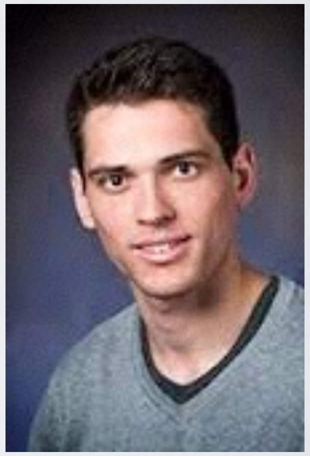
MBL AND TENSOR NETWORKS

Bryan Clark



Xiongjie Yu,
Benjamin Corregea,
David Luitz,
David Pekker,
Vadim Oganesyan,
Gil Rafeal

Outline

MBL in the language of tensor networks

- You can efficiently encode all 2n eigenstates using 2n small matrices!
- Prescription to built 2^n MPS out of these matrices

Using this language, you can choose the 'best' choice for l-bits

- Generate l-bits, effective Hamiltonian
- Look at correlation lengths of different quantities…
- Two correlation lengths

We can find the MPS using two new algorithms

- ES-DMRG: Modified DMRG sweeping* (same algorithm as DMRG-X)
- SIMPS: Energy Targeting using $(H-E)^{-1}$

Entanglement Saturates | P(S) | **Local Excitations**
ETH Violated | Mobility Edge

Beyond MPS: MERA and Huse-Elser states in 2D

Scaling at the transition

Area Law MBL Eigenstates: Bauer, Nayak; Eisert

* Yu, Pekker, BKC; arxiv:1509.01244
* Vedika, Pollmann, Sondhi; arxiv:1509.00483

Two languages for many-body localization

Hamiltonian language: l-bits

$$H = \sum_i \alpha_i \tau_i + \sum_{i,j} \alpha_{ij} \tau_i \tau_j + ...$$

Wave-function language: Tensor Networks

Want a language in which to describe the eigenstates

MPS Ground State:

$$\begin{bmatrix} G_1^{\downarrow} \\ G_1^{\uparrow} \end{bmatrix} \quad \begin{bmatrix} G_2^{\downarrow} \\ G_2^{\uparrow} \end{bmatrix}$$

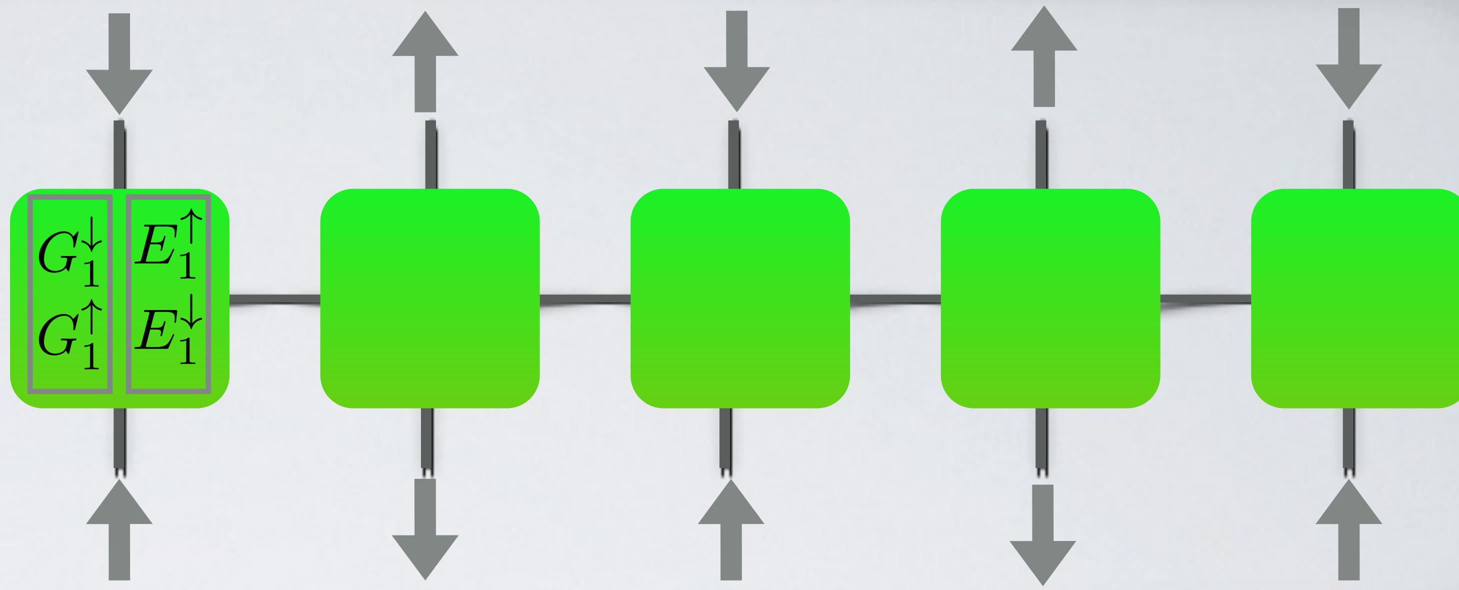
$$G_1 \quad \underline{G_2} \quad \underline{G_3} \quad \dots \quad G_n$$

$$\underline{E_1} \quad E_2 \quad E_3 \quad \dots \quad \underline{E_n}$$

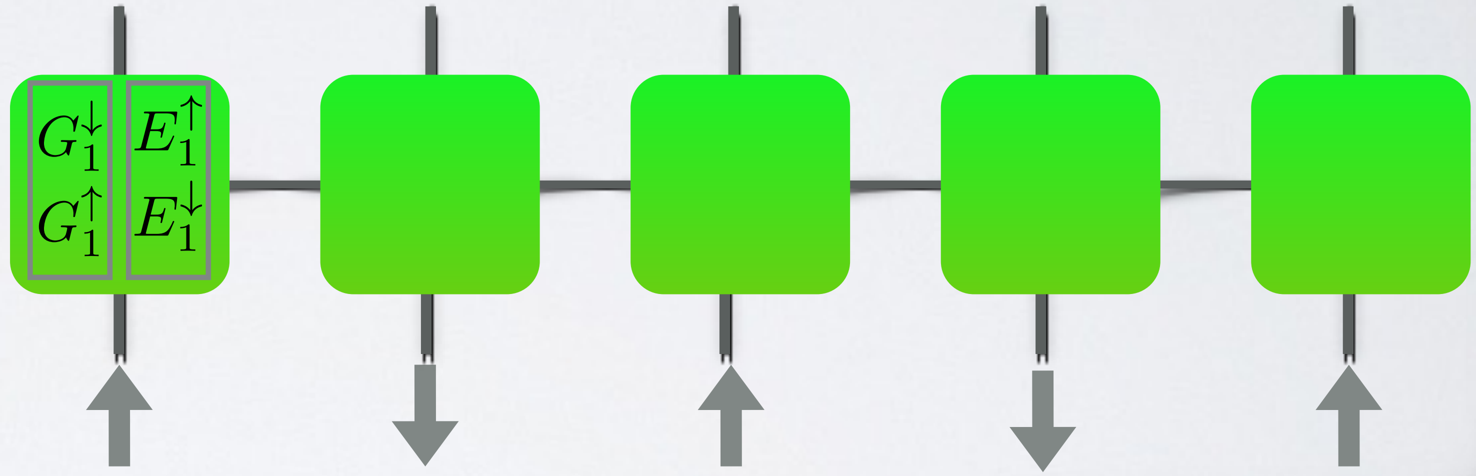
All 2^n eigenstates can be generated by all combinatorial combinations of the G, E matrices.

$\langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \sigma_5 | \hat{O} | \sigma_1' \sigma_2' \sigma_3' \sigma_4' \sigma_5' \rangle$

$G_1^{\downarrow} E_2^{\uparrow} G_3^{\downarrow} E_4^{\uparrow} G_5^{\downarrow}$



$G_1 E_2 G_3 E_4 G_5$

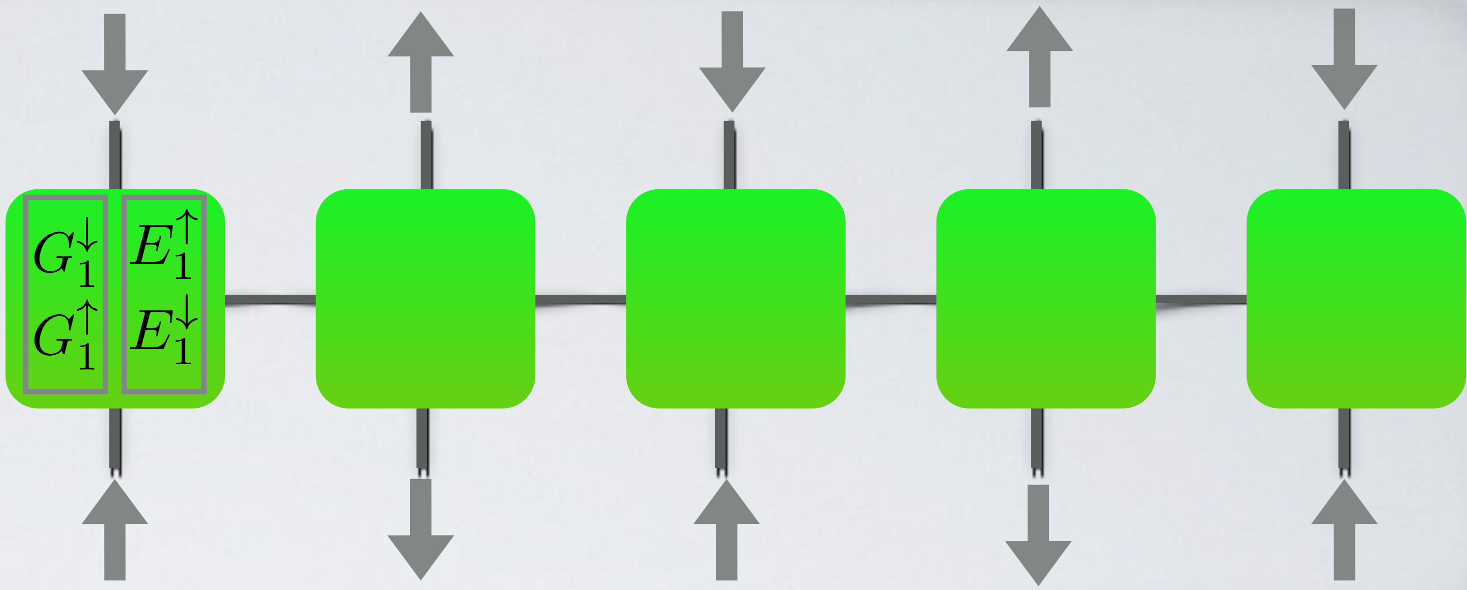


Clark and Pekker: arxiv:1410.2224

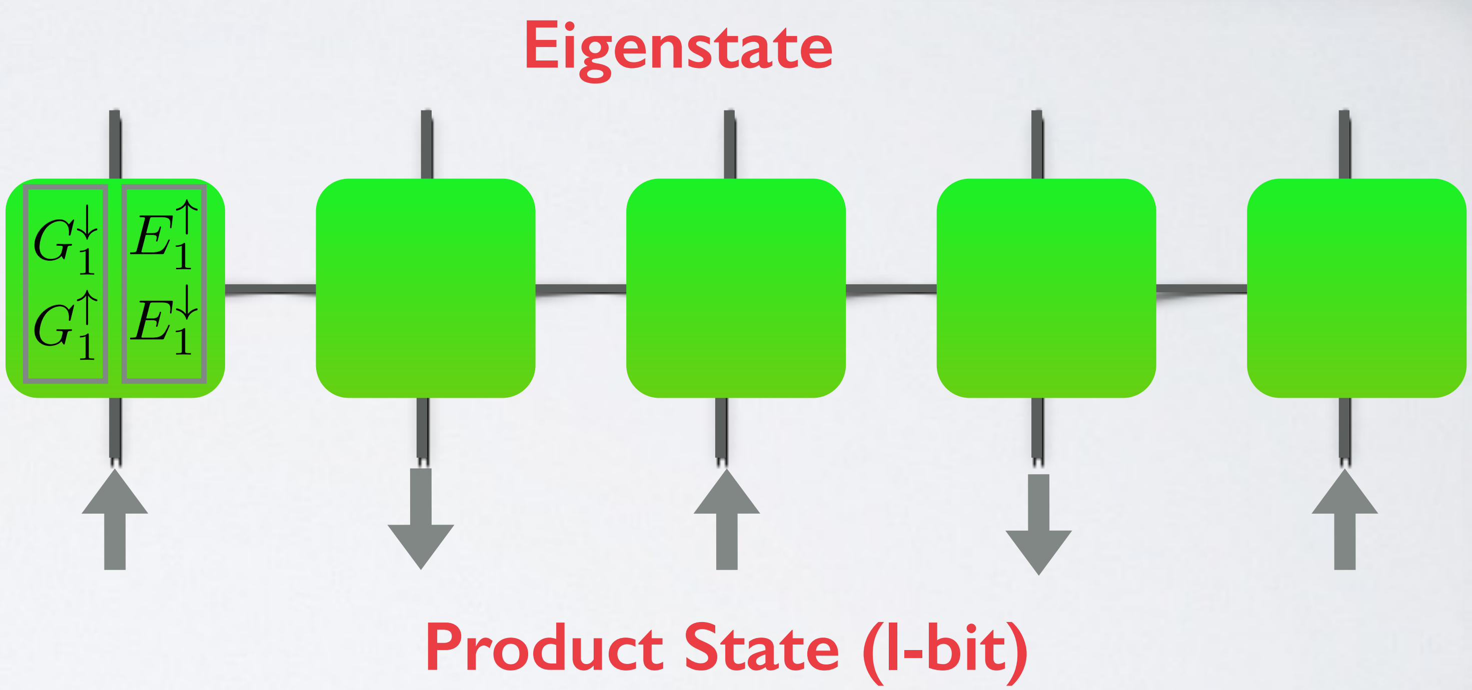
Chandran, Carrasquilla, Kim, Abanin, Vidal: arxiv:1410.0687

$$\langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \sigma_5 | \hat{O} | \sigma_1' \sigma_2' \sigma_3' \sigma_4' \sigma_5' \rangle$$

$$G_1^{\downarrow} E_2^{\uparrow} G_3^{\downarrow} E_4^{\uparrow} G_5^{\downarrow}$$



