Exploring one-particle orbitals in large many-body localized systems

Benjamin Villalonga[1], Xiongjie Yu[1], David J. Luitz[2], Bryan K. Clark[1]

[1] Institute for Condensed Matter Theory and Department of Physics, University of Illinois at Urbana-Champaign
[2] Technical University of Munich

***Physical Review B*, *97*, 104406 (2018)**

ILLINOIS

TECHNISCHE UNIVERSITÄT MÜNCHEN

Many Body Localization (MBL)

Many Body Localization (MBL)

- **Isolated** many-body quantum system
- With **disorder**
- With **interactions**

$$H = -\frac{1}{2}\sum_{i=1}^{L-1}\left(c_i^\dagger c_{i+1} + h.c.\right) + \sum_{i=1}^{L-1} n_i n_{i+1} + \sum_{i=1}^{L} \mu_i n_i \quad \text{with} \quad \mu_i \in [-W, W]$$

(Maps to an isotropic Heisenberg + random field)

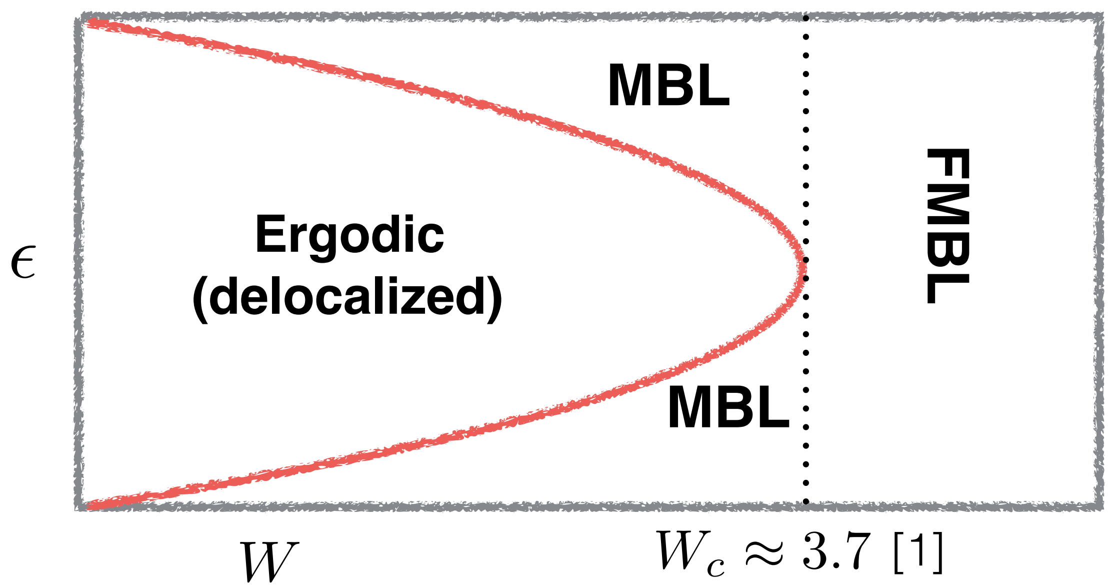
Benjamin Villalonga, UIUC

Many Body Localization (MBL)

- **Isolated** many-body quantum system
- With **disorder**
- With **interactions**

$$H = -\frac{1}{2}\sum_{i=1}^{L-1}\left(c_i^\dagger c_{i+1} + h.c.\right) + \sum_{i=1}^{L-1} n_i n_{i+1} + \sum_{i=1}^{L} \mu_i n_i \quad \text{with} \quad \mu_i \in [-W, W]$$

(Maps to an isotropic Heisenberg + random field)



[1] Luitz, D. J., Laflorencie, N., & Alet, F. (2015). *Physical Review B*, *91*(8), 081103.

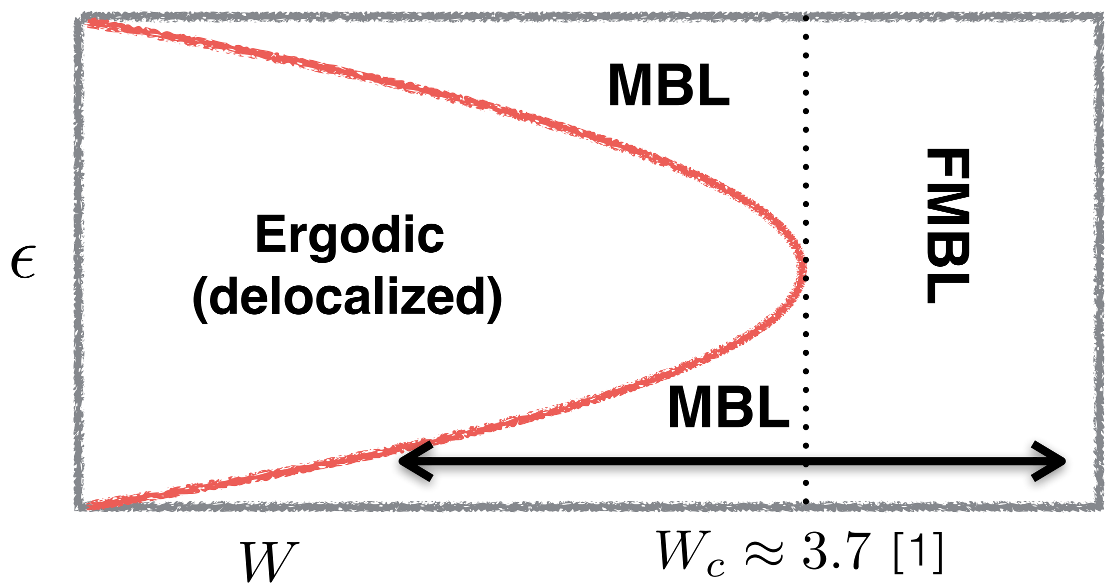
Benjamin Villalonga, UIUC

Many Body Localization (MBL)

- **Isolated** many-body quantum system
- With **disorder**
- With **interactions**

$$H = -\frac{1}{2}\sum_{i=1}^{L-1}\left(c_i^\dagger c_{i+1} + h.c.\right) + \sum_{i=1}^{L-1} n_i n_{i+1} + \sum_{i=1}^{L} \mu_i n_i \qquad \text{with} \qquad \mu_i \in [-W, W]$$

(Maps to an isotropic Heisenberg + random field)



[1] Luitz, D. J., Laflorencie, N., & Alet, F. (2015). *Physical Review B*, *91*(8), 081103.

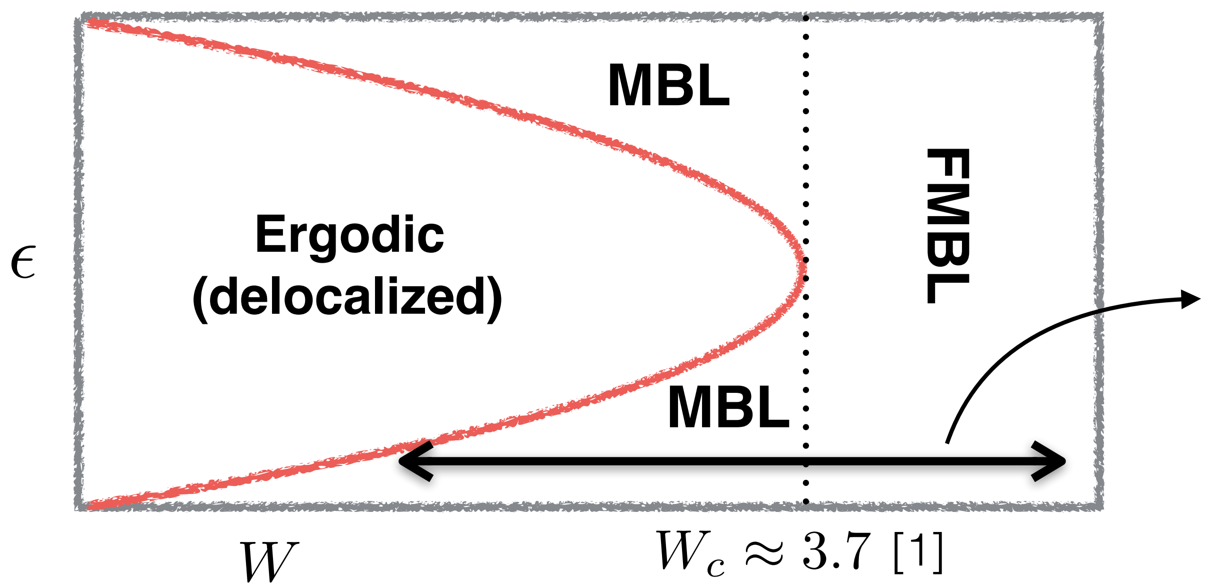
Benjamin Villalonga, UIUC

Many Body Localization (MBL)

- **Isolated** many-body quantum system
- With **disorder**
- With **interactions**

$$H = -\frac{1}{2}\sum_{i=1}^{L-1}\left(c_i^\dagger c_{i+1} + h.c.\right) + \sum_{i=1}^{L-1} n_i n_{i+1} + \sum_{i=1}^{L} \mu_i n_i \quad \text{with} \quad \mu_i \in [-W, W]$$

(Maps to an isotropic Heisenberg + random field)



Build approximate integrals of motion from eigenstates

[1] Luitz, D. J., Laflorencie, N., & Alet, F. (2015). *Physical Review B*, *91*(8), 081103.

Benjamin Villalonga, UIUC

One-Particle Orbitals (OPO)

Eigenstate $|\psi\rangle \rightarrow \rho_{ij} \equiv \left\langle c_i^\dagger c_j \right\rangle$ One particle density matrix

Benjamin Villalonga, UIUC

One-Particle Orbitals (OPO)

Eigenstate $|\psi\rangle \rightarrow \rho_{ij} \equiv \left\langle c_i^\dagger c_j \right\rangle$ One particle density matrix

- Eigenvalues [2]
- Eigenvectors (OPOs)

  (and their number operators)

[2] Bera, S., Schomerus, H., Heidrich-Meisner, F., & Bardarson, J. H. (2015). *Physical review letters*, *115*(4), 046603.

