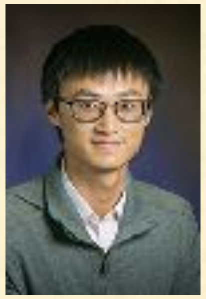
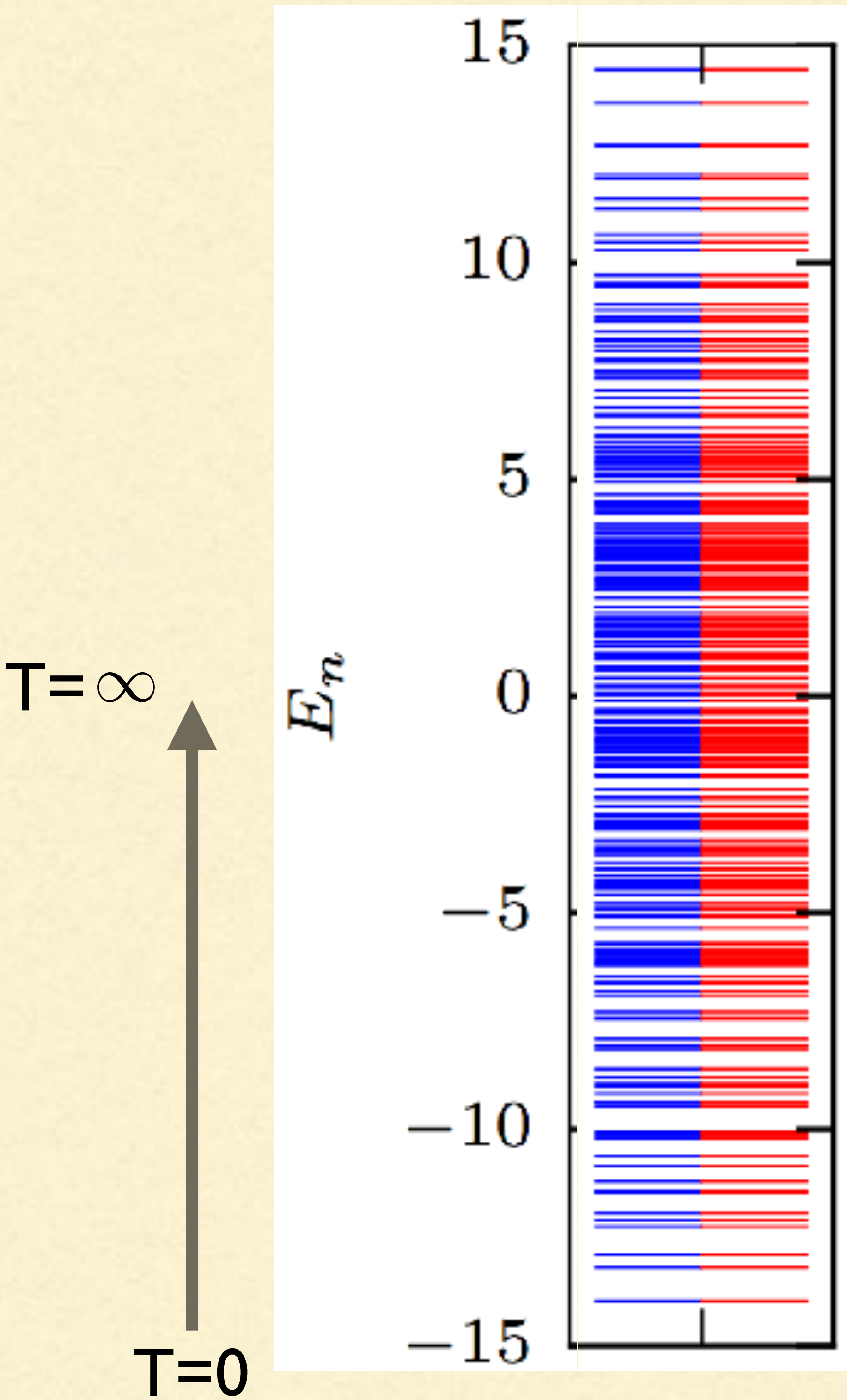
INFINITE TEMPERATURE QUANTUM MECHANICS: HOW SHORT QUANTUM CIRCUITS LEAD TO THE BREAKDOWN OF STATISTICAL PHYSICS AND OTHER STORIES ABOUT MANY-BODY LOCALIZATION

Bryan K Clark





This is a talk about the interplay between quantum mechanics and statistical mechanics.



Statistical Mechanics

Quantum Mechanics

Statistical Mechanics 101

A hot cup of coffee eventually equilibrates to the temperature of the room. It thermalizes

Is their another option?

Are there cases where subsystems don't thermalize?

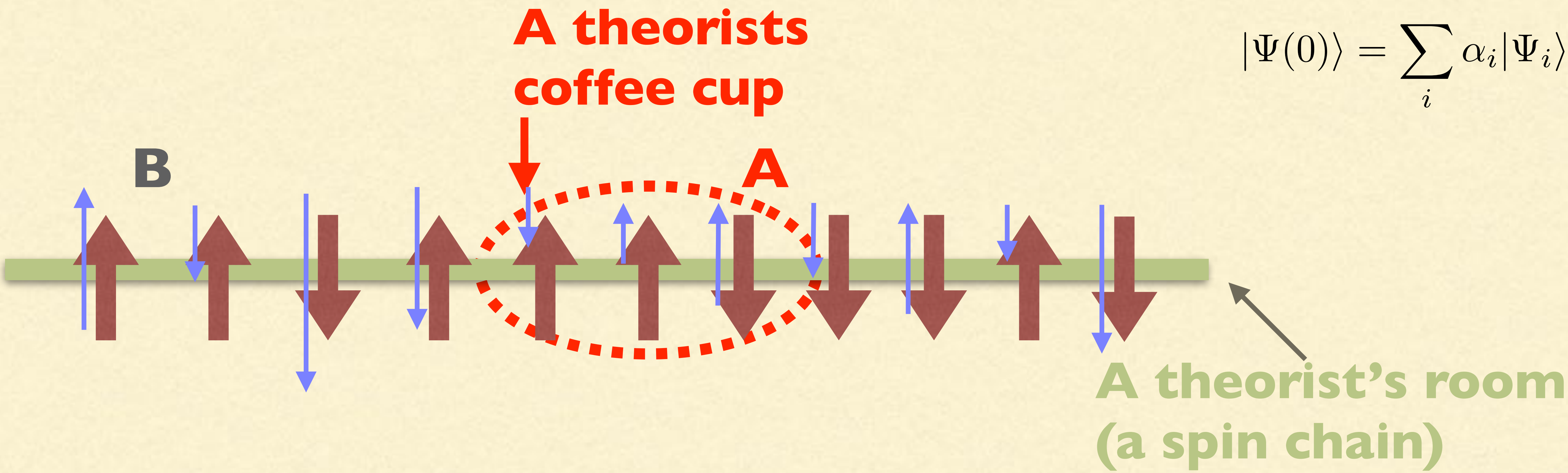
yes…particularly systems with interaction + disorder

Anderson (1957)

Basko, Aleiner and Altshuler, Annals of Physics (2006)

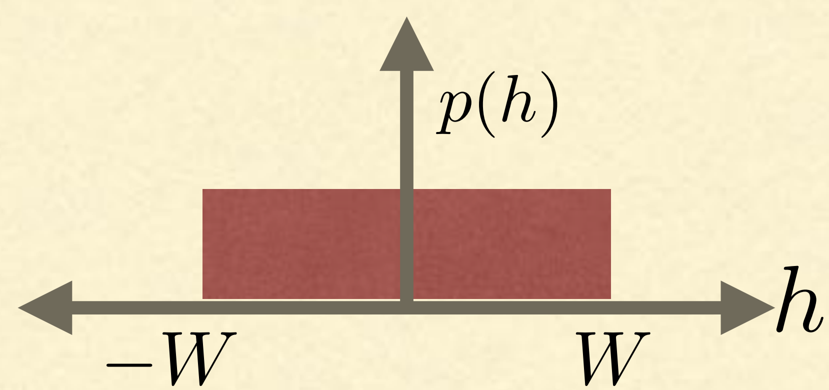
Pal and Huse. *PRB* 82.17 (2010): 174411.

$$|\Psi(0)\rangle = \sum_i \alpha_i |\Psi_i\rangle$$



H = interaction + disorder

$$H = \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + h_i S_i^z \right)$$

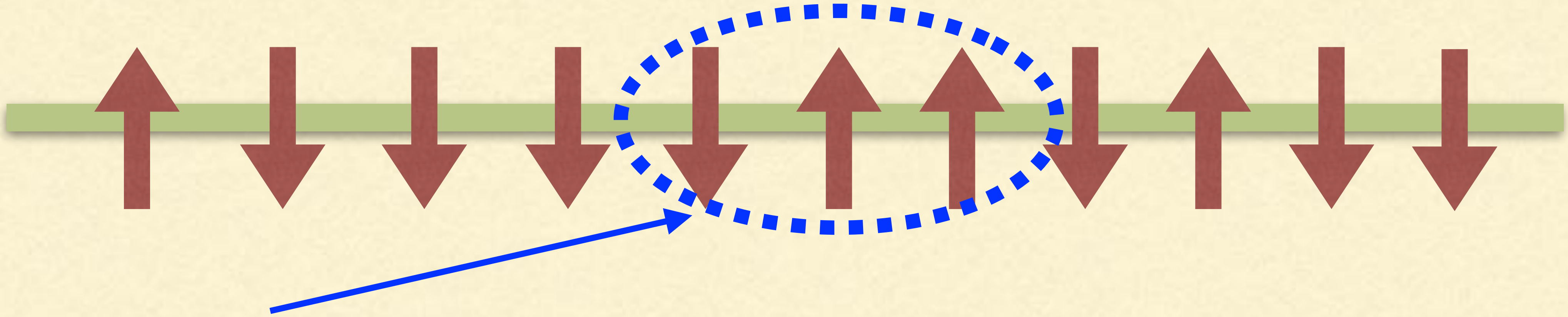


$$h \in [-W, W]$$

$$p(h_i) = \frac{1}{2W}$$

Something we will measure a lot: $\langle S_6^z \rangle$

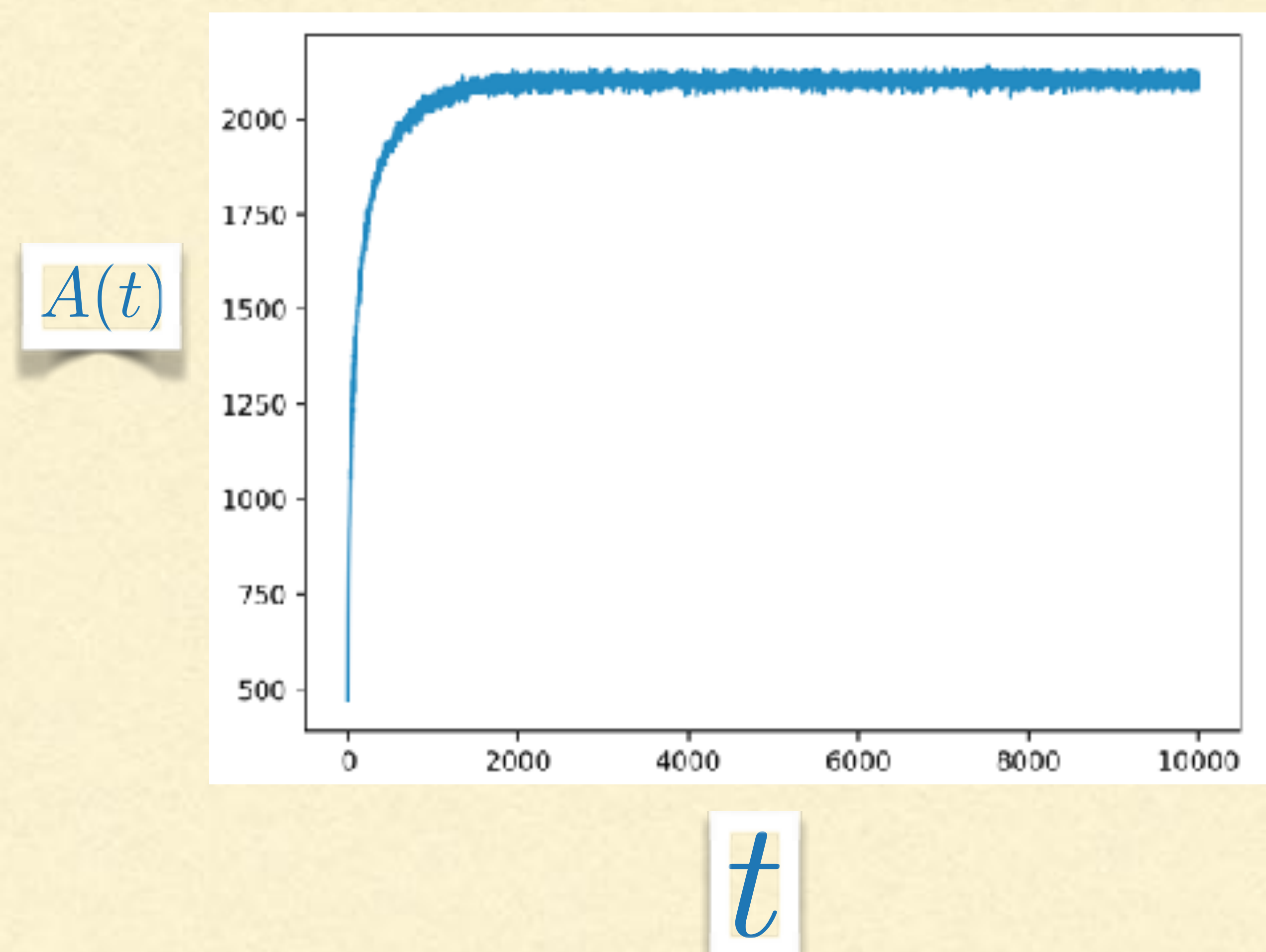
Equilibration means…



$$|\Psi(0)\rangle = \sum_i \alpha_i |\Psi_i\rangle$$

Local observable A on this subsystem $A(t) = \langle \Psi(t)|\hat{A}|\Psi(t)\rangle$

your forget where you started.



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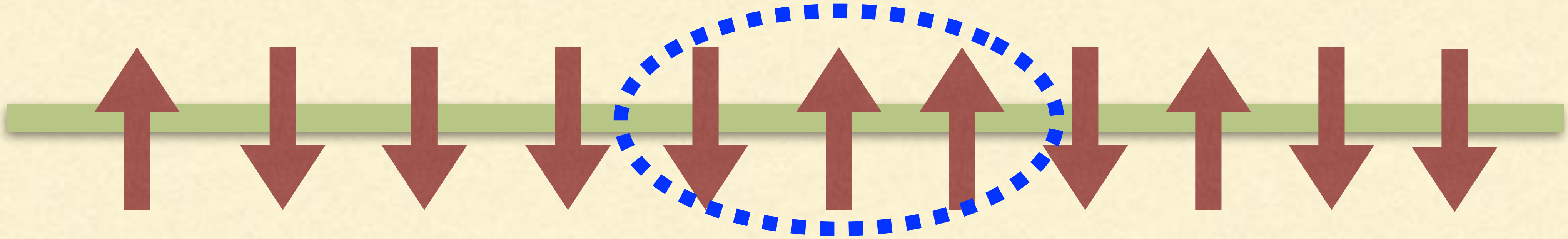
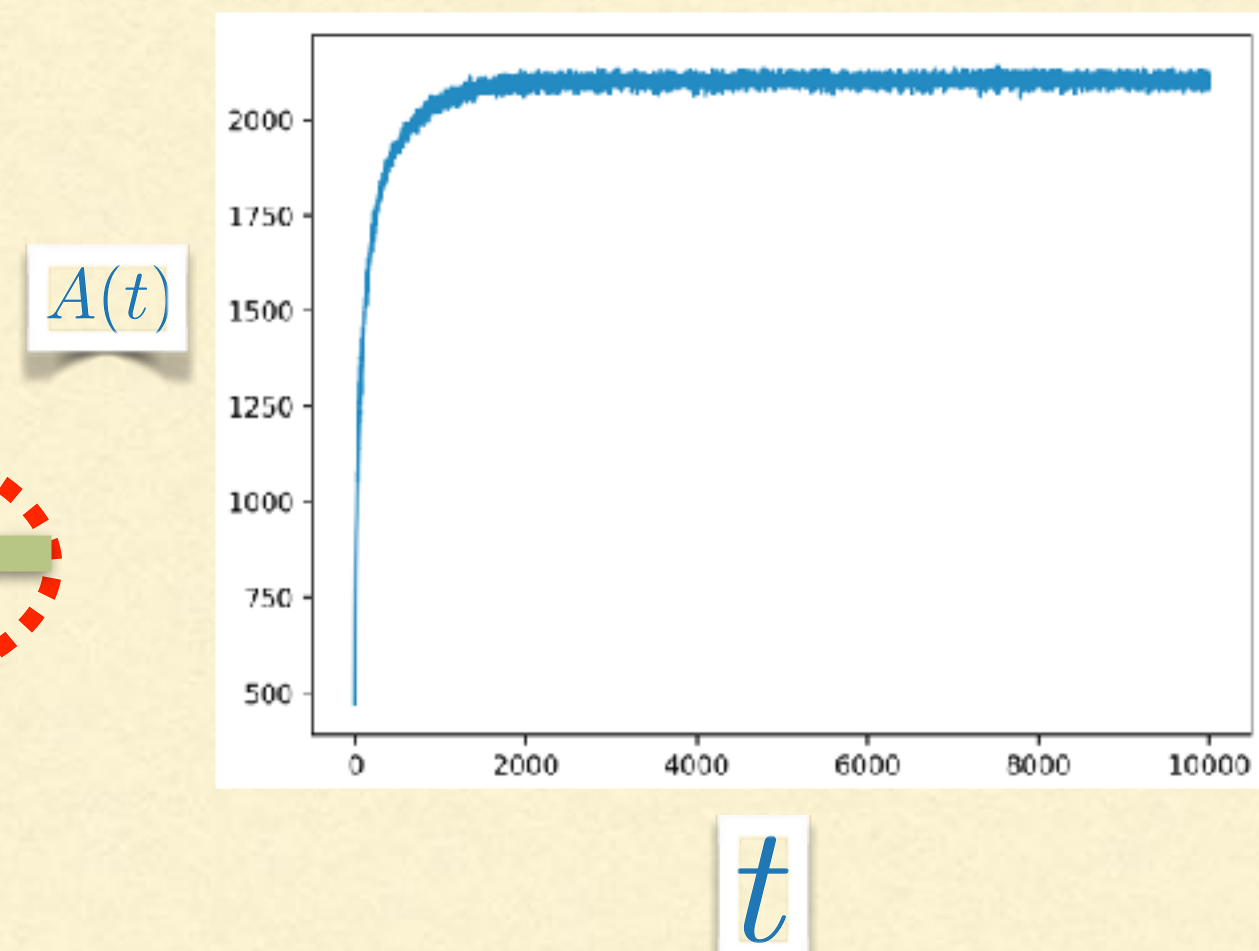


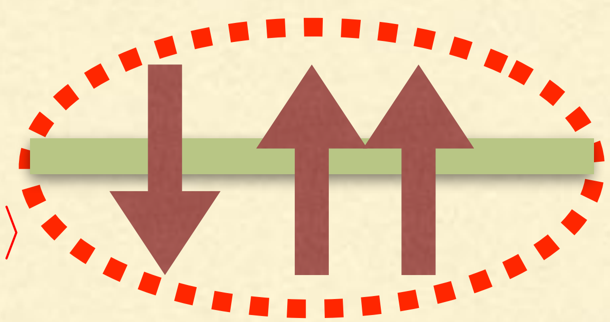
$$|\Psi(0)\rangle = \sum_i \alpha_i |\Psi_i\rangle$$

you get the thermal value of the observables.

