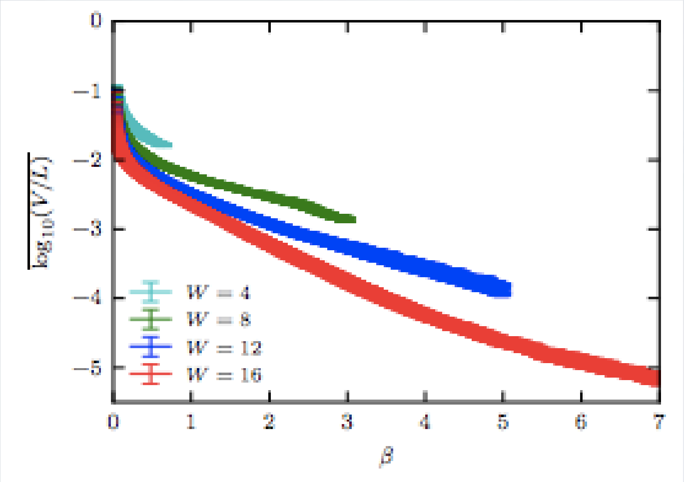
THE MOTHER OF ALL STATES ON THE KAGOME LATTICE

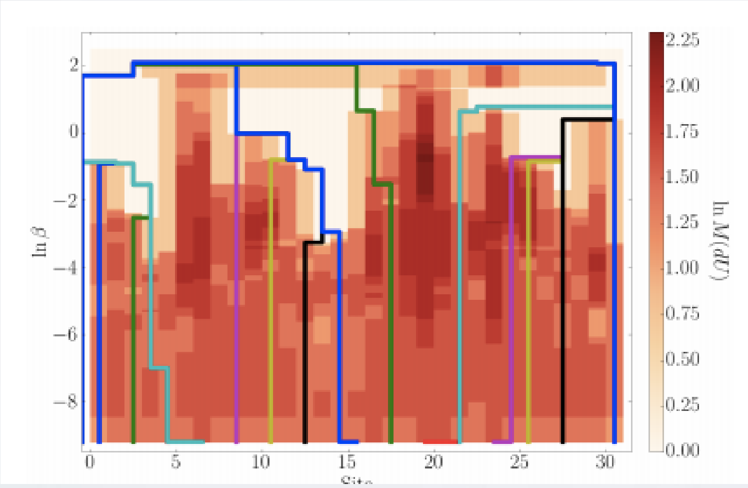
University of Illinois at Urbana Champaign
with Hitesh Changlani, Dmitrii Kochkov, Krishna Kumar, Eduardo Fradkin