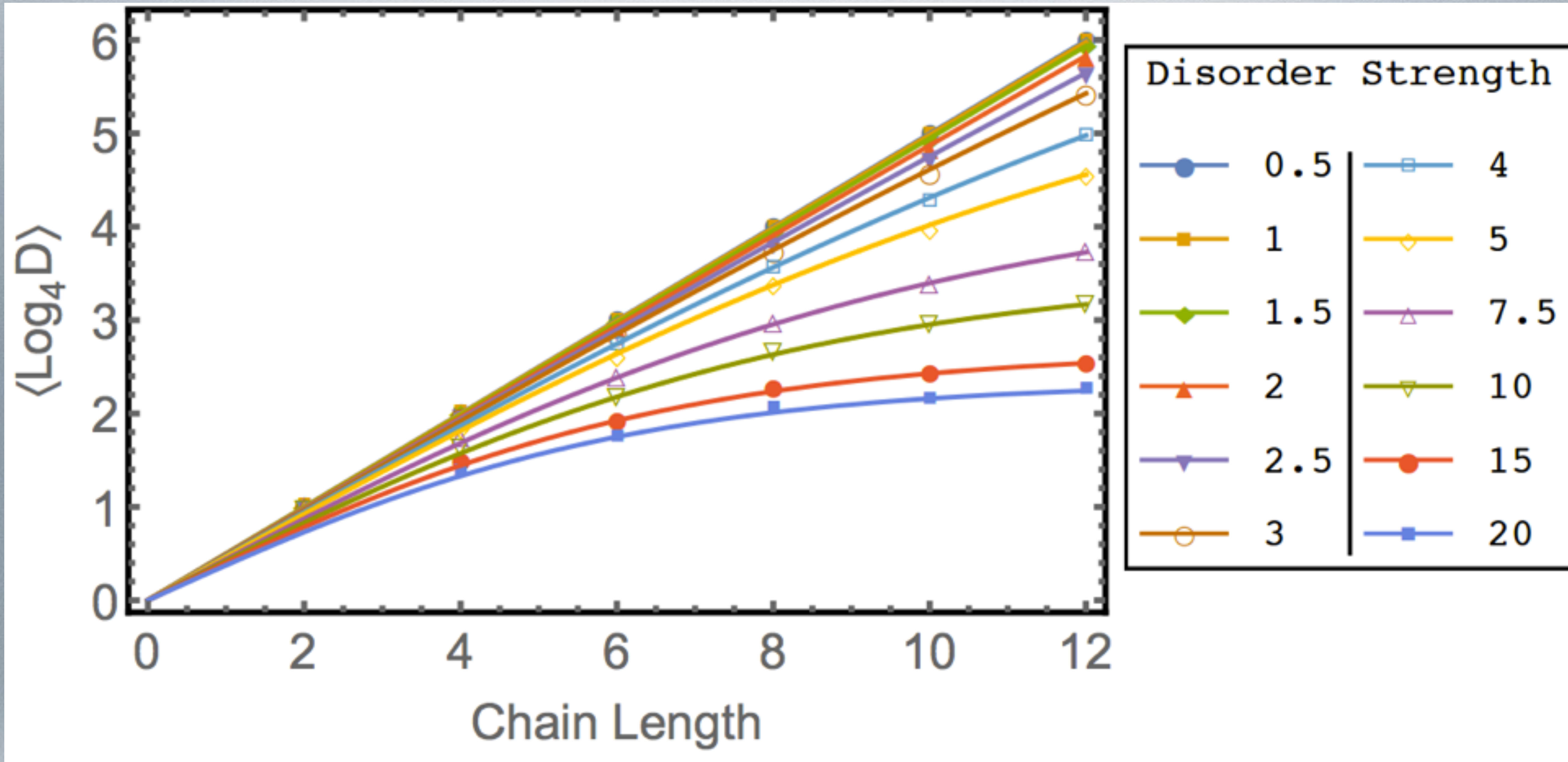
$$G_1 E_2 G_3 E_4 G_5$$



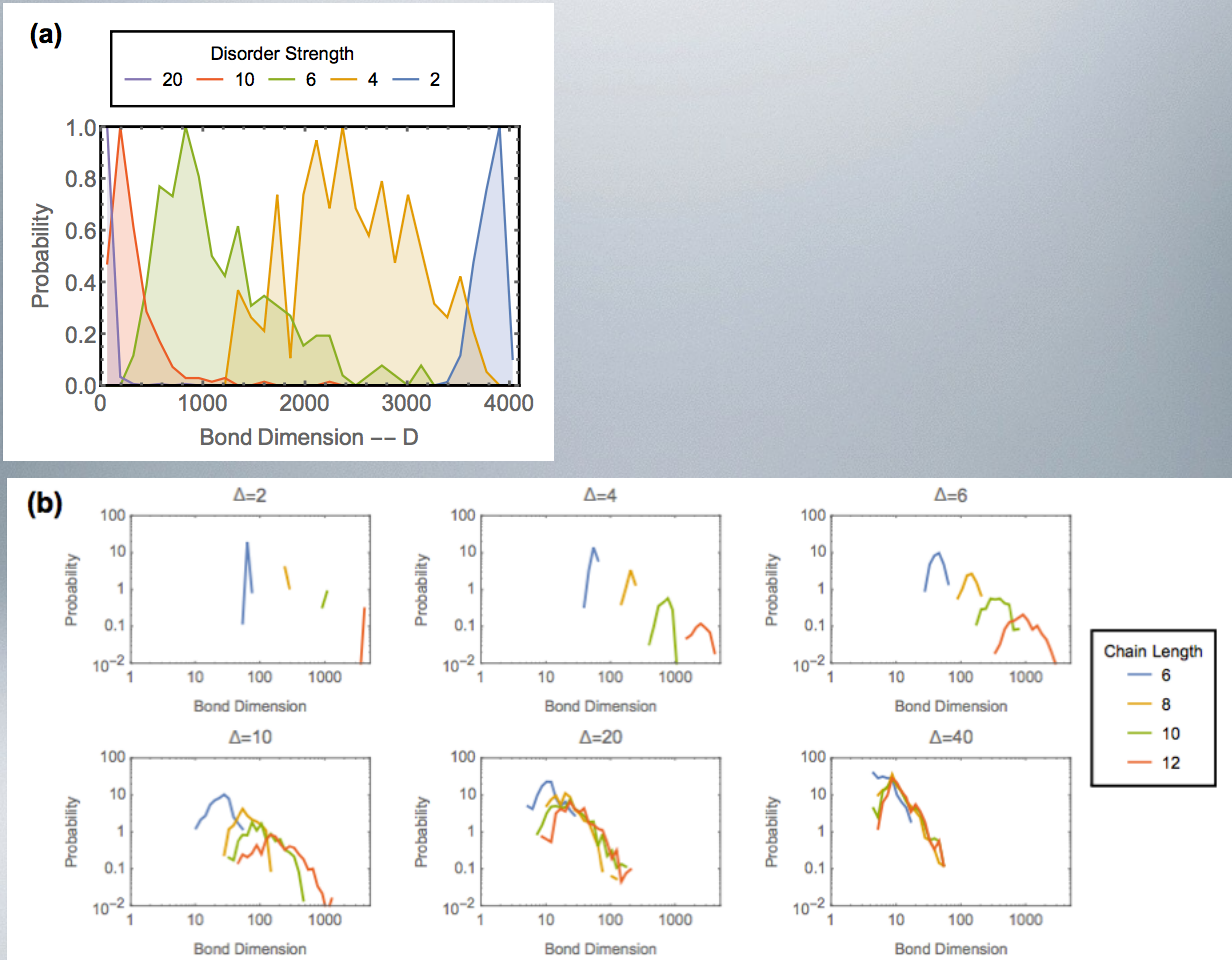
$$UHU^{\dagger} = H_D$$

$$U : |i\rangle \rightarrow |e_i\rangle$$

MPO takes l-bit space to physical space



⟨Log4D⟩
Chain Length
0
1
2
3
4
5
6
0
2
4
6
8
10
12
Disorder Strength
0.5
1
1.5
2
2.5
3
4
5
7.5
10
15
20


(a)
Disorder Strength
20
10
6
4
2
Probability
Bond Dimension -- D
(b)
Δ=2
Δ=4
Δ=6
Δ=10
Δ=20
Δ=40
Probability
Bond Dimension
Chain Length
6
8
10
12

Let's check this....

$\hat{H} \longrightarrow \hat{U} \longrightarrow$ MPO

What's the bond dimension?

U maps product states to eigenstates.

Many such mappings

Which one we pick is important!

Conserved quantum number: $\tau_i = U\sigma_z^i U^\dagger$

Hamiltonian: $H_D = UHU^\dagger$

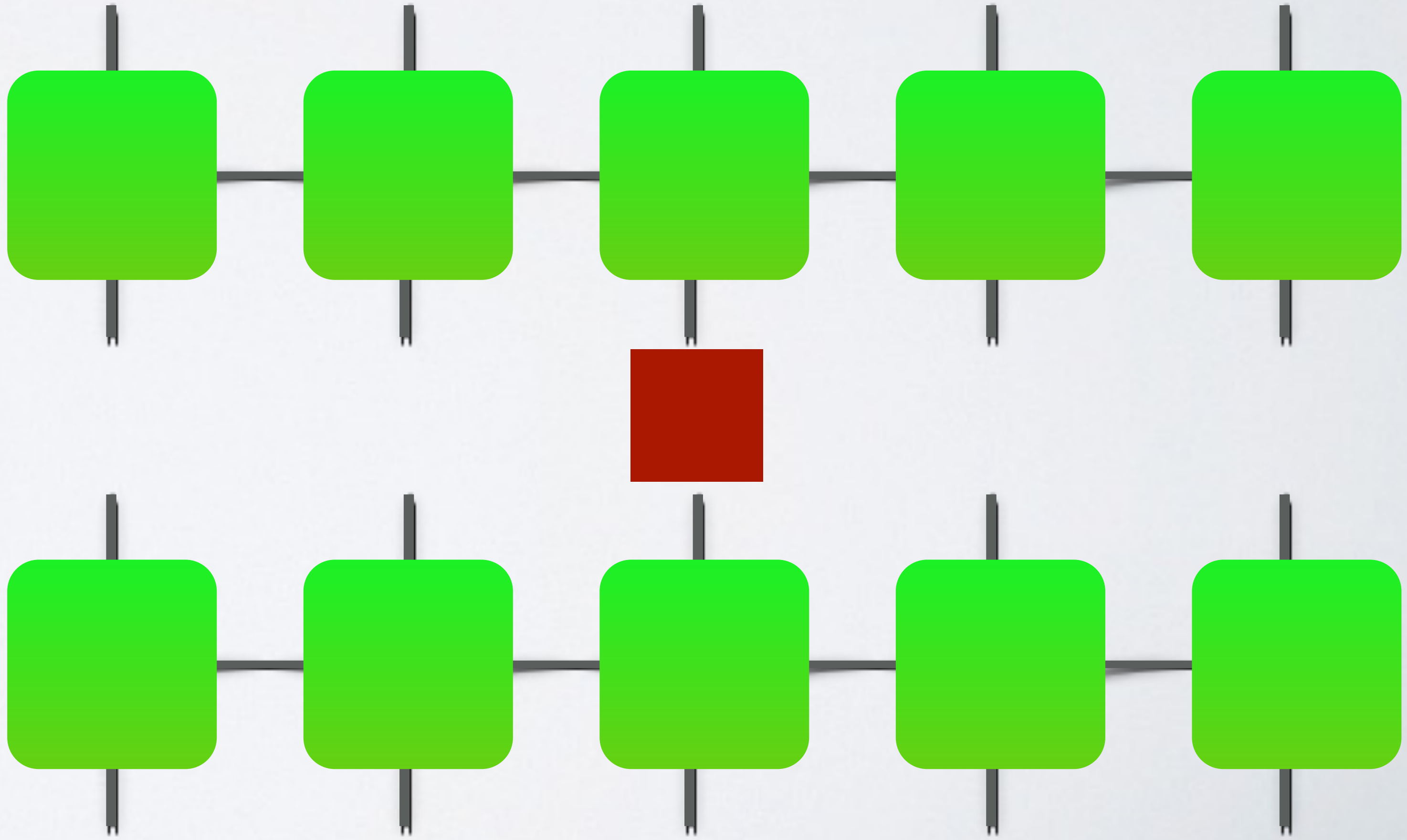
From this you get α_{ij}

Many ways to represent τ and H and α_{ij} are all fixed by U

In U, you get to choose which product states match to which eigenstates and the phases of the eigenstates

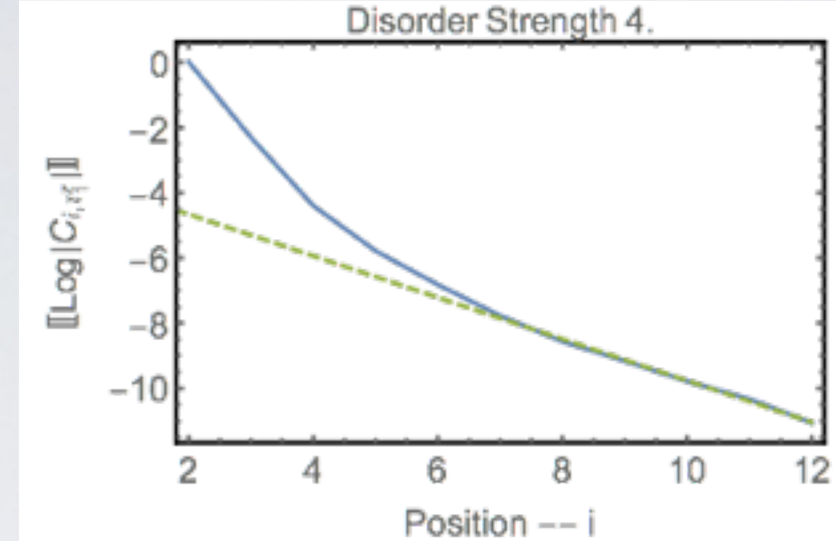
- ☐ bipartite matching
- ☐ jacobi rotation
- ☐ **Wegner flow**

$$\frac{dH}{dT} = [H, [H_0, V]]$$



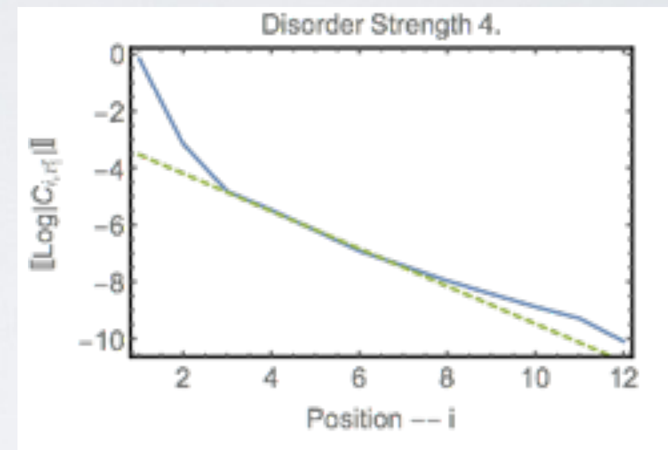
$$H = \sum_i \alpha_i \tau_i + \sum_{i,j} \alpha_{ij} \tau_i \tau_j + \ldots$$

decays exponential



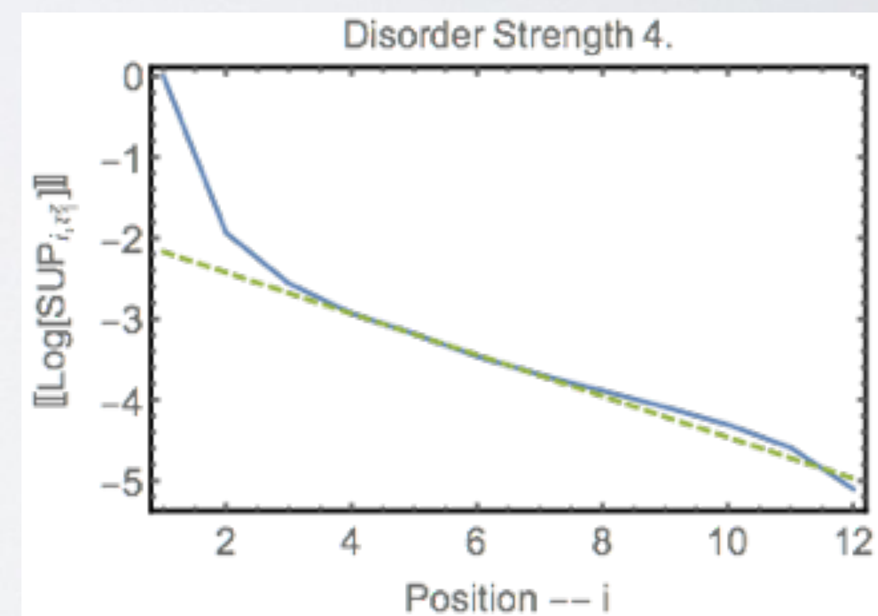
$$\tau_i = \sum_u a_i \sigma_z^i + \sum_{i,j} a_{ij} \sigma_i \sigma_j + \ldots$$

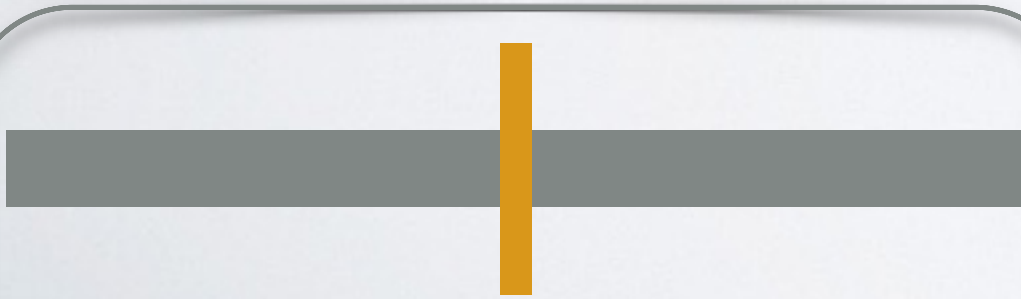
decays exponential



τ_i is mainly supported on i (and it's nearby sites)

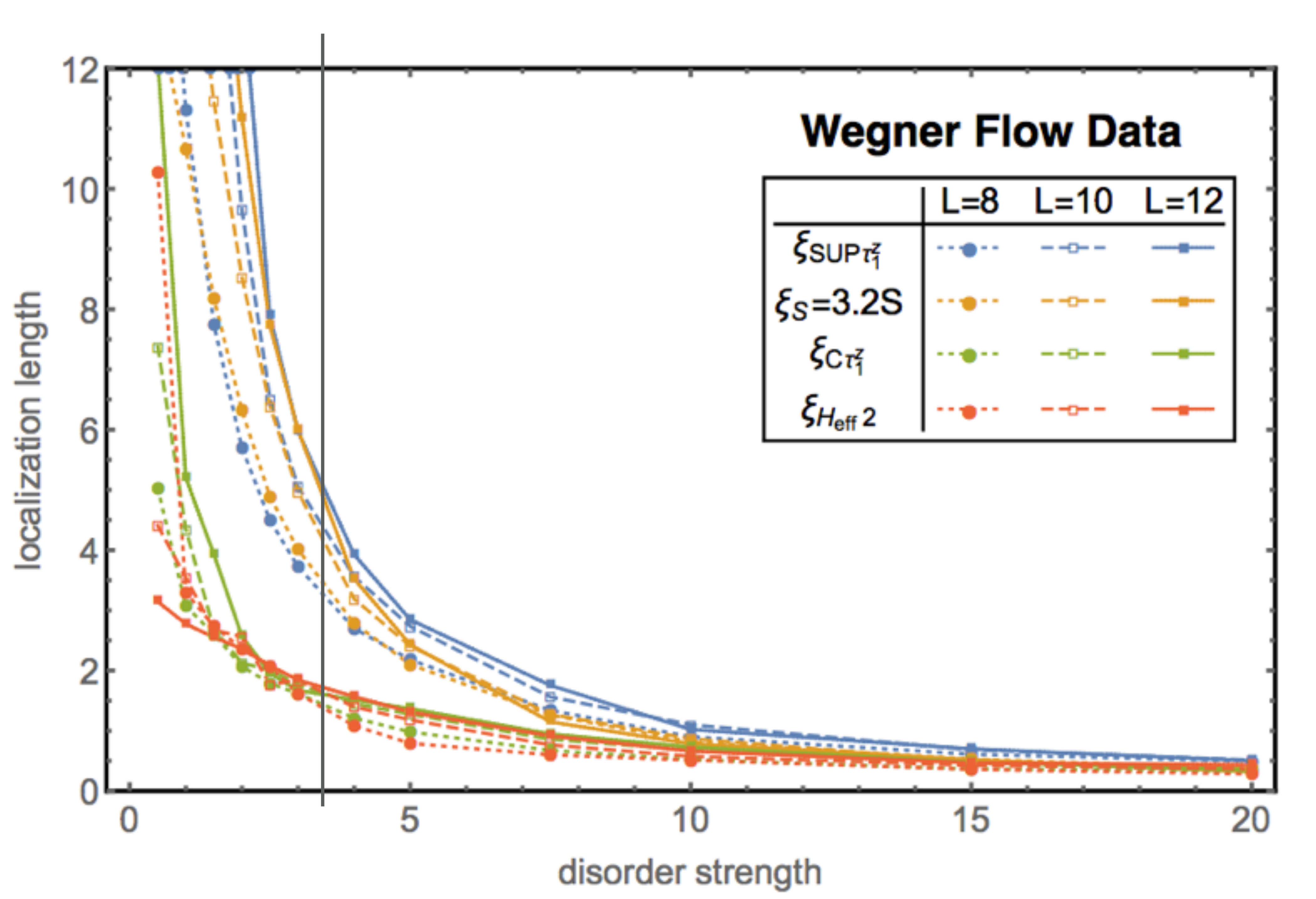
Consider how quickly it loses support: (1 - support)