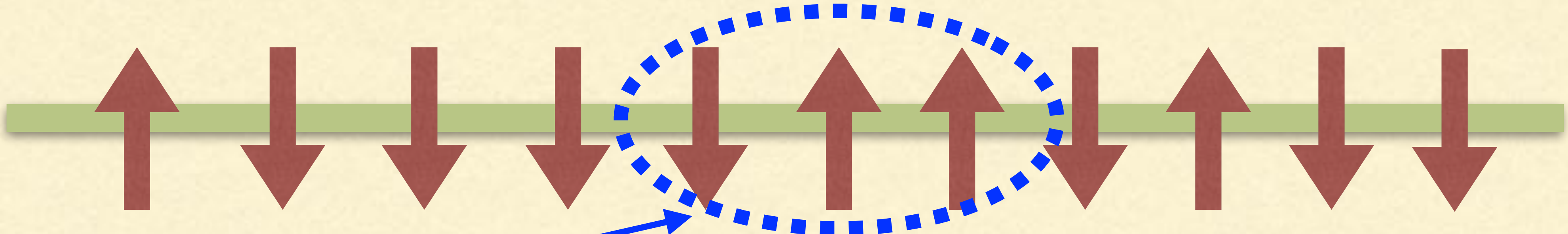


Finite temperature observables

$$A_{\text{thermal}} = \frac{1}{Z}\sum_i \exp[-\beta E_i]\langle\Psi_i|\hat{A}|\Psi_i\rangle$$



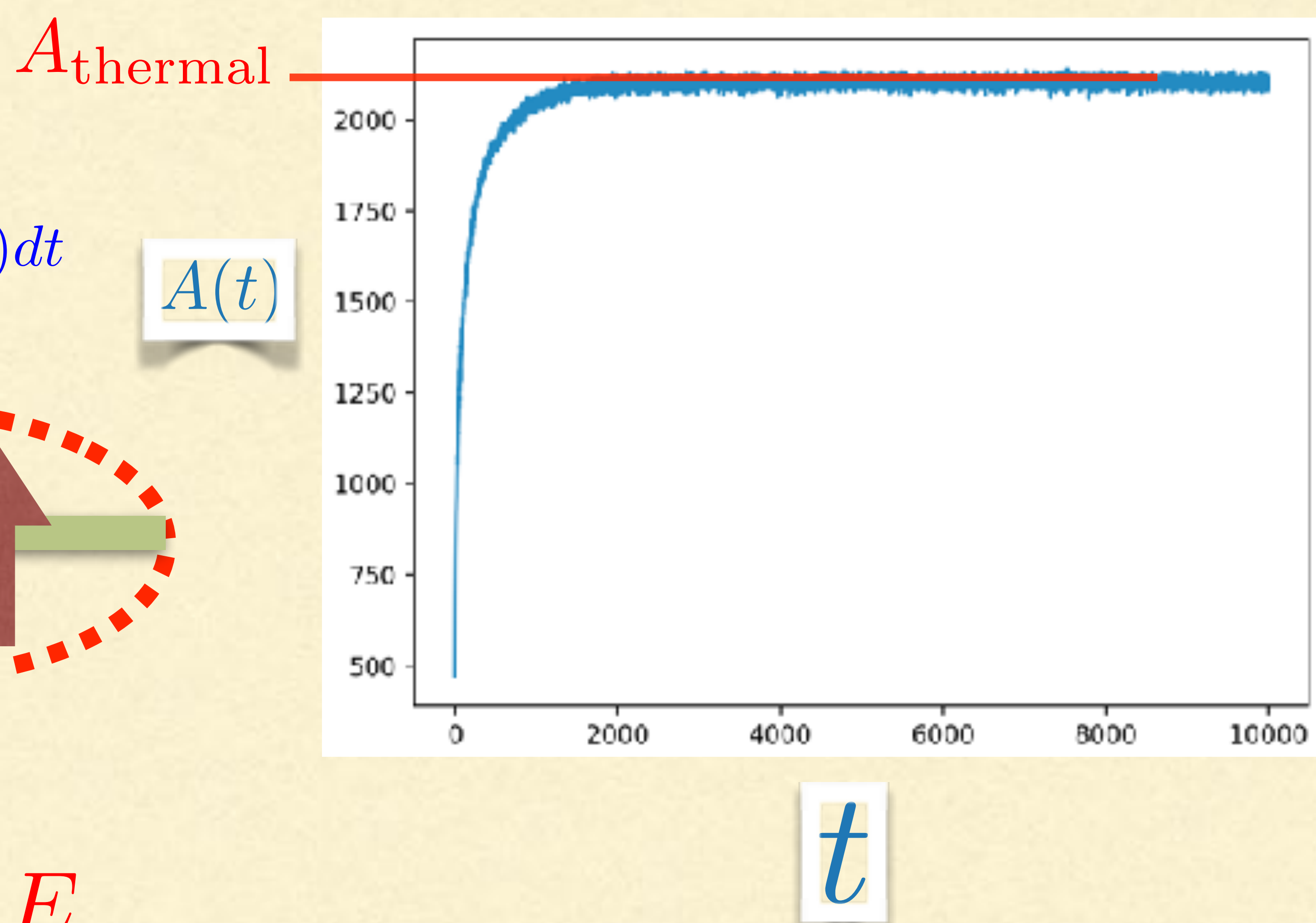
Equilibration means…



$$|\Psi(0)\rangle = \sum_i \alpha_i |\Psi_i\rangle$$

Local observable A on this subsystem $A(t) = \langle \Psi(t)|\hat{A}|\Psi(t)\rangle$

$$A_{\text{thermal}} = \frac{1}{T}\int_t^{t+T} A(t)dt$$



Finite temperature observables

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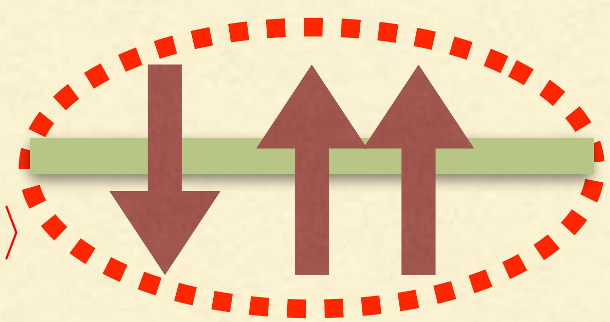
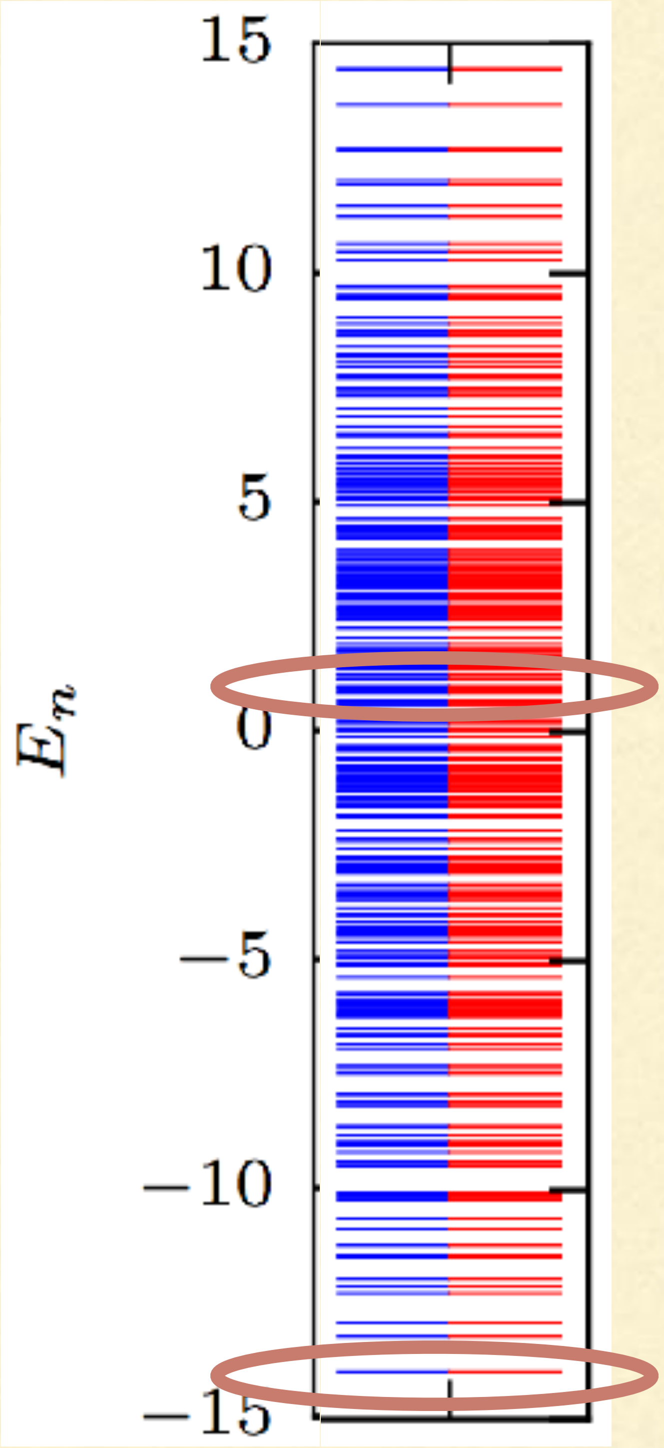


*Choose β such that $\mathrm{Tr}[H\rho_{\text{thermal}}(\beta)] = E$

Failure to thermalize must mean something goes wrong with the eigenstates

Spectrum of H

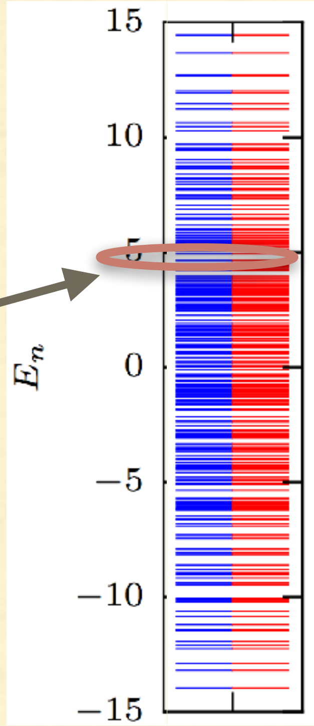


The relevant information is encoded in the eigenstates in the middle of the spectrum.

For quantum phases we look at the ground state.

Eigenstate Thermalization Hypothesis

Equilibration means…you forget where you started.



Start with $|\Psi(0)\rangle = \sum_i \alpha_i |\Psi_i\rangle$ concentrated around fixed E.

$\langle\Psi(0)|\hat{A}|\Psi(0)\rangle = \sum_{ij} \alpha_i^* \alpha_j \langle\Psi_i|\hat{A}|\Psi_j\rangle$ ⟶ Depends heavily on α_i

To equilibrate, the observable A(t), at large t, needs forget where we started.

$$\langle\Psi(t)|A|\Psi(t)\rangle = \sum_{ij} \alpha_i^* \alpha_j \exp[-it(E_i - E_j)] \langle\Psi_i|\hat{A}|\Psi_j\rangle$$

If $t \gg (E_i - E_j)$ then $i \neq j$ terms average to zero. ⟶

$$\langle\Psi(t)|A|\Psi(t)\rangle = \sum_i |\alpha_i|^2 \langle\Psi_i|\hat{A}|\Psi_i\rangle$$

If $\langle\Psi_i|\hat{A}|\Psi_i\rangle \approx \langle\Psi_j|\hat{A}|\Psi_j\rangle$ then we forget the initial conditions.

Deutsch, PRA 43, 2046 (1991).
M. Srednicki, PRE 50, 888 (1994).
Rigol, Dunjko, Olshanii, Nature 452, 854 (2008).

Eigenstate Thermalization Hypothesis

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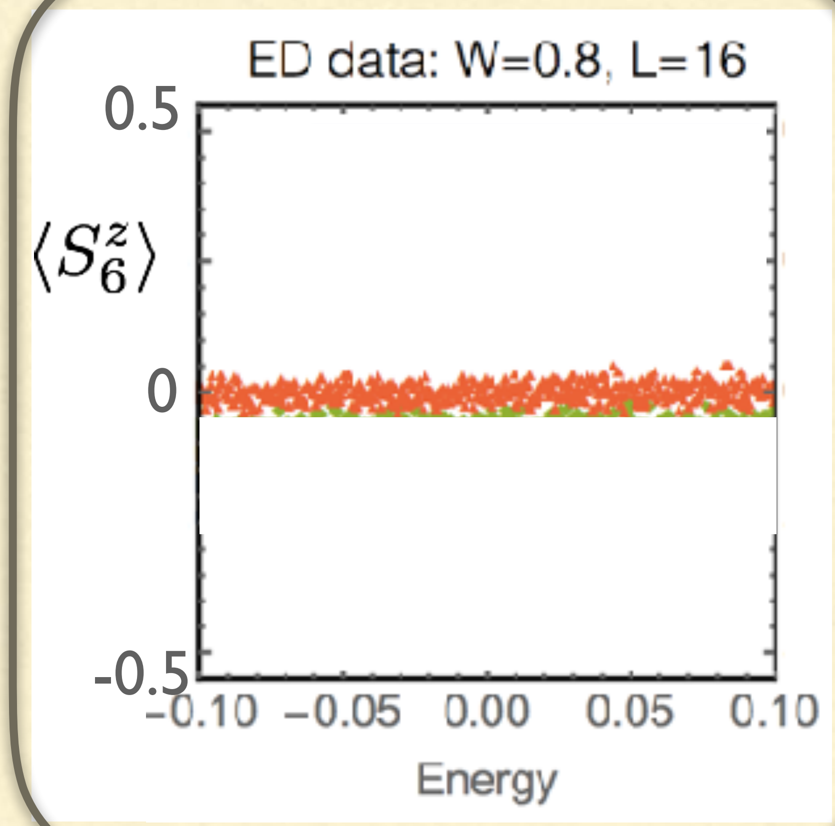
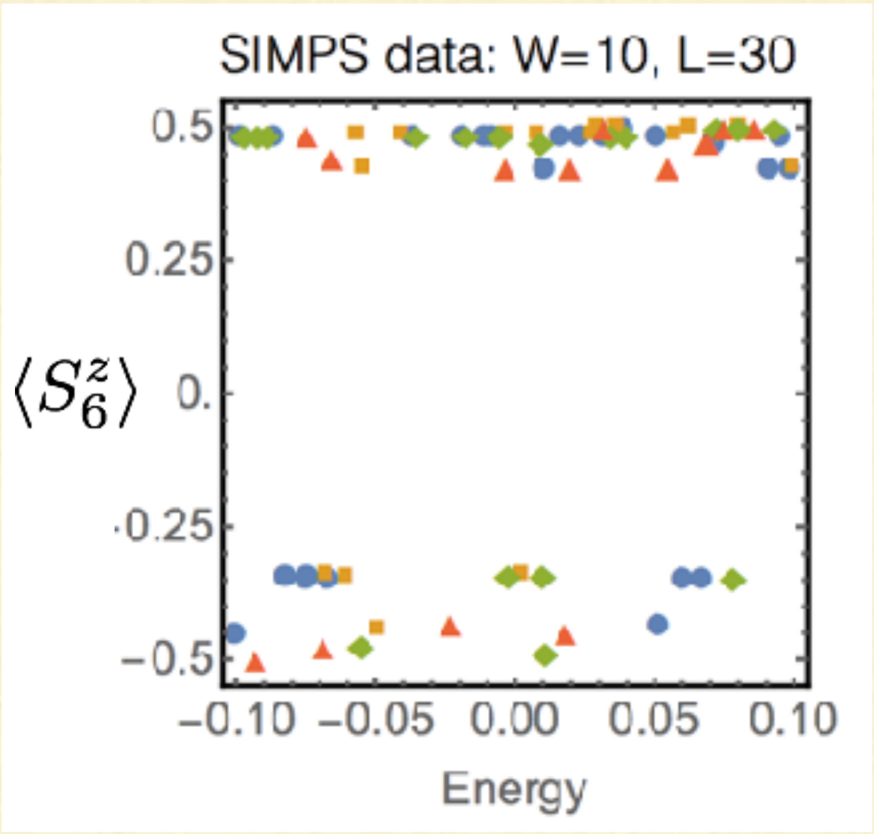
Ergodic