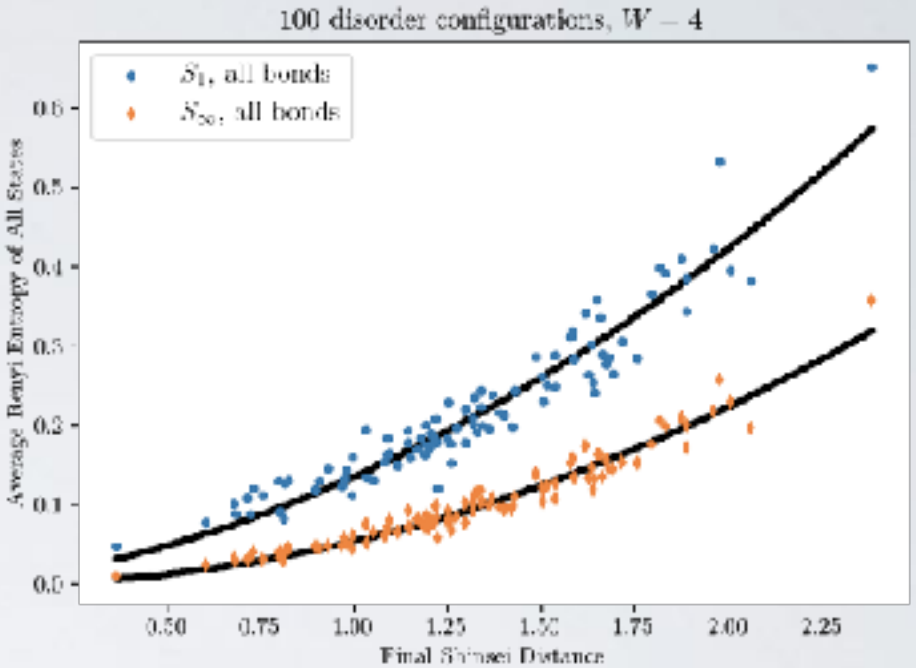

Microsoft Research - Station Q

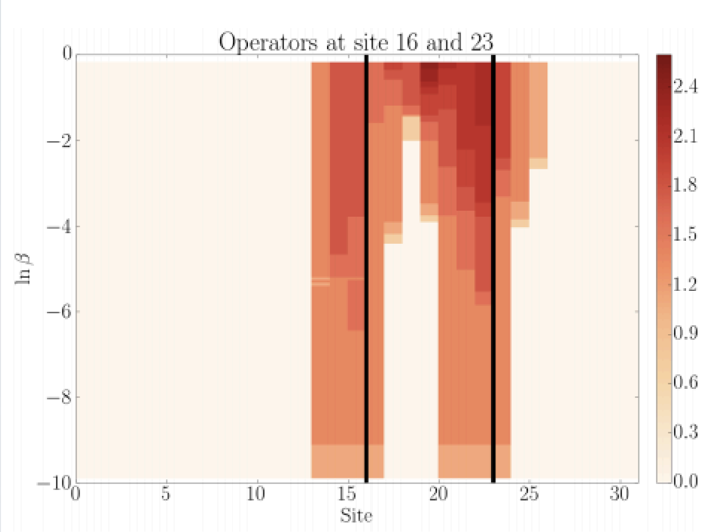
The talk I'm not giving....

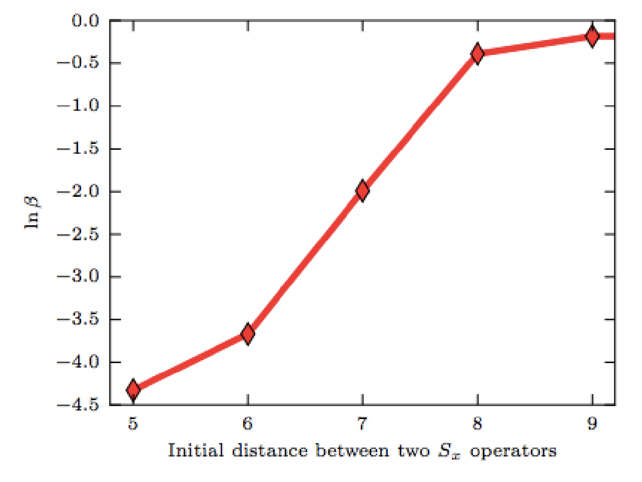
Many-body localization, quantum circuits, and holography