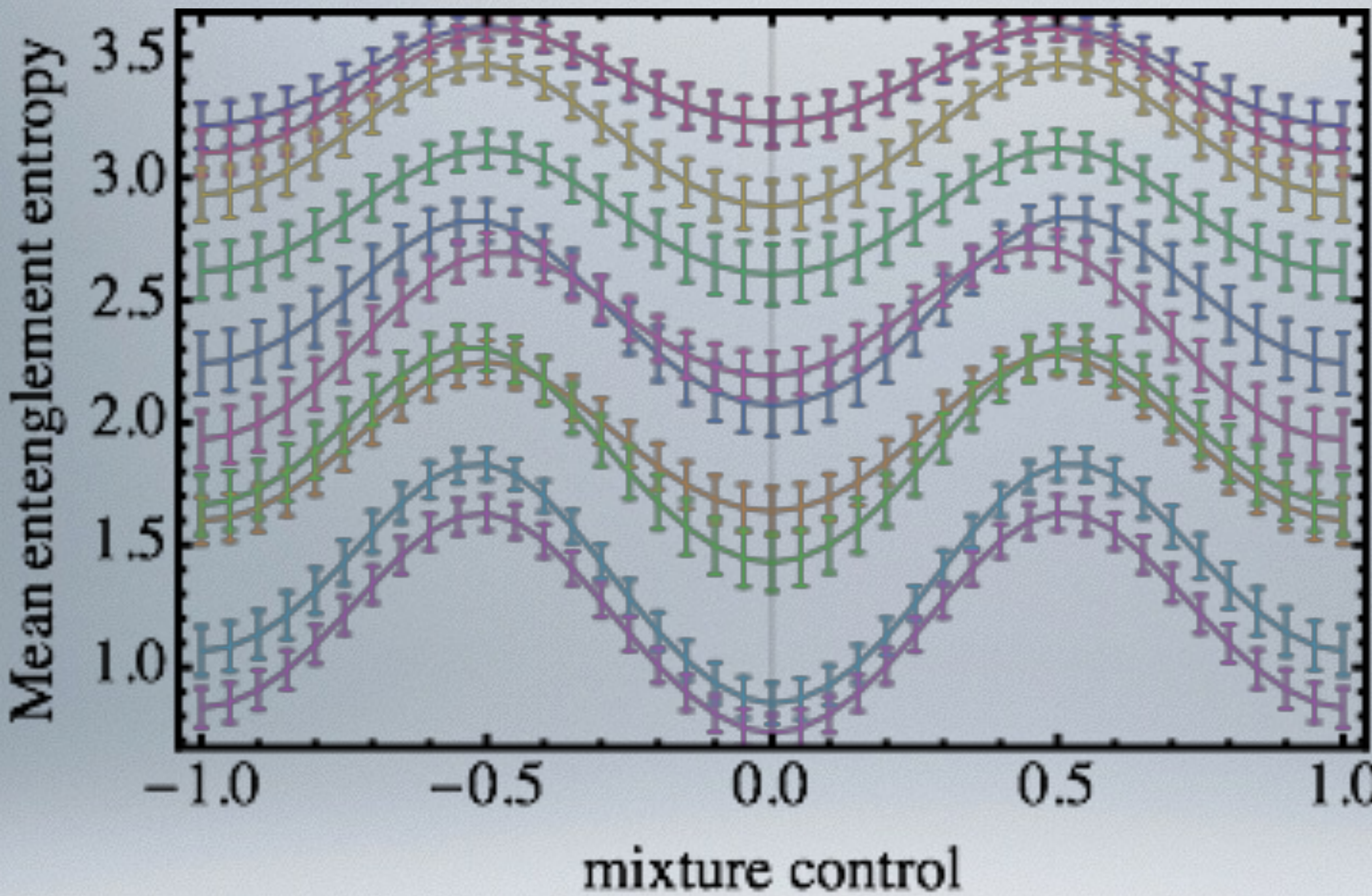
Entanglement (scaled)

Pekker, BKC, Vadim, Gil (unpublished)
Vedika, Huse (unpublished)



Wegner Flow Data
L=8
L=10
L=12
$\xi_{SUP\tau_1^z}$
ξ_S=3.2S
$\xi_{C\tau_1^z}$
$\xi_{H_{eff}}$ 2
localization length
disorder strength
0
2
4
6
8
10
12
0
5
10
15
20

Acquiring Eigenstates



The evolution of two algorithms…

interesting to see how our two algorithms has evolved over the years

	Modified DMRG Sweeping	Energy Targeting
APS 2014 / Dresden 2014	ES-DMRG	$(H-E)^2$
APS 2015		$(H-E)^{-1}$ SIMPS
Today	ES-DMRG	$(H-E)^{-1}$

Tomorrow → **MERA and Huse-Elser states**

ES-DMRG + SIMPS	Yu, Pekker, BKC; arxiv:1509.01244
ES-DMRG	Vedika, Pollmann, Sondhi; arxiv:1509.00483
$(H-E)^2$ + tricks	Lim, Sheng; arxiv:1510.08145
$(H-E)^2$	Kennes and Karrasch arxiv:1511.02205

Getting a MPS

Dresden: May 2014

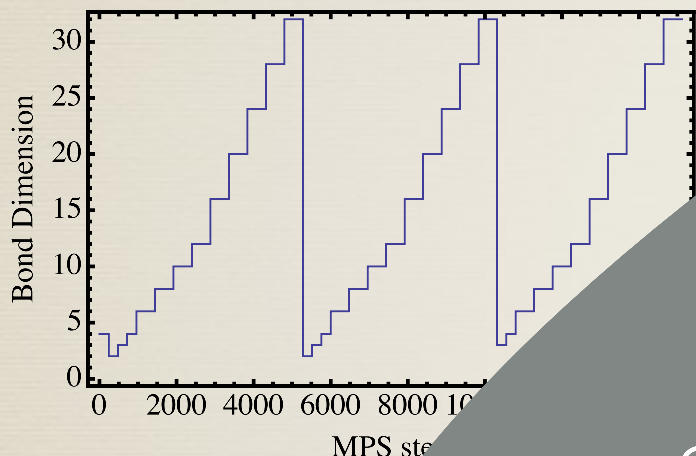
MPS are a good representation. How do we get them?

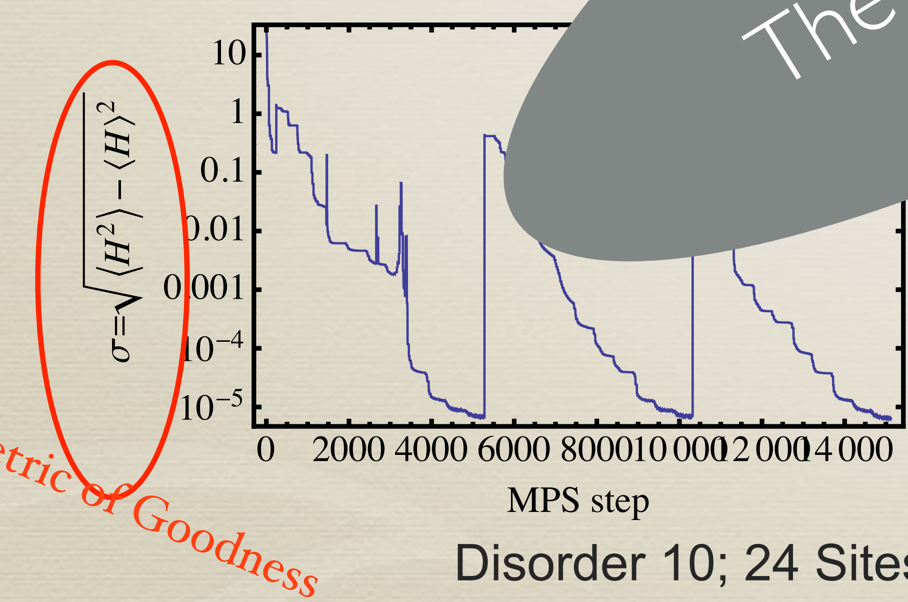
DMRG on $(H - E)^2$
+
artificially drop bond-dimension during run.

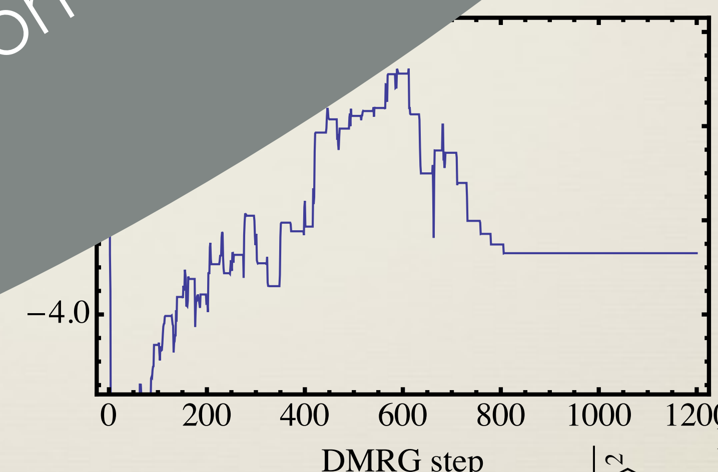
Typical DMRG: For a site produce an effective Hamiltonian H' and solve for the ground state of ...

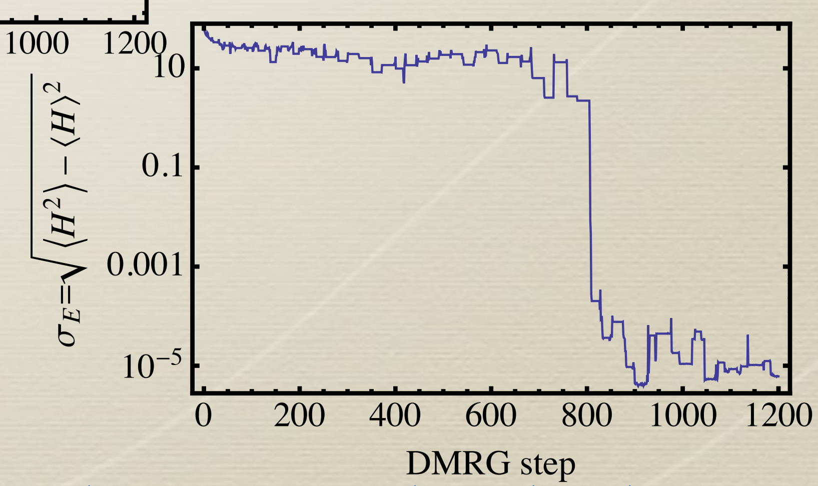
M... ...te produce an ... and choose the ... st to the current ... e.

The evolution of two algorithms









Disorder 10; 24 Sites
Level Spacing: $1.5*10^{-5}$

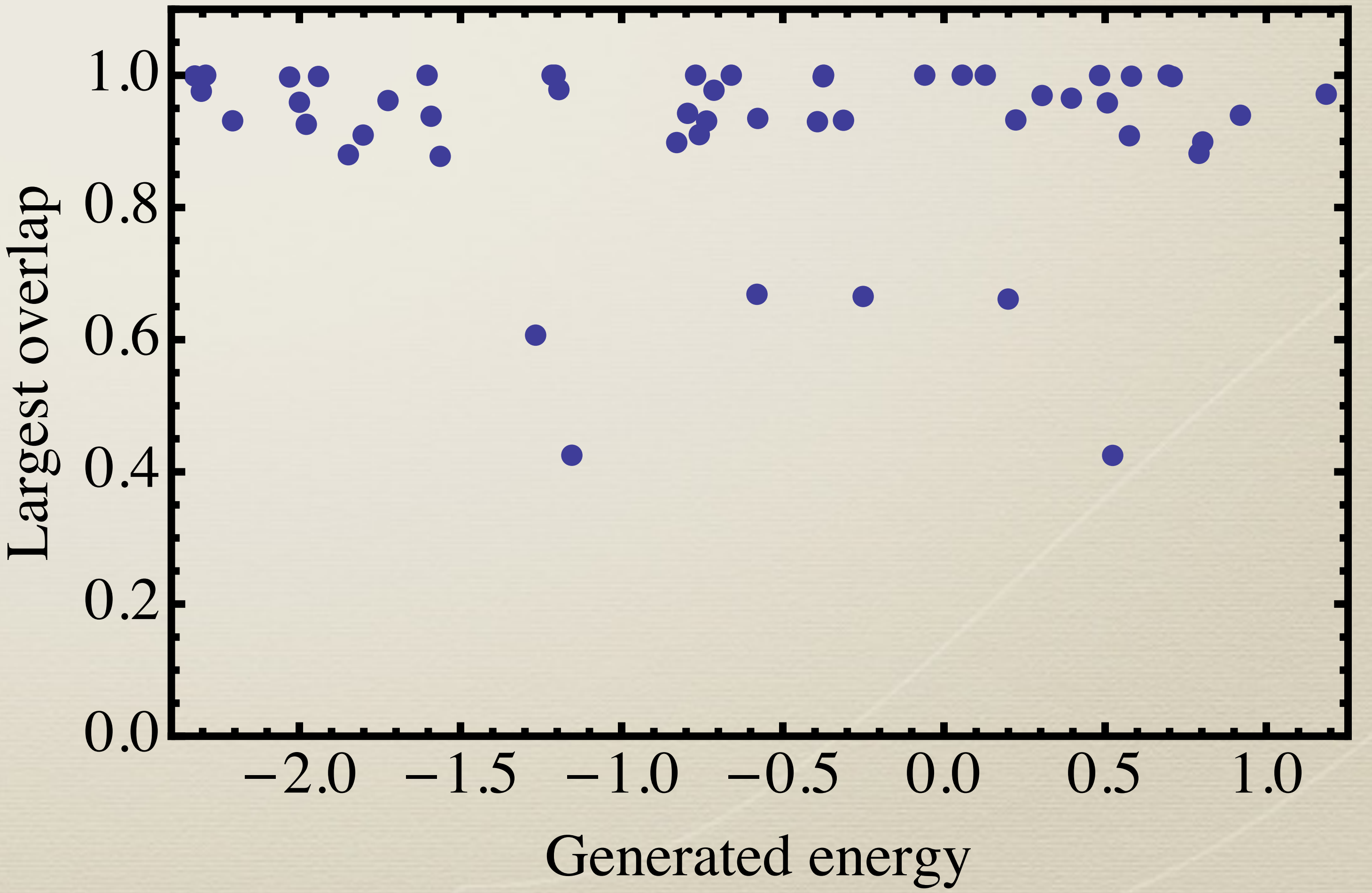
Disorder 20; 24 Sites
Level Spacing: $3*10^{-5}$

* Also time evolution variants of these; easier to 'analyze' but general experience is diagonalization approach tends to be more accurate.

Other Eigenstates

New Approach: For a site produce an effective Hamiltonian H' and choose the eigenstate of H' closest to the current energy

For a site produce an effective Hamiltonian H' and choose another eigenstate.