MBL

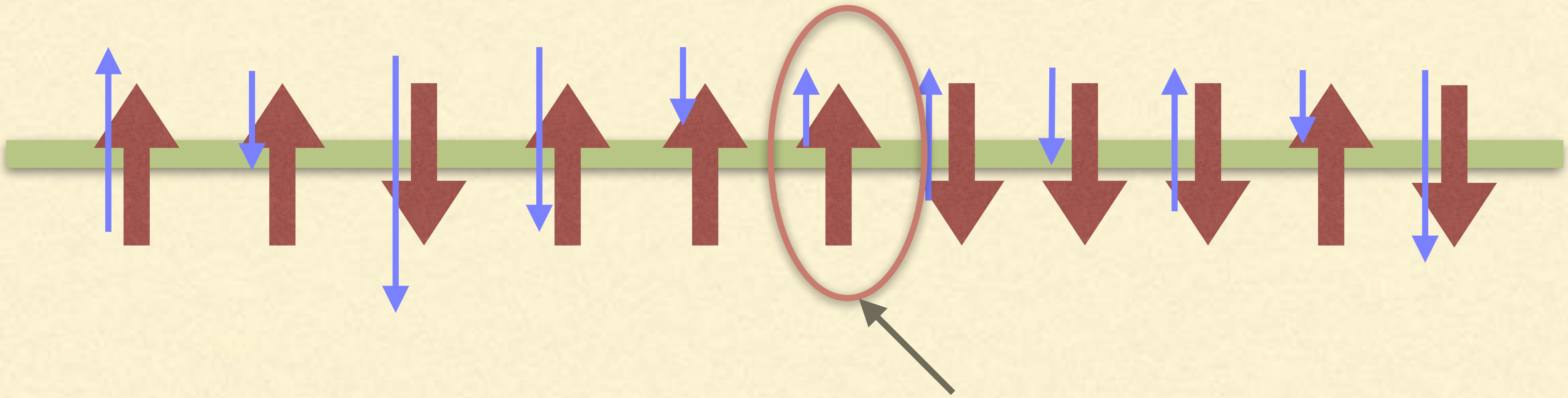


Yu, Pekker, BKC, (1509.01244) PRL 118,017201

Let’s talk about entanglement

Equilibration means… you get the thermal value of the observables.

$$\langle \Psi_i | \hat{A} | \Psi_i \rangle \approx \langle \Psi_j | \hat{A} | \Psi_j \rangle \approx A_{\mathrm{thermal}}$$



Q: At infinite temperature, when measured, how often is this spin up vs down?

A: Up half the time and down half the time

Q: What quantum state, when measured, is up half the time and down half the time?

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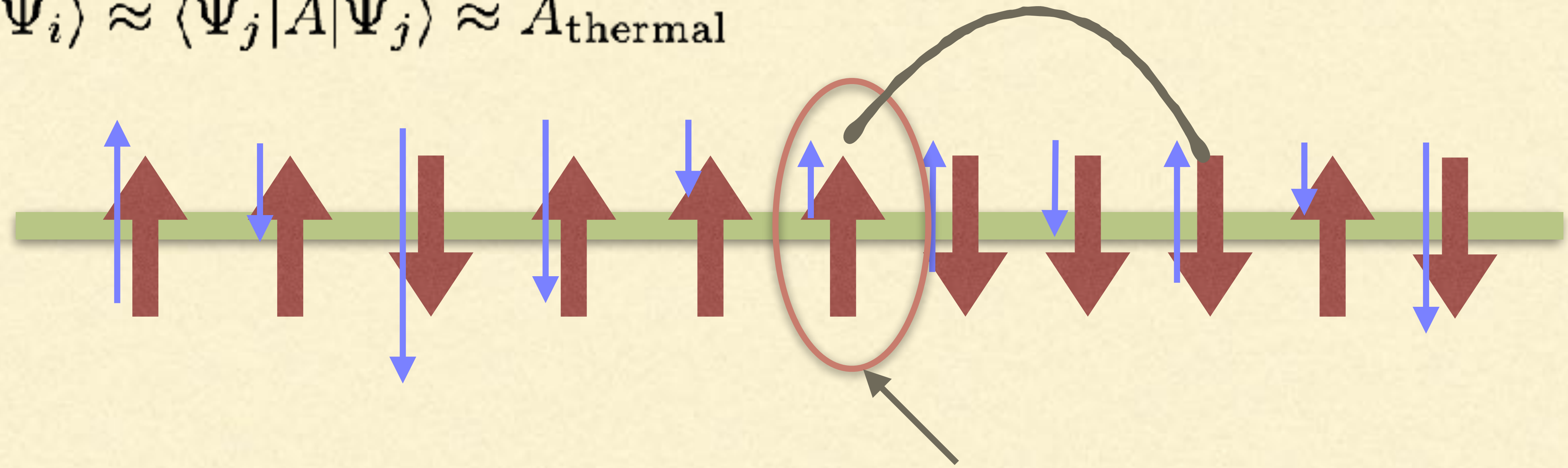

Q: At infinite temperature, when measured, how often is this spin up vs down?

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Q: What quantum state, when measured, is up half the time and down half the time?

A: Entangle it with another spin: $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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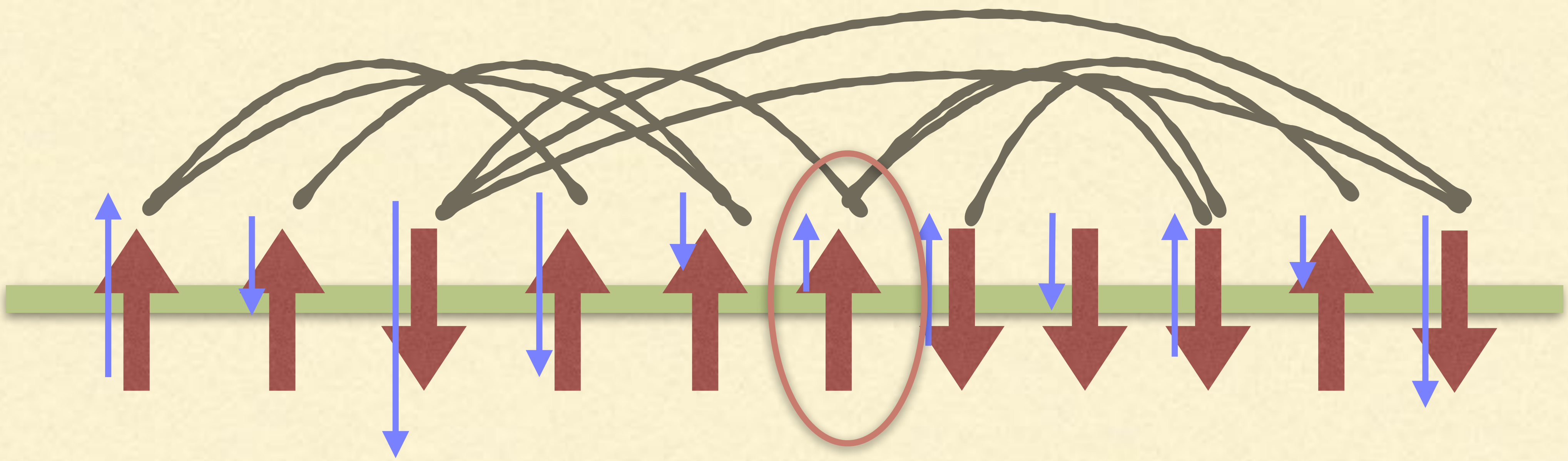

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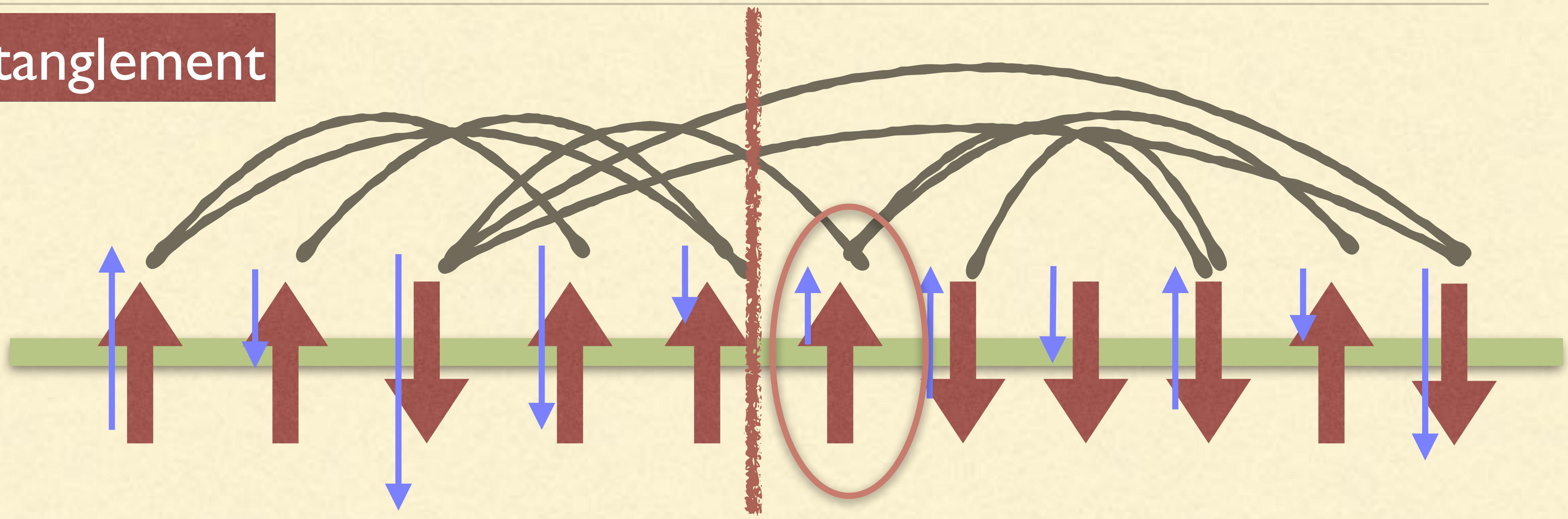
A: Entangle it with another spin: $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

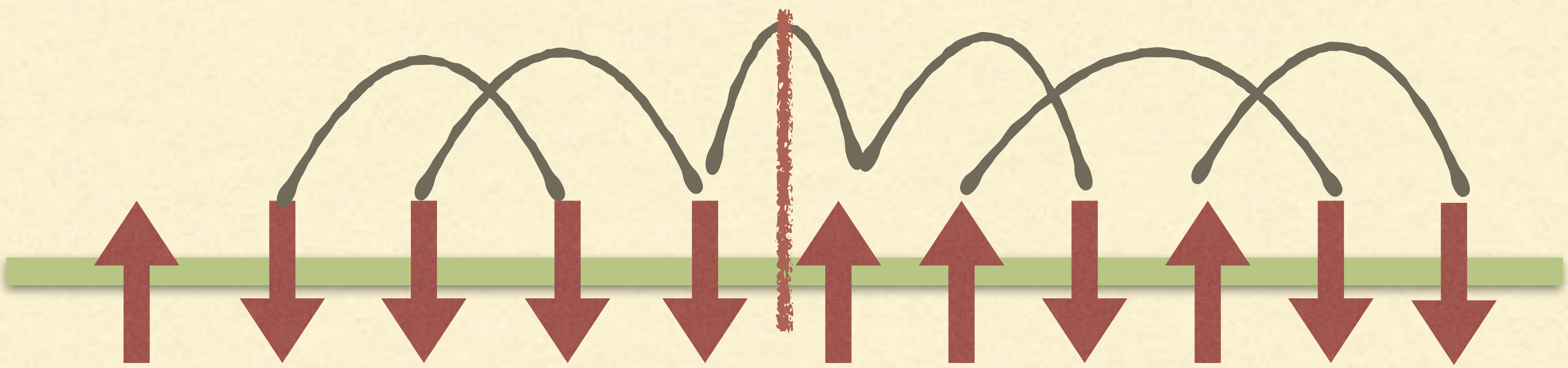
Better is to weakly entangle everyone with everyone else.

Let's talk about entanglement



The entanglement over the half cut goes as $S \propto L$
Volume Law Entanglement

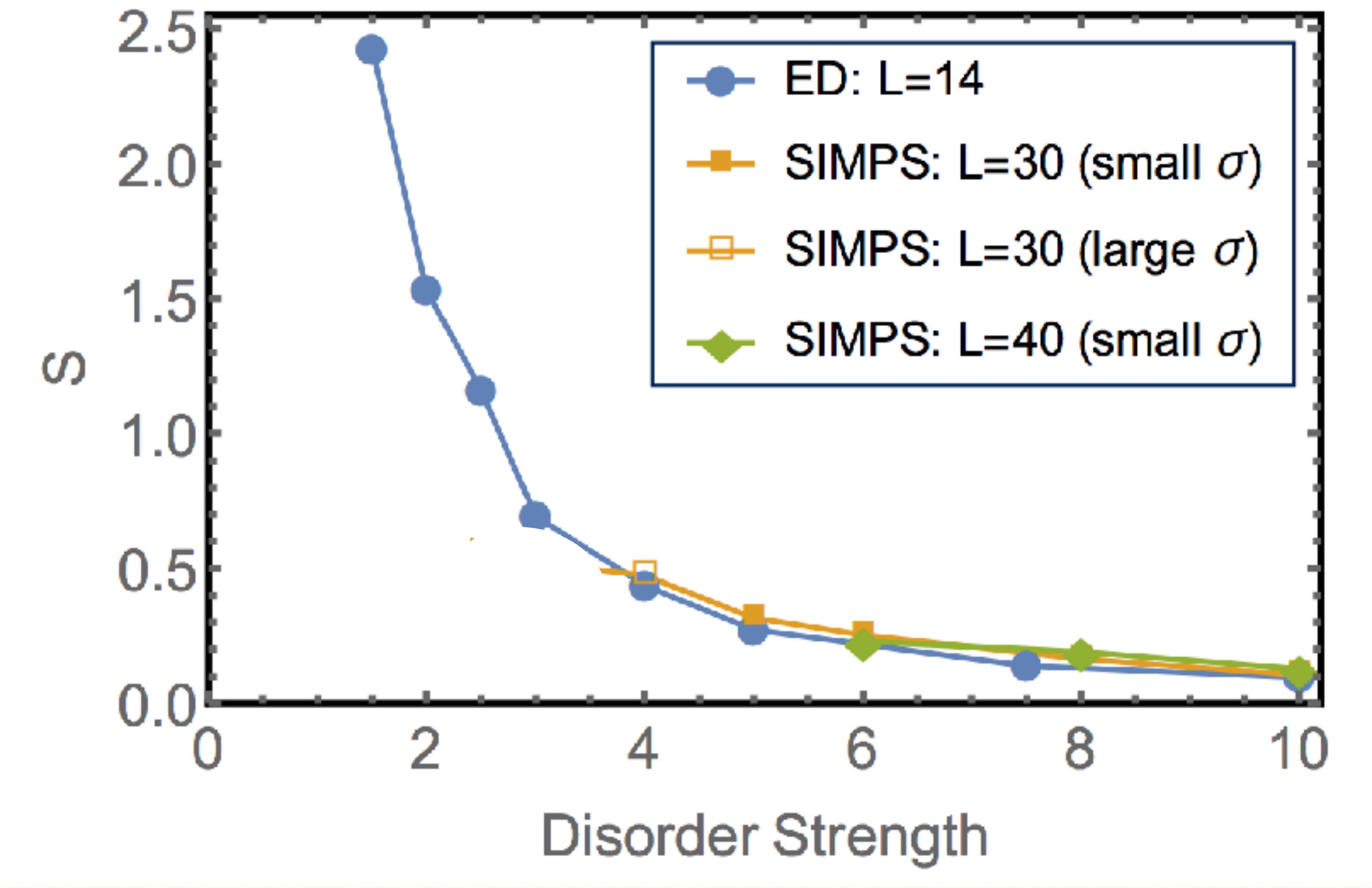
Sanity Check: Most states are highly entangled.
No concept of locality
Entangled states can hide information; forget initial conditions



MBL states and ground states are "weakly entangled" $S \propto O(1)$
Area Law Entanglement

Bauer, and Nayak. JSTAT (2013): P09005.

Let's talk about entanglement



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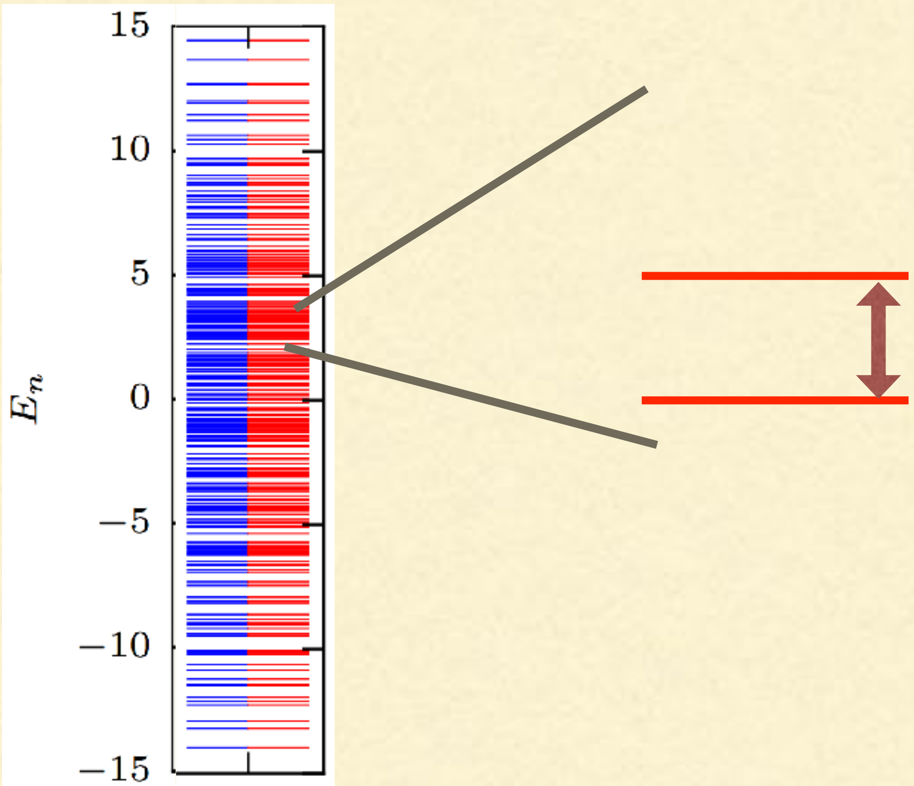
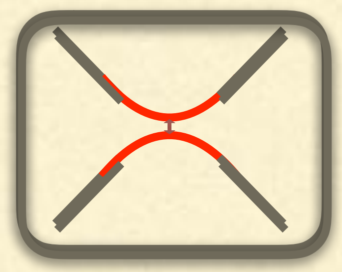
Eigenspectrum Statistics



How far apart is the next eigenstate?
Naive expectation: *Just random*

Typical situation: *Level Repulsion* gives GOE statistics

Level Repulsion:



Ergodic Phase: GOE statistics

MBL Phase: Poisson statistics

Pal and Huse. *PRB* 82.17 (2010): 174411.