Benjamin Villalonga, UIUC

One-Particle Orbitals (OPO)

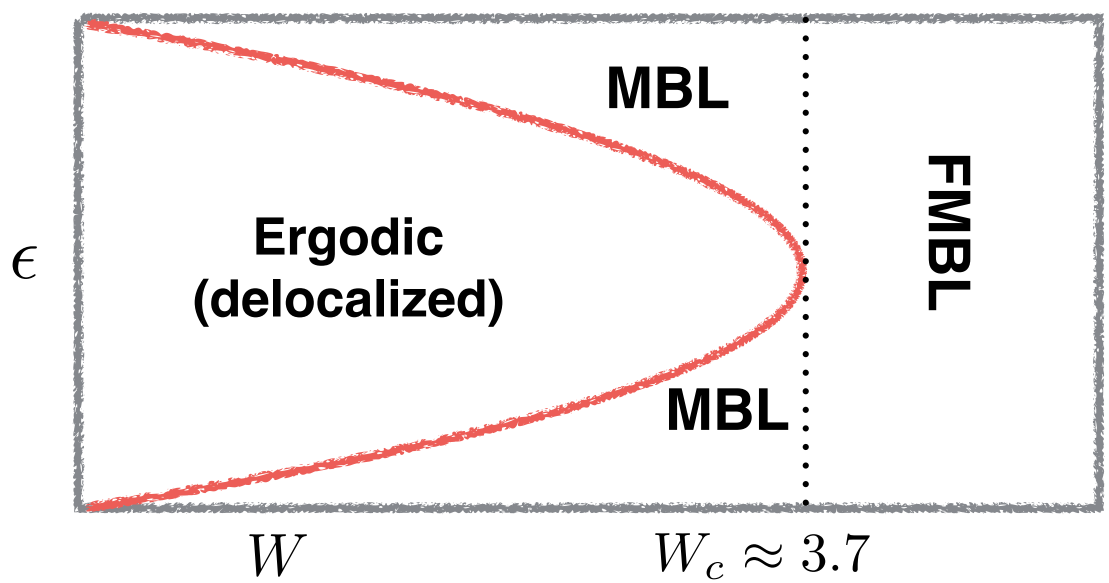
Eigenstate $|\psi\rangle \rightarrow \rho_{ij} \equiv \left\langle c_i^\dagger c_j \right\rangle$ One particle density matrix

- Eigenvalues [2]
- Eigenvectors (OPOs)

  (and their number operators)

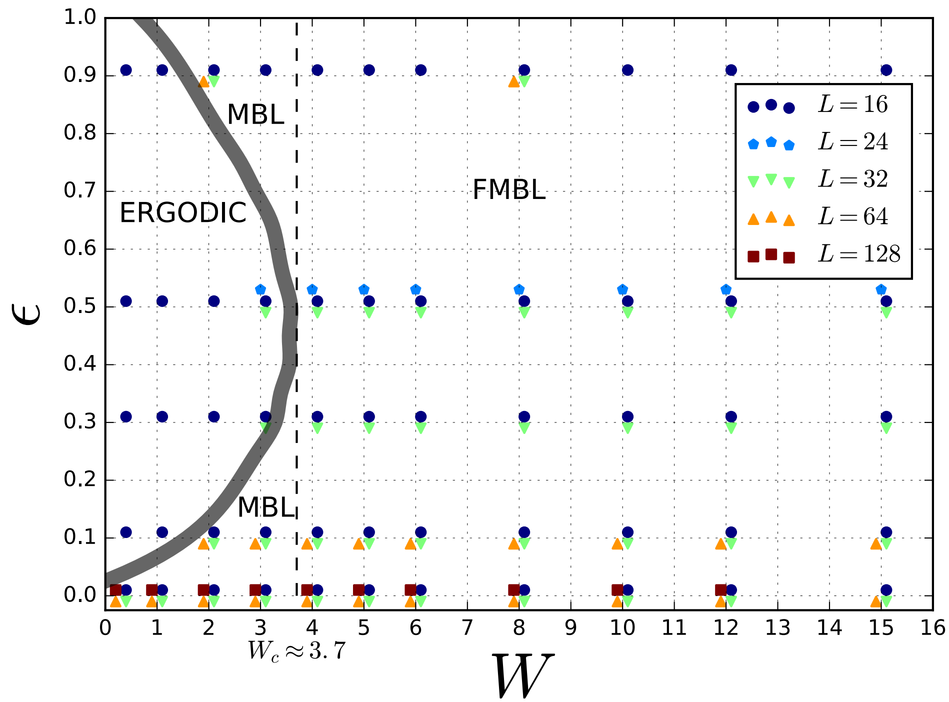
→

- Exact integrals of motion in strong disorder limit
- One-particle approximation elsewhere

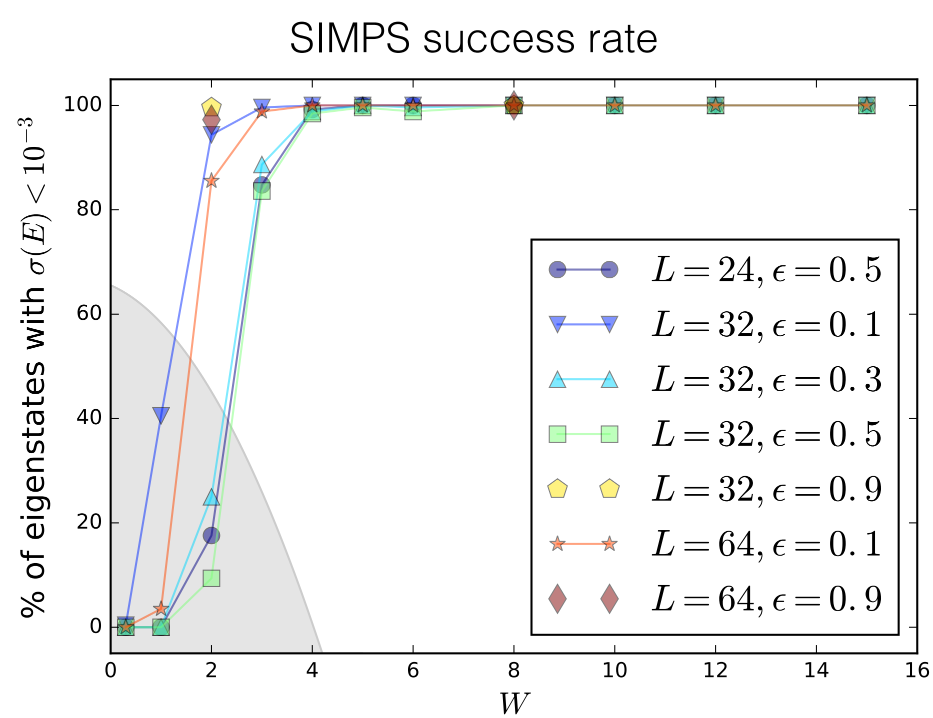


[2] Bera, S., Schomerus, H., Heidrich-Meisner, F., & Bardarson, J. H. (2015). *Physical review letters*, *115*(4), 046603.

Numerics



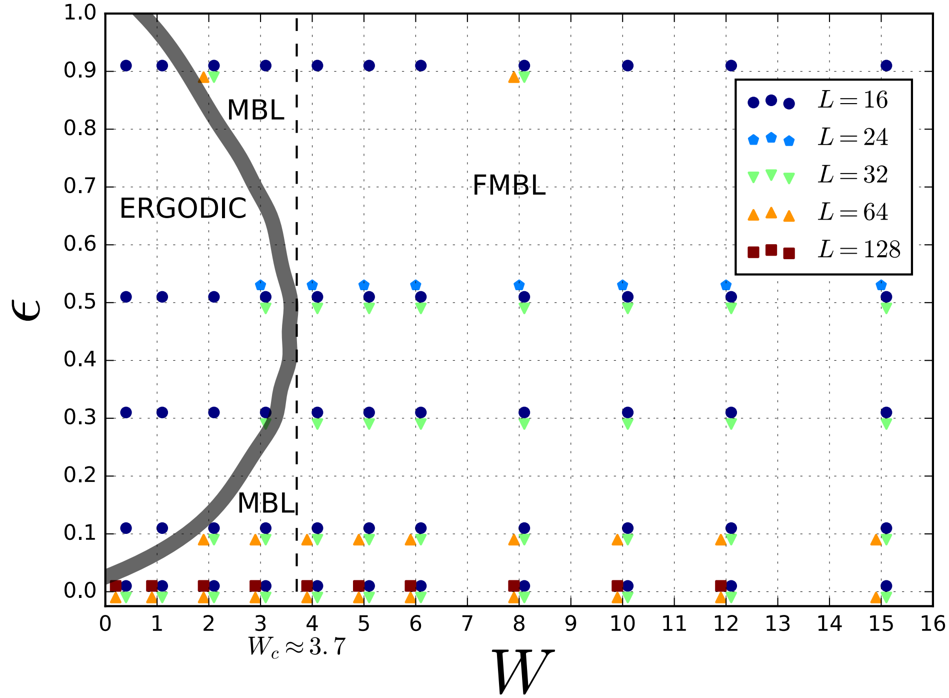
We use Shift and Invert MPS (**SIMPS**) [3]



[3] Xiongjie Yu, David Pekker, & Bryan K. Clark (2015). *Physical review letters*, 118, 017201.

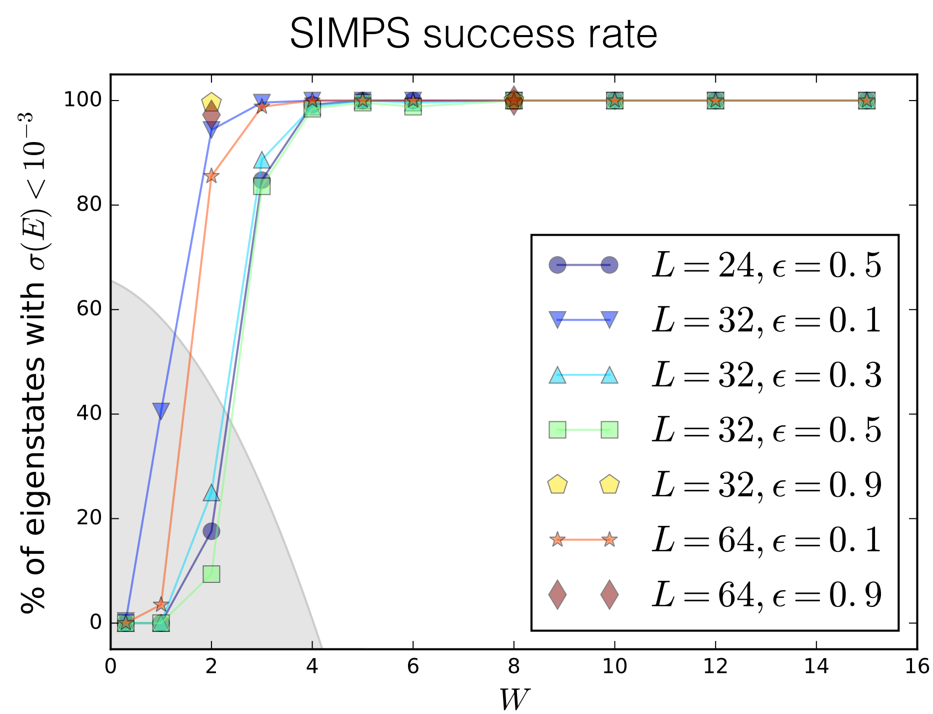
Benjamin Villalonga, UIUC

Numerics



We use Shift and Invert MPS (**SIMPS**) [3]