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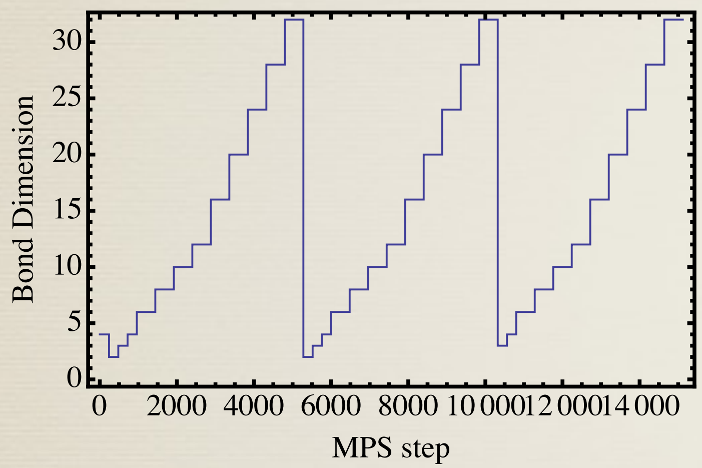
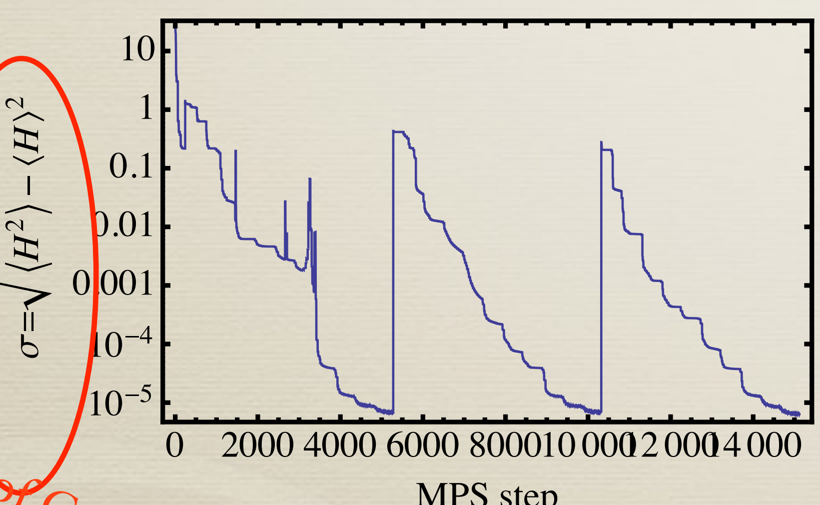




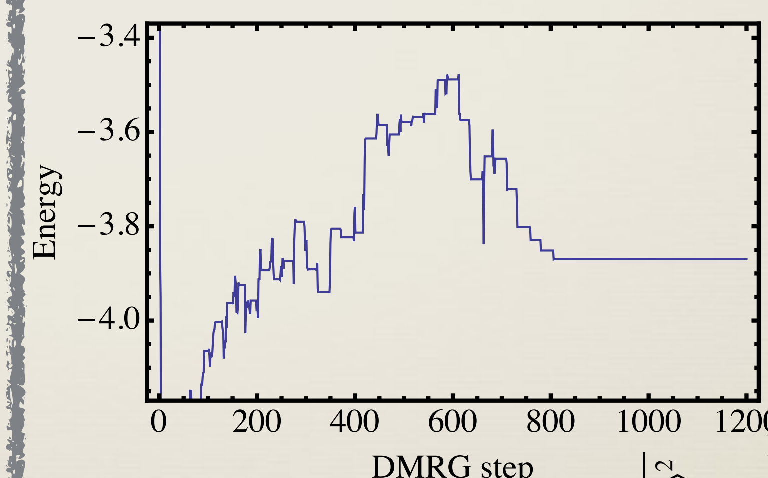
Metric of Goodness

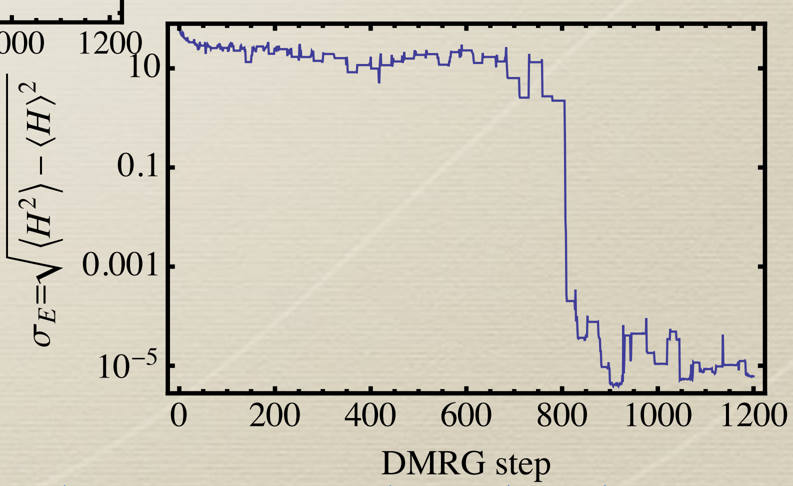
Disorder 10; 24 Sites

Level Spacing: $1.5*10^{-5}$

Typical DMRG: For a site produce an effective Hamiltonian H' and solve for the ground state of H'

Modified DMRG: For a site produce an effective Hamiltonian H' and choose the eigenstate of H' closest to the current energy of your state.





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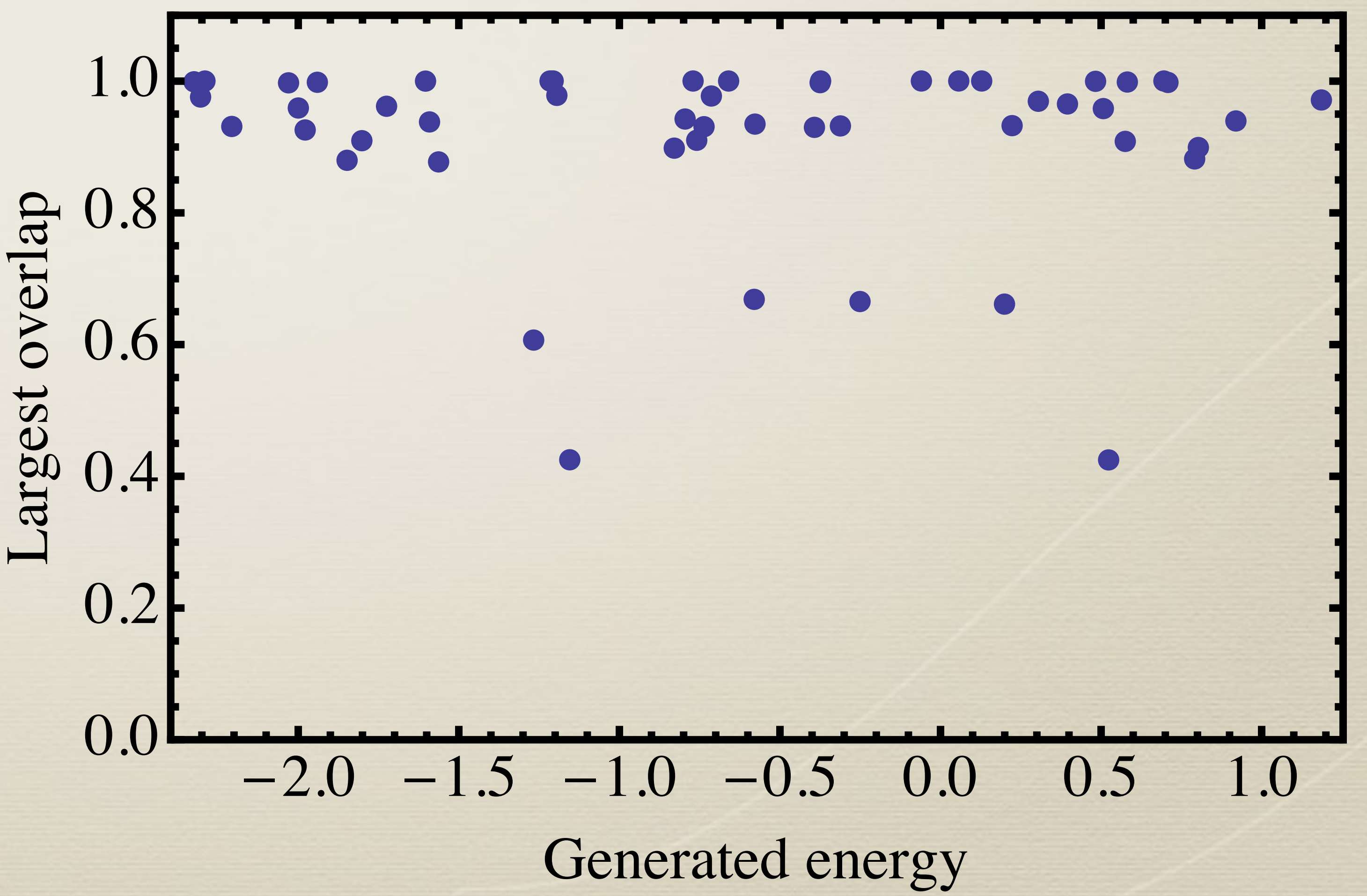
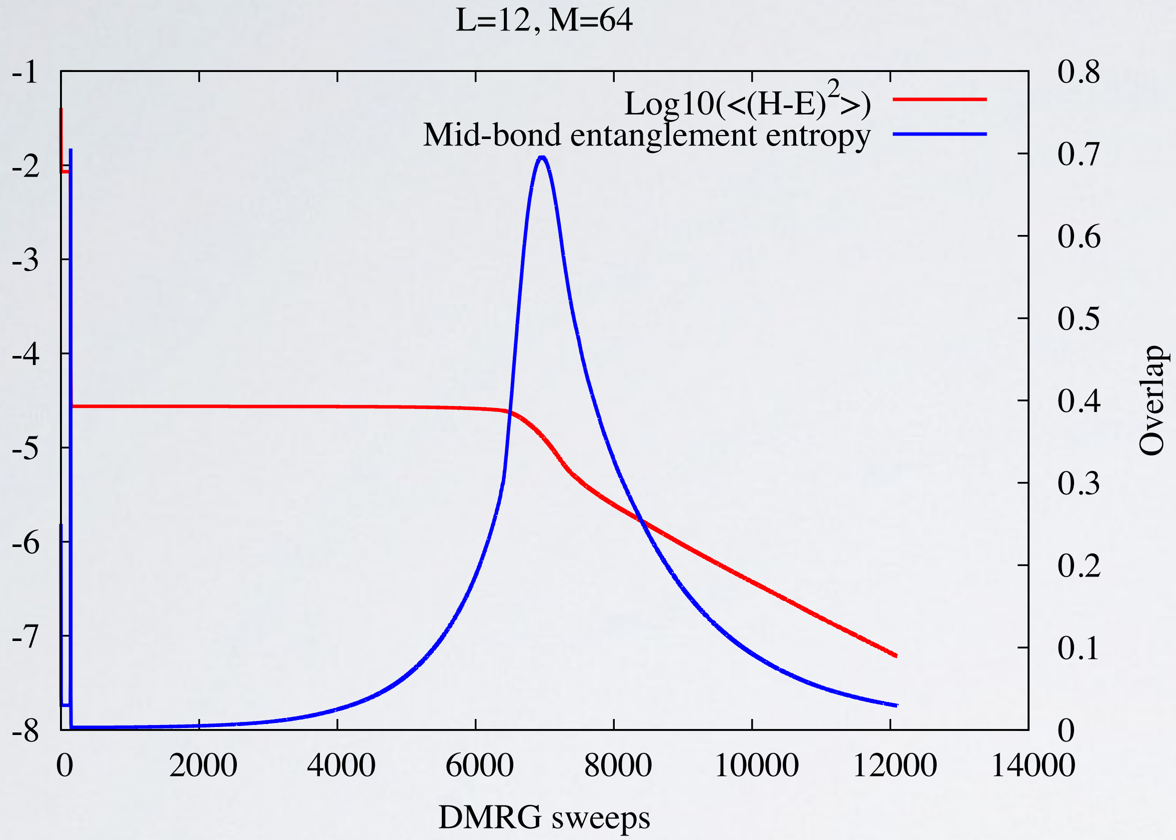
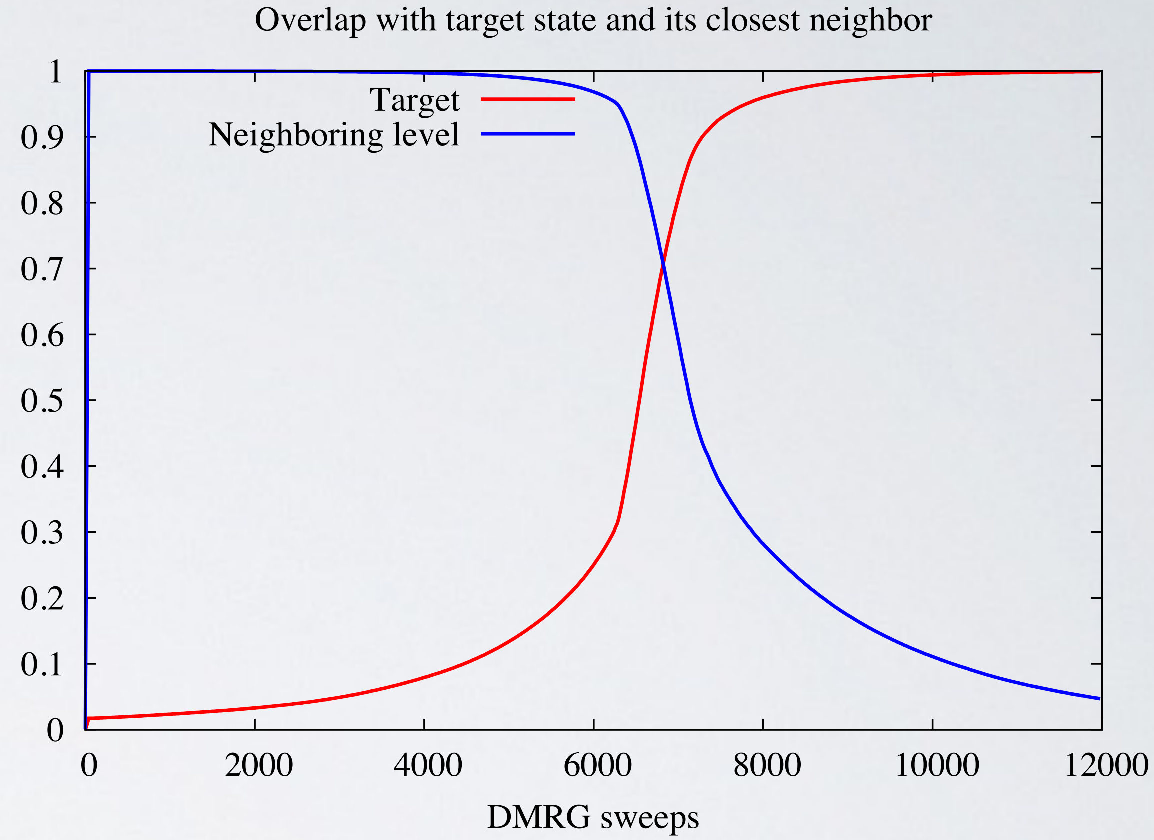
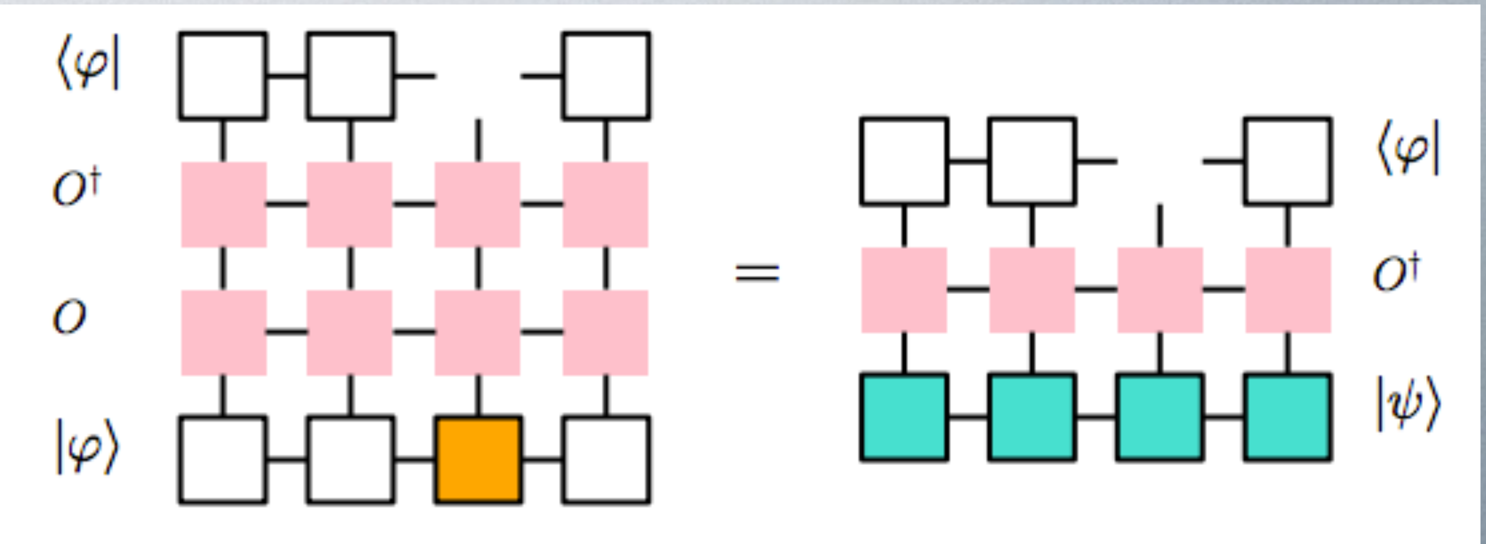


