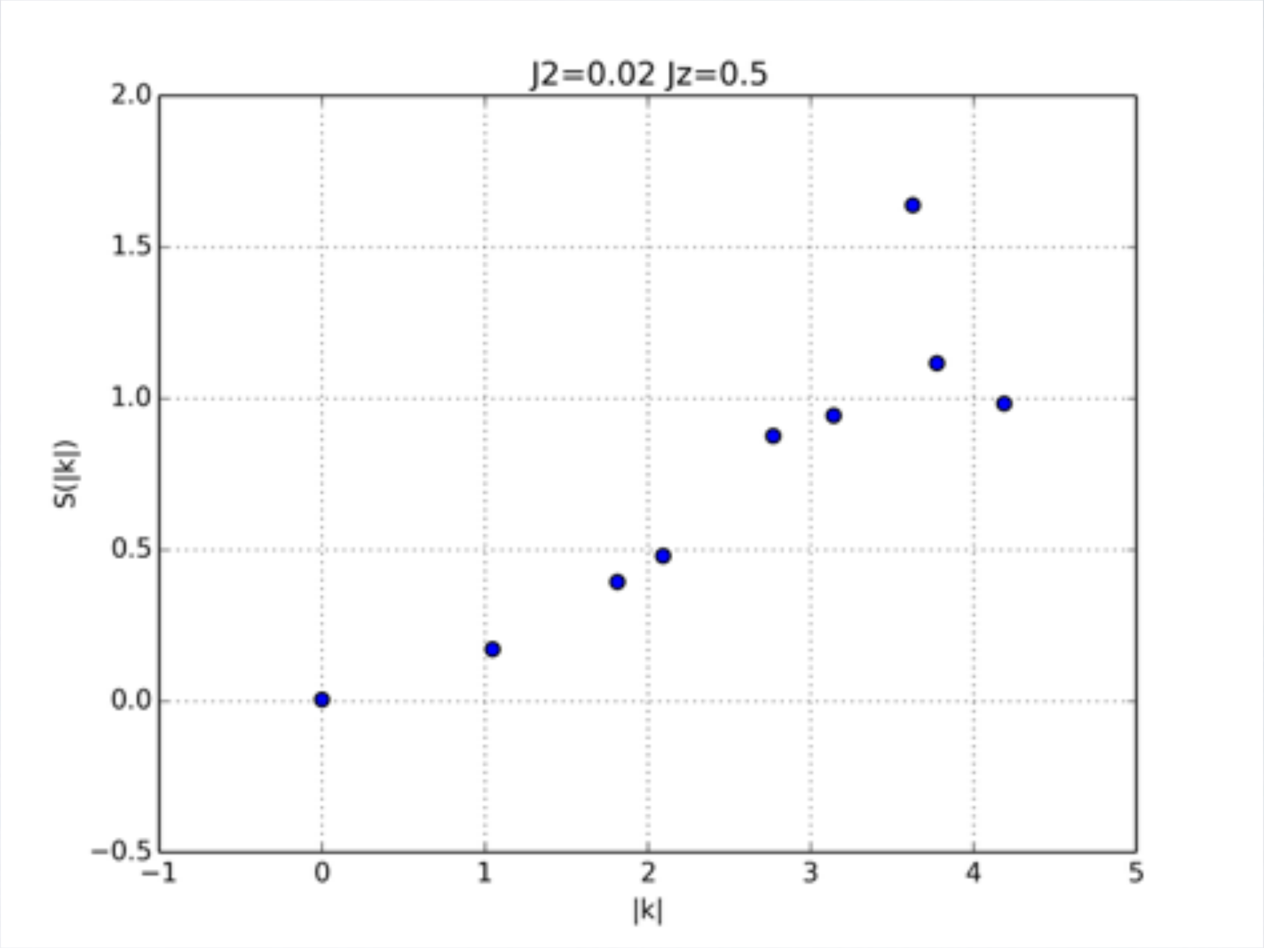
The “Herbertsmithite” spin-liquid

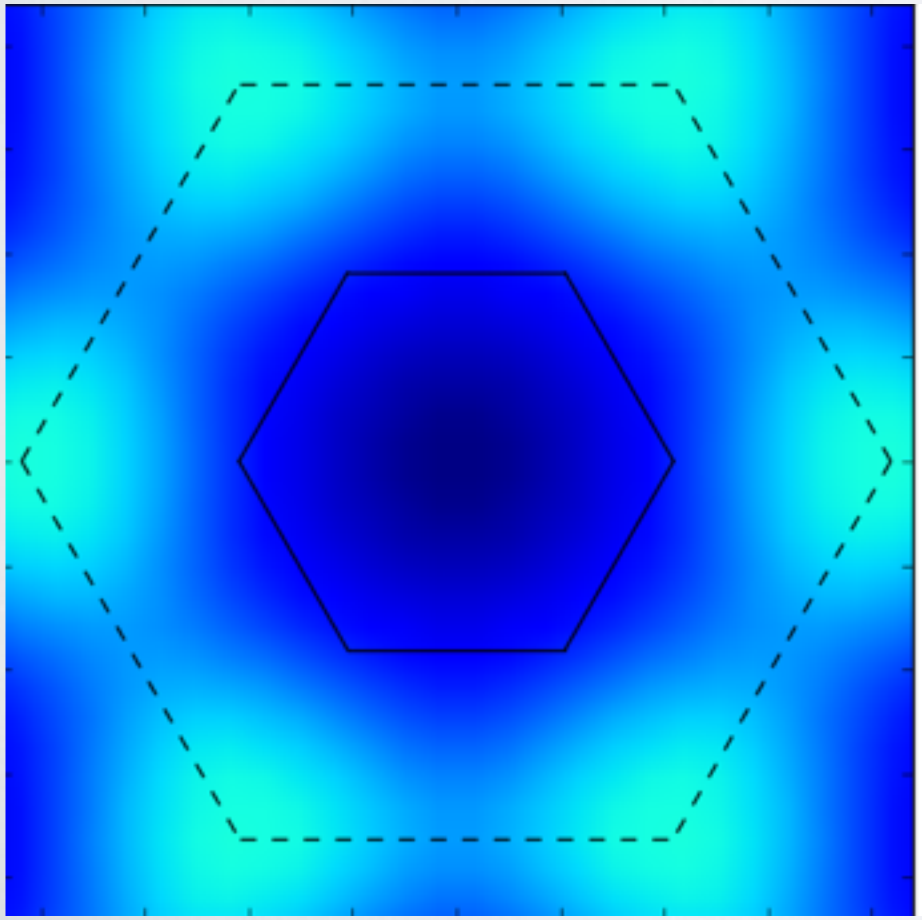