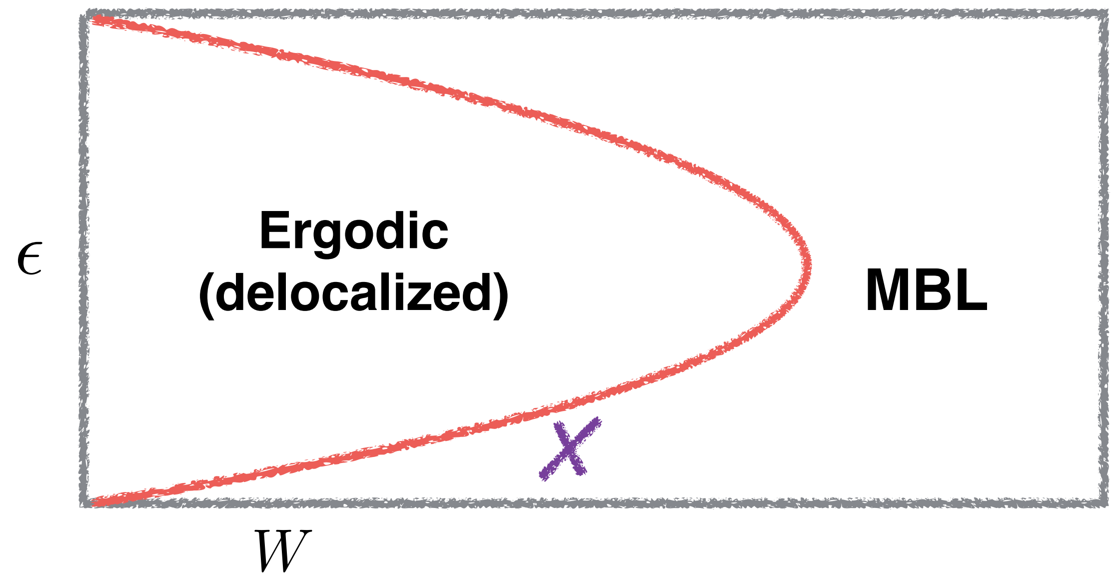
Can see the mobility edge up to L=64



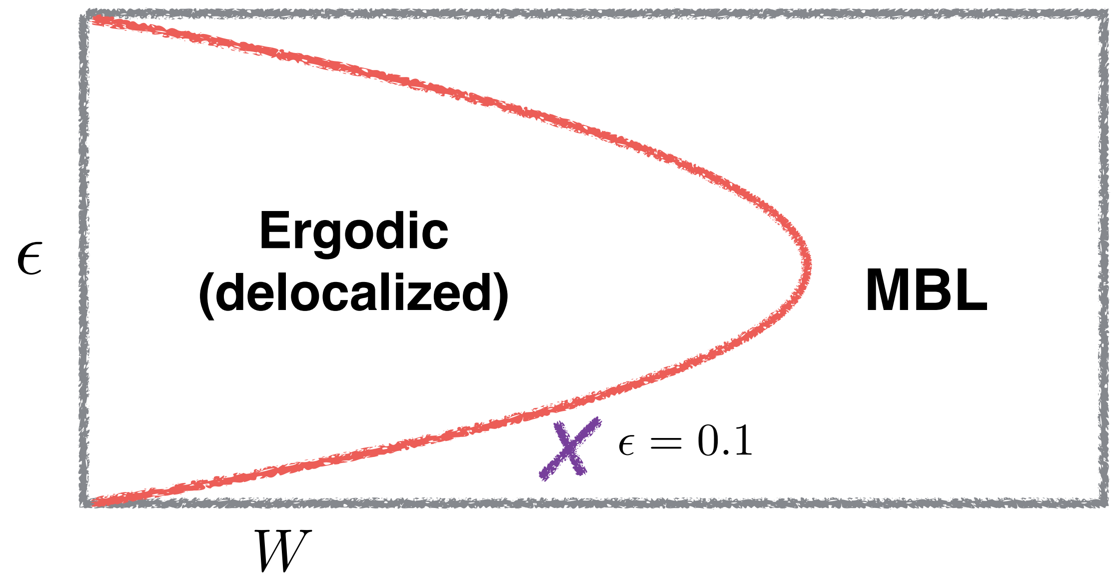
[3] Xiongjie Yu, David Pekker, & Bryan K. Clark (2015). *Physical review letters*, 118, 017201.

OPOs are universal across energy spectrum I

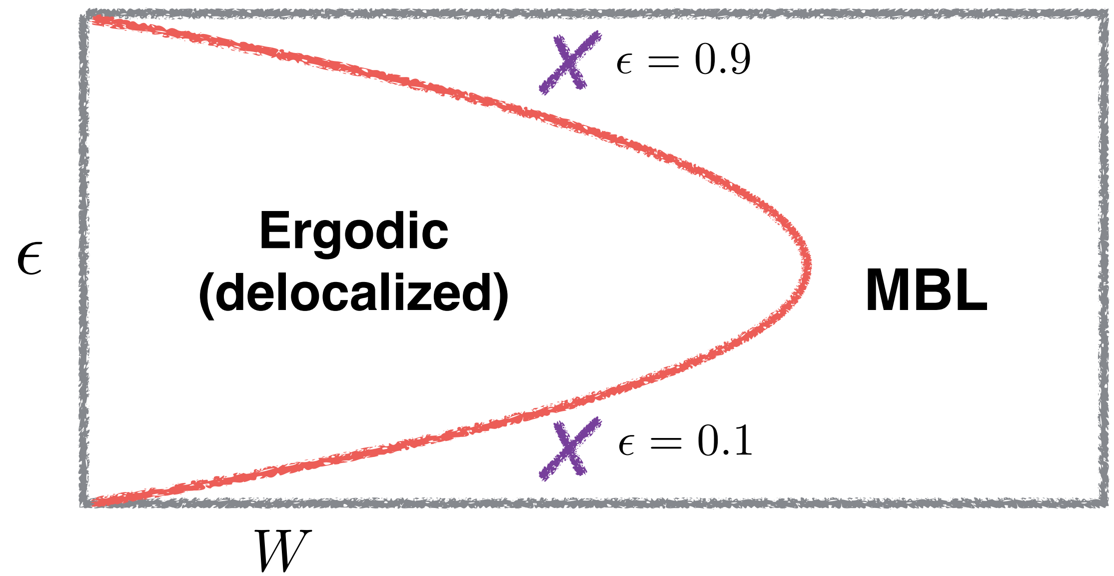
OPOs are universal across energy spectrum I