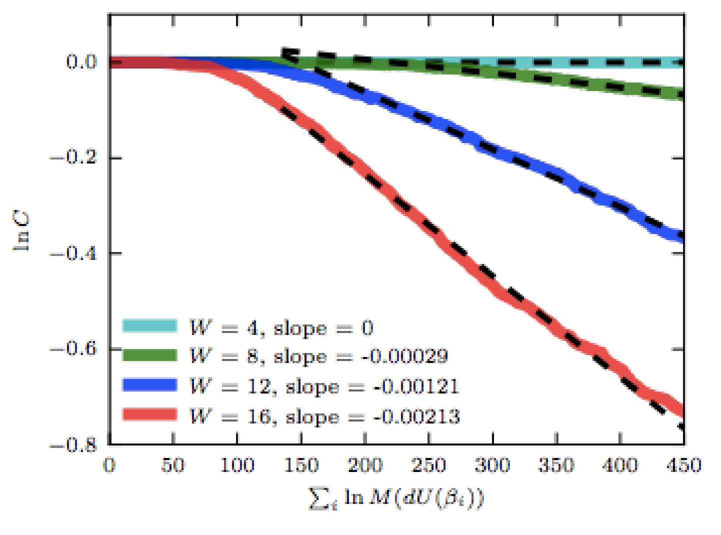


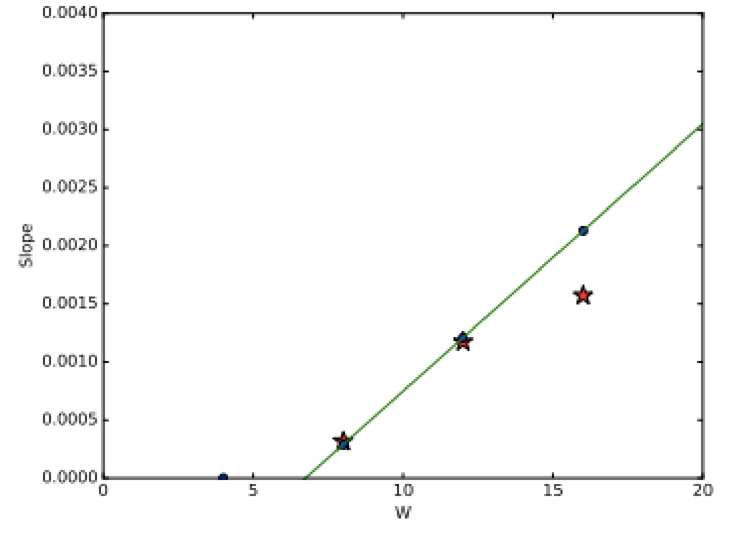


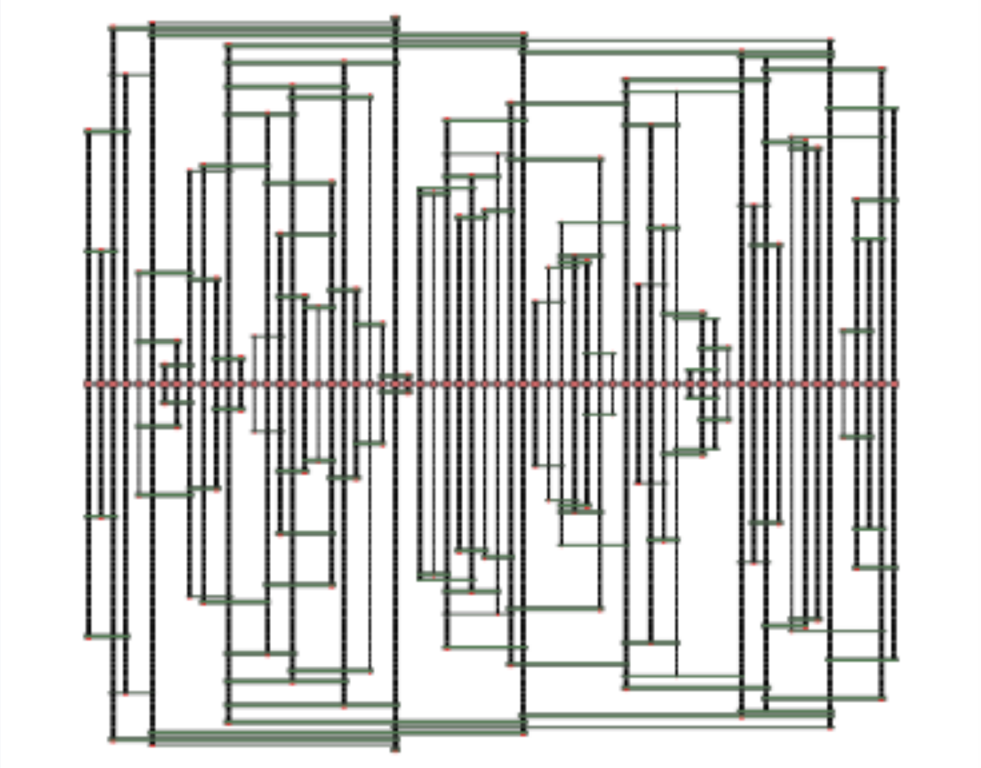




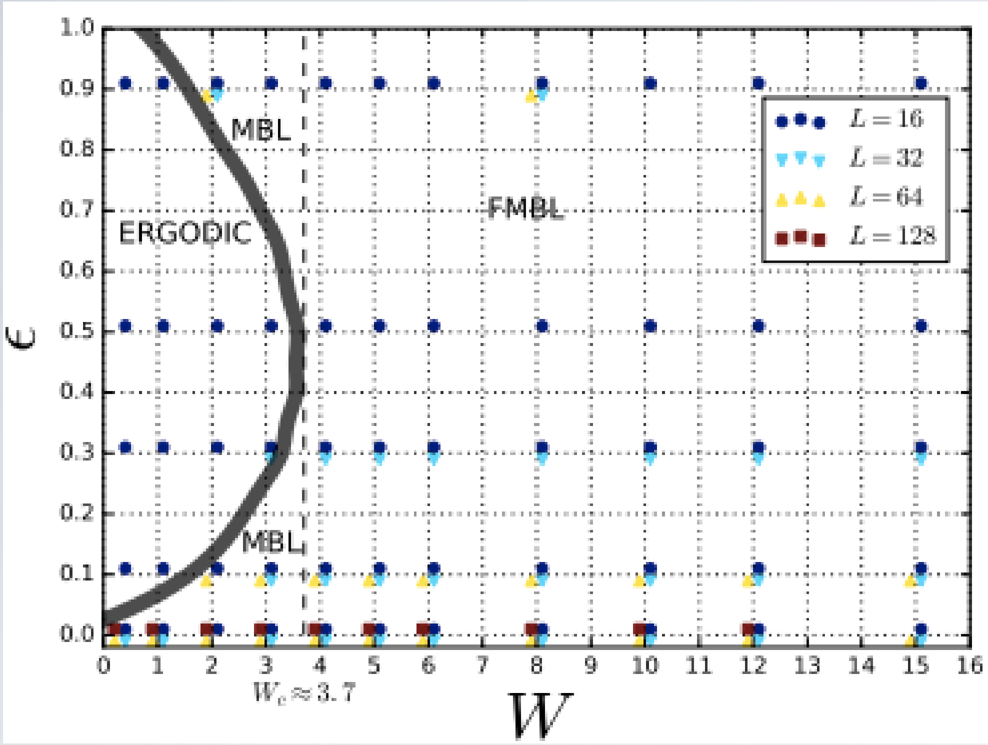