An algorithmic interlude: How do we get eigenstates to see if something weird has happened to them?

$L = 16$ Use exact diagonalization

$L = 22$ Use shift and invert*

$L \geq 30$ A no-go theorem exists for inter-level spacing at the level of machine precision.

* Luitz, Laflorencie, and Alet .PRB (2015): 081103.

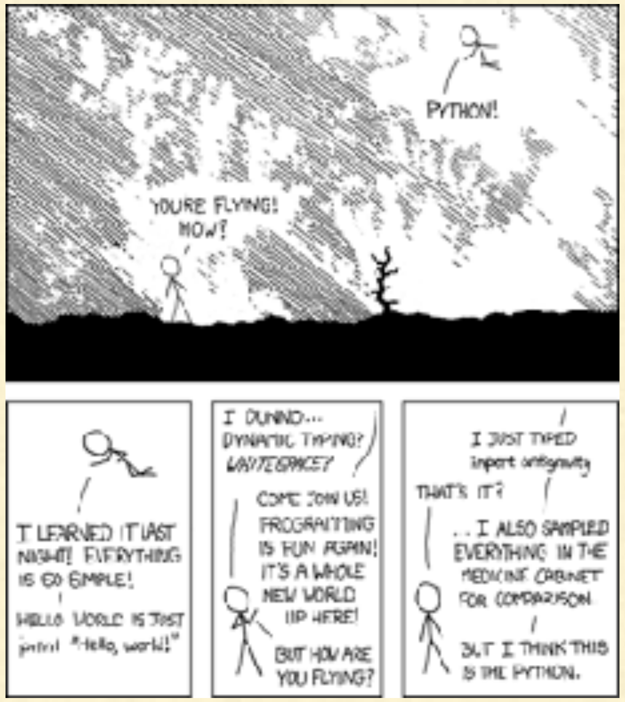
A no-go theorem

Goal: *Find an eigenstate.*

Program: *Find an quantum state with zero variance at energy E*

$$\sqrt{\langle\Psi|H^2|\Psi\rangle - |\langle\Psi|H|\Psi\rangle|^2}$$

Linear superposition of eigenstates have zero (up to machine resolution) variance. Essentially a no-go theorem.



https://xkcd.com/353/

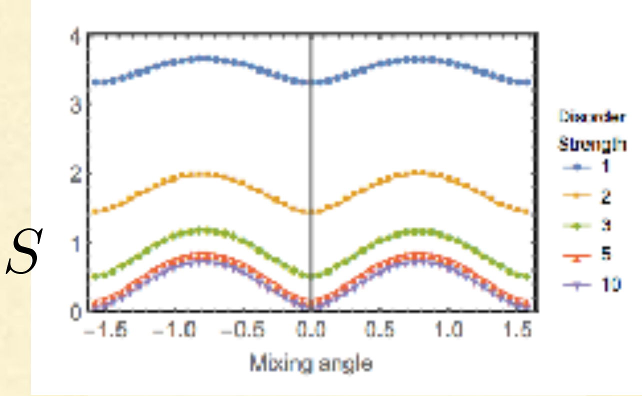
Goal: *Find an eigenstate*

New Program: *Find the quantum state with zero variance at energy E and is locally minimal in entanglement.*

Q: *Why does this work?*

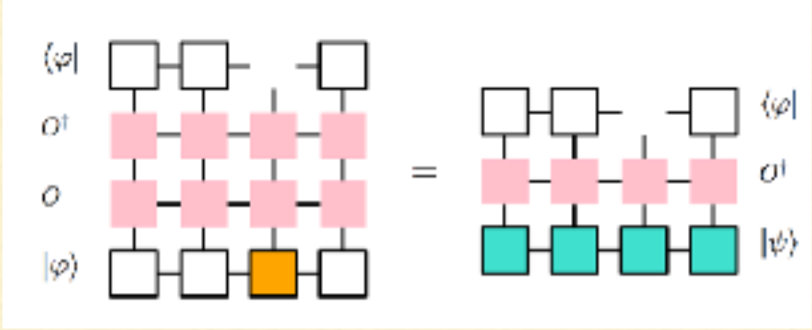
A:

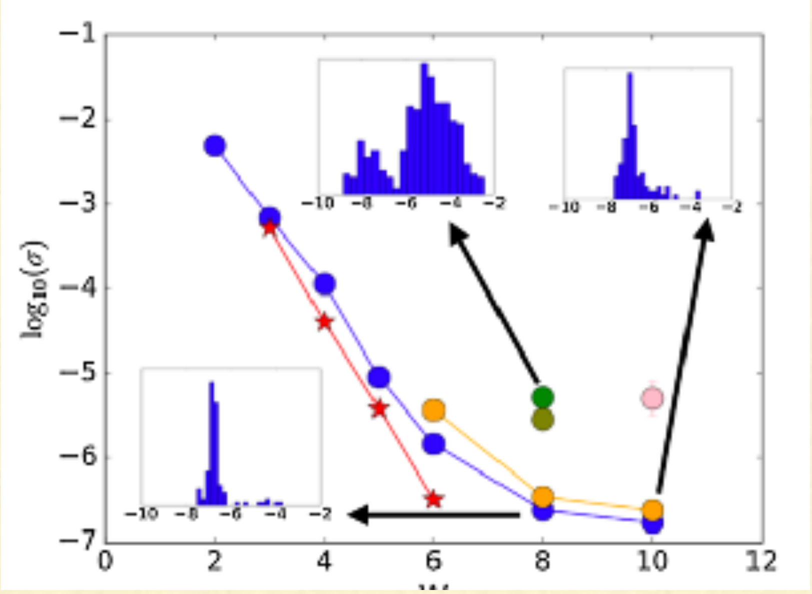
S



$$|\Psi\rangle = \cos(\alpha)|\Psi_i\rangle + \sin(\alpha)|\Psi_{i+1}\rangle$$

SIMPS





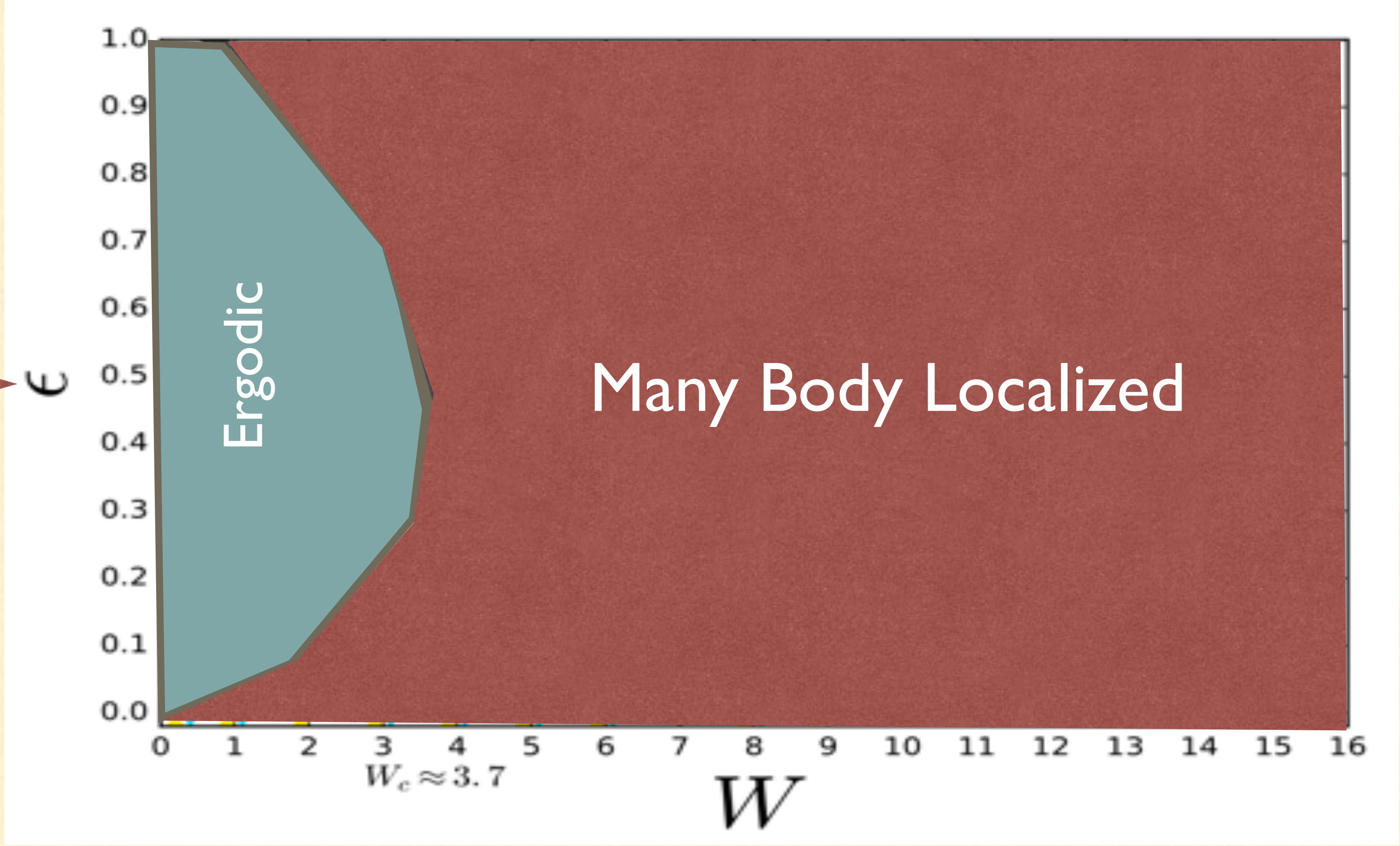
End of algorithmic interlude

Yu, Pekker, BKC, (1509.01244) PRL 118,017201

Properties of Eigenstates

Ergodic	MBL
ETH - Eigenstates equal thermal observables	Eigenstates have disparate observables
Volume Law Entanglement	Area Law Entanglement
Eigenspectrum obeys GOE statistics	Eigenspectrum obeys poisson statistics

infinite temperature



Luitz, Laflorencie, and Alet .PRB (2015): 081103.

Our goal is now to figure out a unifying story for how we get unusual eigenstates.

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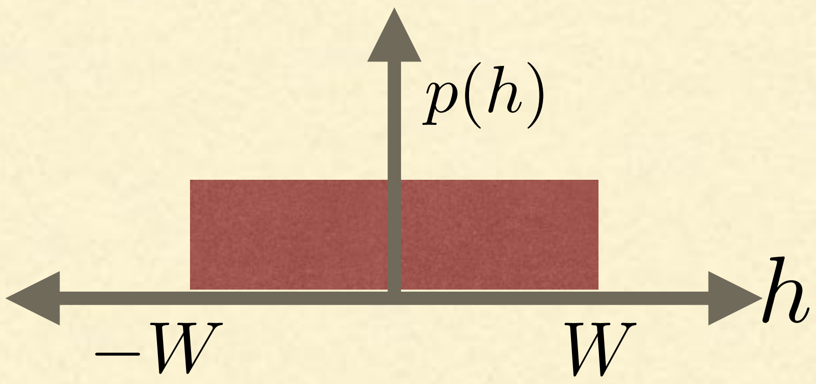
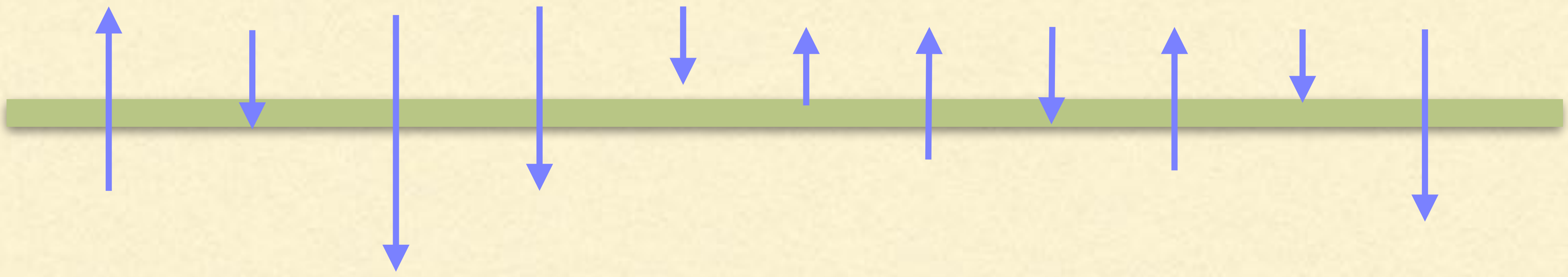
Just disorder…

$$H = \sum_i h_i S_i^z$$

$$h \in [-W, W]$$

$$p(h_i) = \frac{1}{2W}$$





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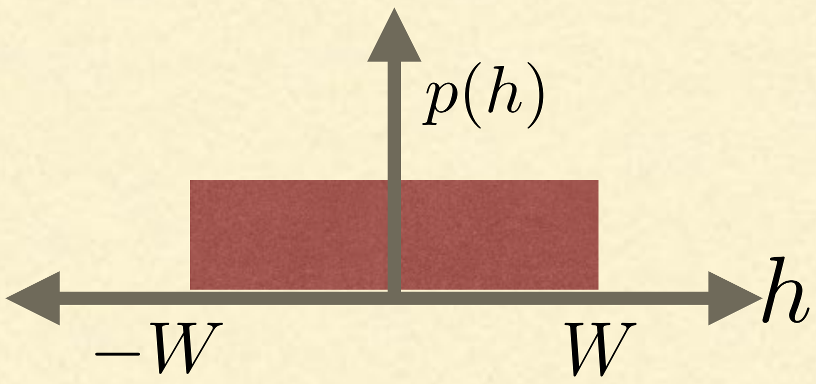
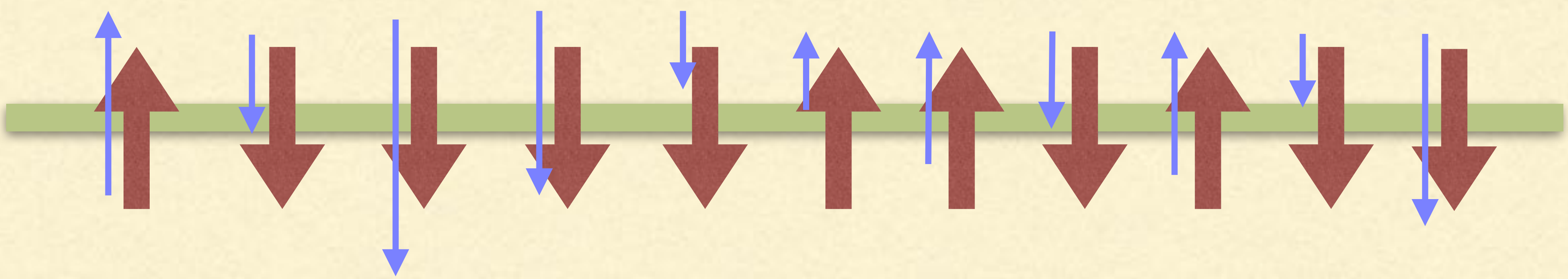


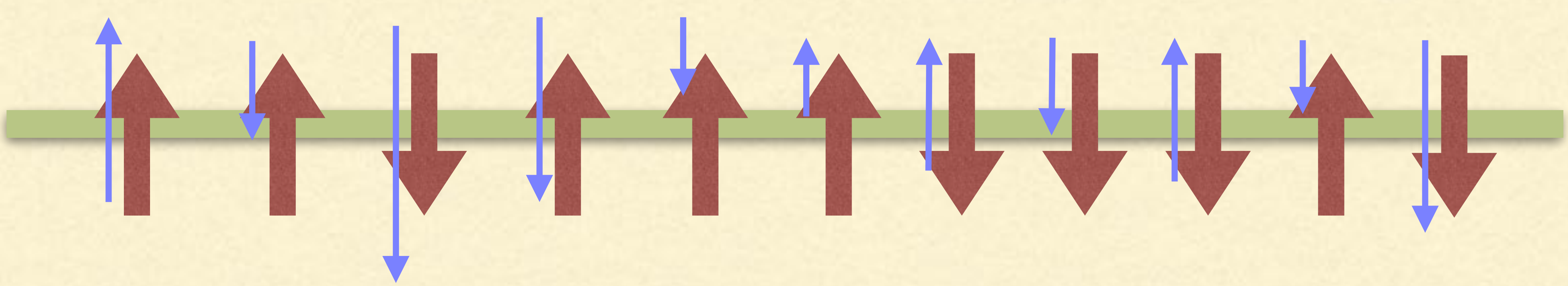


Our goal is now to figure out a unifying story for how we get unusual eigenstates.