OPOs are universal across energy spectrum I

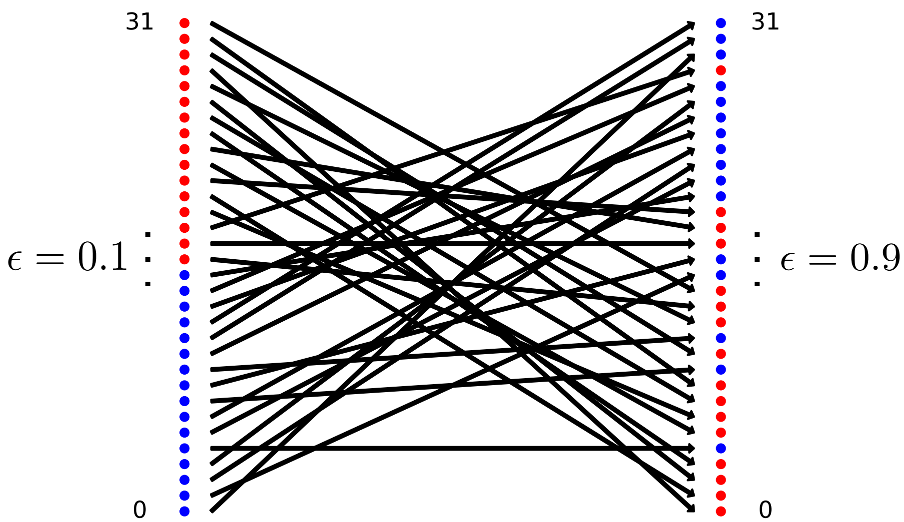


OPOs are universal across energy spectrum I

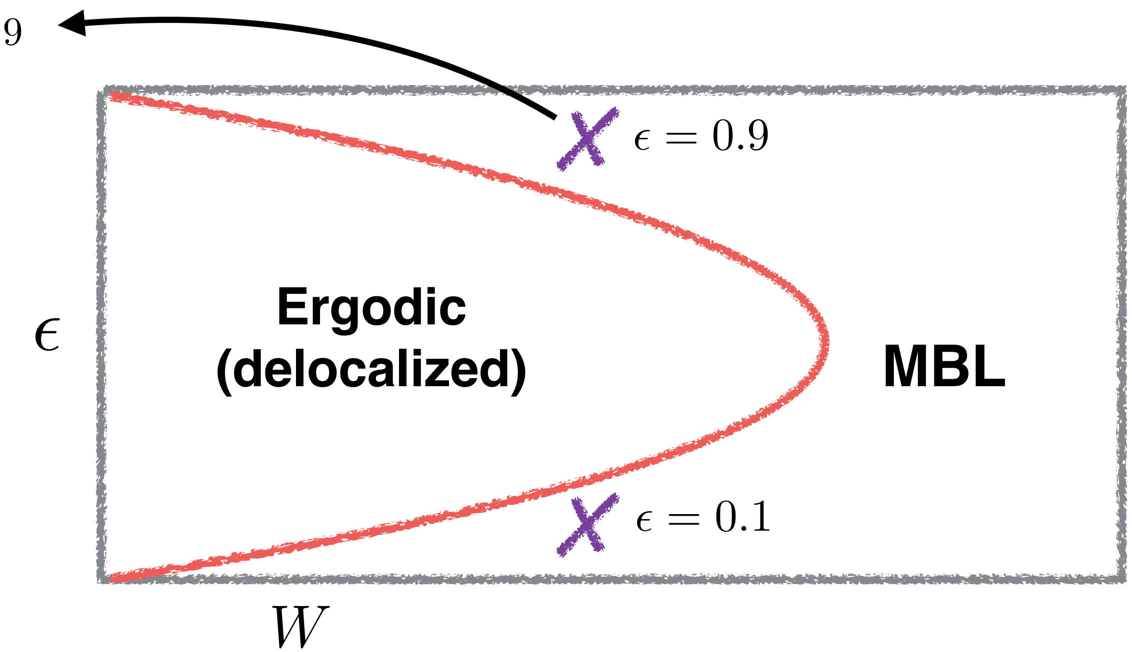


Benjamin Villalonga, UIUC

OPOs are universal across energy spectrum I

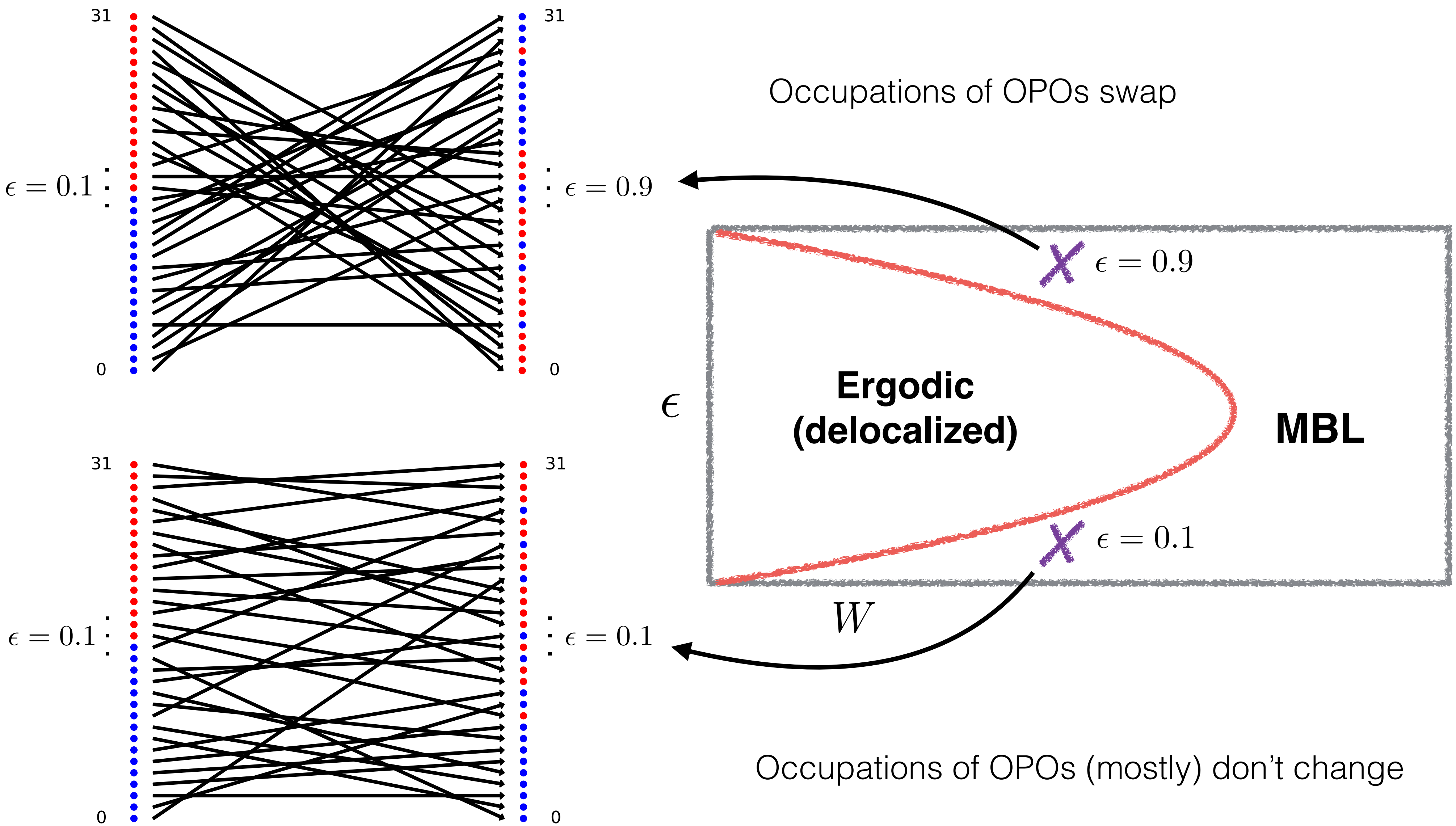


Occupations of OPOs swap



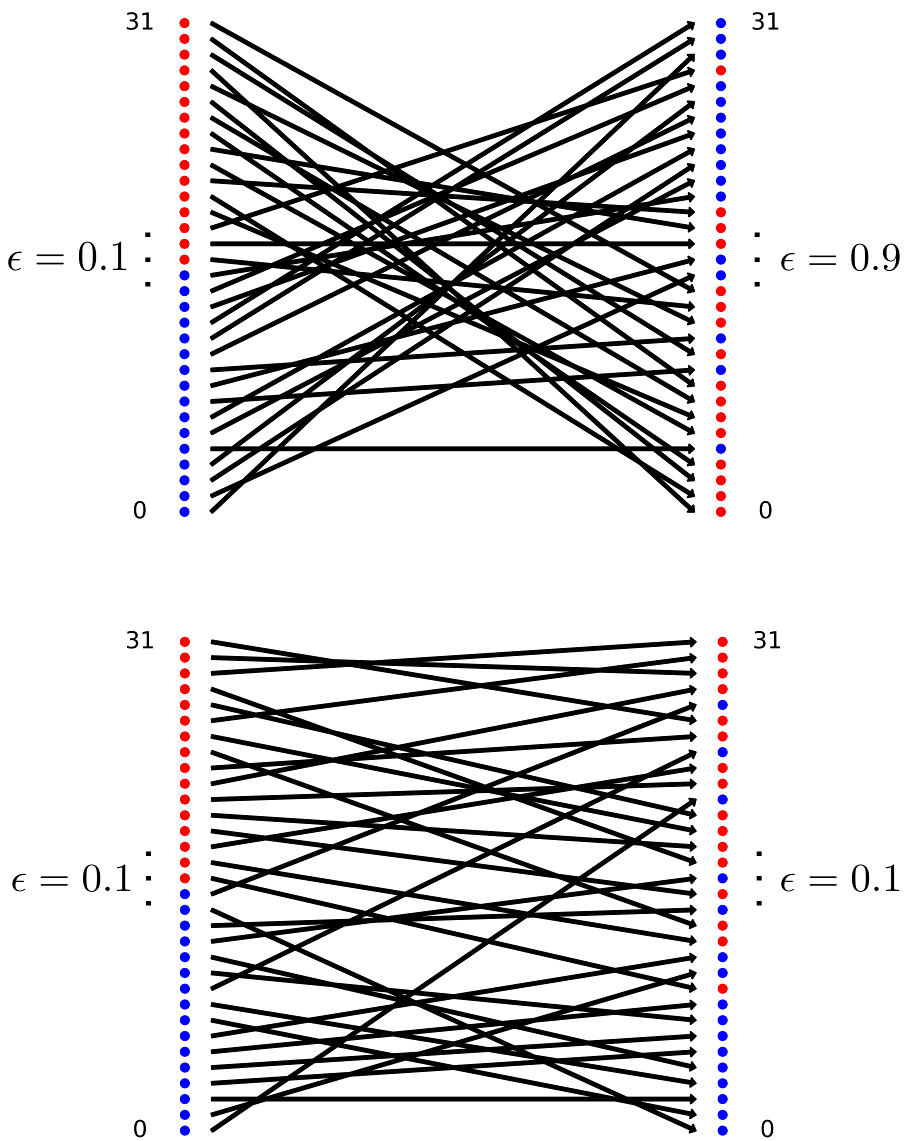
Benjamin Villalonga, UIUC

OPOs are universal across energy spectrum I

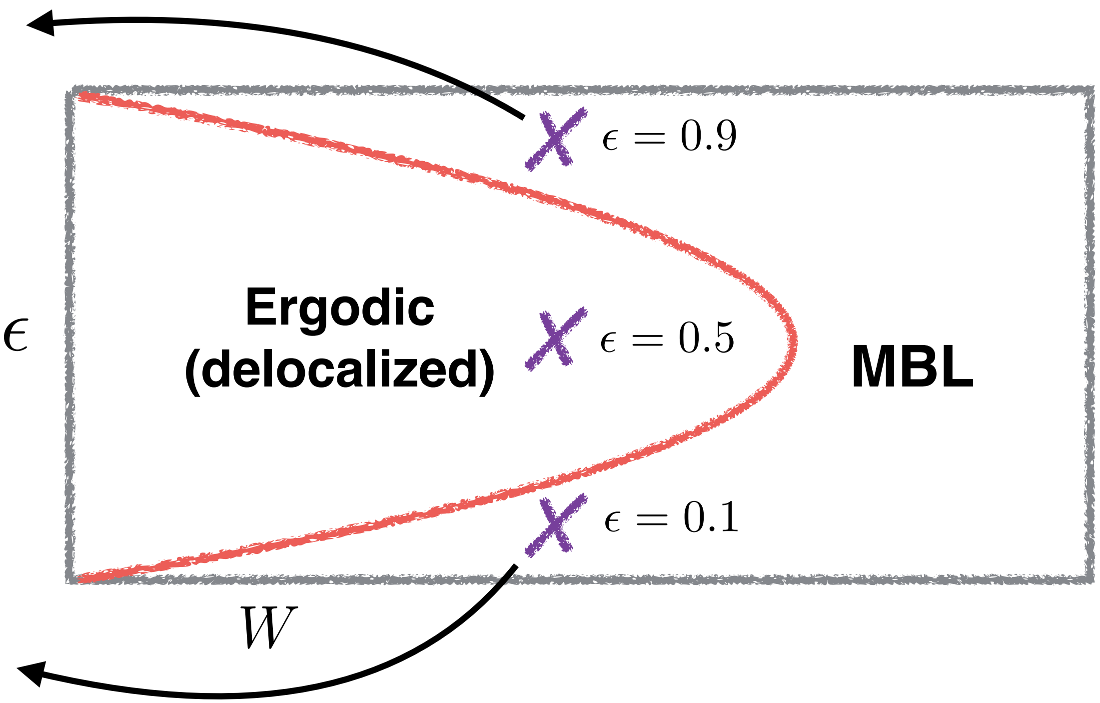


Benjamin Villalonga, UIUC

OPOs are universal across energy spectrum I



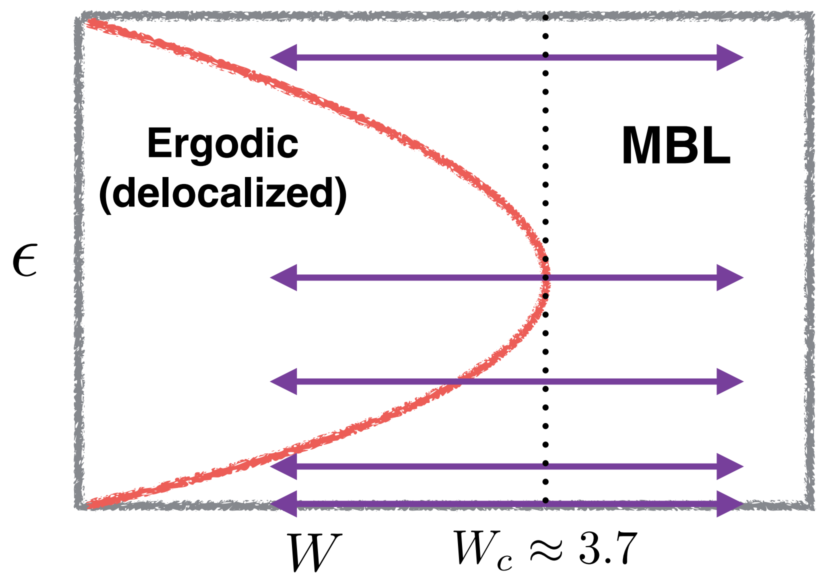
Occupations of OPOs swap



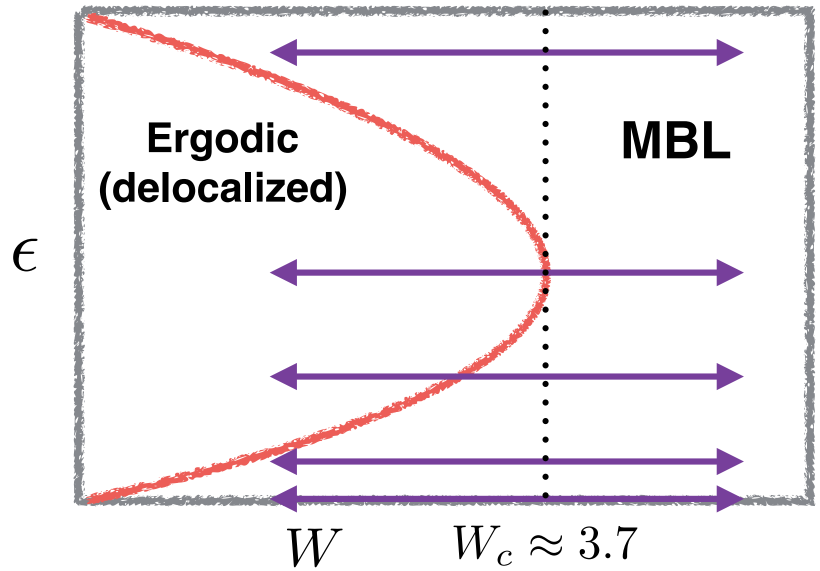
Occupations of OPOs (mostly) don't change