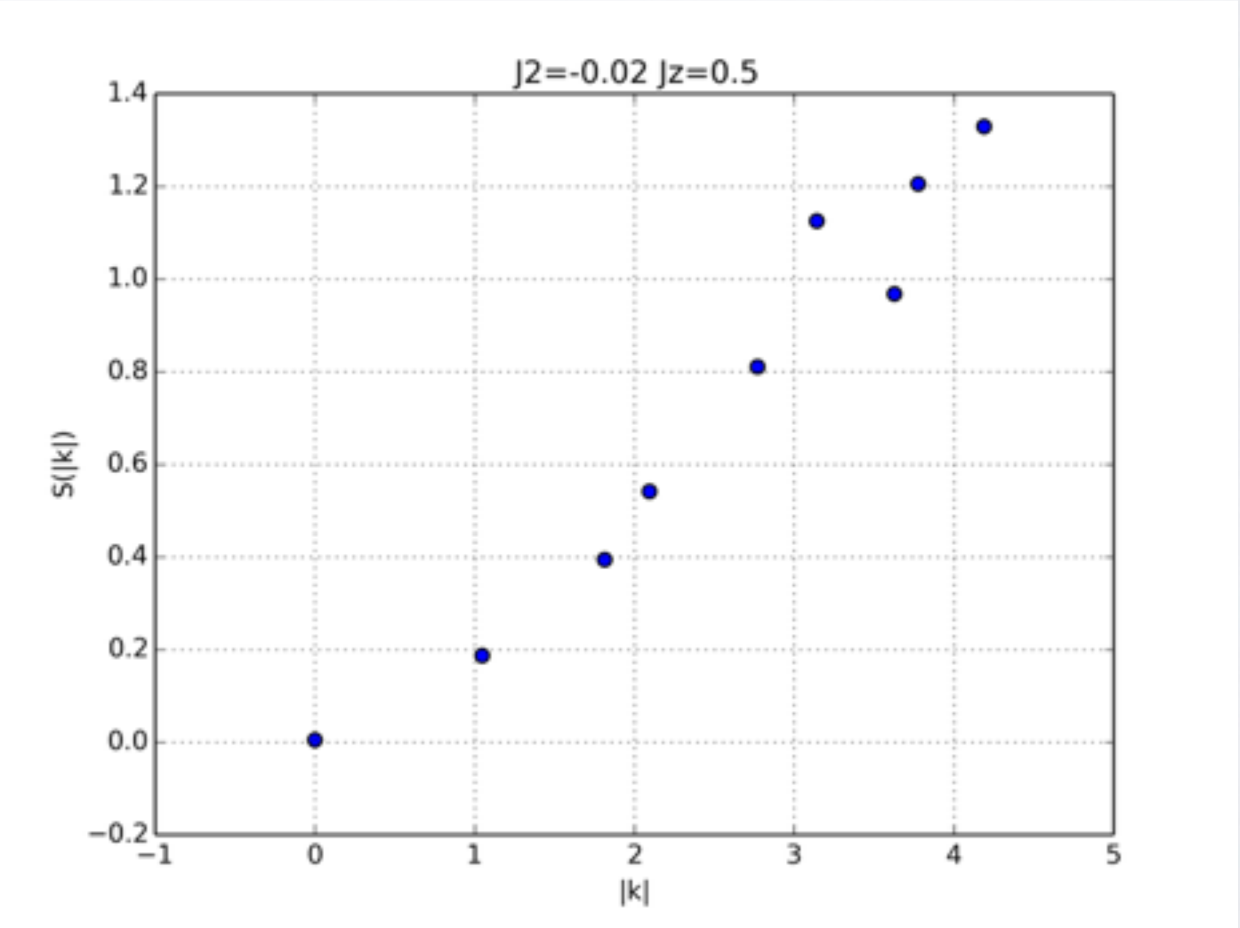


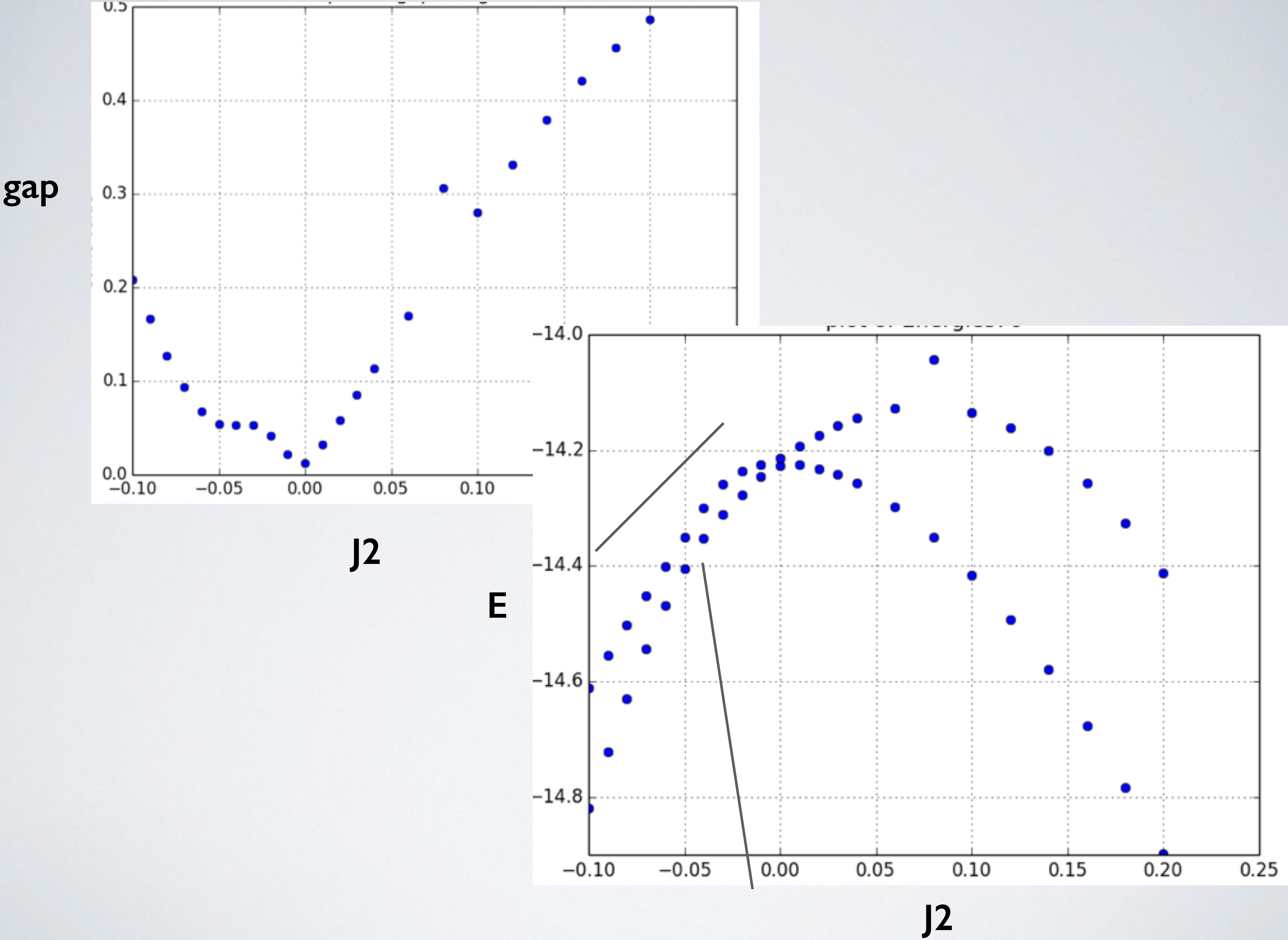
J_2
J_z
q=0
QSL
Ferromagnet
$\sqrt{3}\times\sqrt{3}$
-0.5
0
J2
Sz
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−1.0
−0.5
0.0