No entanglement!

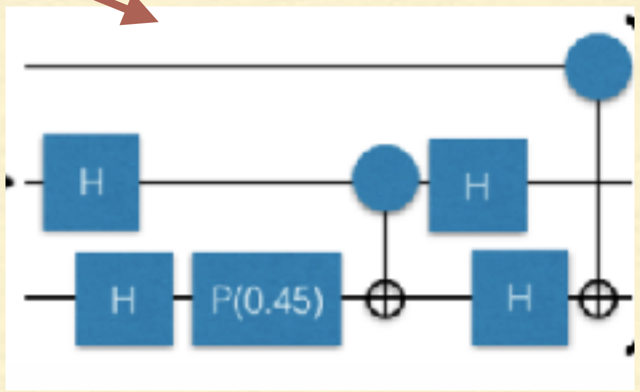
$|\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow\rangle$ and $|\downarrow\downarrow\downarrow\uparrow\downarrow\downarrow\uparrow\uparrow\rangle$ have same energy but different $\langle S_6^z\rangle$

No level repulsion



Why are the eigenstates so weird?

$$UHU^\dagger = H_{\text{diagonal}}$$



There are many different unitary quantum circuits which diagonalize a Hamiltonian H.

Q: What is the circuit depth of the smallest unitary which diagonalizes H?

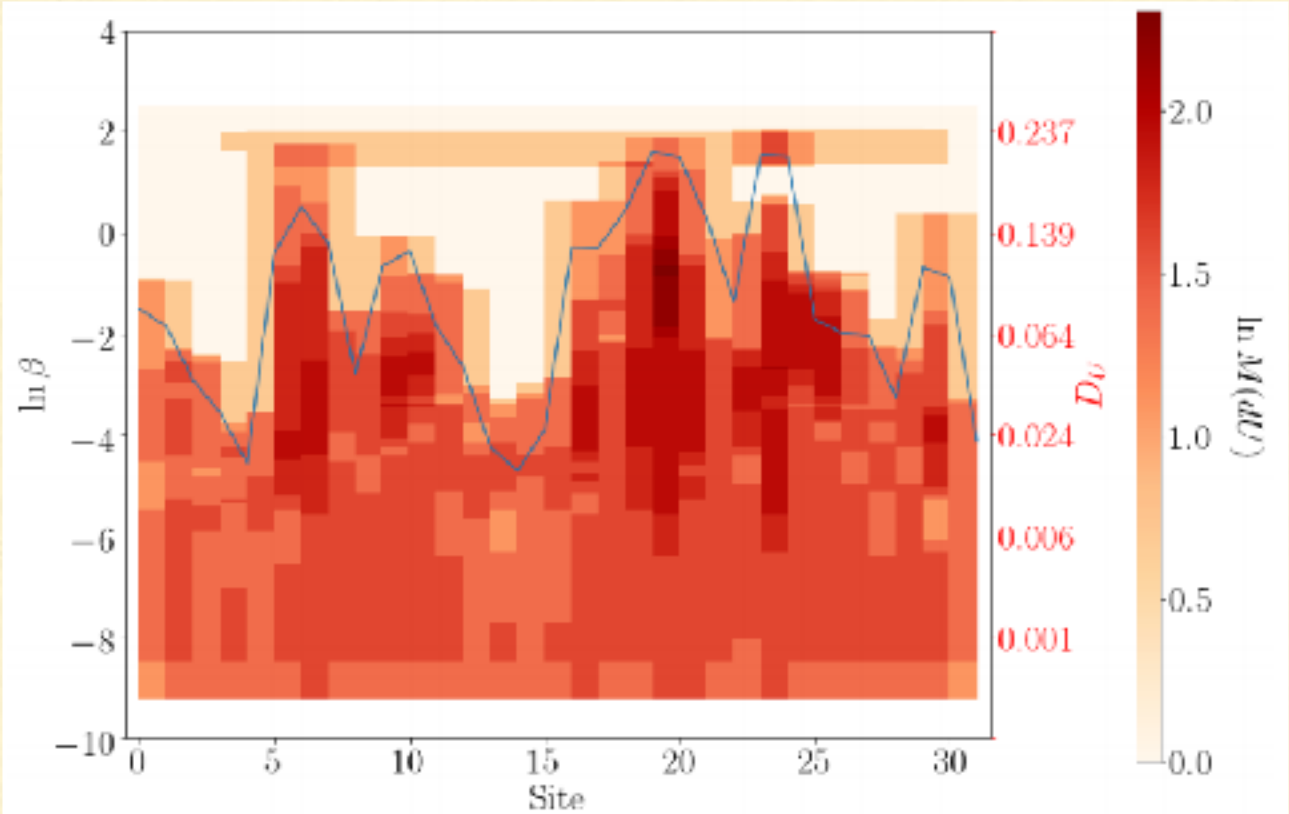
Pekker, BKC (1410.2224) PRB 95.035116

Building the unitary circuit

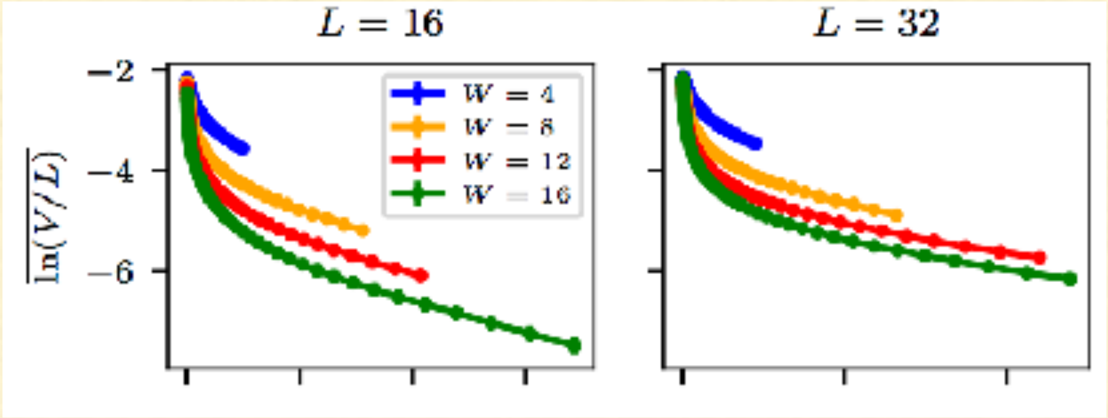
$$\eta(\beta) = [H_D(\beta), H_{OD}(\beta)]$$
$$U(\beta) = e^{i\eta(\beta)}$$
$$H(\beta + \delta\beta) = U(\beta)H(\beta)U^\dagger(\beta)$$

Unitary RG Process

Wegner-Wilson Flow



PEPSification of the flow



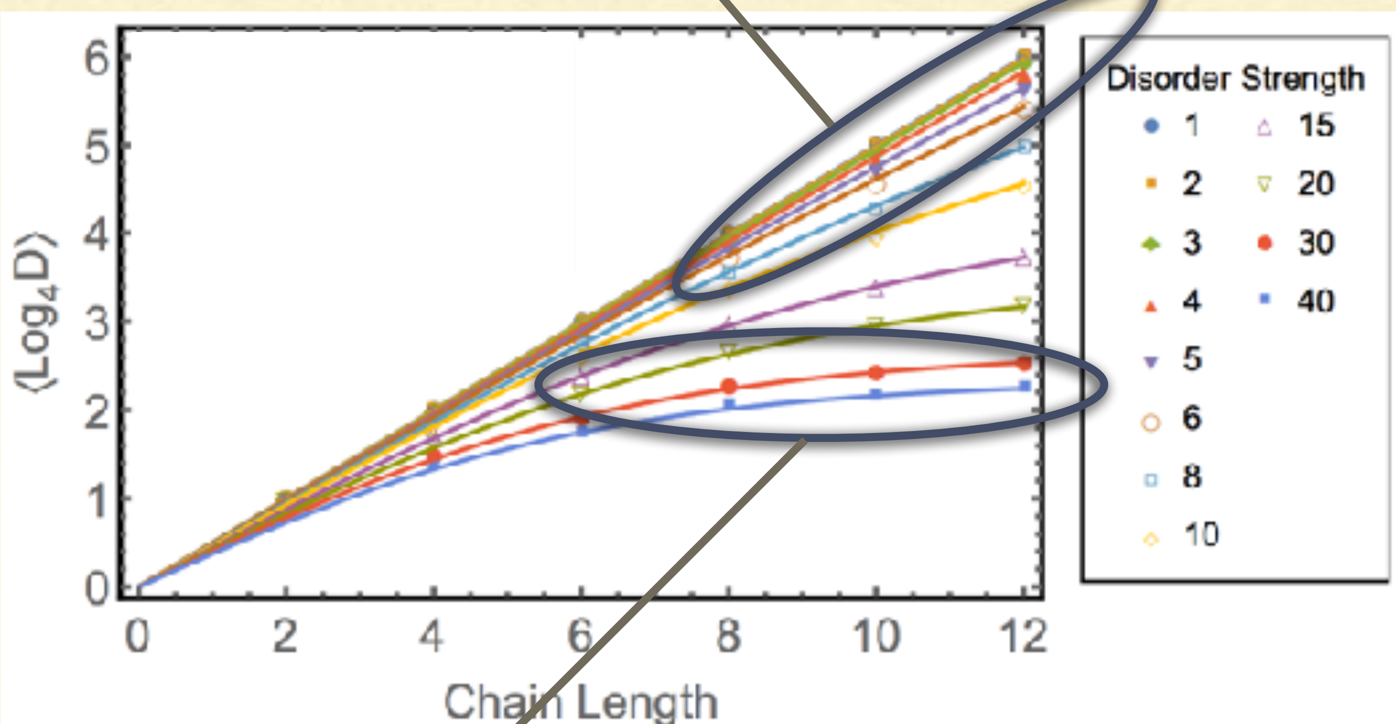
Pollmann, Khemani, Cirac, Sondhi, *PRB*, *94*(4), 041116.

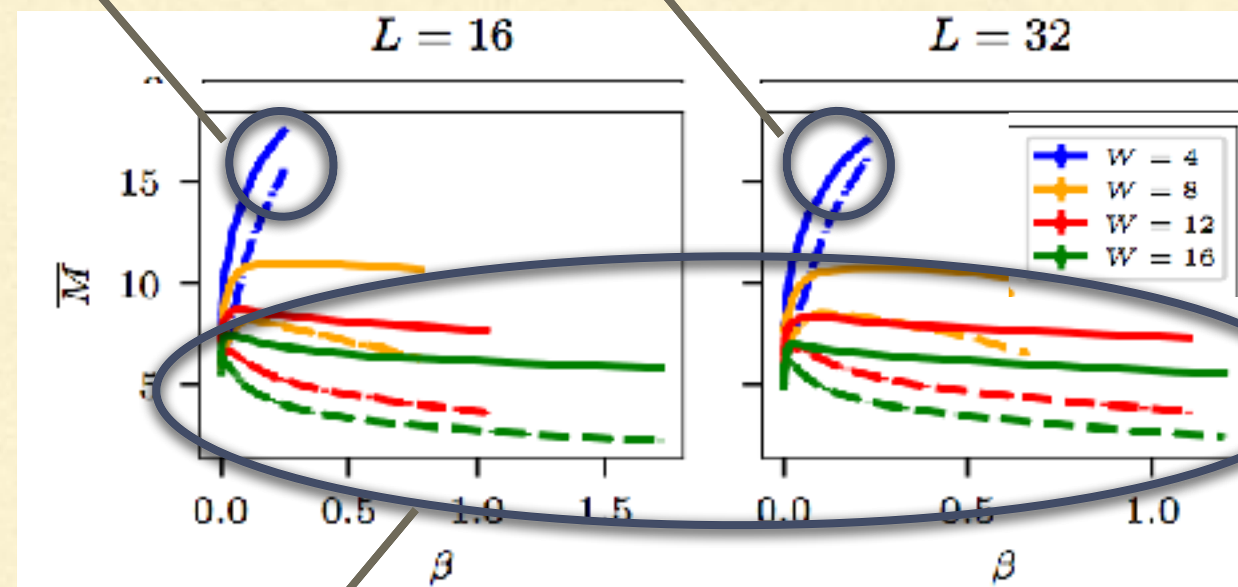
Yu, Pekker, BKC (February 2018)

Wahl, Pal, and Simon. *PRX* 7 2 (2017): 021018.

Why are the eigenstates so weird?