OPOs are universal across energy spectrum II

OPOs are universal across energy spectrum II



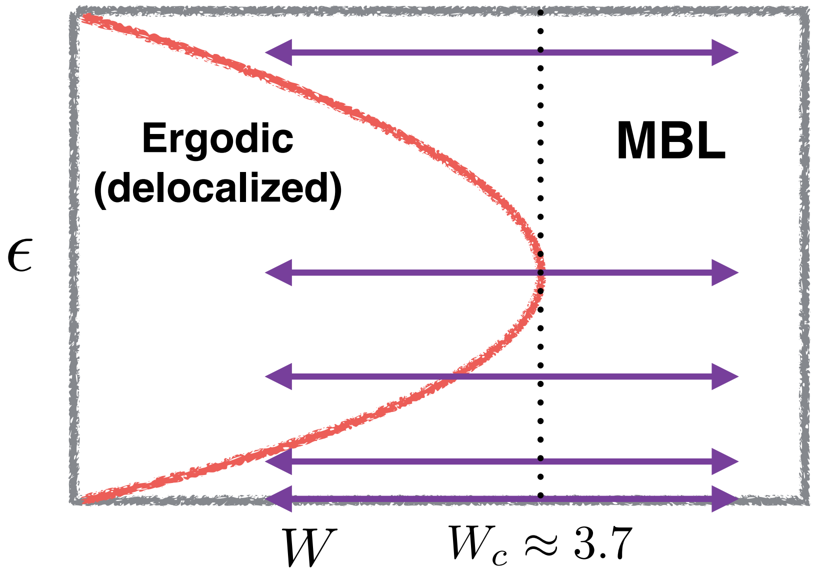
OPOs are universal across energy spectrum II



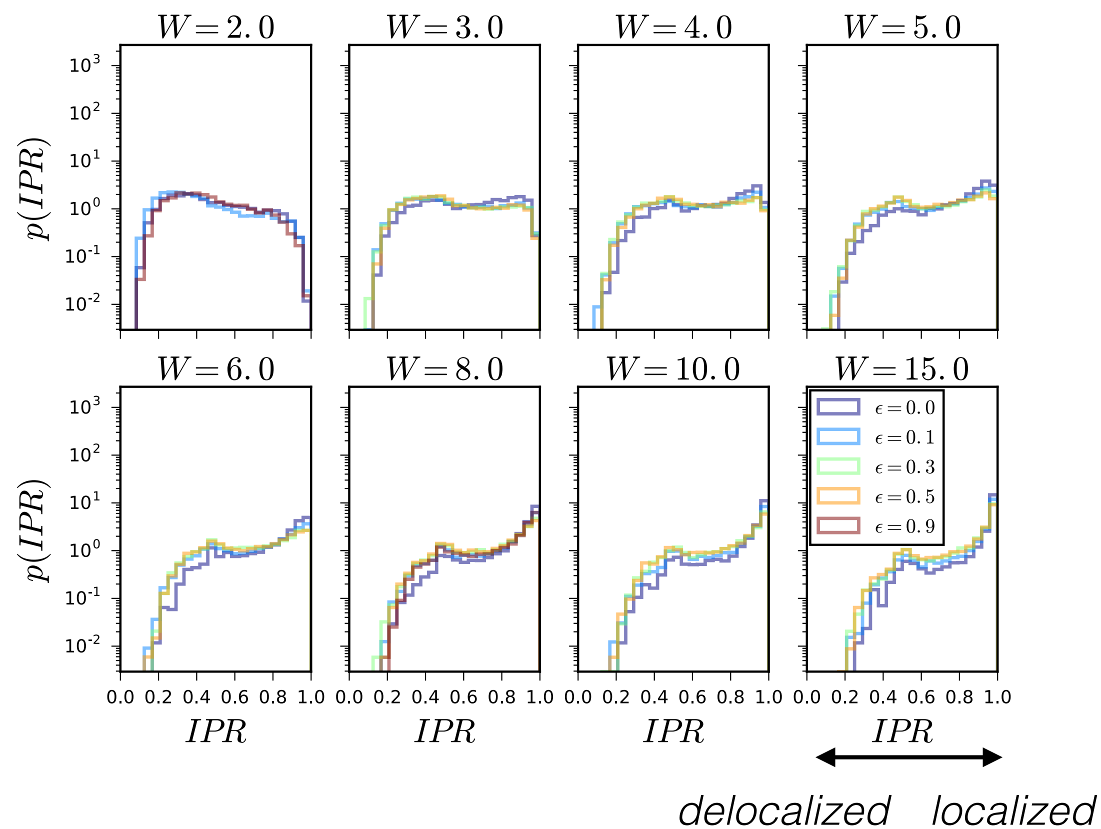
IPR of an OPO → 0 delocalized
IPR of an OPO → 1 localized