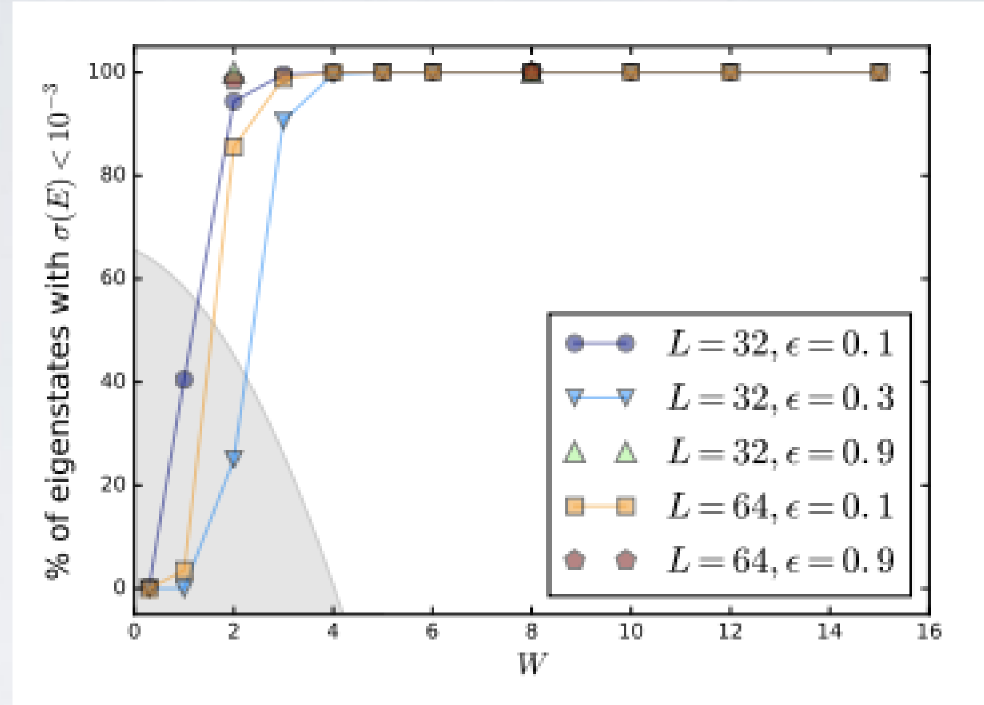


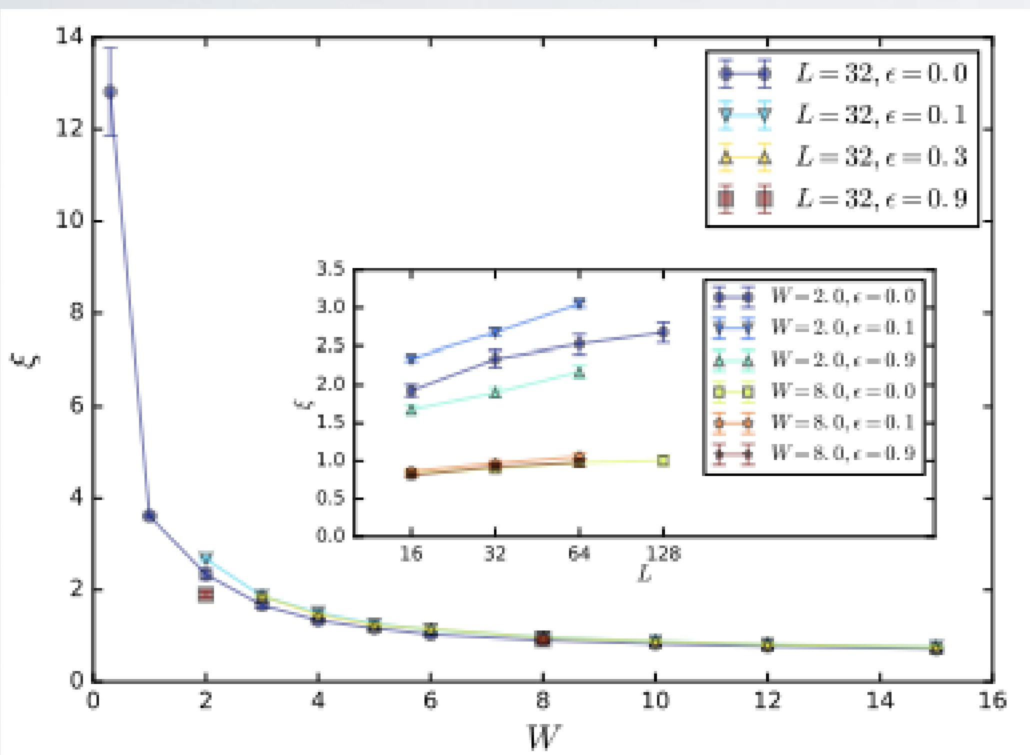


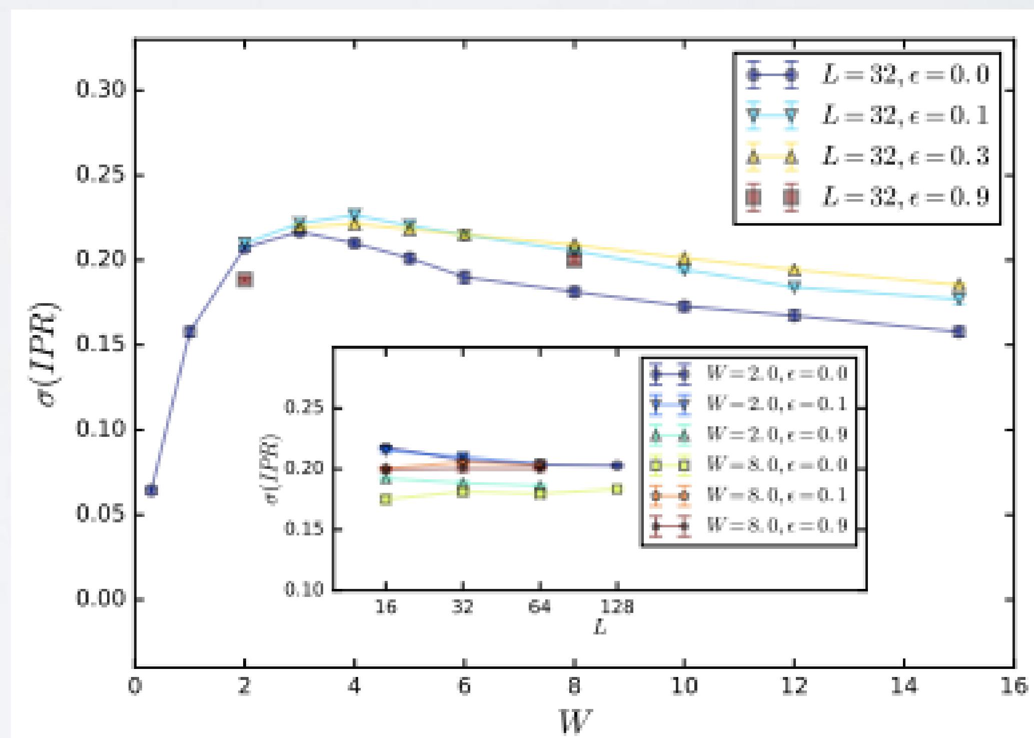


The other MBL talk I'm not giving….