$(H-E)^2$ **Entanglement Barrier**

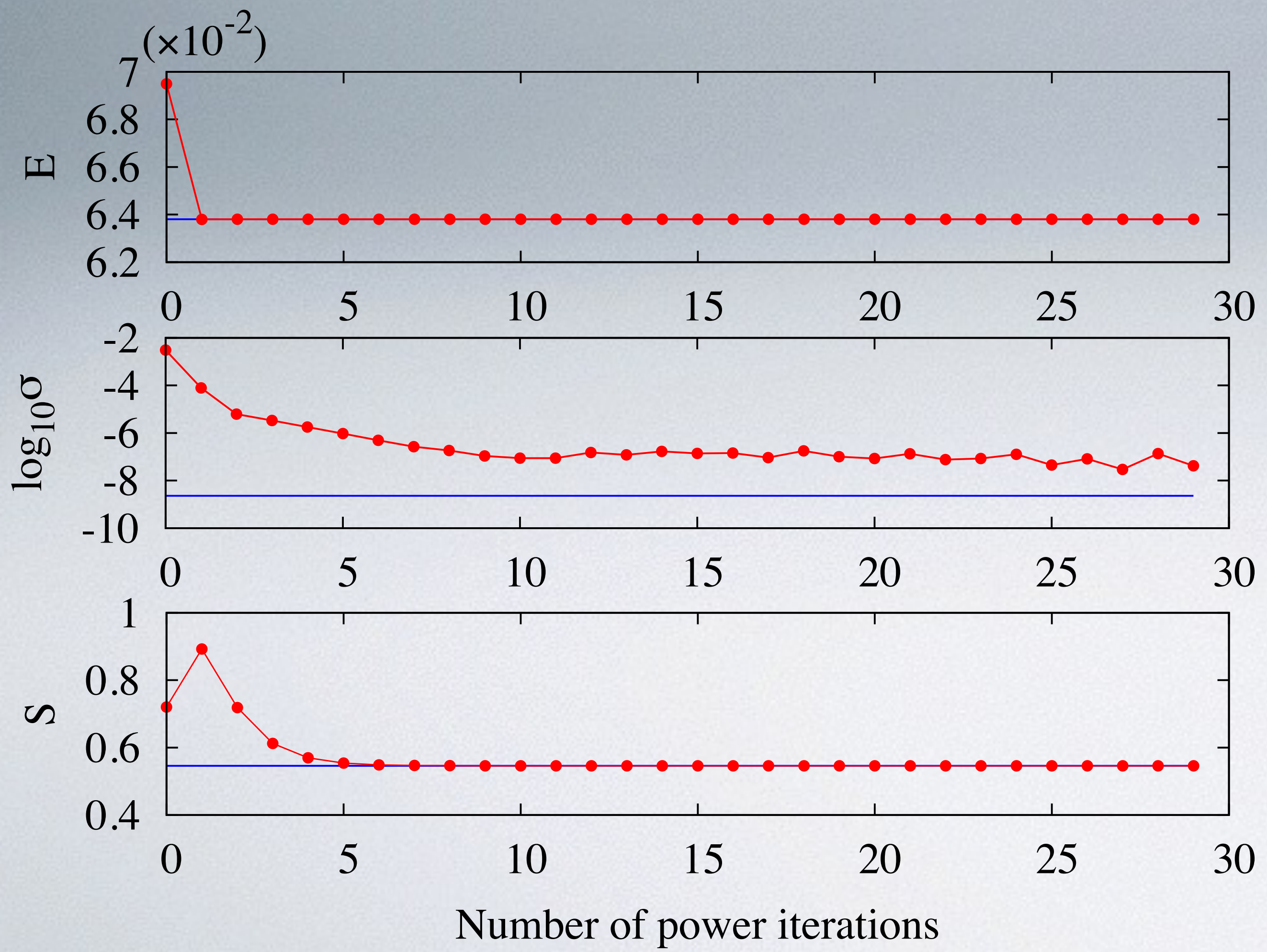




SIMPS $(H-E)^{-1}$



$\langle\varphi|$
$O^\dagger$
O
$|\varphi\rangle$
$=$
$\langle\varphi|$
$O^\dagger$
$|\psi\rangle$

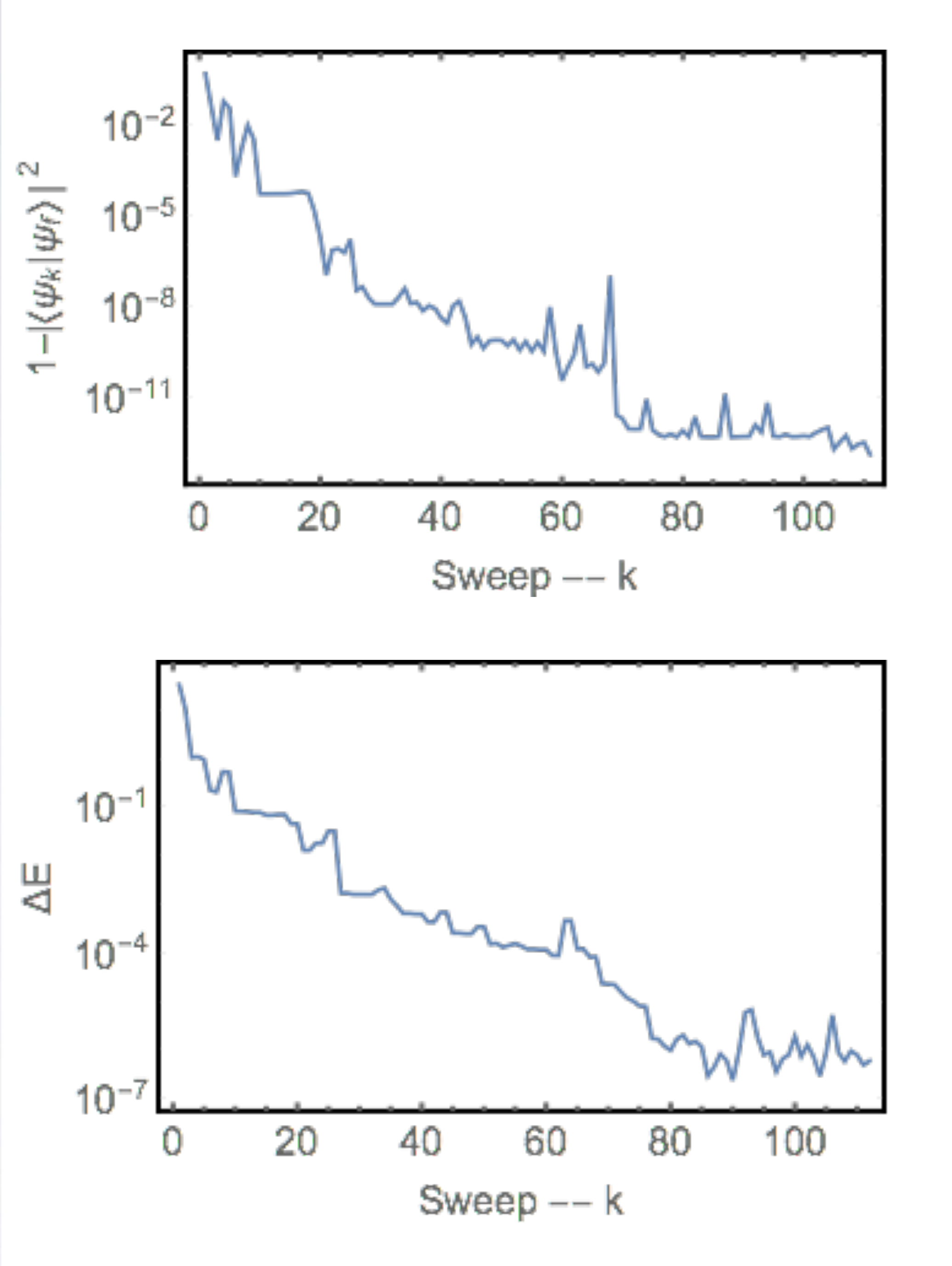


(×10-2)
E
log10σ
S
Number of power iterations

ES DMRG

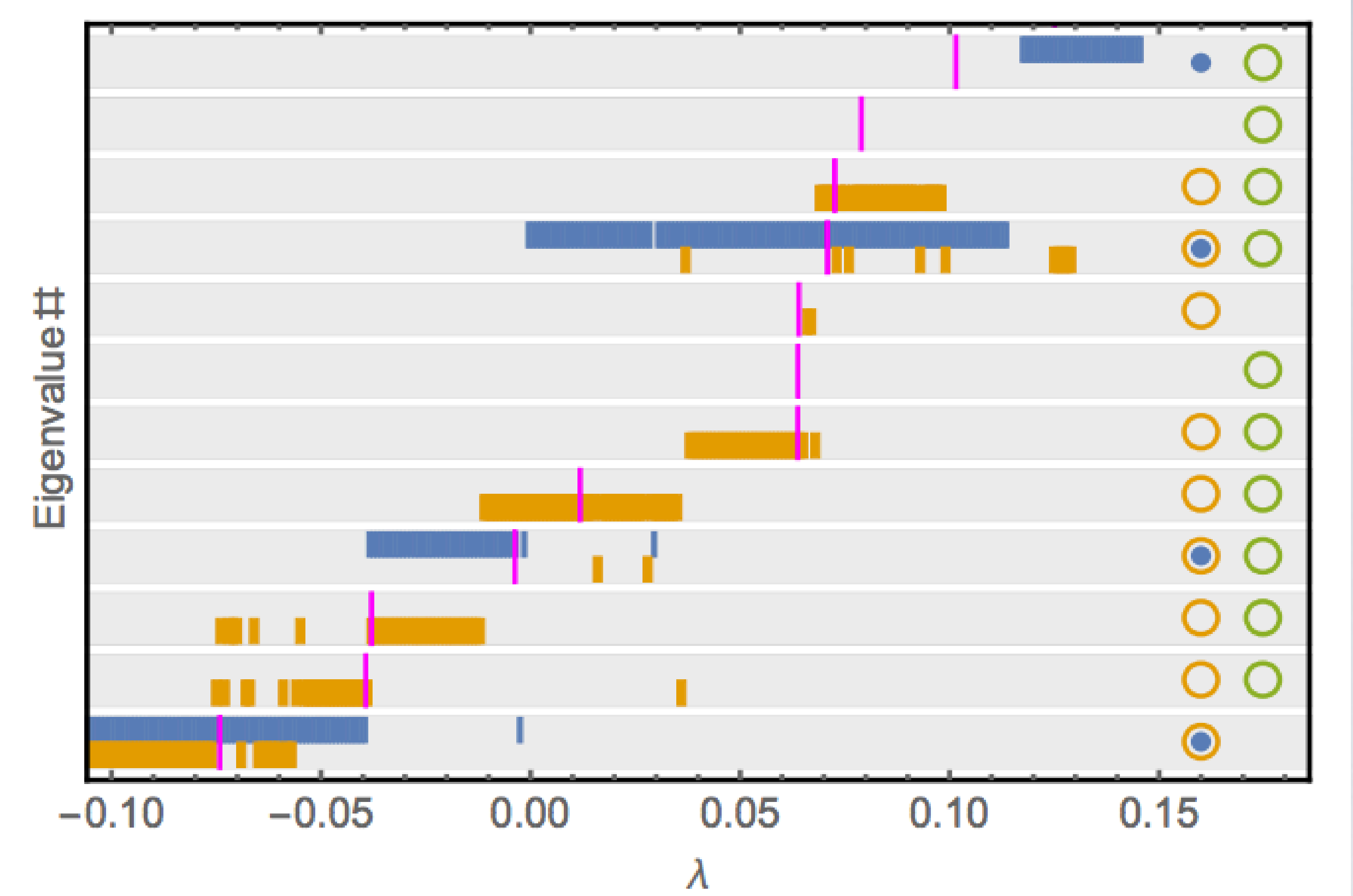
Typical DMRG: For a site produce an effective Hamiltonian H' and solve for the ground state of H'

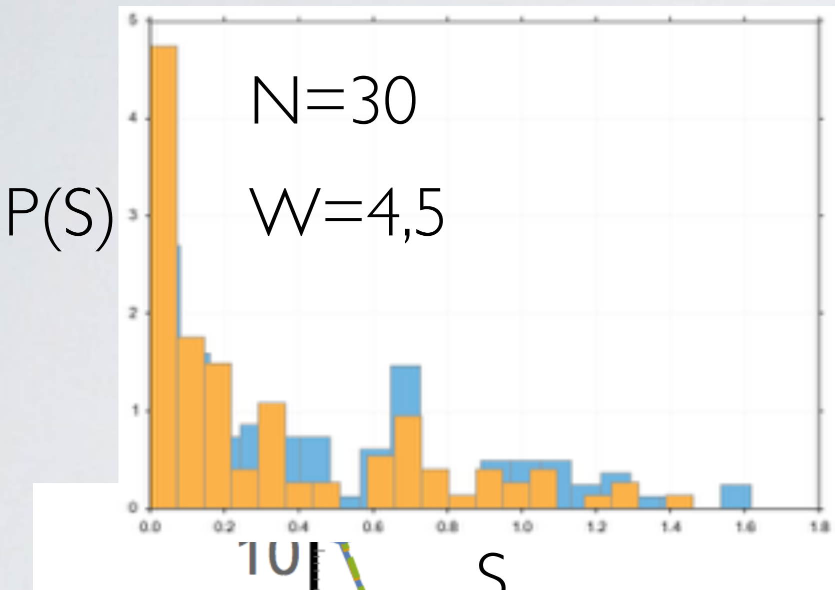
Modified DMRG: For a site produce an effective Hamiltonian H' and choose the eigenstate of H' closest to the current energy of your state.

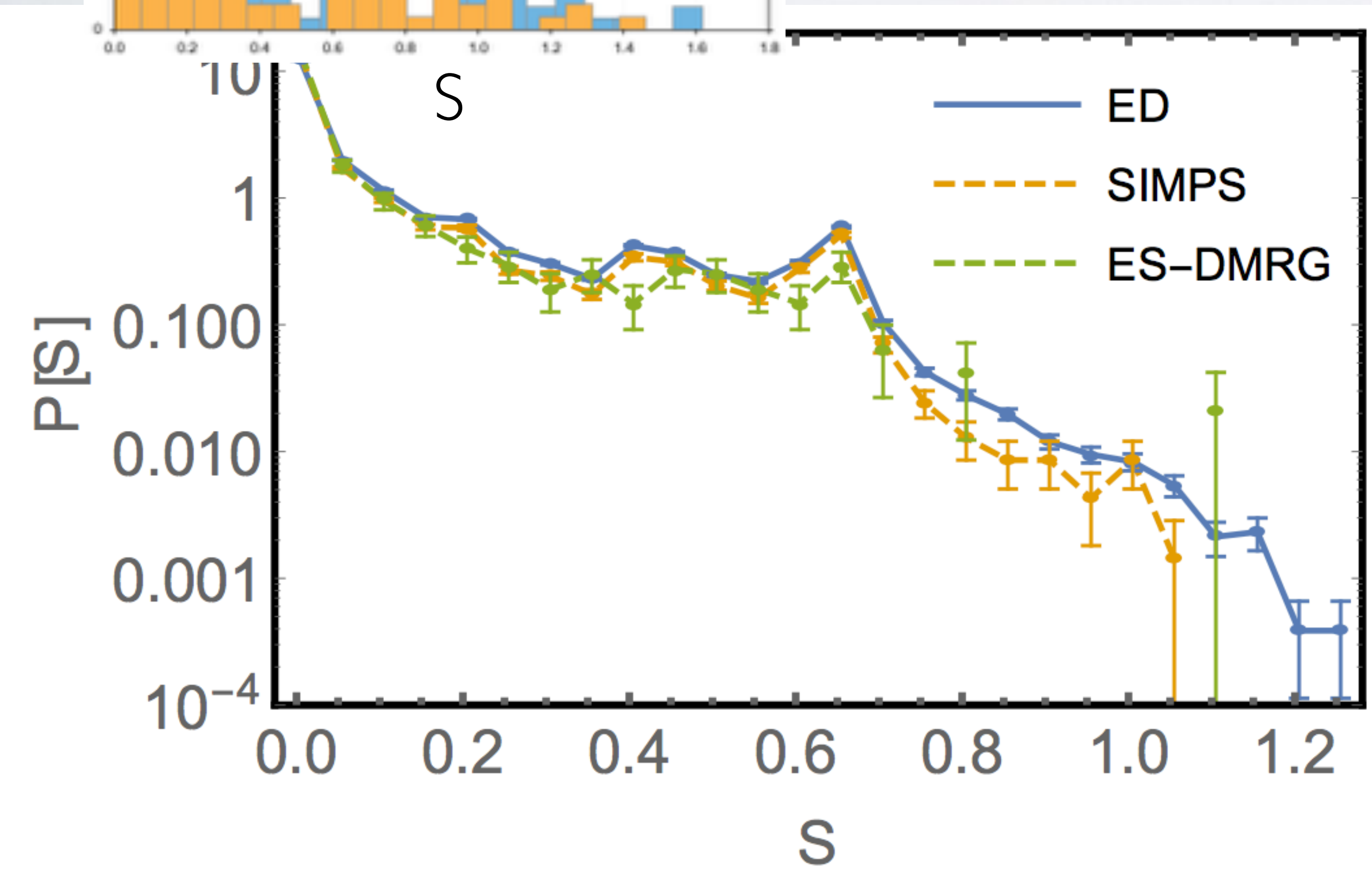


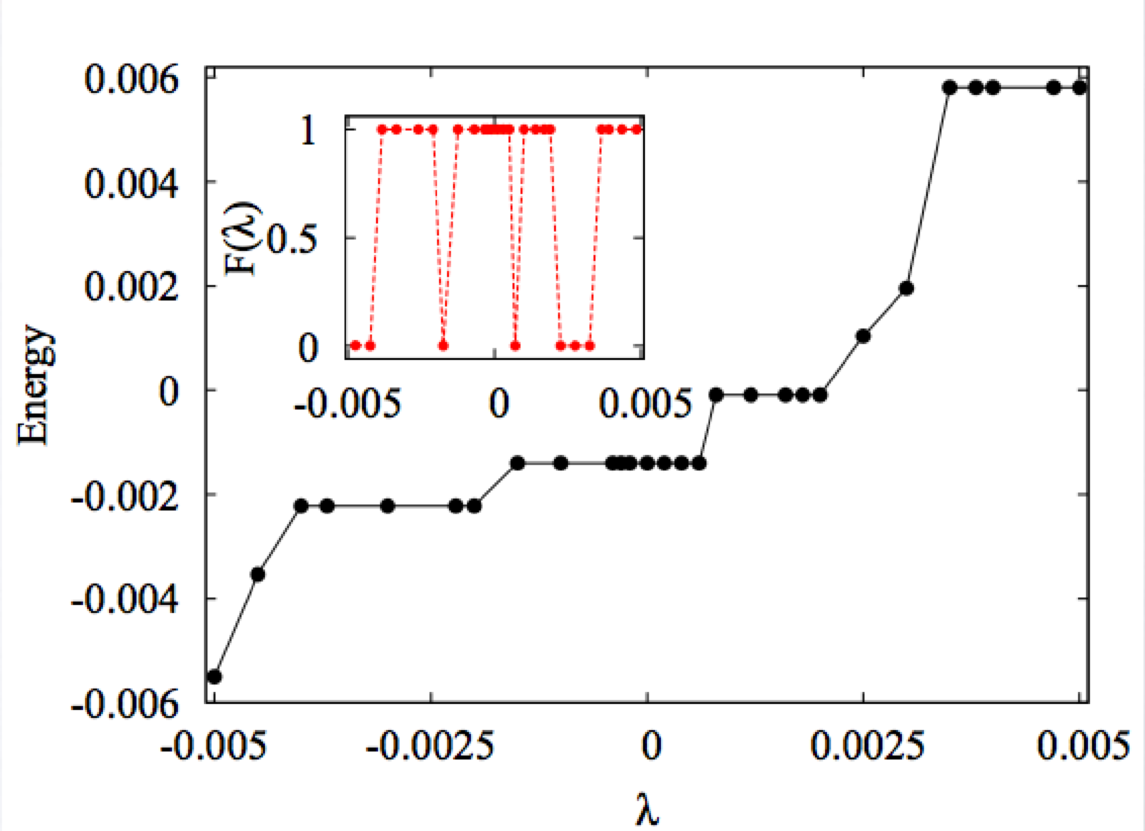
Does it work?