weak disorder = ergodic = linear-depth quantum circuit

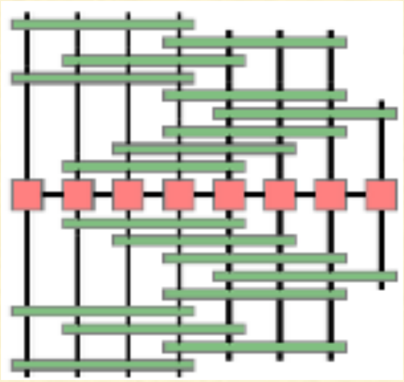




strong disorder = MBL = constant-depth quantum circuit

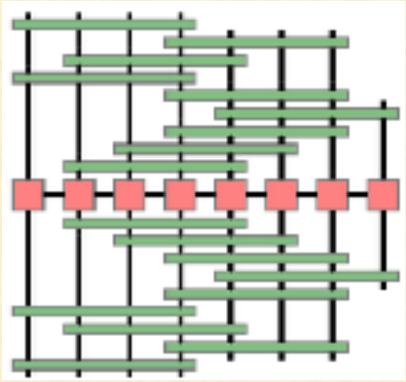
Pekker, BKC (1410.2224) PRB 95.035116

Short Diagonalizing Quantum Circuits



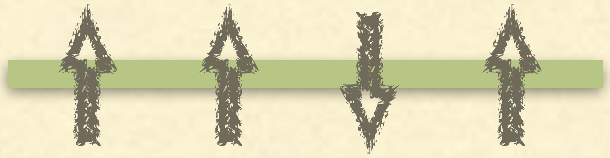
Fully MBL

Short Diagonalizing Quantum Circuits

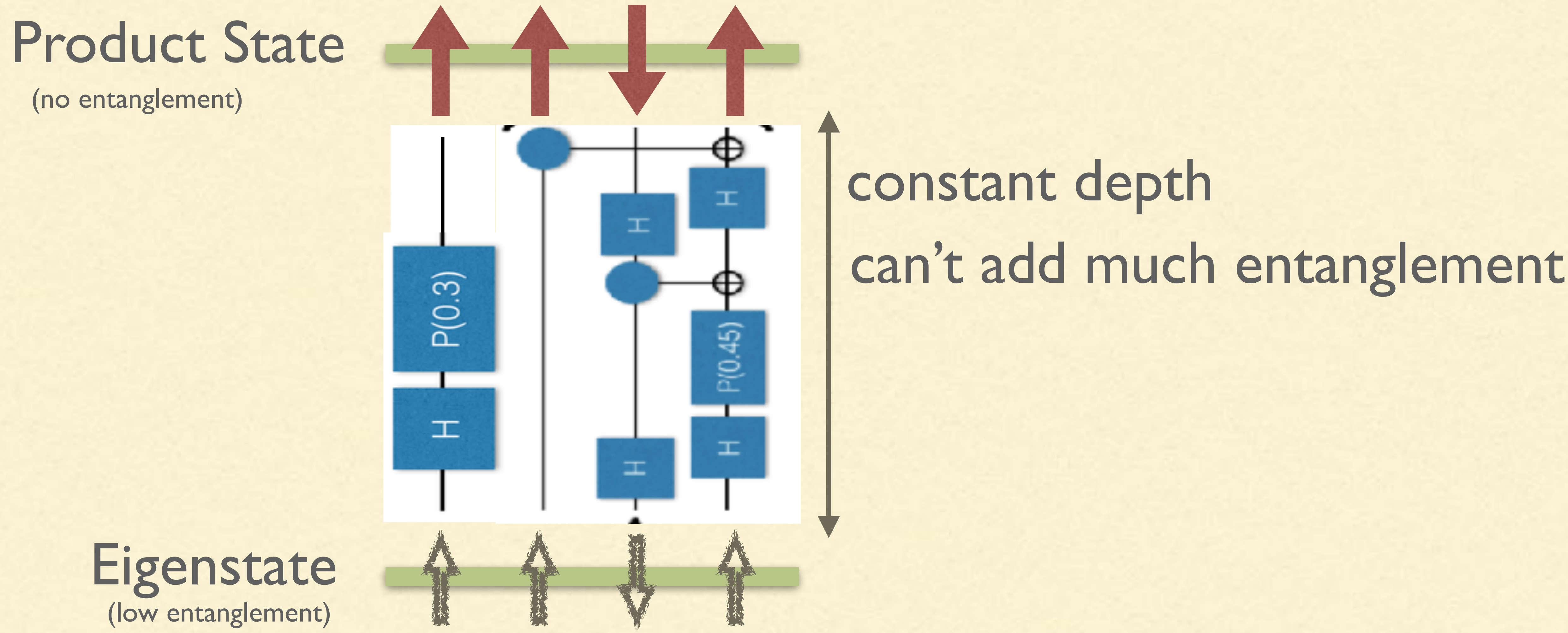


Low Entanglement

Simple Eigenstate Structure



Low Entanglement + Simple Eigenstate Structure



Simple Eigenstate Structure

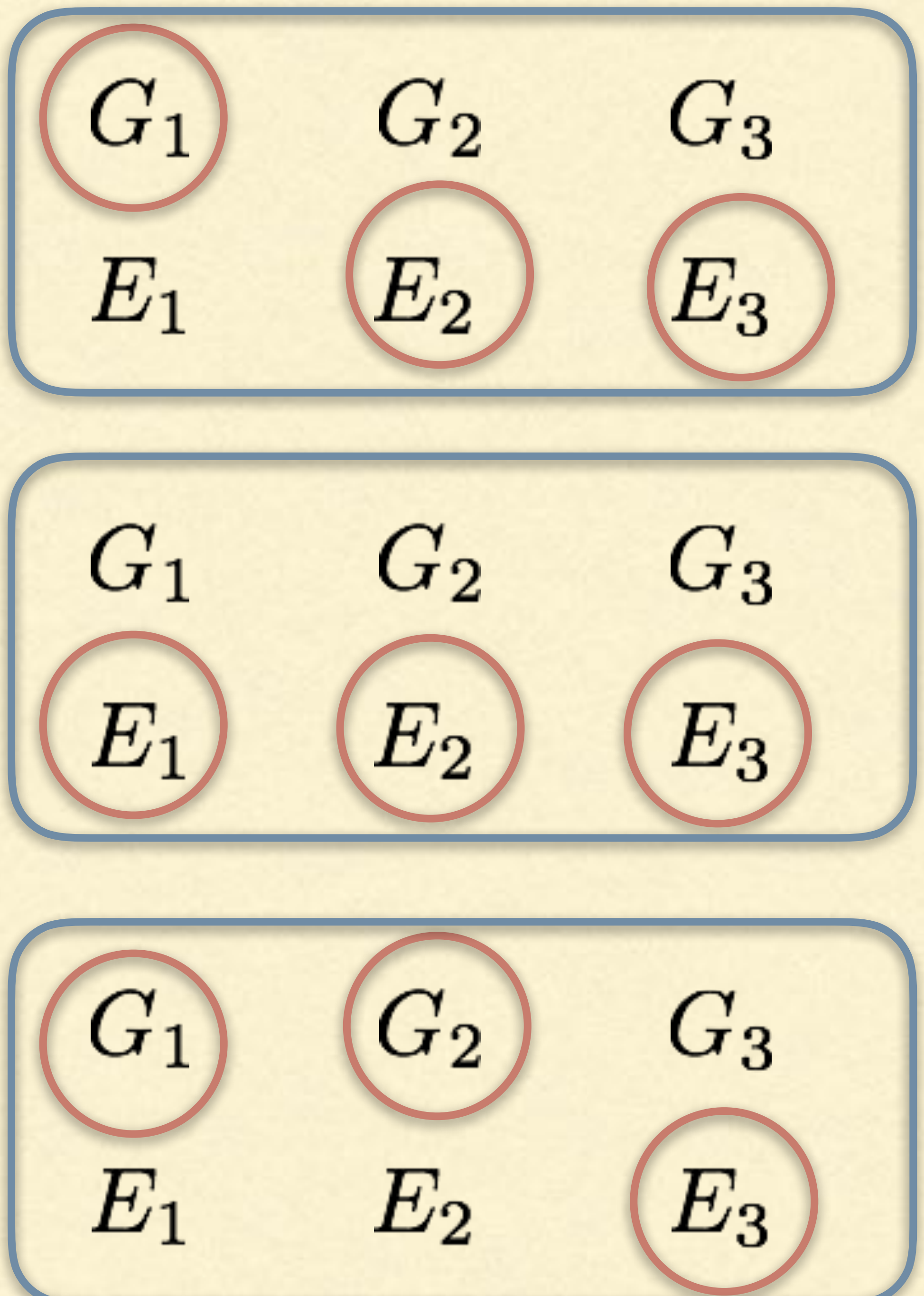
G_i Tensor of $2D^2$ numbers

E_i Tensor of $2D^2$ numbers

Eigenstate one: $|\uparrow\downarrow\downarrow\rangle$ →

Eigenstate two: $|\downarrow\downarrow\downarrow\rangle$ →

Eigenstate three: $|\uparrow\uparrow\downarrow\rangle$ →



#`s for all eigenstates

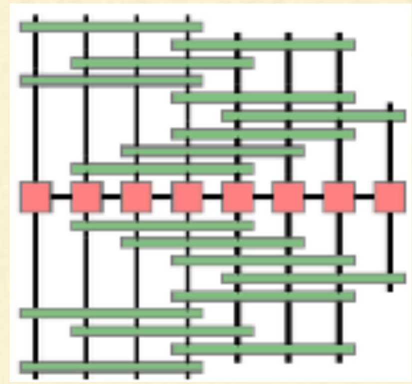
Expected	$2^n 2^n$
Actual	nD^2

Really surprising!

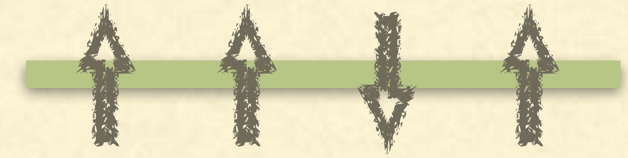
Pekker, BKC (1410.2224) PRB 95.035116

Fully MBL

Short Diagonalizing Quantum Circuits

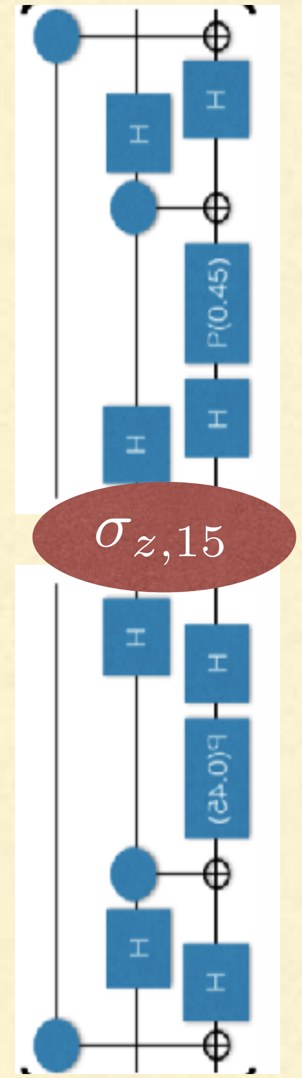


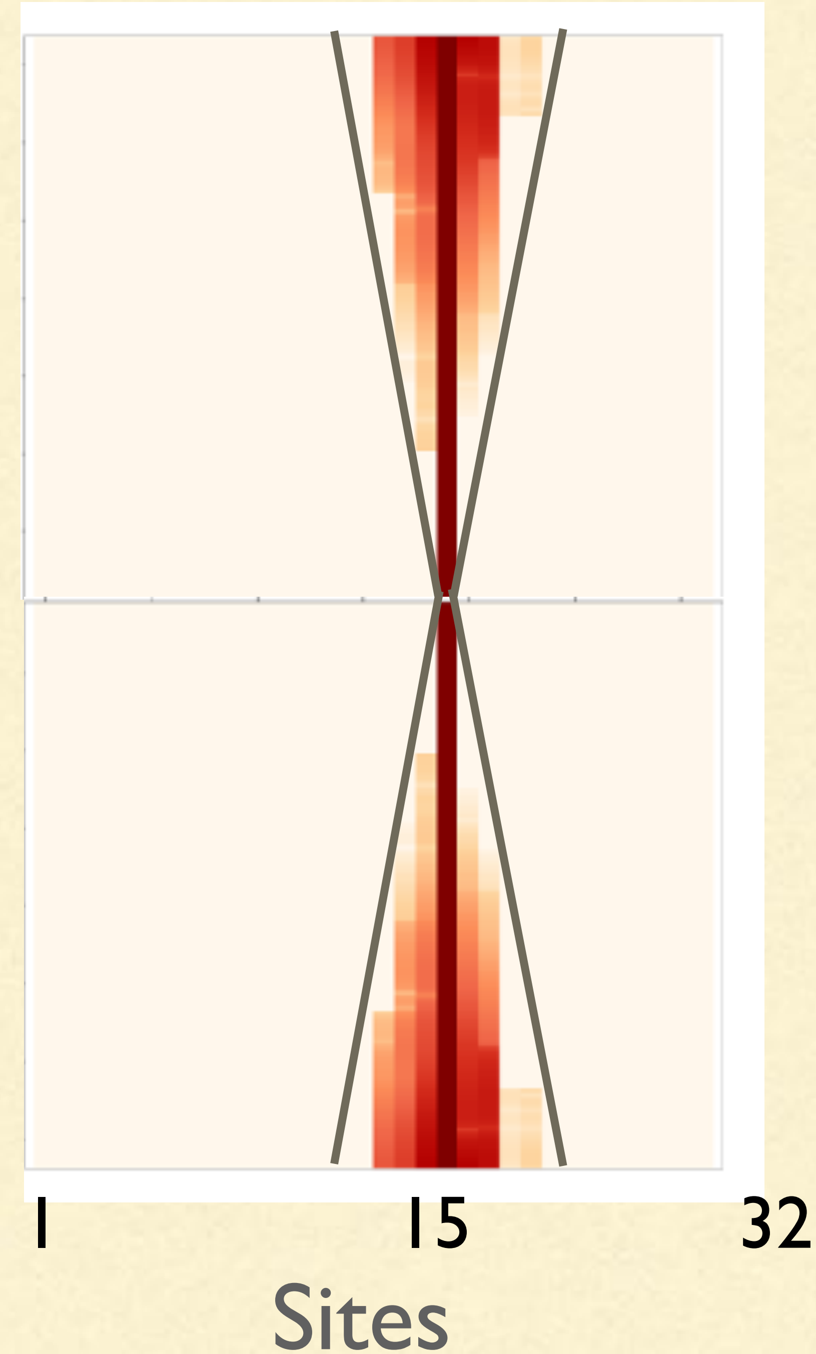
Simple Eigenstate Structure



Circuits have a light cone.

$U\sigma^z_{15}U^\dagger$ is close to σ^z_{15}





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Fully MBL

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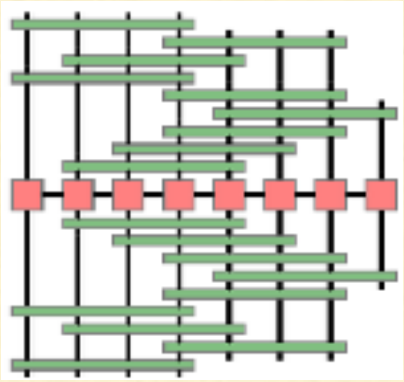
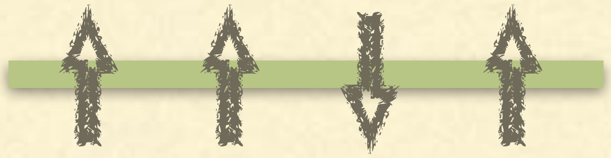


Simple Eigenstate Structure



ℓ-bits

$$[H, \hat{O}_i] = 0$$
$$[\hat{O}_j, \hat{O}_i] = 0$$
$$Tr[\hat{O}_i \hat{O}_j] = 0$$

Huse, Nandkishore, Oganesyan. *PRB* 90.17 (2014): 174202

Pekker, BKC (1410.2224) PRB 95.035116

Chandran, Carrasquilla, Kim, Abanin, and G. Vidal, 1410.0687

ℓ-bits

Some commuting operators

$$\tau_1 = U^\dagger \sigma_{1,z} U$$

$$\tau_2 = U^\dagger \sigma_{2,z} U$$

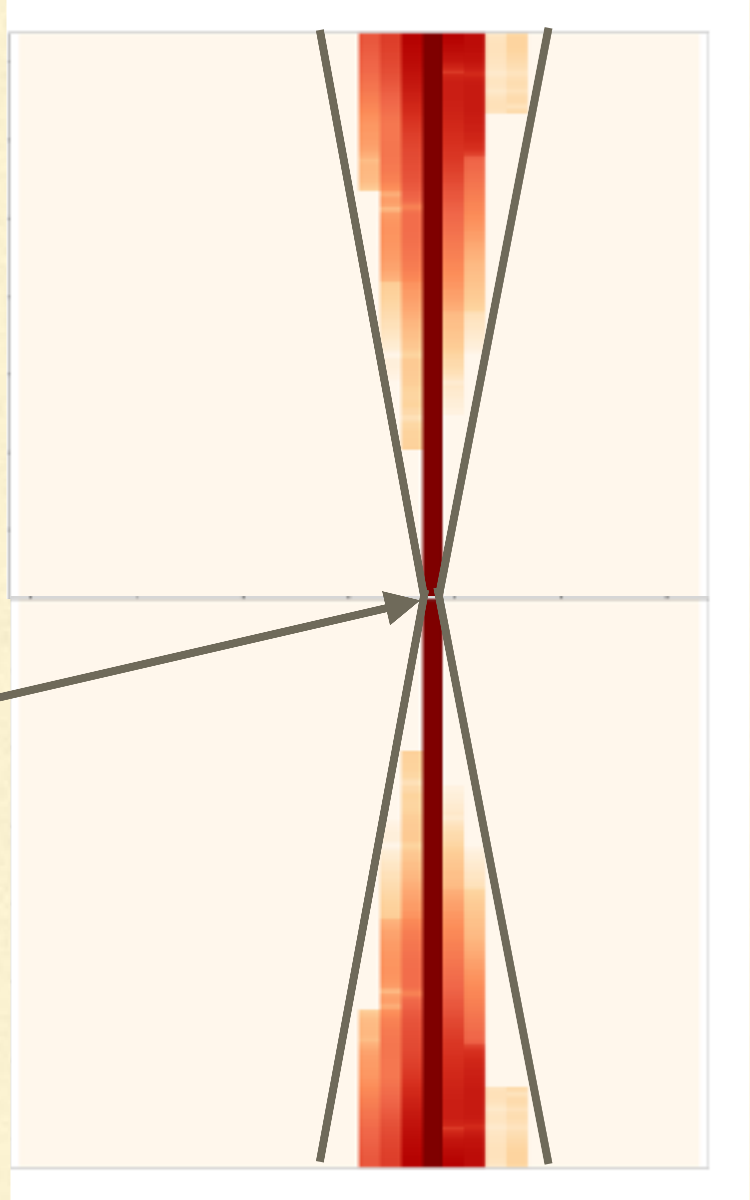
$$\tau_3 = U^\dagger \sigma_{3,z} U$$

$$\vdots$$

$$\tau_{15} = U^\dagger \sigma_{15,z} U$$

$$\vdots$$

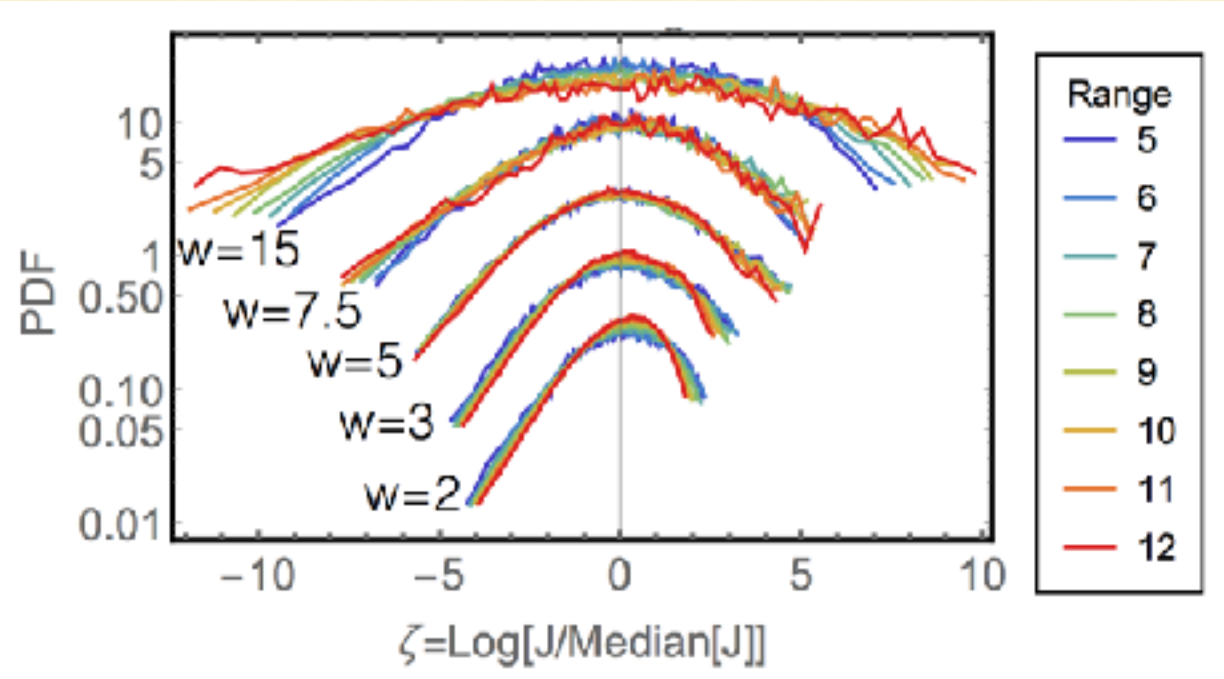
$$\tau_{32} = U^\dagger \sigma_{32,z} U$$