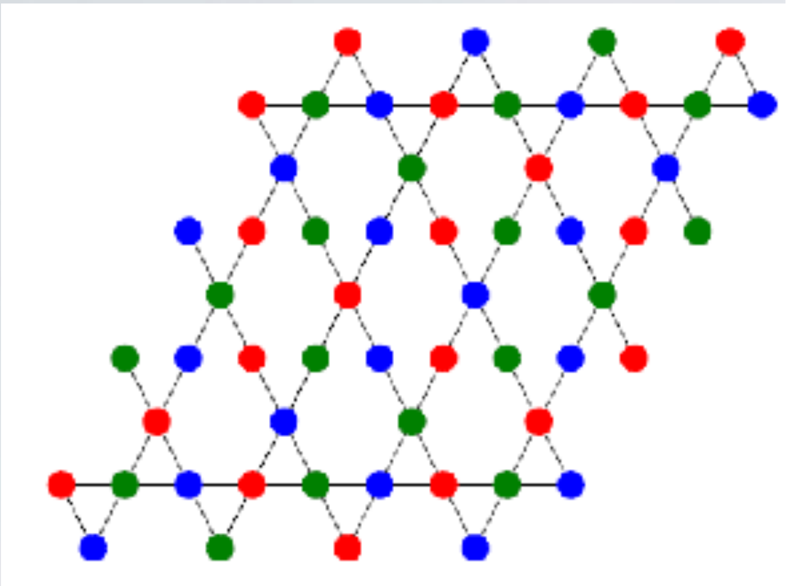




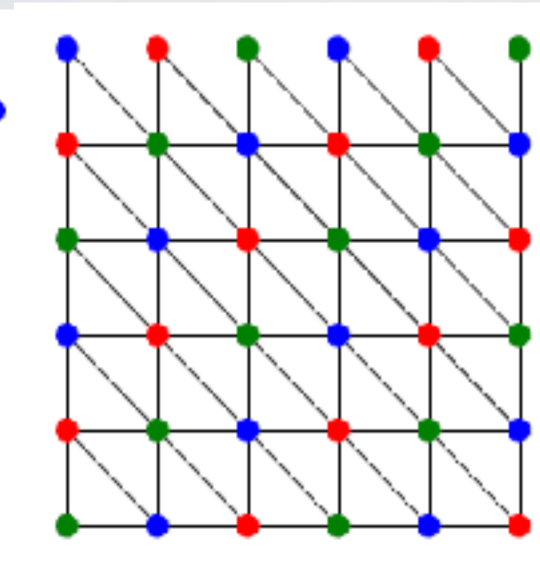


Many-body localization below the mobility edge and one-body density matrices

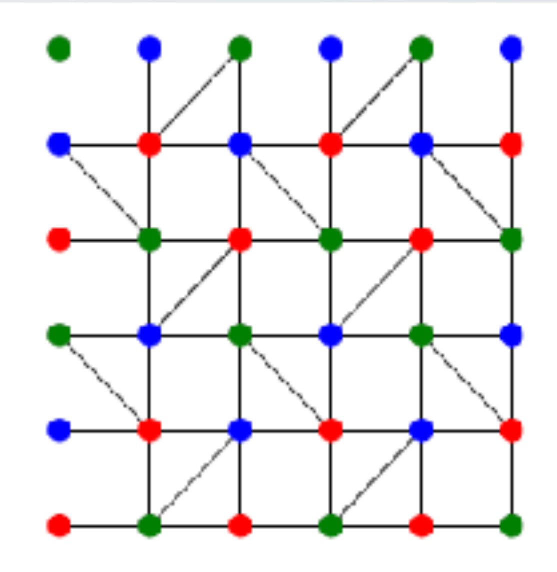
The story of frustrated magnetism is really the story of insulating materials with spin degrees of freedom which live on a non-bipartite lattice.