The question: Do you get an eigenstate?









How well does it work?

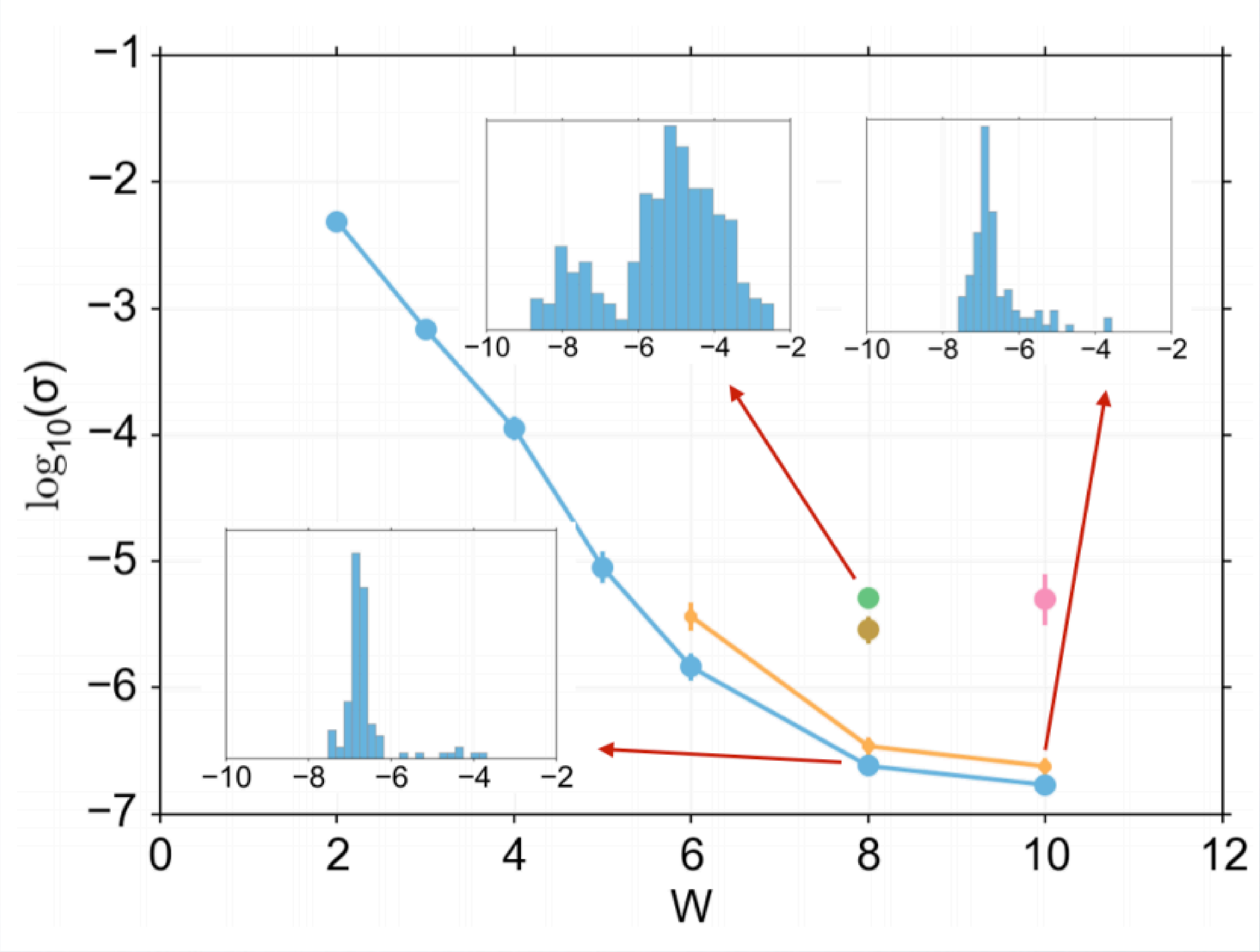
N=100 ES-DMRG

N=30 - SIMPS M=20

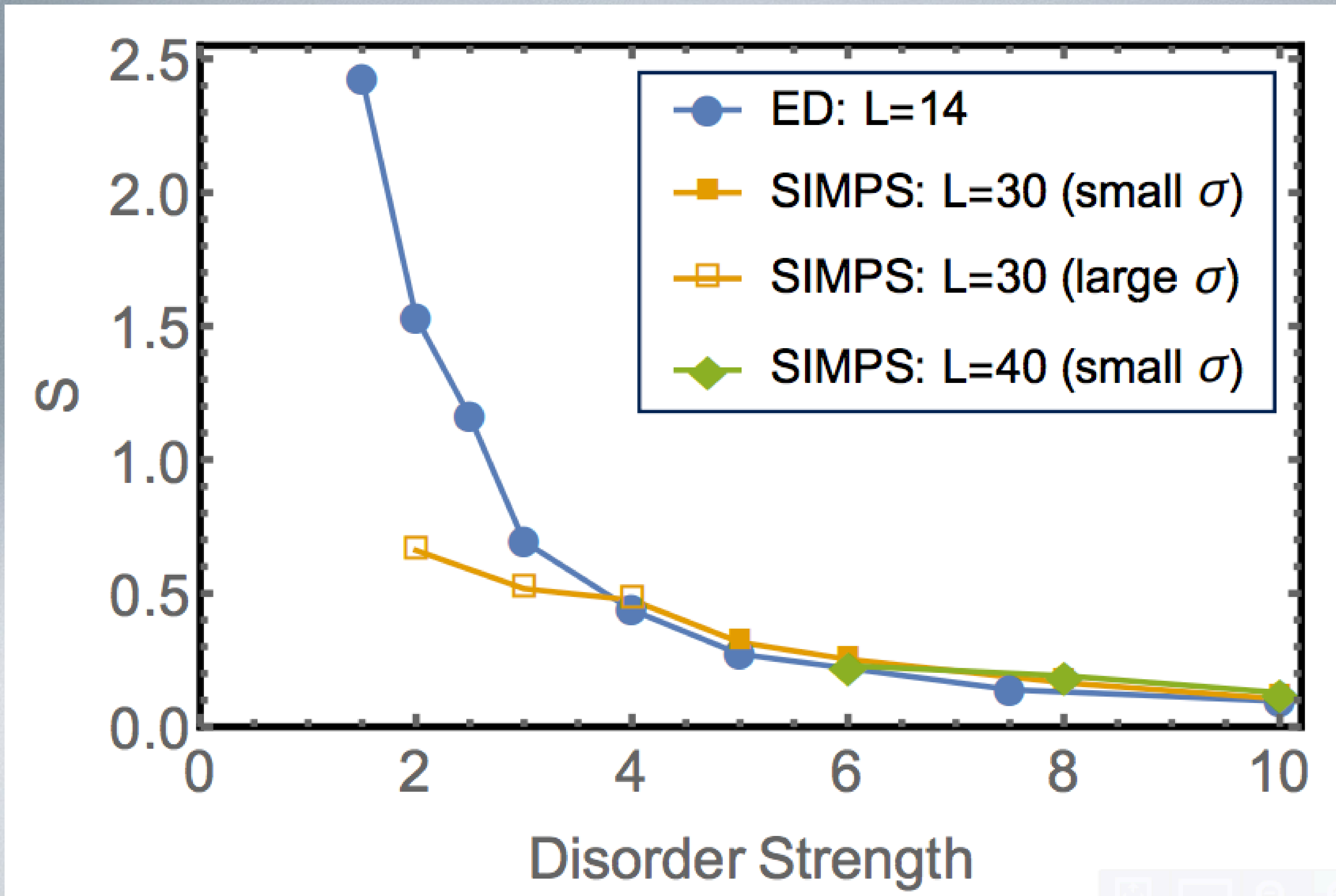
N=30 - ES-DMRG M=20

N=30 - SIMPS
M=60

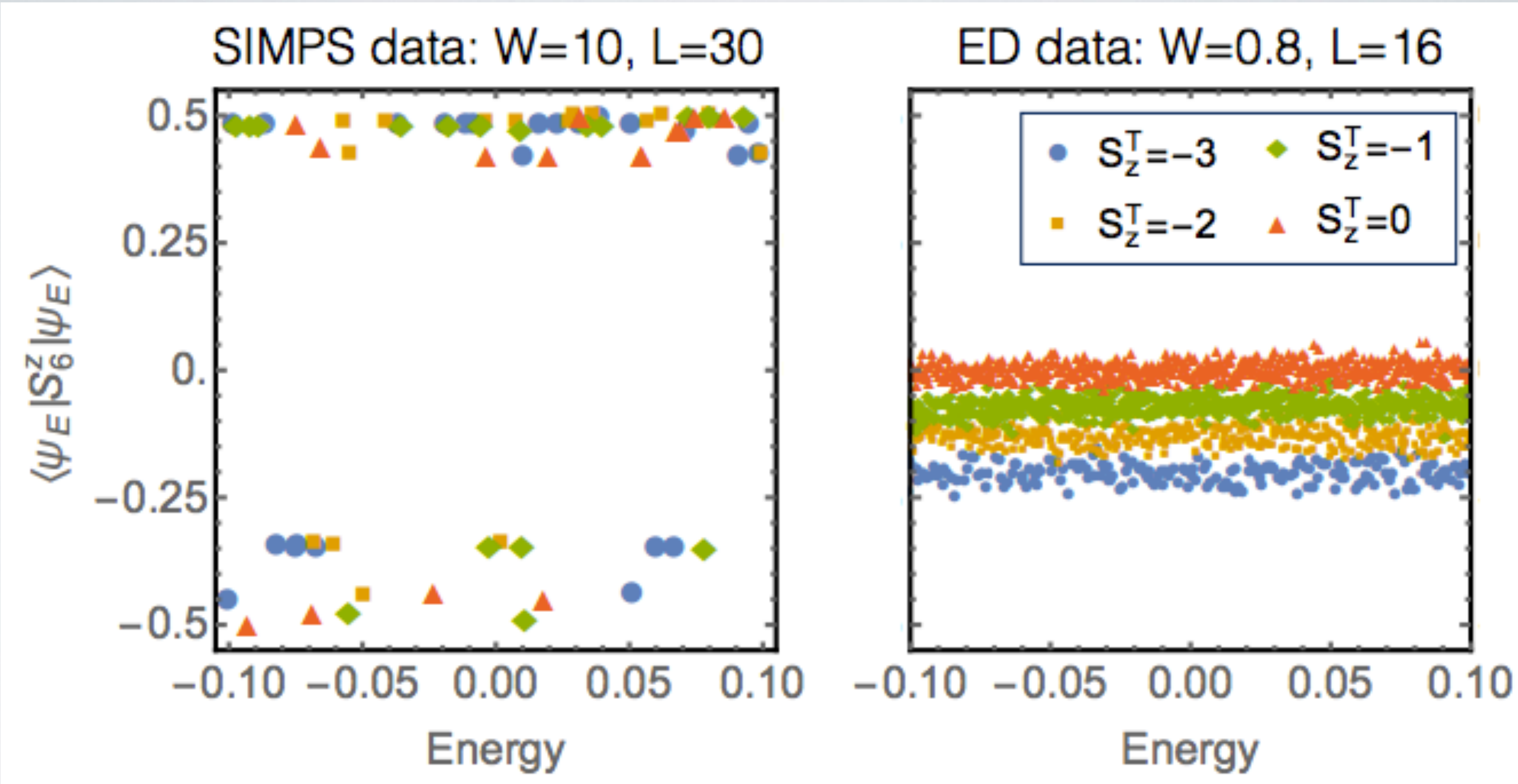
N=40 - SIMPS
M=60



How does entanglement scale?