Benjamin Villalonga, UIUC

OPOs are universal across energy spectrum II

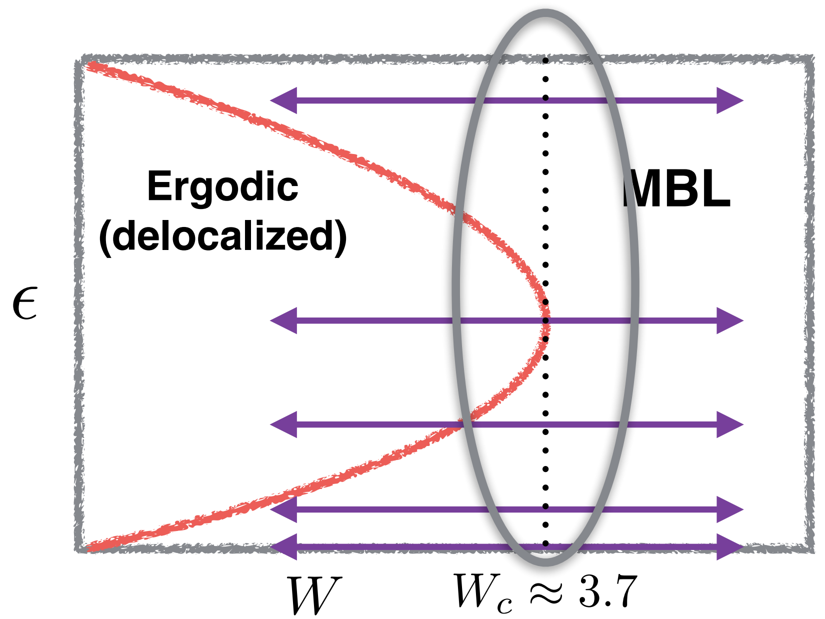


IPR of an OPO → 0 delocalized
IPR of an OPO → 1 localized

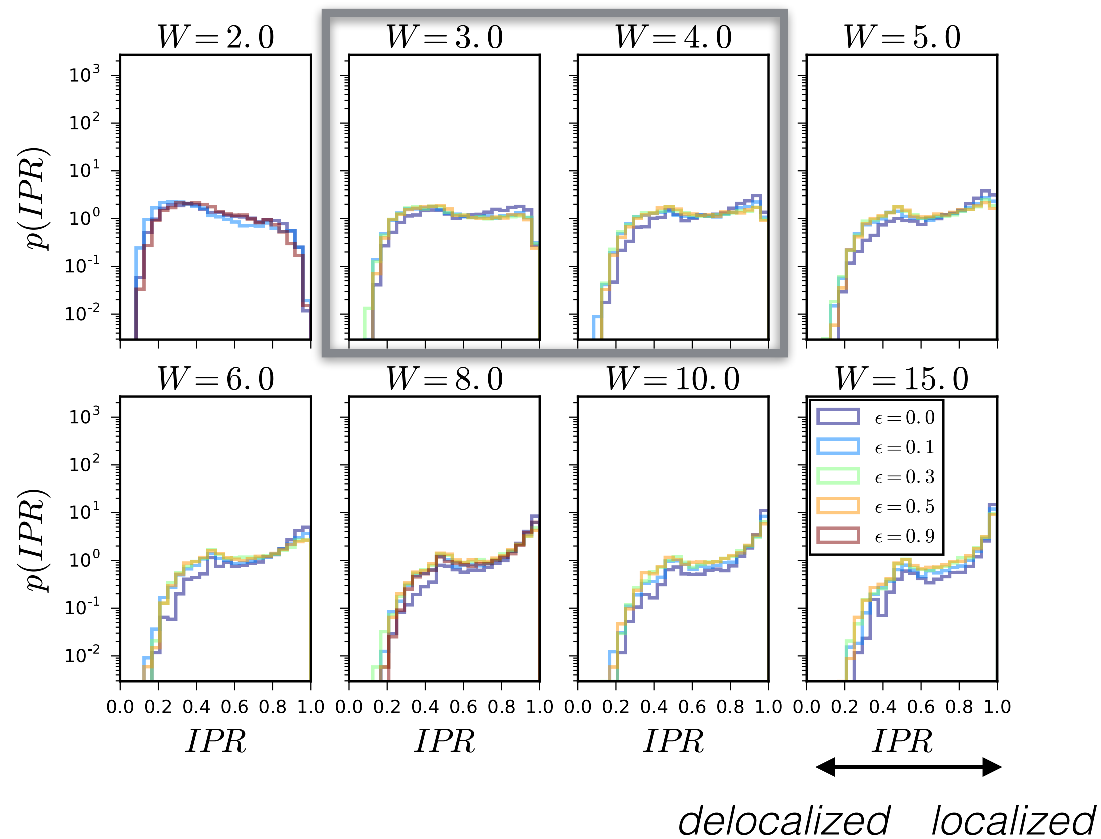


Benjamin Villalonga, UIUC

OPOs are universal across energy spectrum II

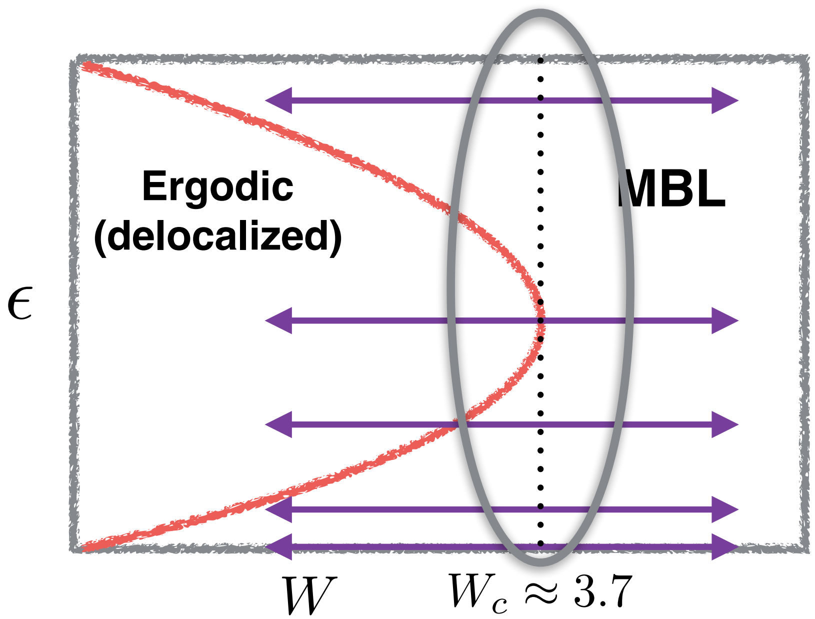


IPR of an OPO → 0 delocalized, 1 localized

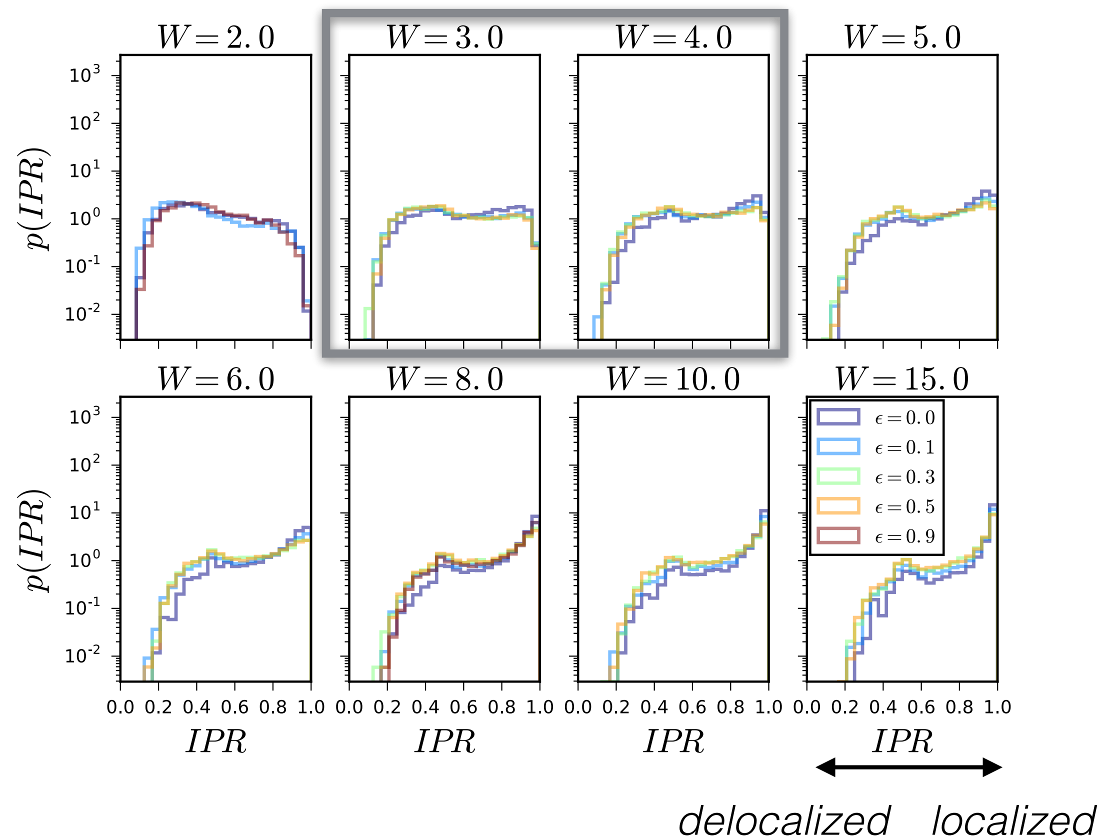


Benjamin Villalonga, UIUC

OPOs are universal across energy spectrum II



IPR of an OPO → 0 delocalized
IPR of an OPO → 1 localized



MBL eigenstates are aware of the transition happening at a different energy

Universal behavior at strong disorder

Benjamin Villalonga, UIUC

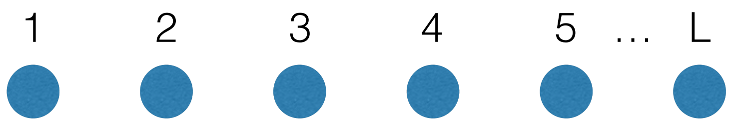
Universal behavior at strong disorder

Approximate integral of motion

Benjamin Villalonga, UIUC

Universal behavior at strong disorder

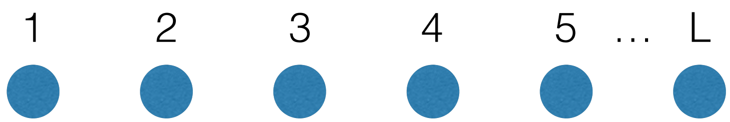
Approximate integral of motion



Benjamin Villalonga, UIUC

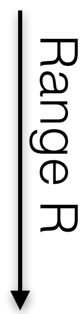
Universal behavior at strong disorder

Approximate integral of motion



Sum over operators that couple all pairs of sites:

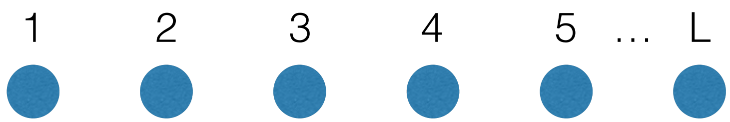
" f[1,2] + f[2,3] + f[3,4] + … +
f[1,3] + f[2,4] + f[3,5] + … +
… +
f[1,L-1] + f[2,L] +
f[1,L] "



Benjamin Villalonga, UIUC

Universal behavior at strong disorder

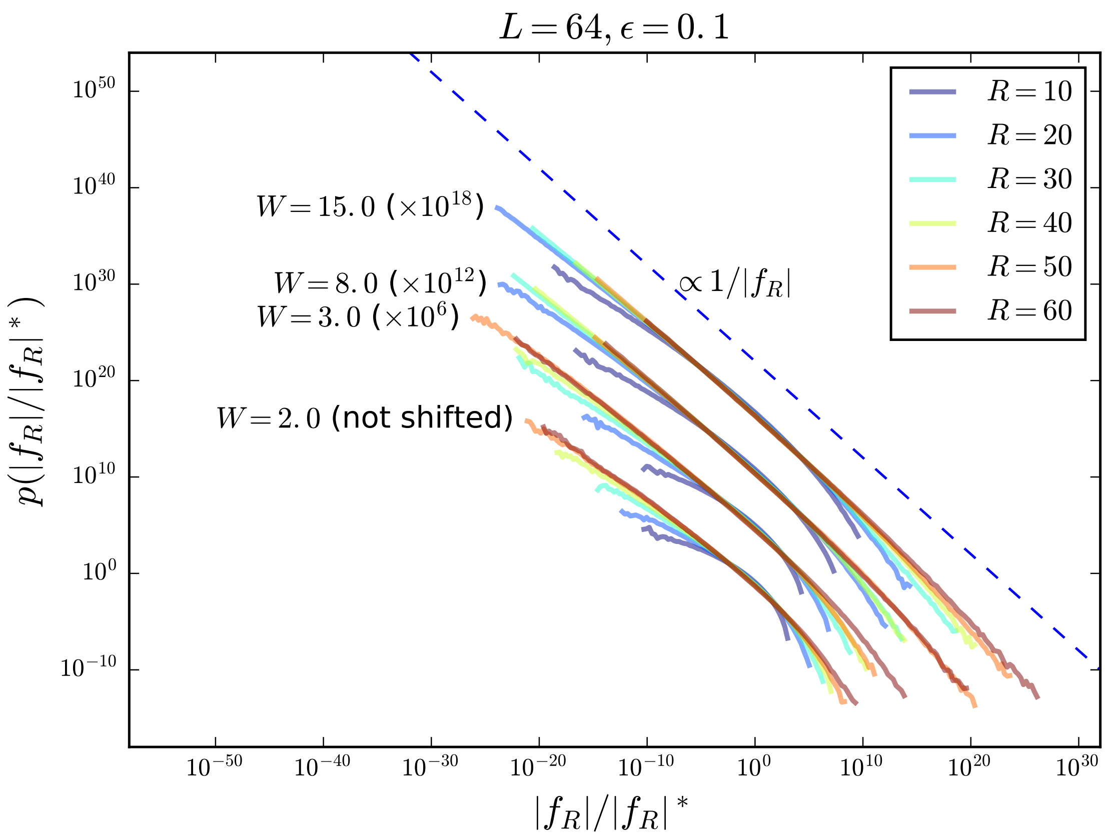
Approximate integral of motion



Sum over operators that couple all pairs of sites:

" f[1,2] + f[2,3] + f[3,4] + … +
f[1,3] + f[2,4] + f[3,5] + … +
… +
f[1,L-1] + f[2,L] +
f[1,L] "

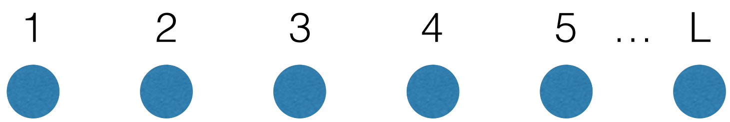
Range R



Benjamin Villalonga, UIUC

Universal behavior at strong disorder

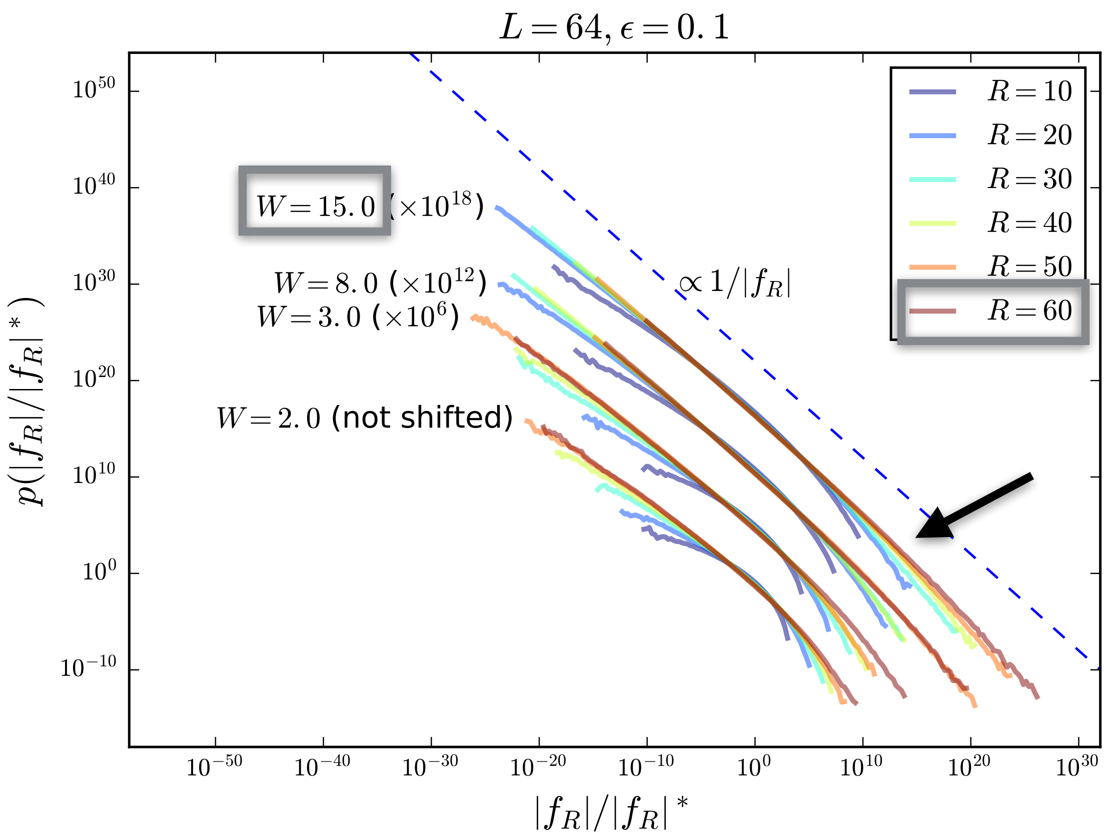
Approximate integral of motion



Sum over operators that couple all pairs of sites:

" f[1,2] + f[2,3] + f[3,4] + … +
f[1,3] + f[2,4] + f[3,5] + … +
… +
f[1,L-1] + f[2,L] +
f[1,L] "

Range R

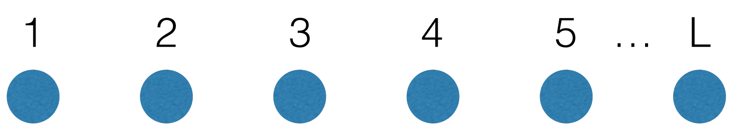


At large disorder, a large range coupling constant taken at random has a magnitude following a *universal* **1/f** distribution