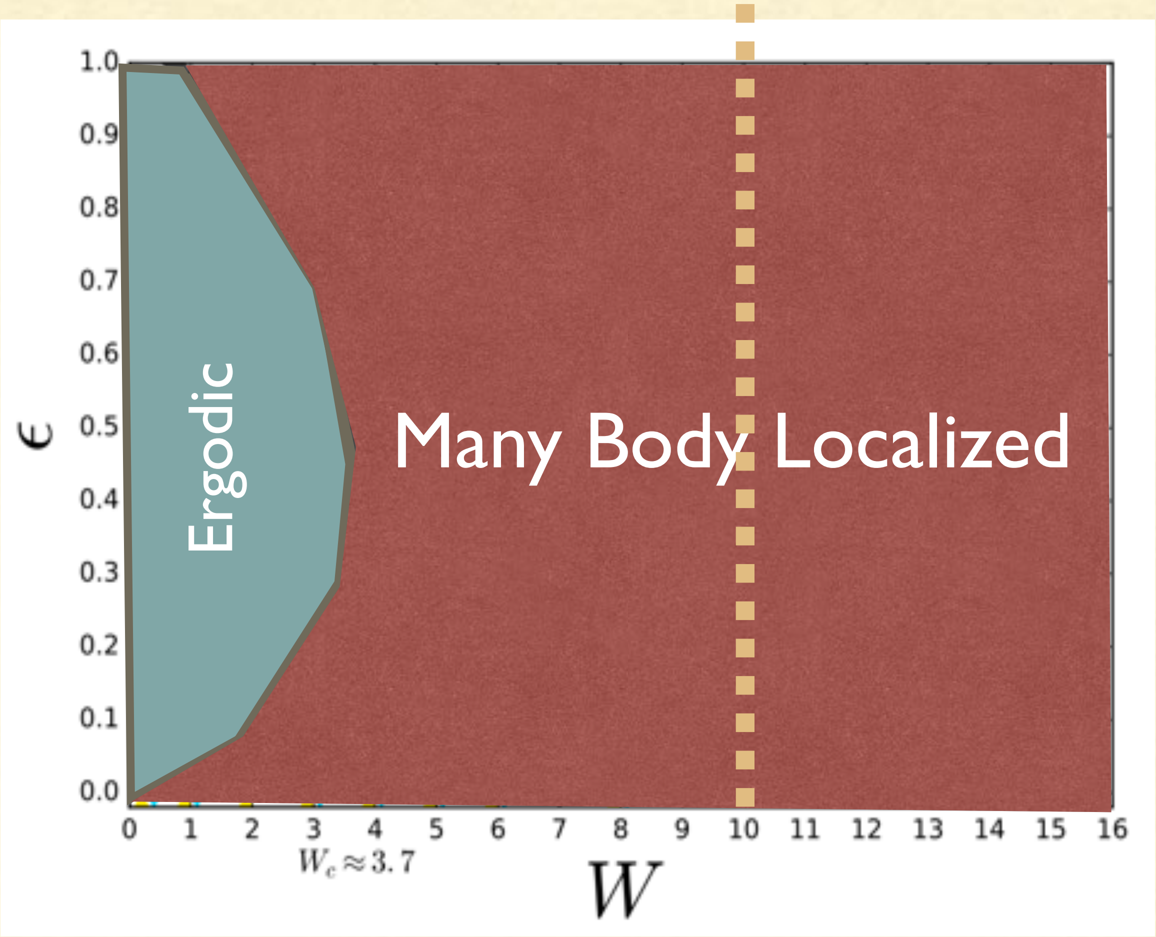
A local commuting operator

Yu, Pekker, BKC (February 2018)

$$H = \sum_i J_i \tau_i^z + \sum_{ij} J_{ij} \tau_i^z \tau_j^z + \sum_{ijk} J_{ijk} \tau_i^z \tau_j^z \tau_k^z + \ldots$$



Deep in the MBL phase, as $|i-j| \to \infty$ the probability that J_{ij} is of strength 10^k is independent of k



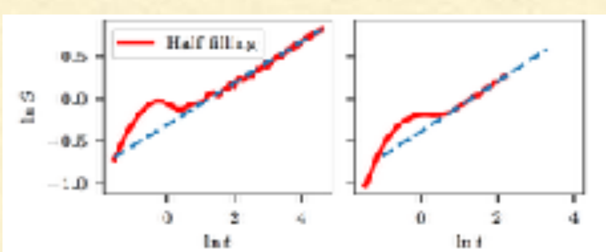
Pekker, BKC, Oganesyan, Rafael PRL (2017)

Open Challenges:

Mobility Edge

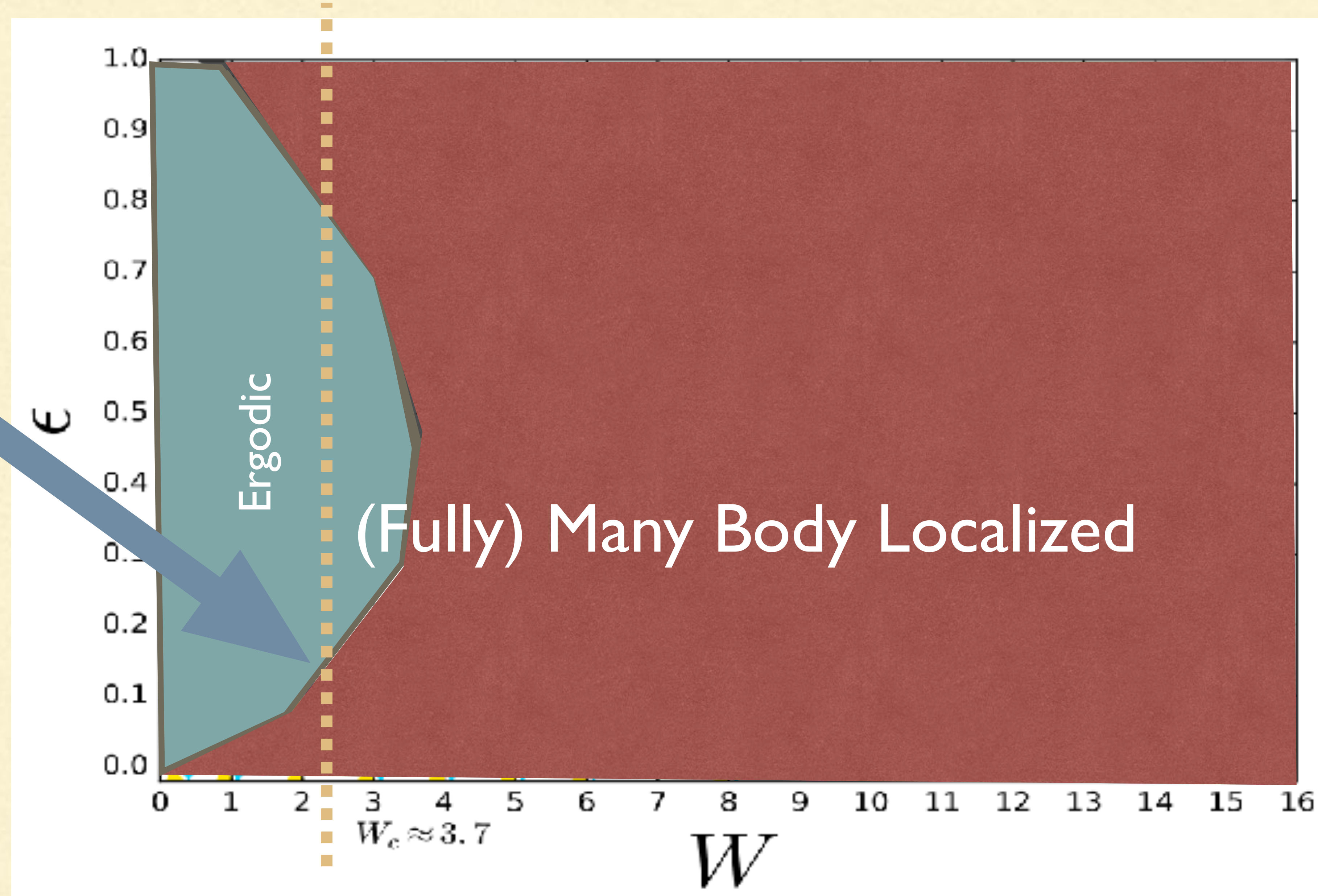
Transition

Beyond MBL



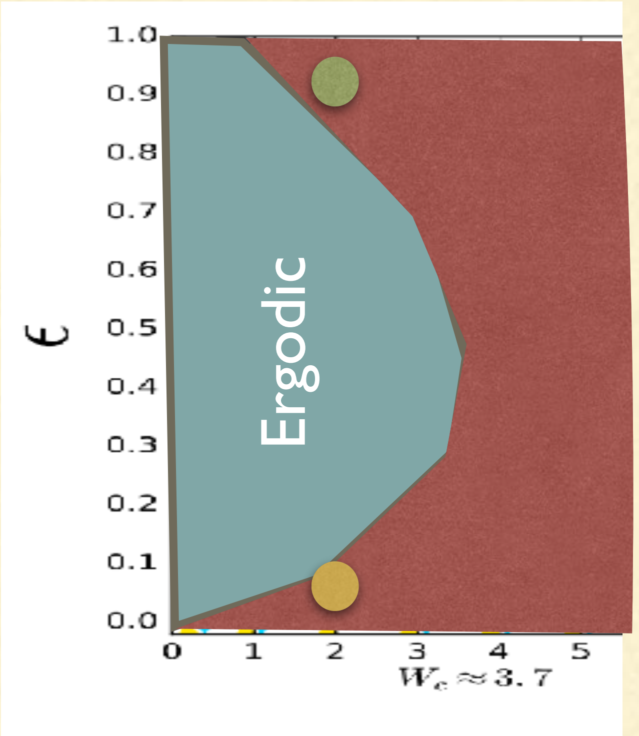
Eigenstates which are logarithmically entangled which equilibrate strangely.

Yu, Luo, Clark (Feb 2018)



Mobility Edge

Single eigenstates know about the whole spectrum.

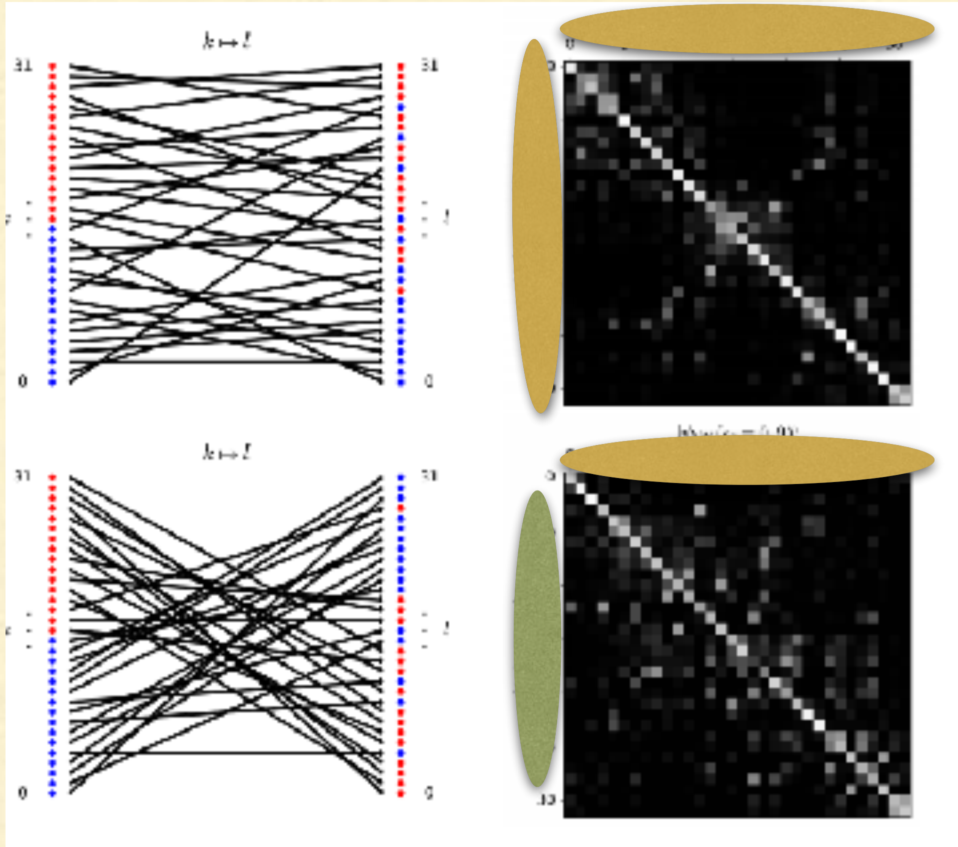


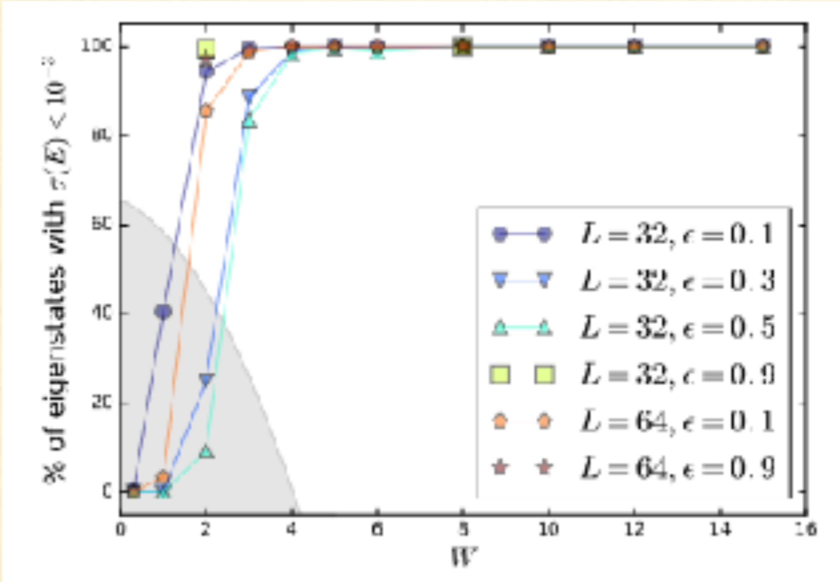
MBL Eigenstate

$\langle c_i^\dagger c_j \rangle$

eigenvectors

ϕ_i

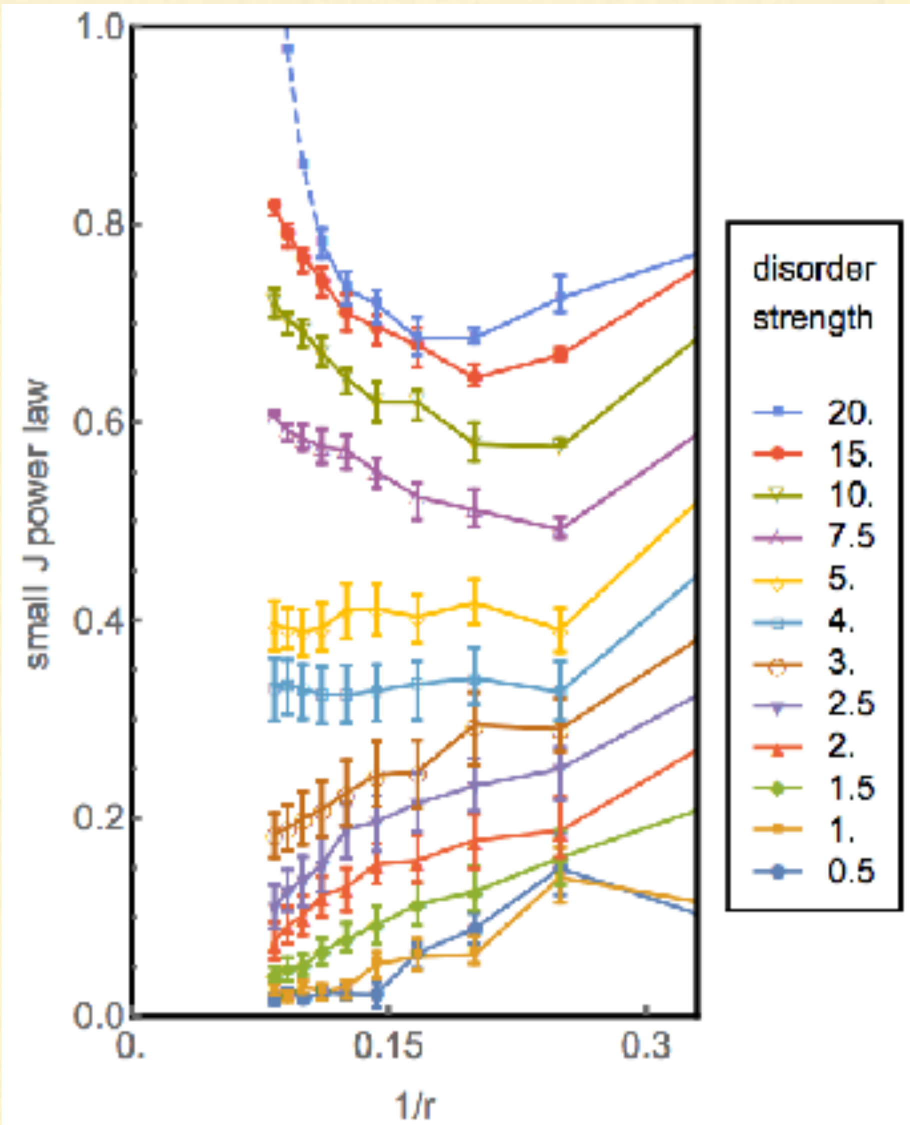




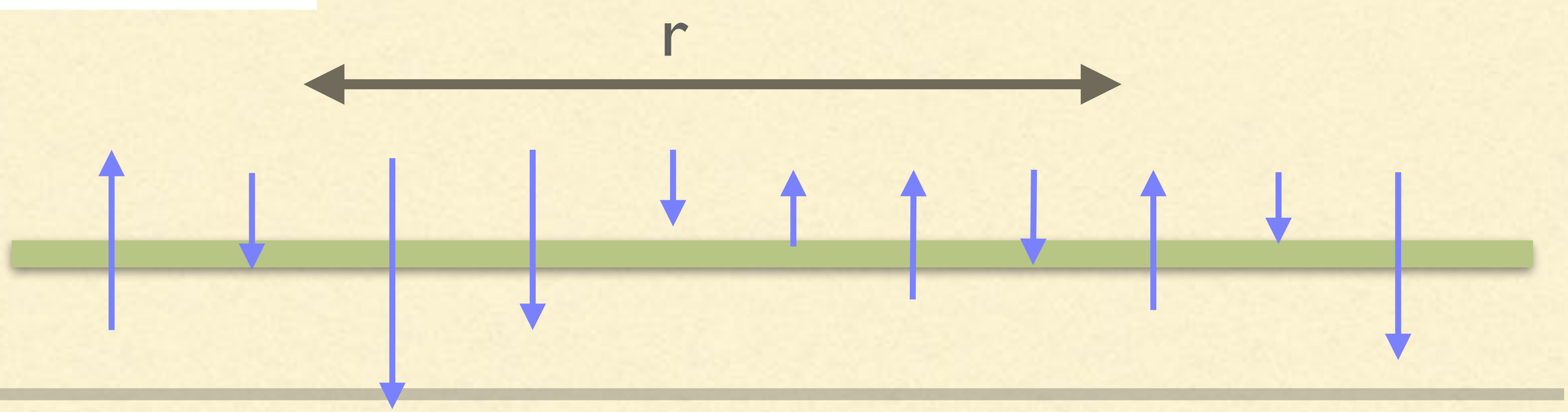
Villalonga, Yu, Luitz, BKC (1710.05036)