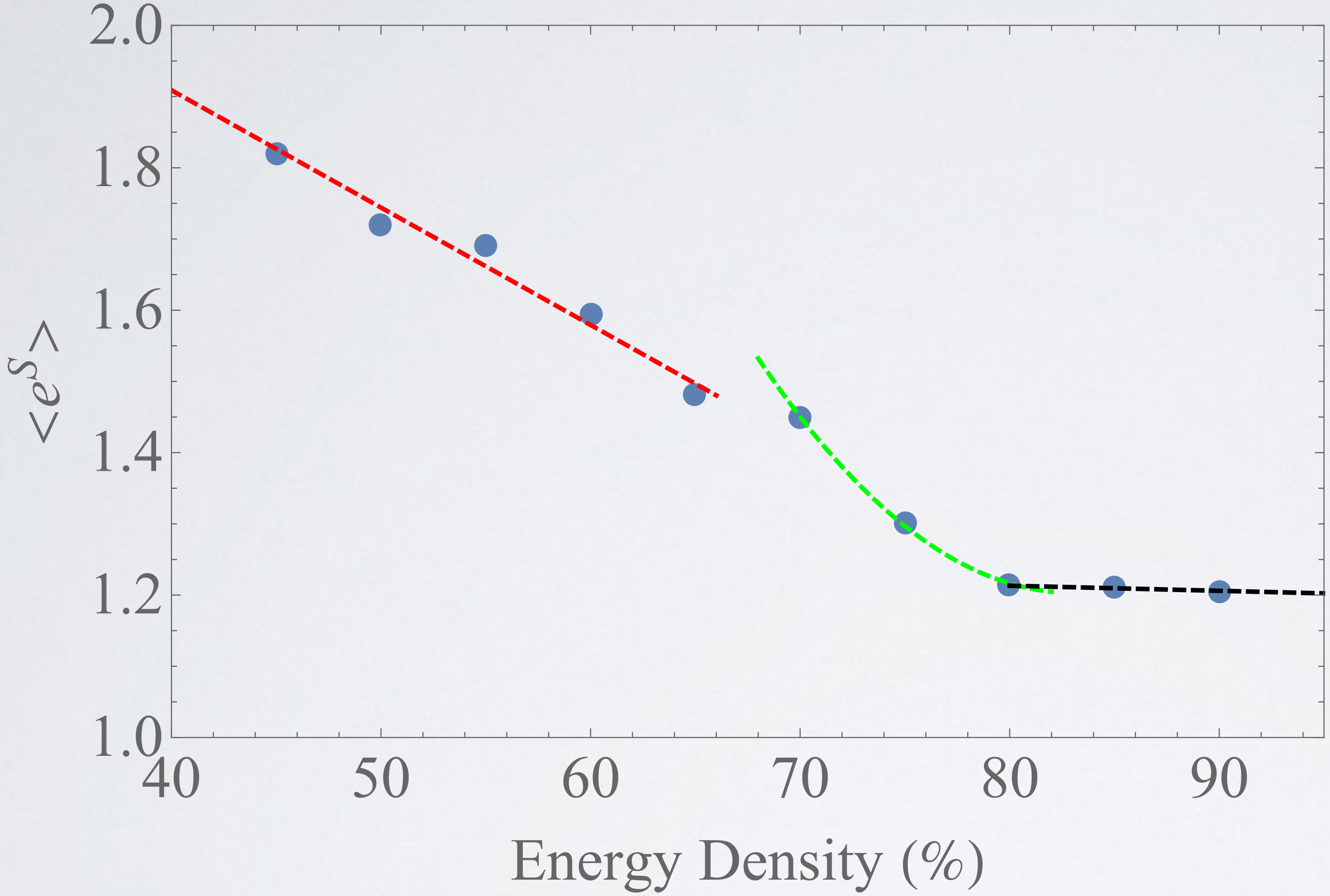
ETH

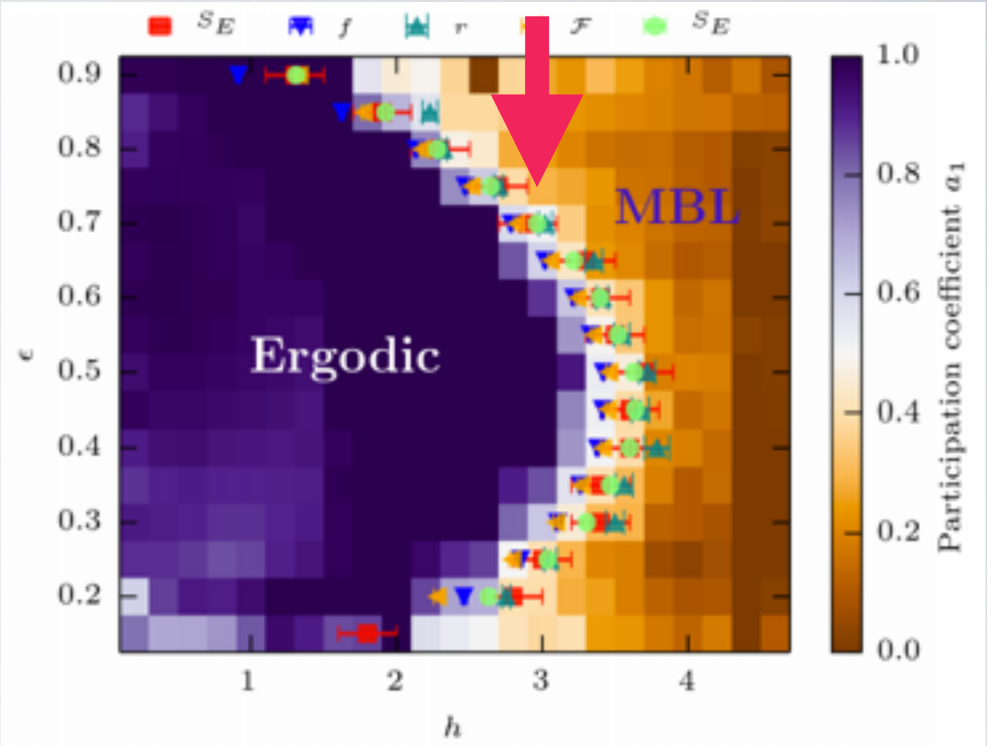


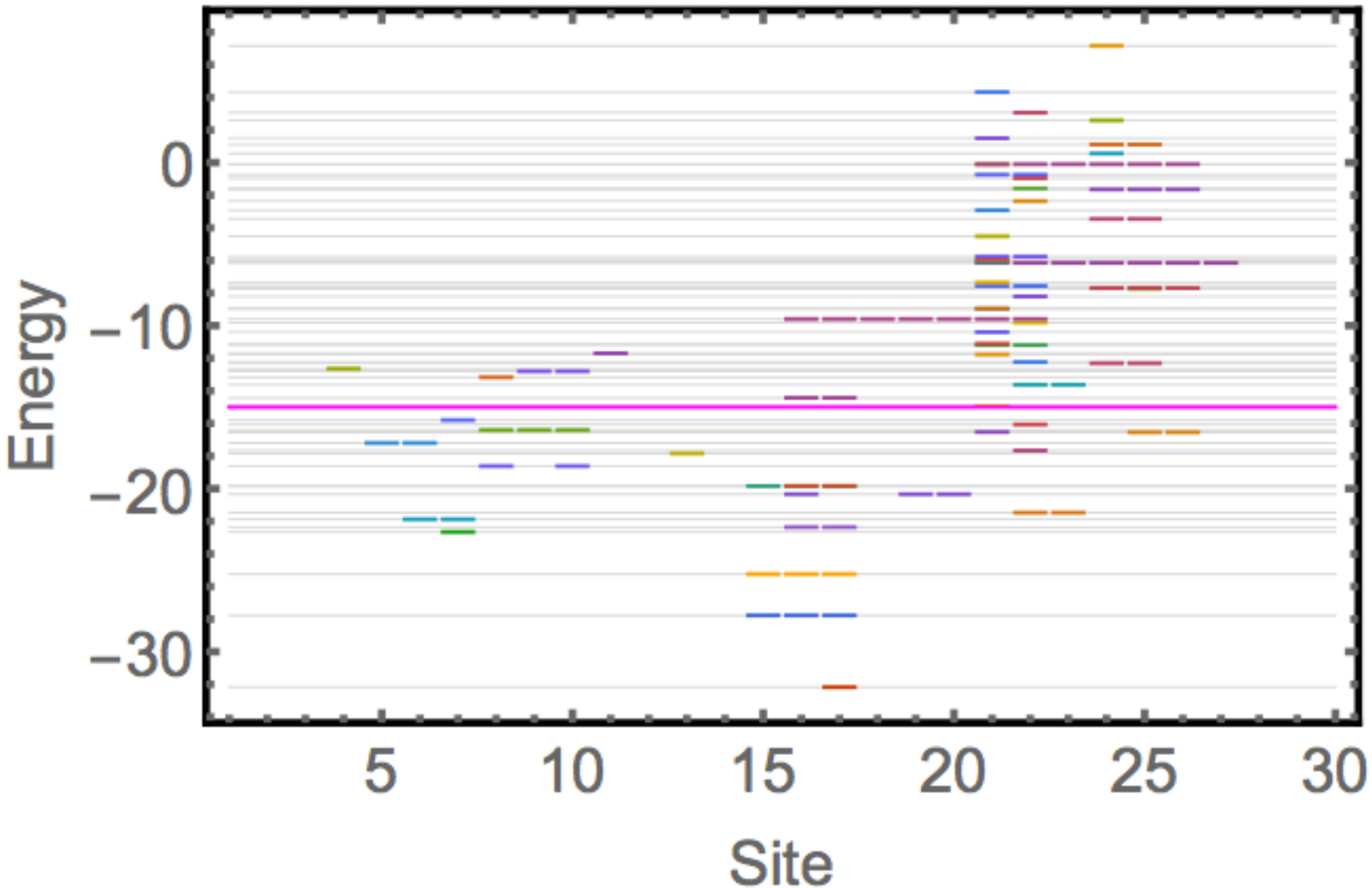
SIMPS data: W=10, L=30
ED data: W=0.8, L=16
$\langle\psi_E|S_6^z|\psi_E\rangle$
0.5
0.25
0.
−0.25
−0.5
−0.10 −0.05 0.00 0.05 0.10
Energy
S_z^T=−3
S_z^T=−1
S_z^T=−2
S_z^T=0

What can we see

Mobility Edge:

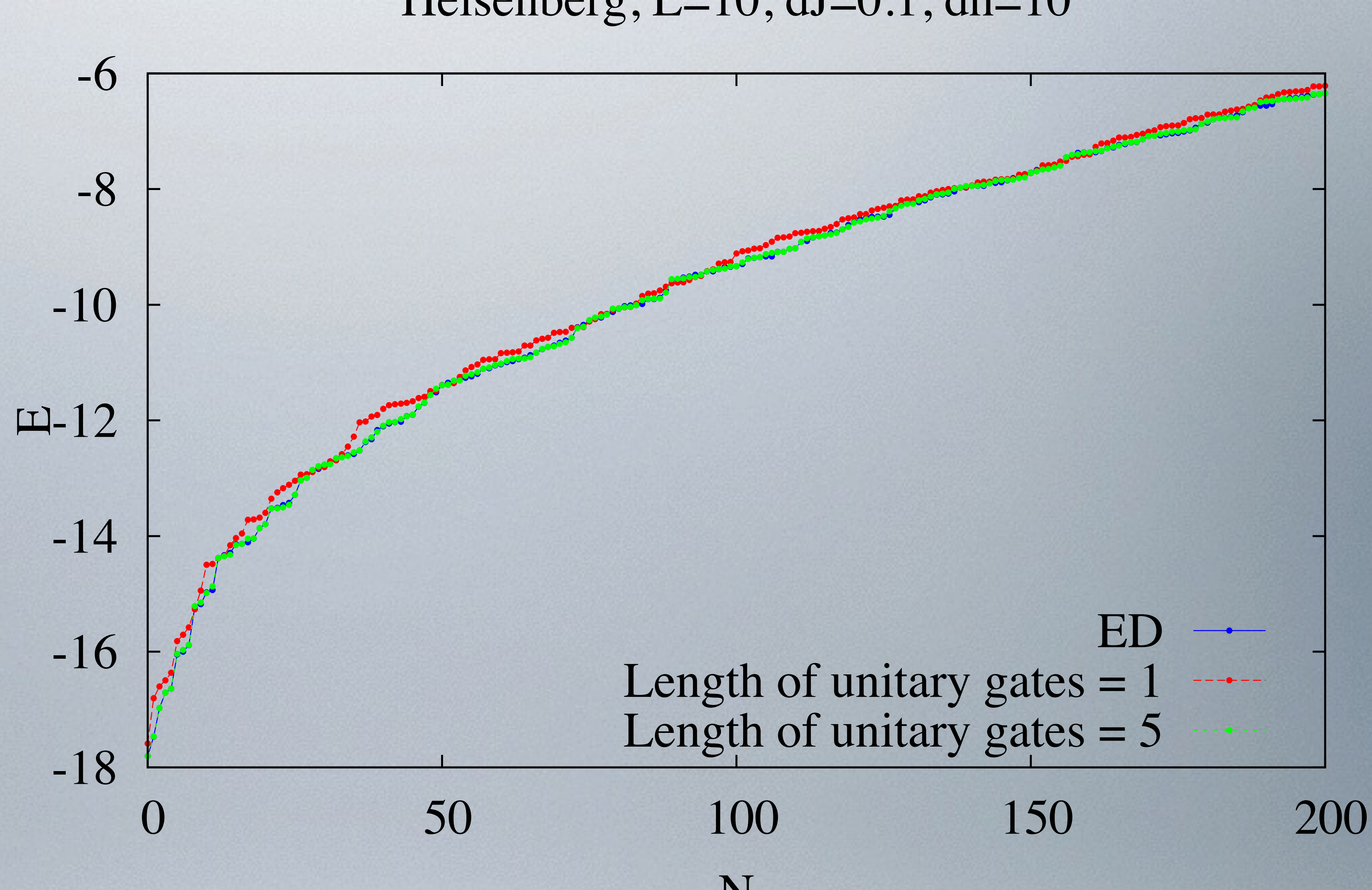


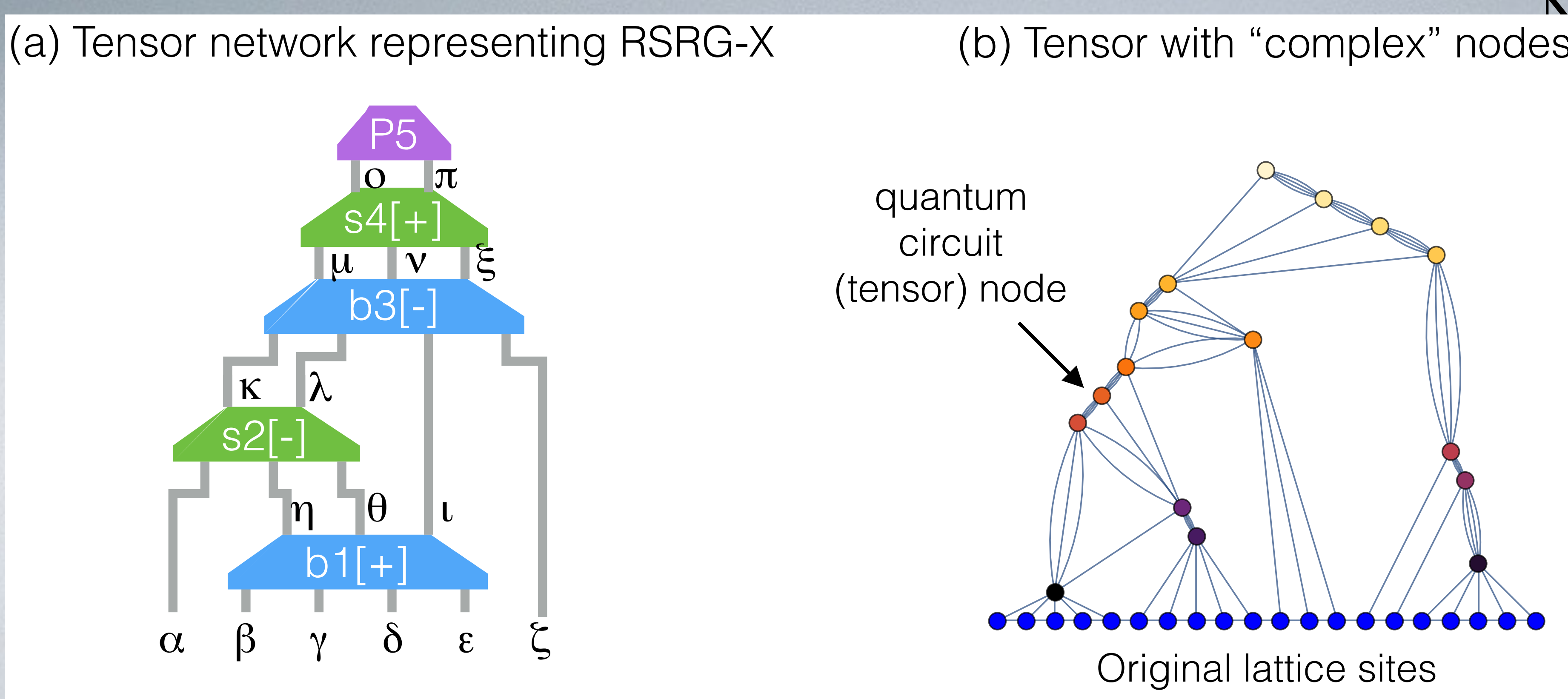




Energy
0
−10
−20
−30
5
10
15
20
25
30
Site

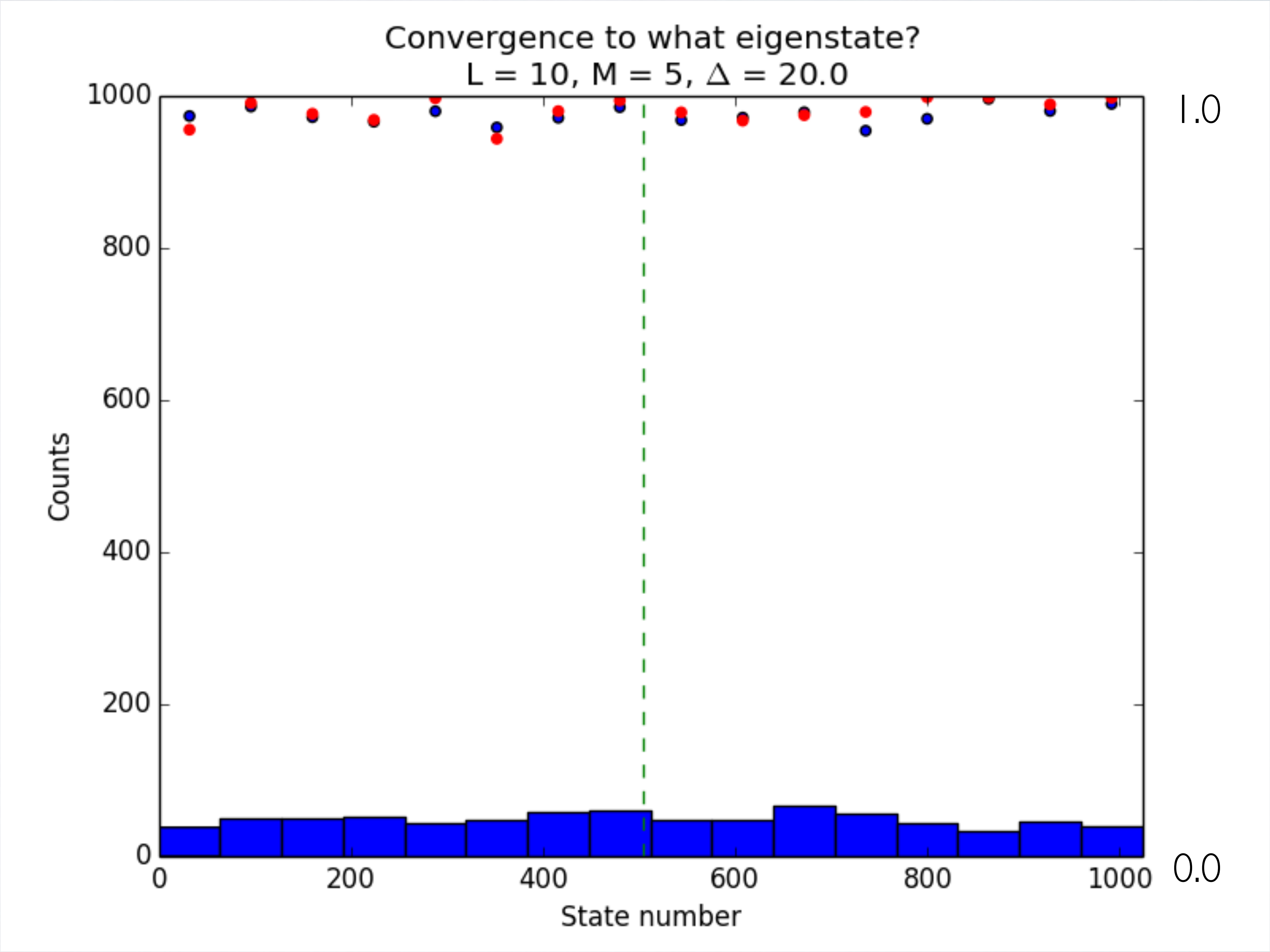
MERA


E
-6
-8
-10
-12
-14
-16
-18
0
50
100
150
200
ED
Length of unitary gates = 1
Length of unitary gates = 5