Benjamin Villalonga, UIUC

Universal behavior at strong disorder

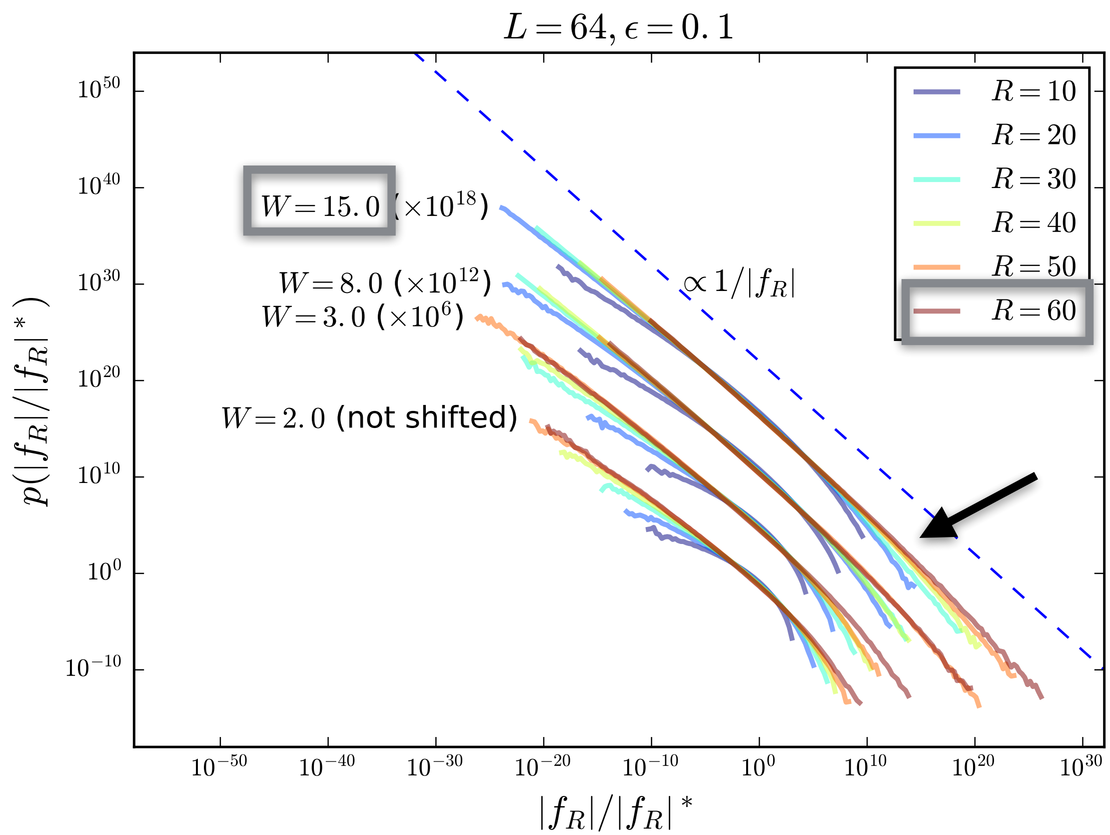
Approximate integral of motion



Sum over operators that couple all pairs of sites:

" f[1,2] + f[2,3] + f[3,4] + … +
f[1,3] + f[2,4] + f[3,5] + … +
… +
f[1,L-1] + f[2,L] +
f[1,L] "

Range R



At large disorder, a large range coupling constant taken at random has a magnitude following a *universal* **1/f** distribution

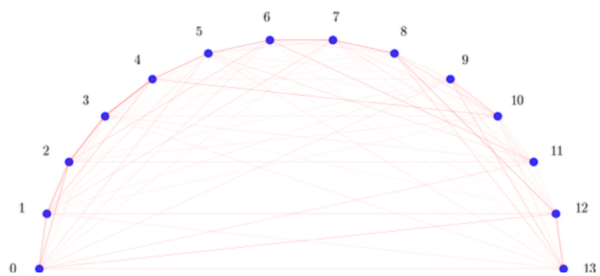
[4] D. Pekker, B. K. Clark, V. Oganesyan, and G. Refael (2017). Phys. Rev. Lett. 119, 075701.

Benjamin Villalonga, UIUC

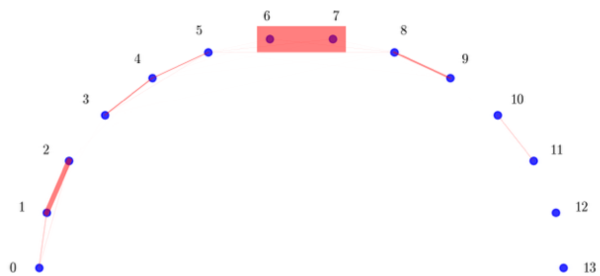
Resonances at the transition

Quantum mutual information [5]

Ergodic



MBI



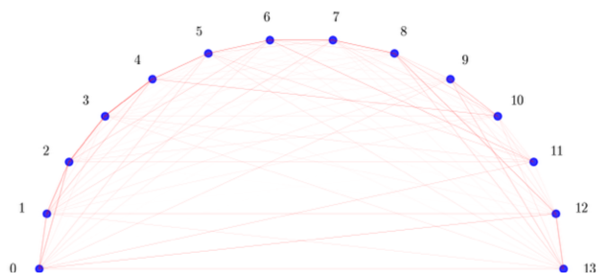
[5] G. De Tomasi, S. Bera, J. H. Bardarson, and F. Pollmann (2017). Phys. Rev. Lett. 118, 016804.

Benjamin Villalonga, UIUC

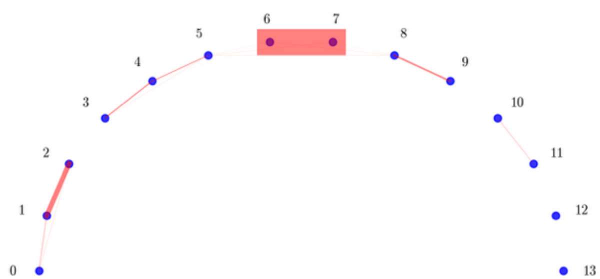
Resonances at the transition

Quantum mutual information [5]

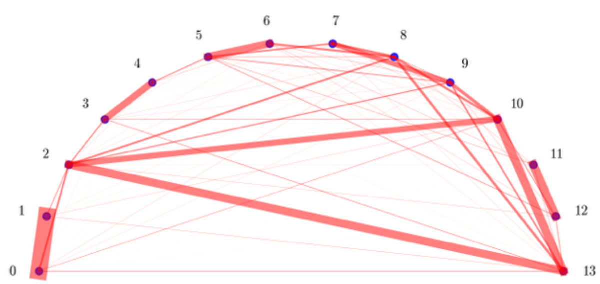
Ergodic



MBI



Transition



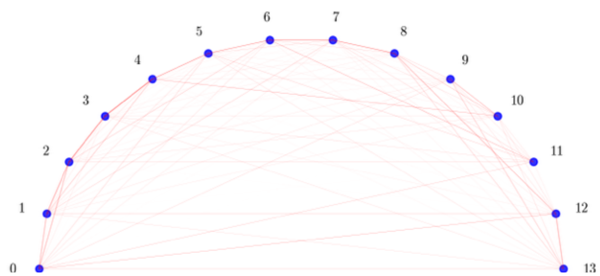
[5] G. De Tomasi, S. Bera, J. H. Bardarson, and F. Pollmann (2017). Phys. Rev. Lett. 118, 016804.

Benjamin Villalonga, UIUC

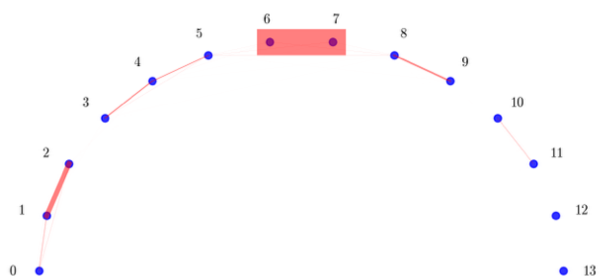
Resonances at the transition

Quantum mutual information [5]

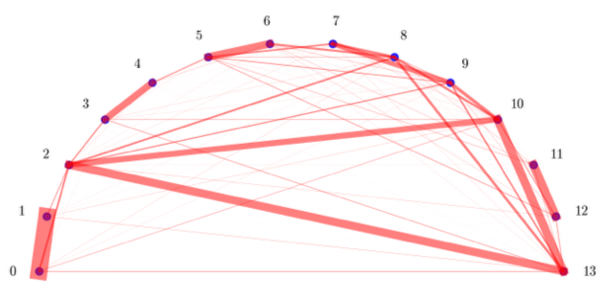
Ergodic

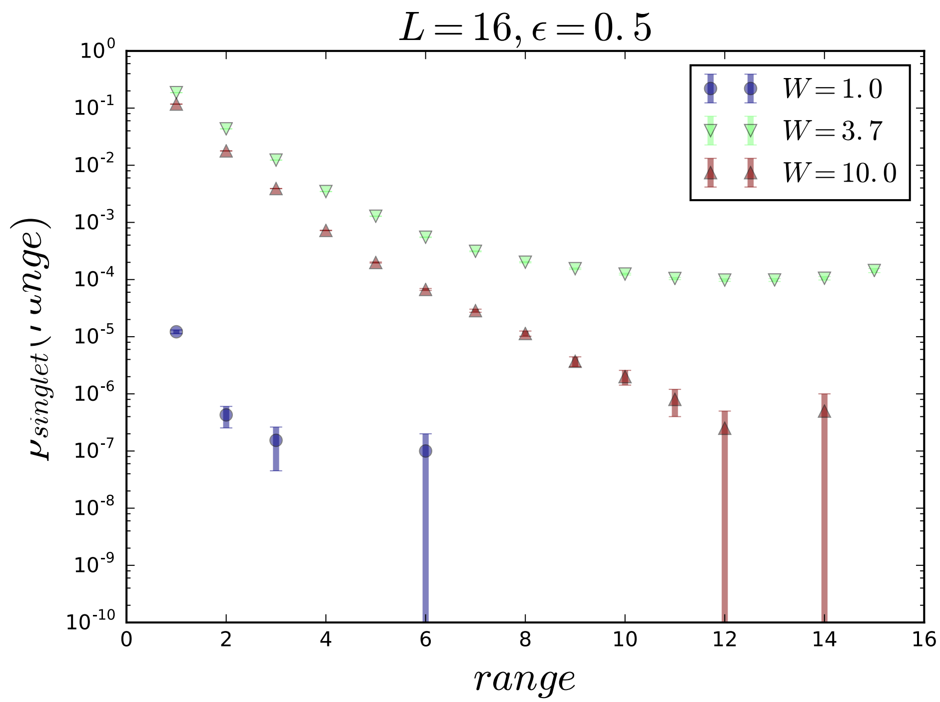


MBI



Transition





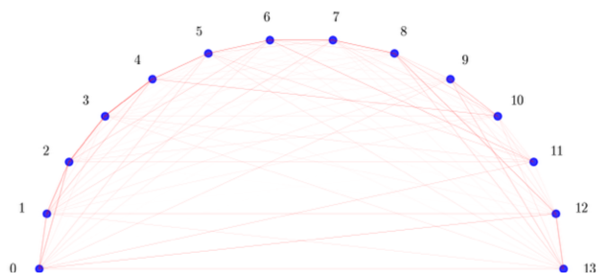
[5] G. De Tomasi, S. Bera, J. H. Bardarson, and F. Pollmann (2017). Phys. Rev. Lett. 118, 016804.