Transition

$$H = \sum_i J_i \tau_i^z + \sum_{ij} J_{ij}\tau_i^z\tau_j^z + \sum_{ijk} J_{ijk}\tau_i^z\tau_j^z\tau_k^z + \ldots$$

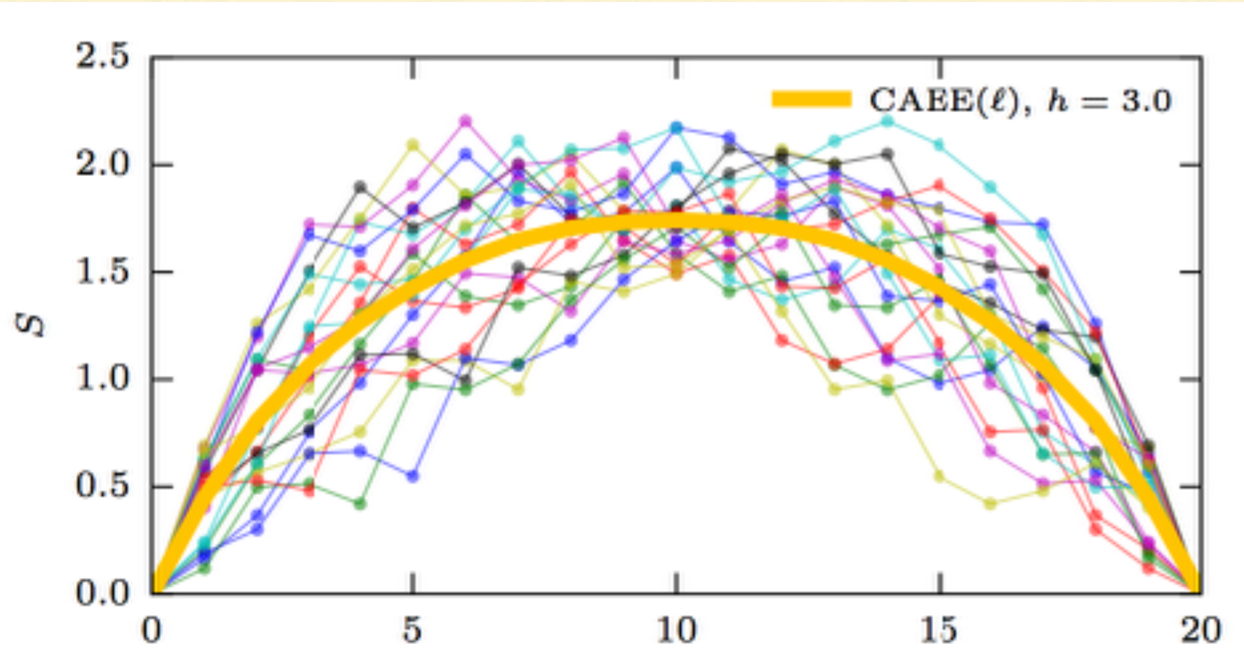


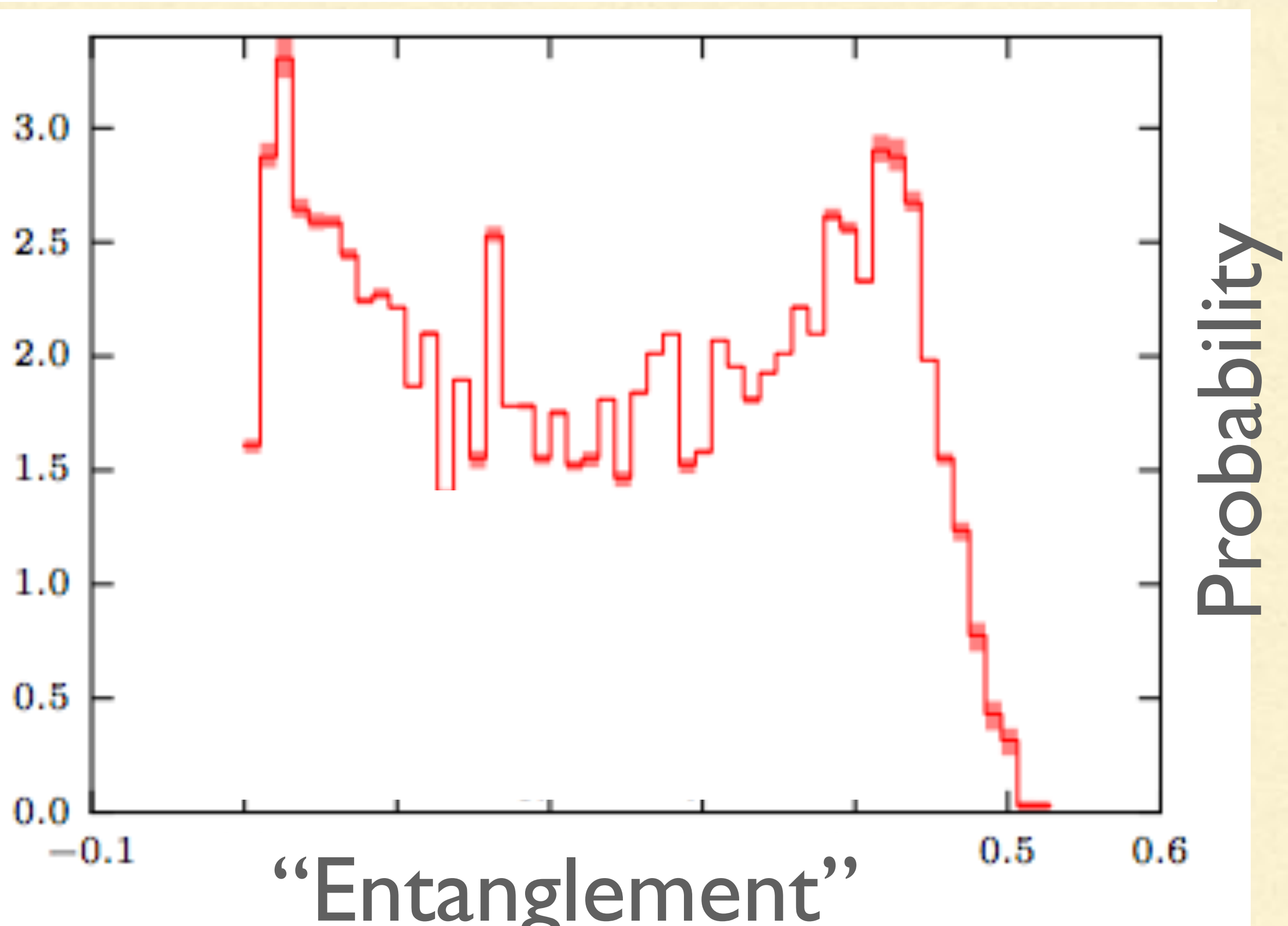
Probability distribution of coupling constants of the l-bit Hamiltonians are scale invariant at the transition.



Pekker, BKC, Oganesyan, Rafaela PRL (2017)

Transition



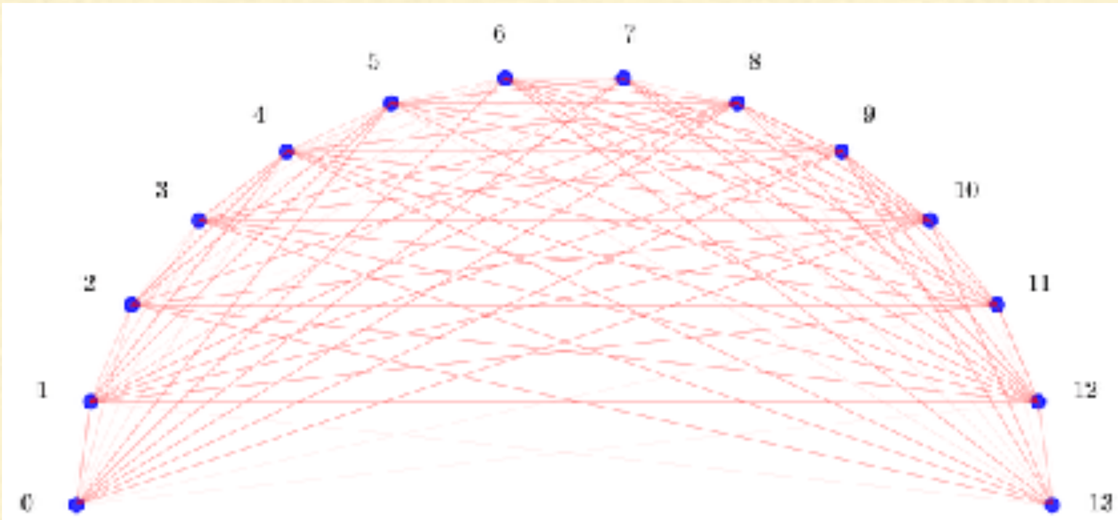


Eigenstates at the transition have both area law and volume law states

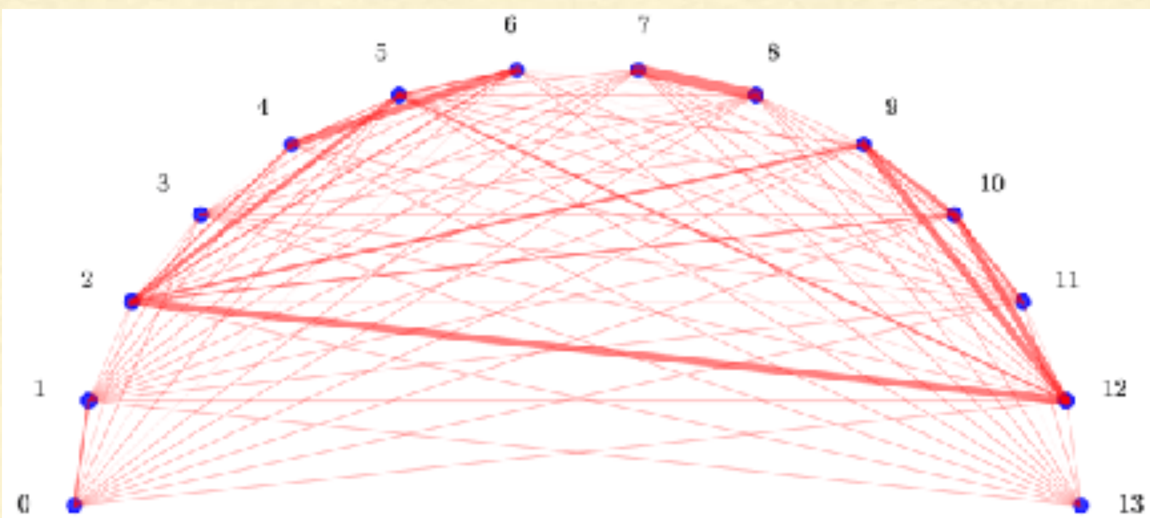
Yu, Luitz, BKC PRB 94, 184202

Transition

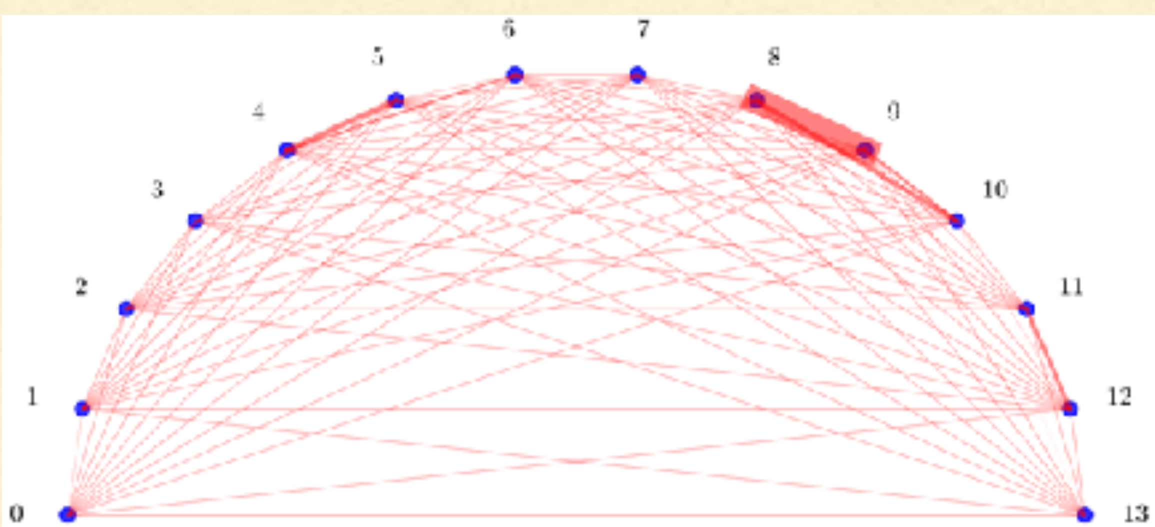
W=1.0

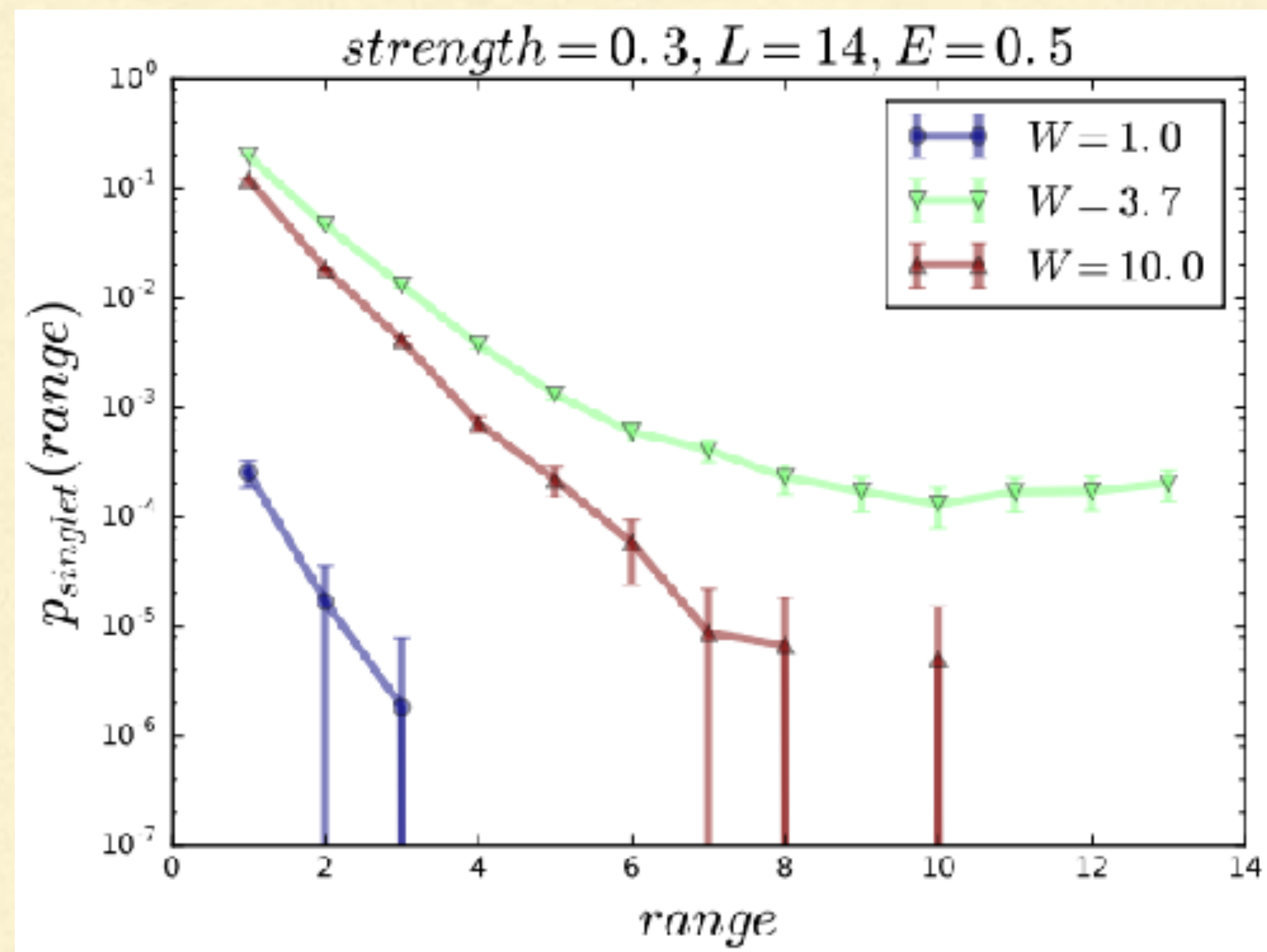


W=3.7



W=10





De Tomasi, Bera, Bardarson, Pollmann, PRL 118, 016804 (2017)

Villalonga, BKC (2018)

Some Hamiltonians don't thermalize
because their eigenstates are pathological
because you can diagonalize the Hamiltonian with short circuits
which implies low entangled, a compact eigenstate structure
and l-bits.

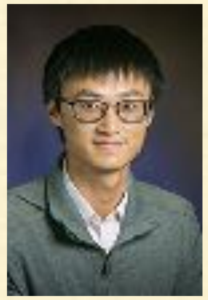
MBL eigenstates know about the mobility edge.
Bimodality at the transition
Scale-invariance at the transition

Algorithms/new observables:

SIMPS to get eigenstates
Tensor-network wegner-flow to get unitary tensor networks
CAEE window-averaging to convexify.
Universal approximate l-bits from eigenstates.

Acknowledgements

MBL





Frustrated
Magnetism

Finite T Algorithms
Quantum Information

Inverse approaches
to finding parent
Hamiltonians

Superconductivity