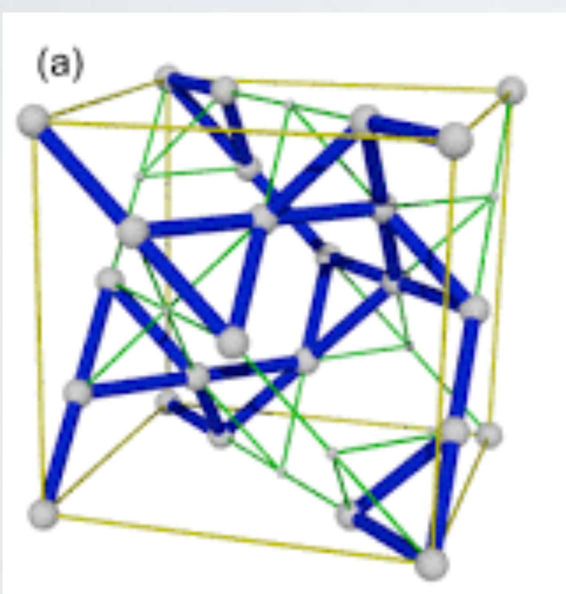
Kagome



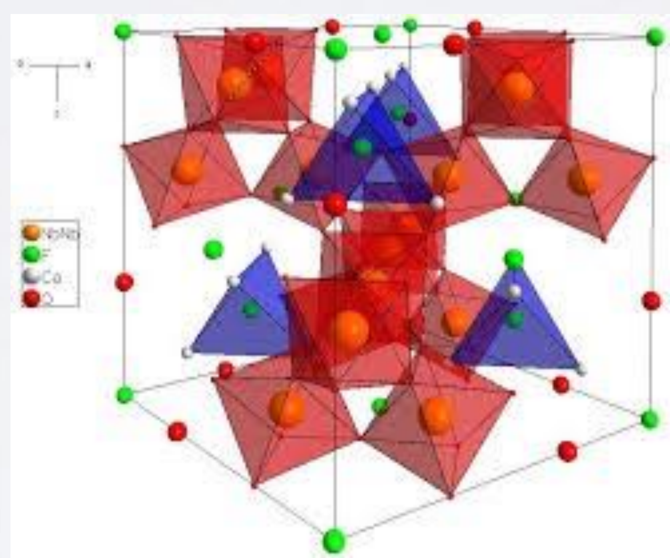
Triangular



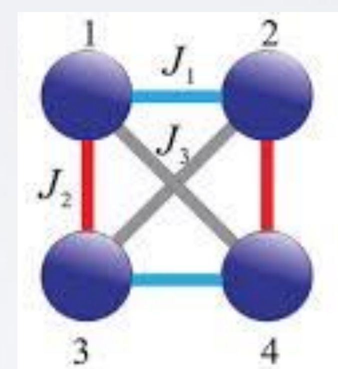
Shastry-Sutherland



Hyperkagome

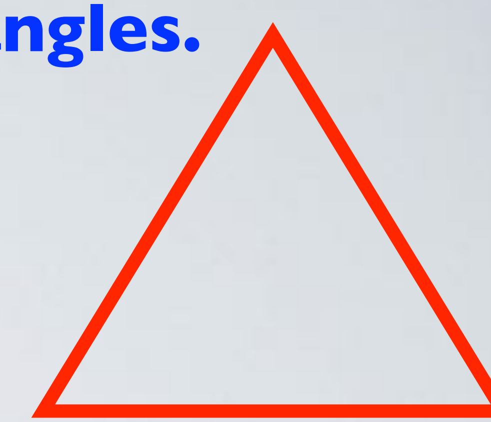


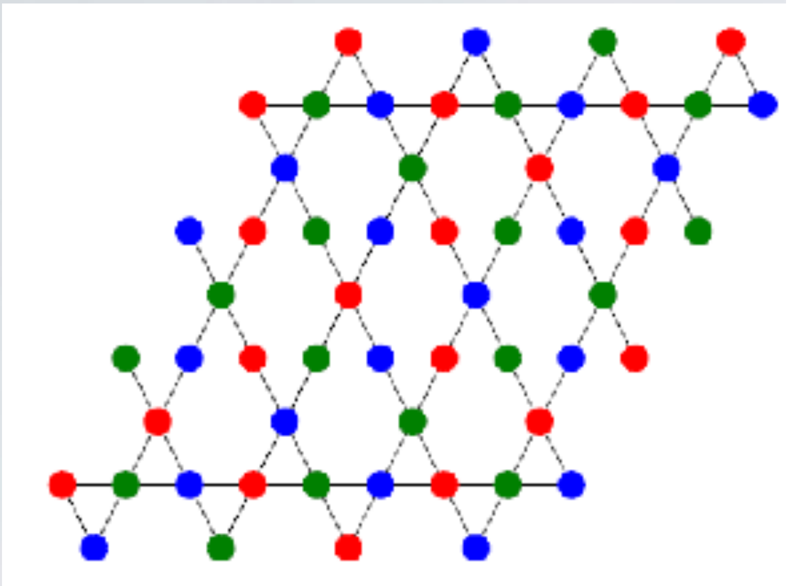
Pyrochlore



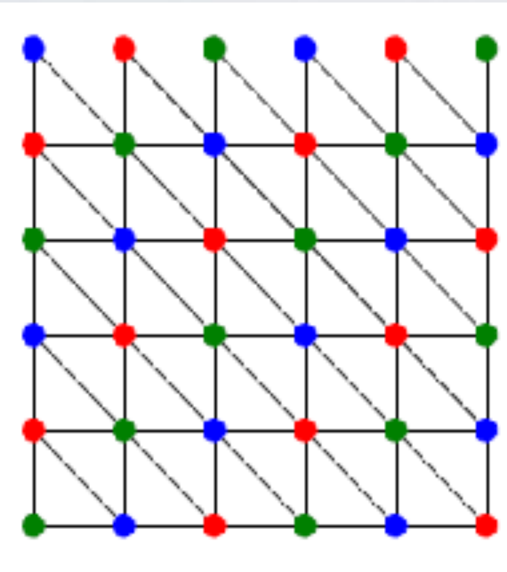
J1-J2 square

The story of frustrated magnetism is really the story of triangles.