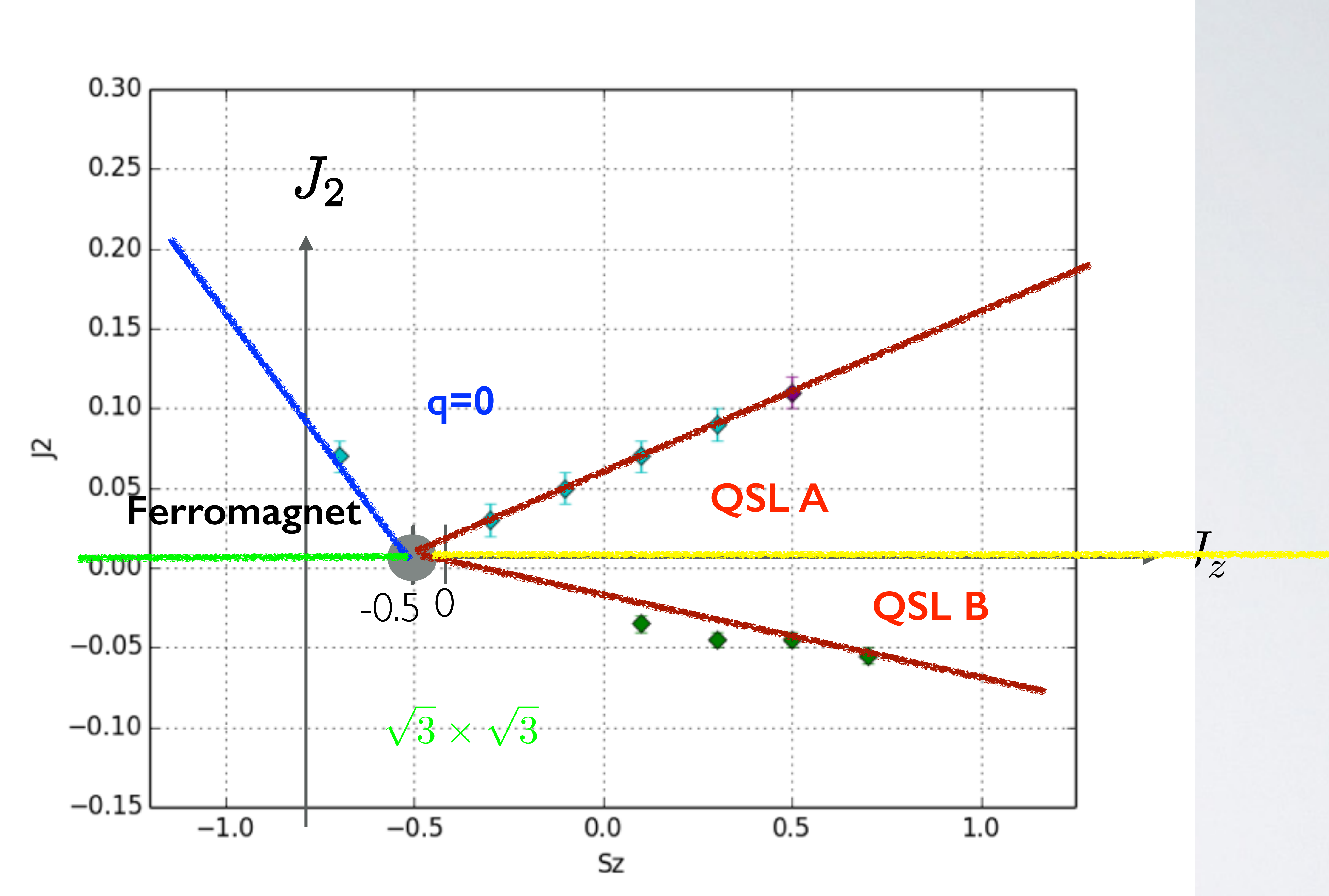

J2=0.02 Jz=0.5
S(|k|)
|k|
2.0
1.5
1.0
0.5
0.0
−0.5
−1
0
1
2
3
4
5




J2=-0.02 Jz=0.5
S(|k|)
|k|
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−1
0
1
2
3
4
5

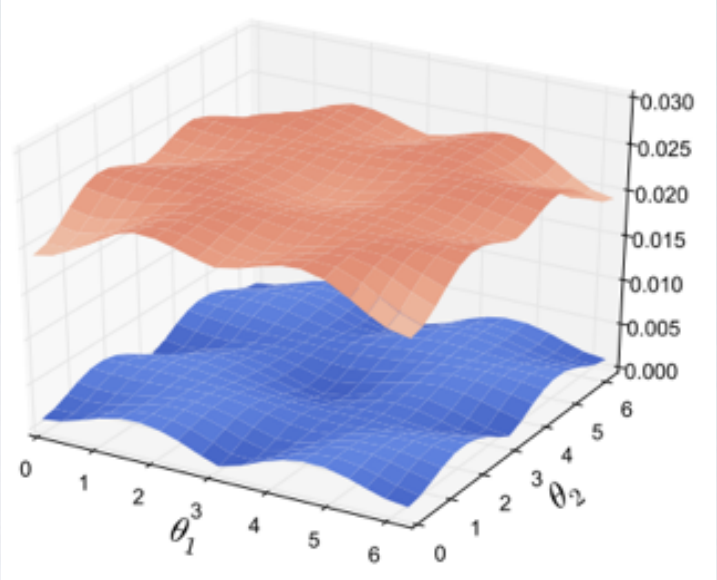

gap
0.0
0.1
0.2
0.3
0.4
0.5
−0.10
−0.05
0.00
0.05
0.10
J2
E
−14.0
−14.2
−14.4
−14.6