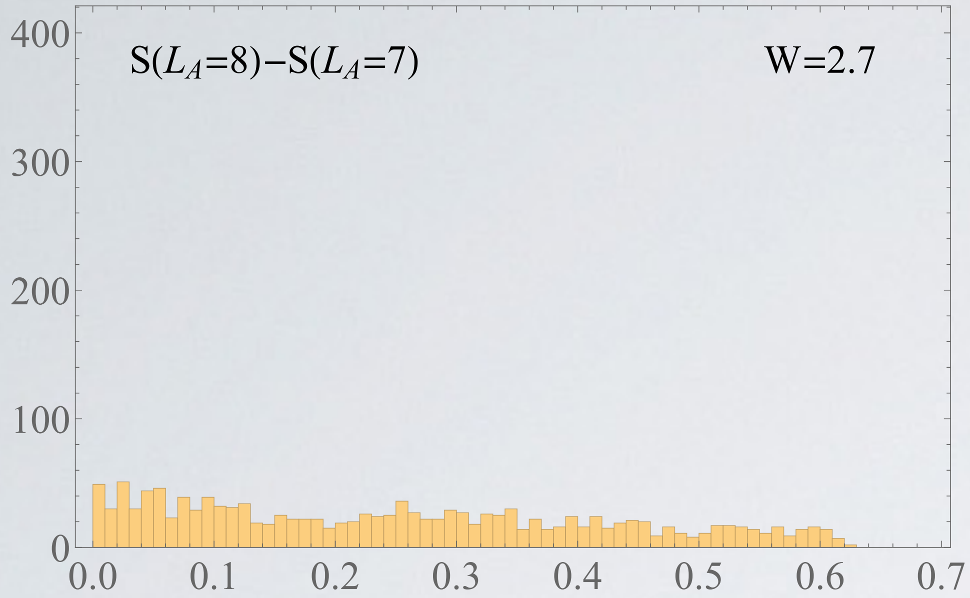

(a) Tensor network representing RSRG-X
P5
ο π
s4[+]
μ ν ξ
b3[-]
κ λ
s2[-]
η θ ι
b1[+]
α β γ δ ε ζ
(b) Tensor with “complex” nodes
quantum
circuit
(tensor) node
Original lattice sites

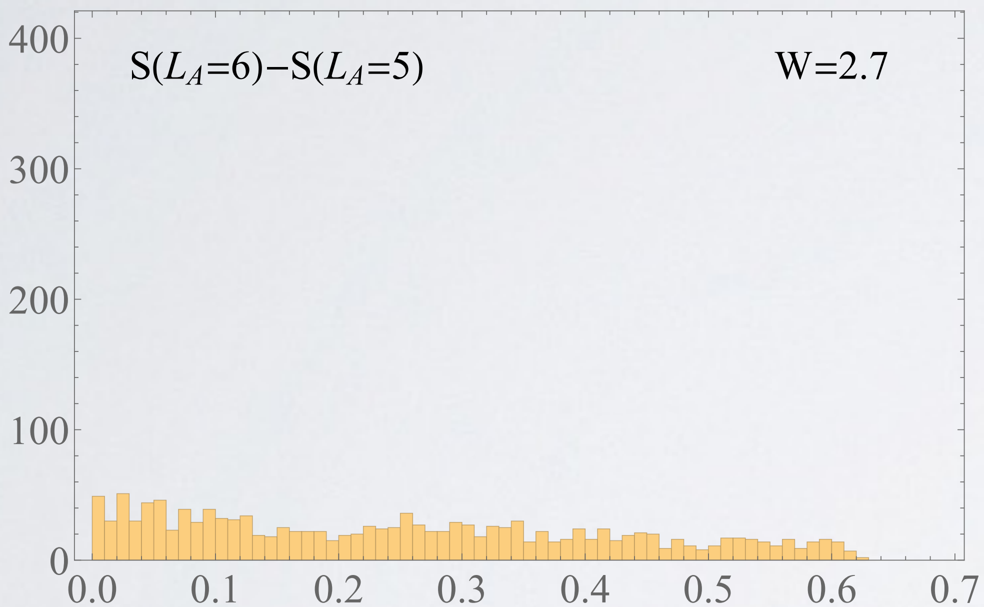
Is their any hope for two-dimensions?

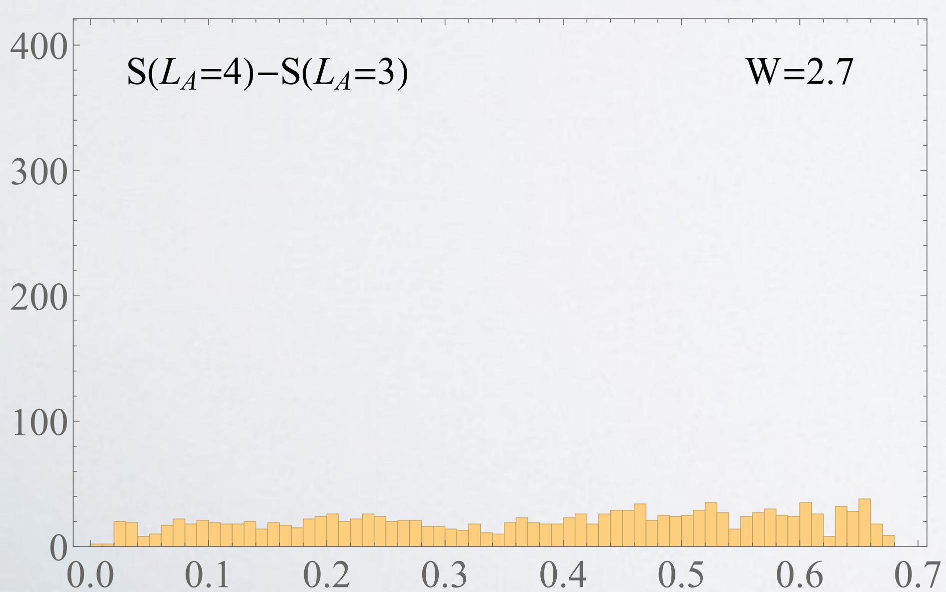
Huse-Elser states

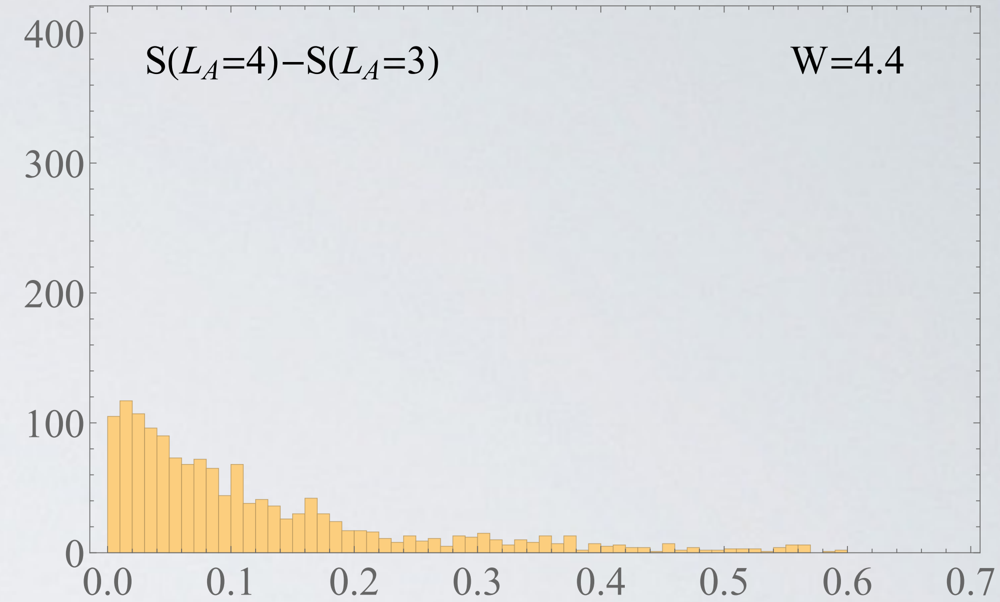


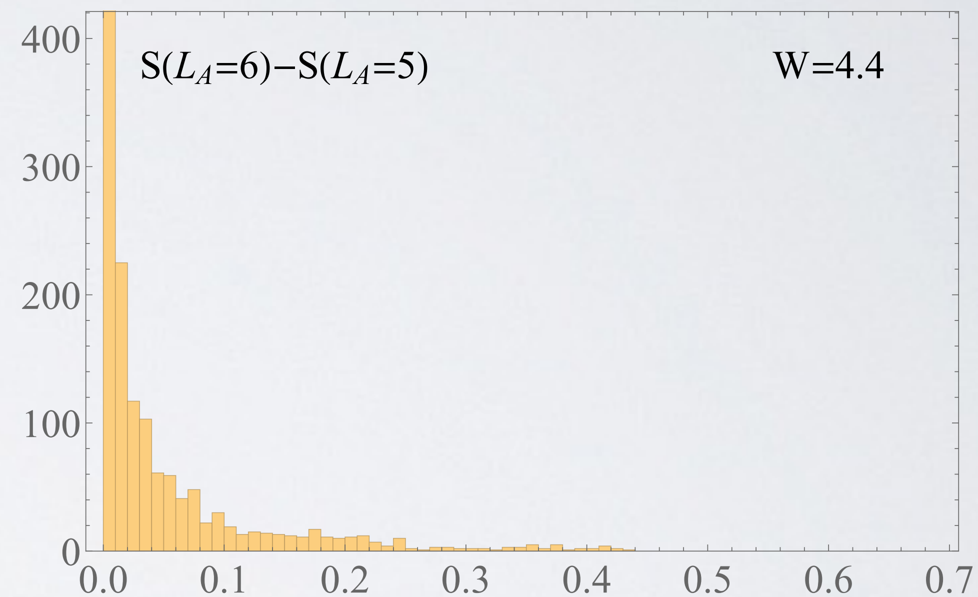
subadditivity

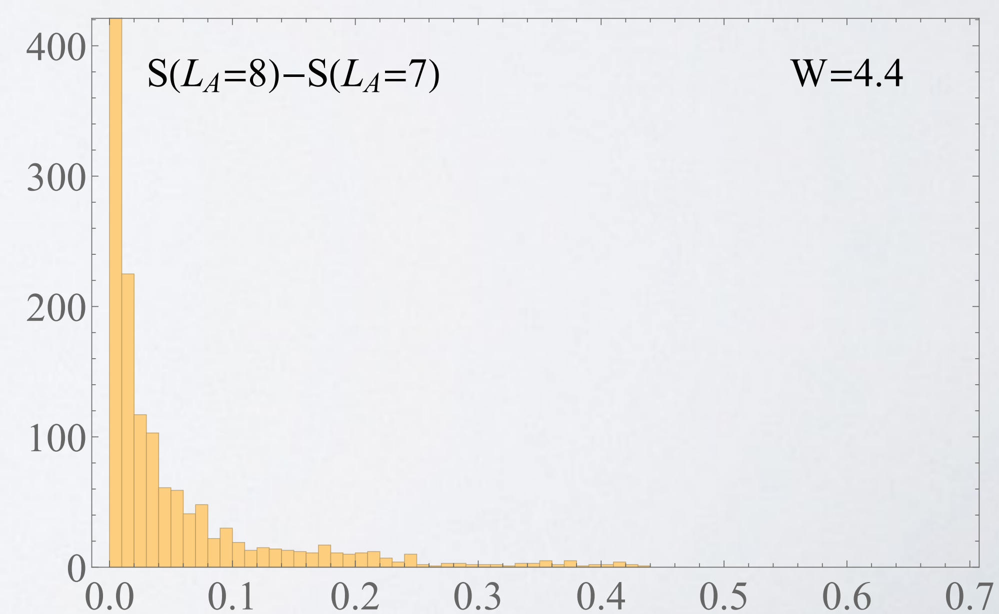


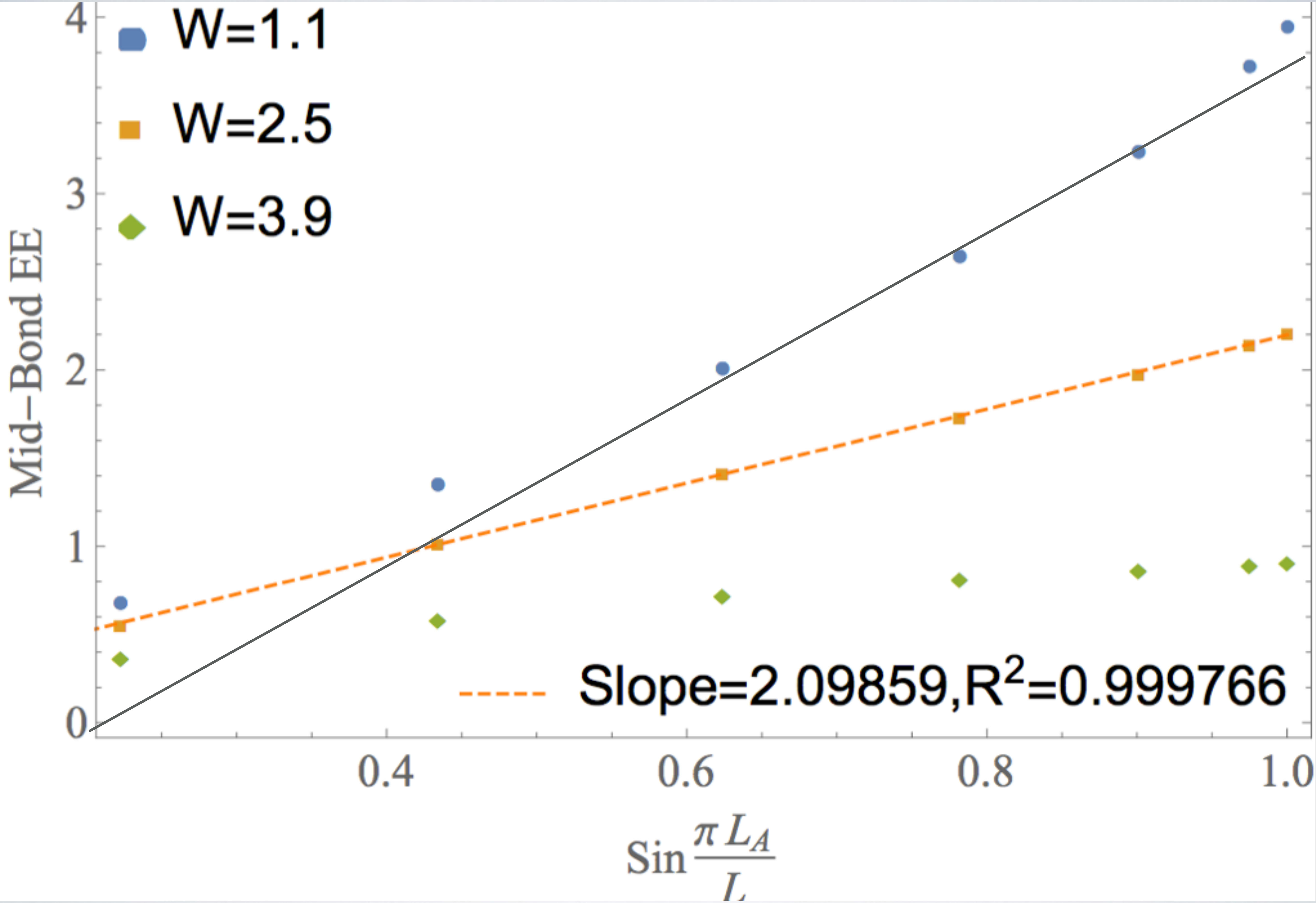











W=1.1
W=2.5
W=3.9
Slope=2.09859,R^2=0.999766
Mid-Bond EE
0
1
2
3
4
0.4
0.6
0.8
1.0
$\mathrm{Sin}\frac{\pi L_A}{L}$

Conclusion

MBL in the language of tensor networks

You can efficiently encode all 2n eigenstates using 2n small matrices!

Prescription to built 2^n MPS out of these matrices

Using this language, you can choose the 'best' choice for l-bits

Look at correlation lengths of different quantities…

Two correlation lengths

We can find the MPS using two new algorithms

ES-DMRG: Modified DMRG sweeping* (same algorithm as DMRG-X)

SIMPS: Energy Targeting using $(H-E)^{-1}$

Beyond MPS: MERA and Huse-Elser states in 2D

Identifying the transition

Area Law MBL Eigenstates: Bauer, Nayak; Eisert

* Yu, Pekker, BKC; arxiv:1509.01244
* Vedika, Pollmann, Sondhi; arxiv:1509.00483