Benjamin Villalonga, UIUC

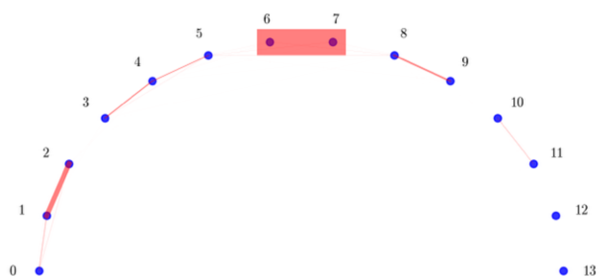
Resonances at the transition

Quantum mutual information [5]

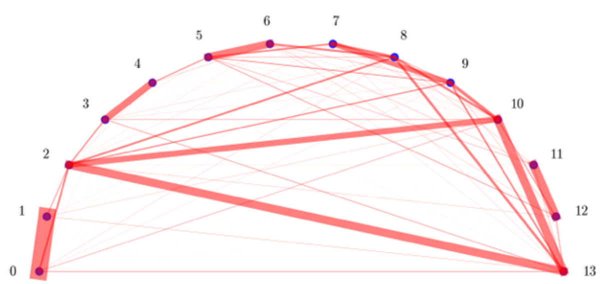
Ergodic

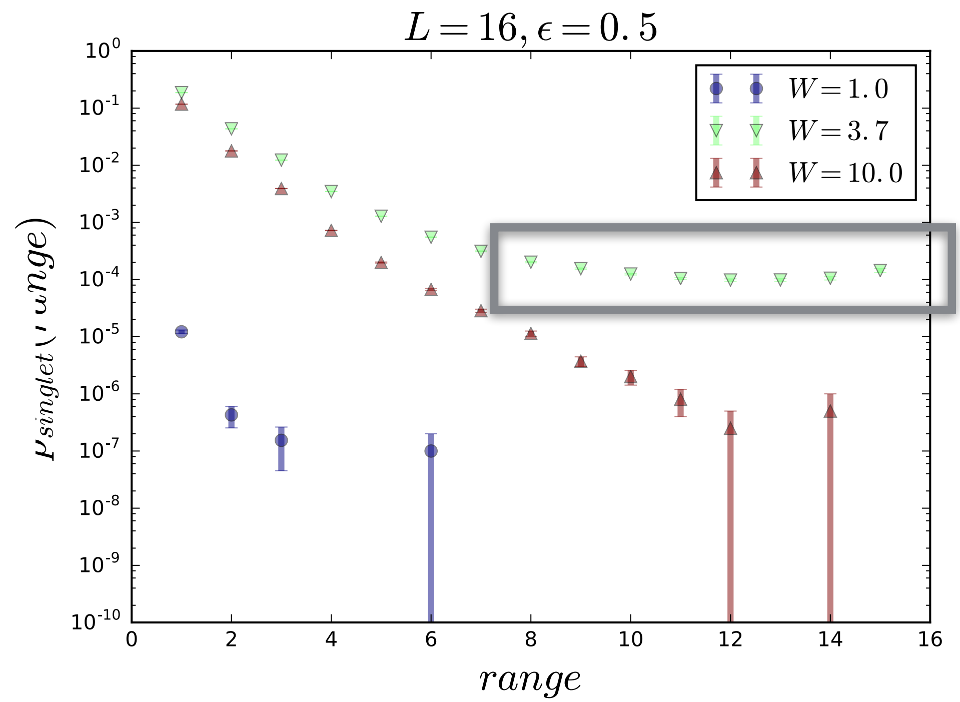


MBI



Transition





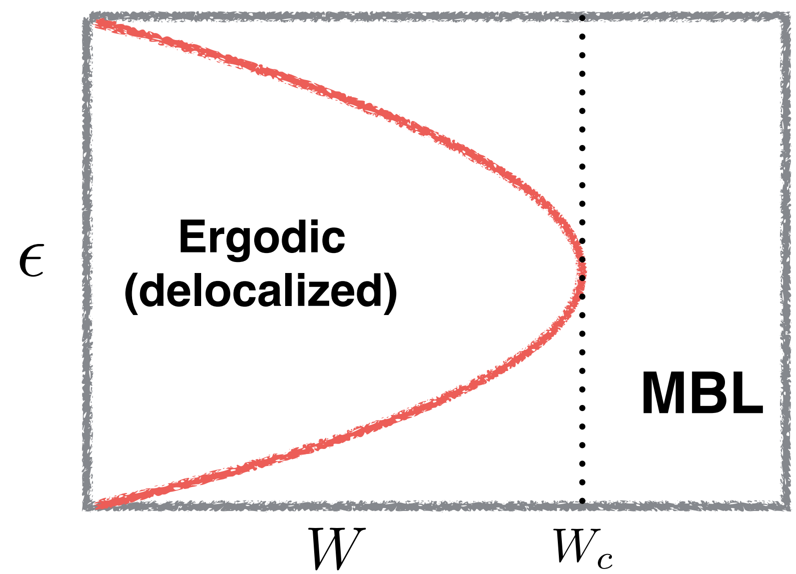
The probability of finding long range resonances is *scale invariant* at the transition

[5] G. De Tomasi, S. Bera, J. H. Bardarson, and F. Pollmann (2017). Phys. Rev. Lett. 118, 016804.

Benjamin Villalonga, UIUC

Conclusions

- Find a quasi-universal (across the energy spectrum) set of approximate integrals of motion (OPOs).
- Occupations of the OPOs are correlated to the energy of the eigenstates.
- MBL eigenstates under the mobility edge are aware of the transition happening at higher energies
- Universal distribution of the magnitude of the long-range coupling constants of the approximate integrals of motion at strong disorder.

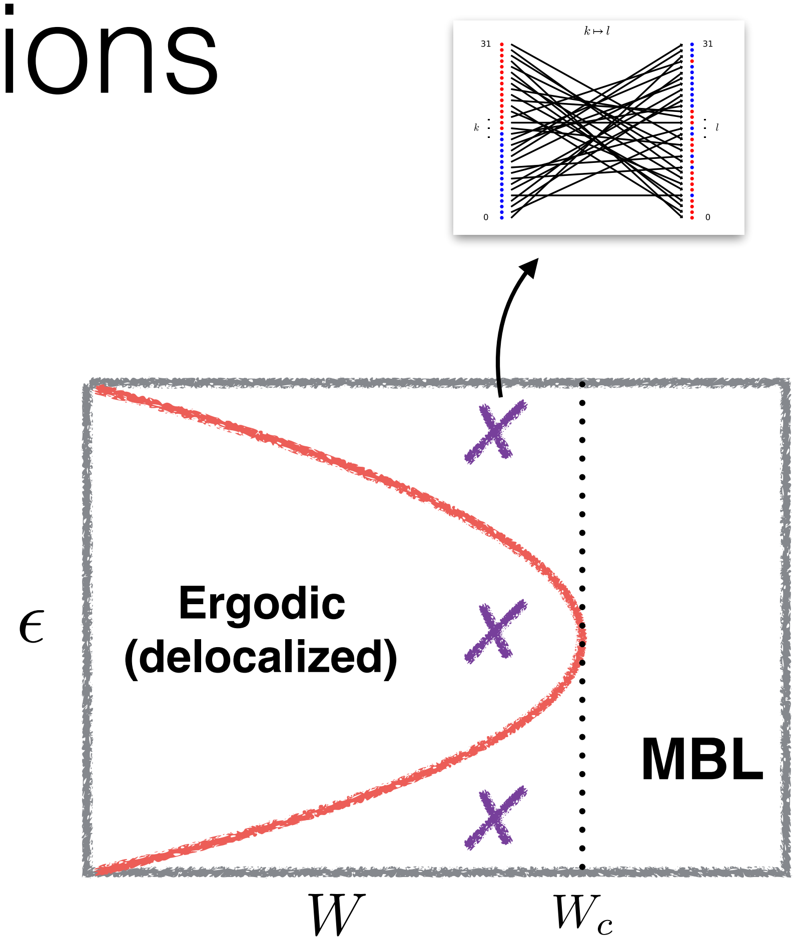


***Physical Review B, 97*, 104406 (2018)**

Benjamin Villalonga, UIUC

Conclusions

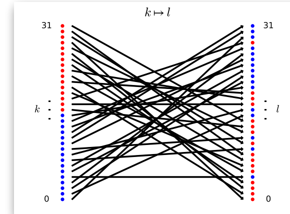
- Find a quasi-universal (across the energy spectrum) set of approximate integrals of motion (OPOs).
- Occupations of the OPOs are correlated to the energy of the eigenstates.
- MBL eigenstates under the mobility edge are aware of the transition happening at higher energies
- Universal distribution of the magnitude of the long-range coupling constants of the approximate integrals of motion at strong disorder.



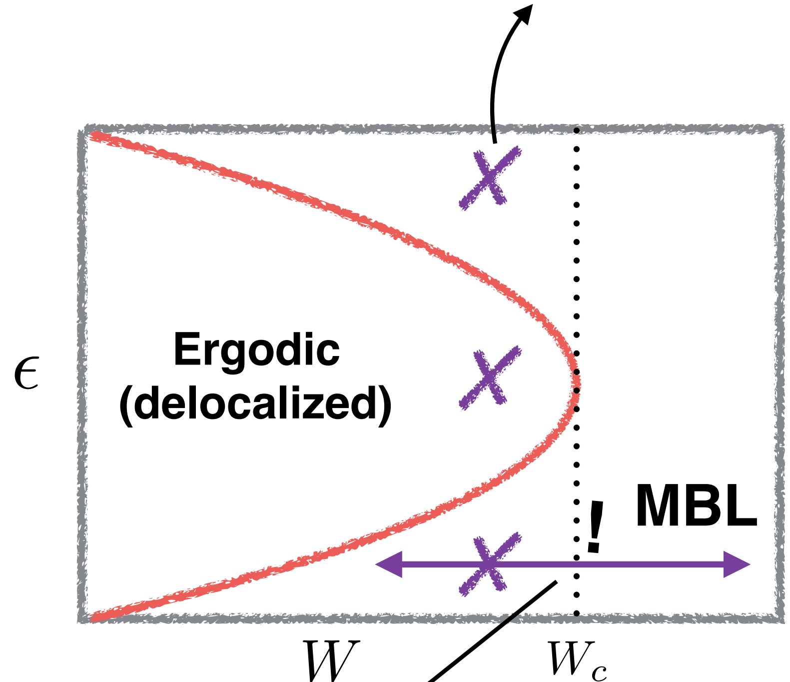
***Physical Review B, 97*, 104406 (2018)**

Benjamin Villalonga, UIUC

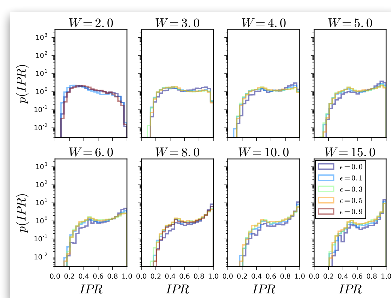
Conclusions



- Find a quasi-universal (across the energy spectrum) set of approximate integrals of motion (OPOs).
- Occupations of the OPOs are correlated to the energy of the eigenstates.
- MBL eigenstates under the mobility edge are aware of the transition happening at higher energies
- Universal distribution of the magnitude of the long-range coupling constants of the approximate integrals of motion at strong disorder.



***Physical Review B, 97*, 104406 (2018)**



Benjamin Villalonga, UIUC

Conclusions

- Find a quasi-universal (across the energy spectrum) set of approximate integrals of motion (OPOs).
- Occupations of the OPOs are correlated to the energy of the eigenstates.
- MBL eigenstates under the mobility edge are aware of the transition happening at higher energies
- Universal distribution of the magnitude of the long-range coupling constants of the approximate integrals of motion at strong disorder.

***Physical Review B, 97*, 104406 (2018)**