−14.8
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20
0.25
J2

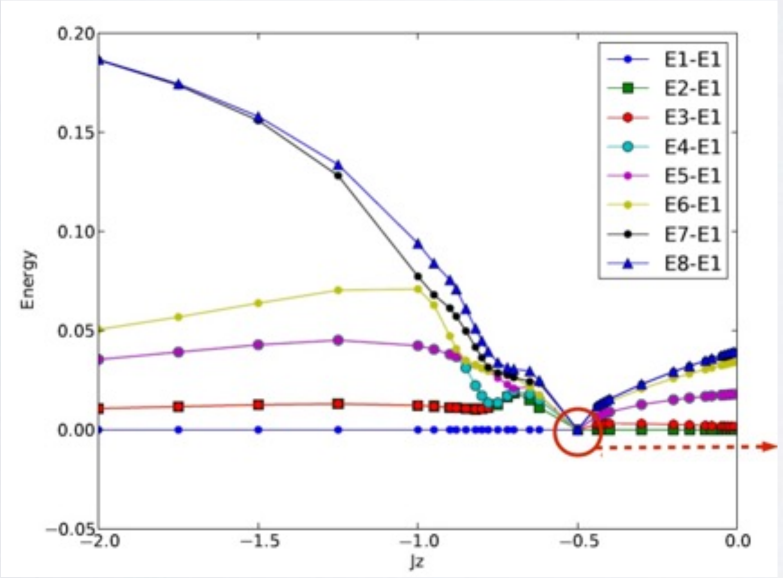
The “Herbertsmithite” spin-liquid



Summary

New chiral spin liquid.





Macroscopic Degeneracy in the kagome XXZ model

$$H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$

Connected to everyone's spin liquid.