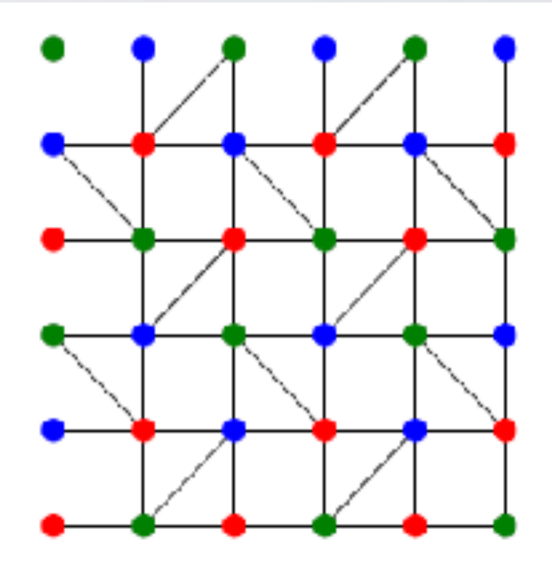




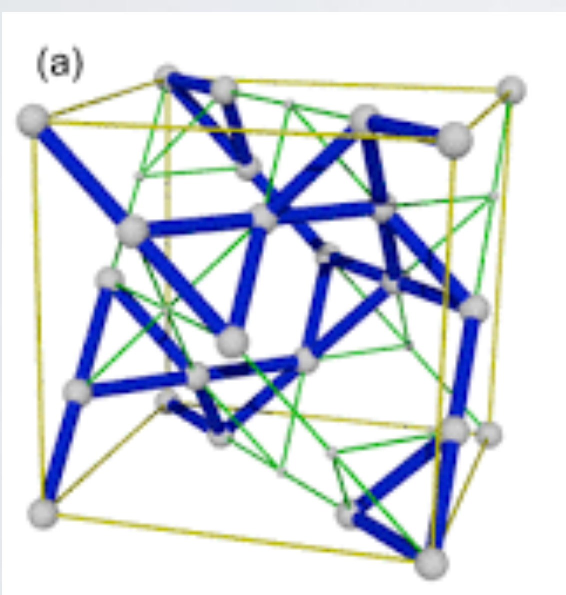
Kagome



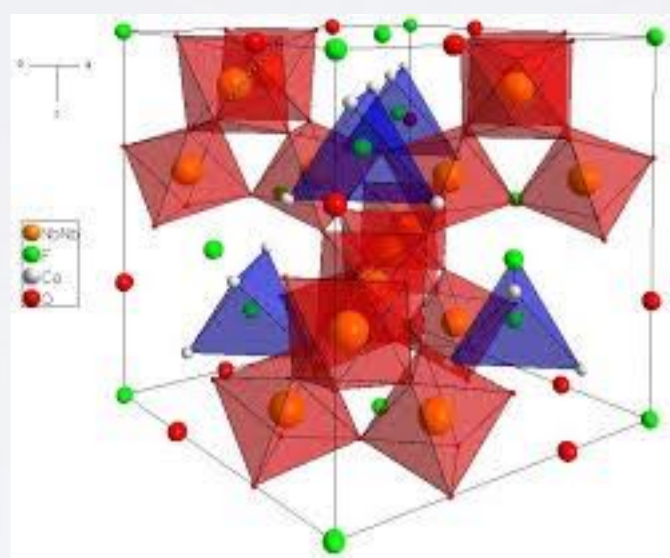
Triangular



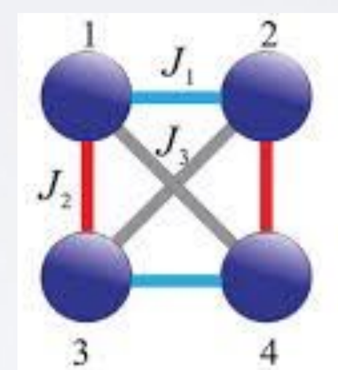
Shastry-Sutherland



Hyperkagome



Pyrochlore



J1-J2 square

The history of frustrated magnetism started in 1973

when Phil Anderson suggested that the n.n. Heisenberg model on the triangular lattice wasn't a neel state (frustration!)

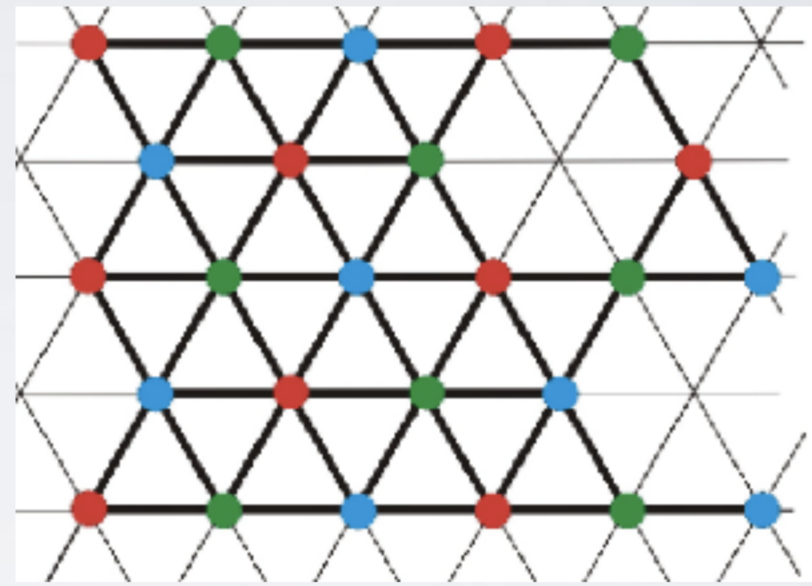
Spin 1/2 quantum Hamiltonian's

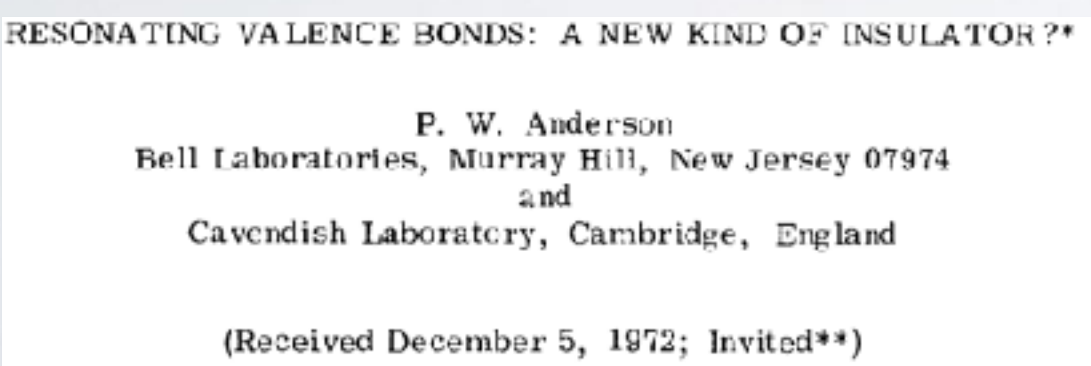
$$H_{xy} = \sum_{\langle i,j \rangle} S_i^x S_j^x + S_i^y S_j^y$$

$$H_{xxz} = H_{xy} + J_z \sum_{ij} S_i^z S_j^z$$

$$J_z = 1$$

Ground State





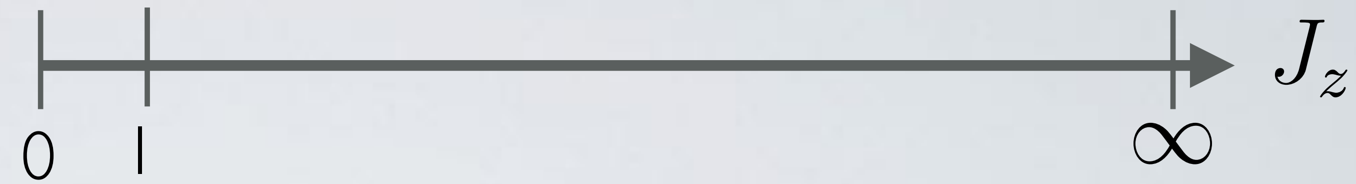
RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR?*

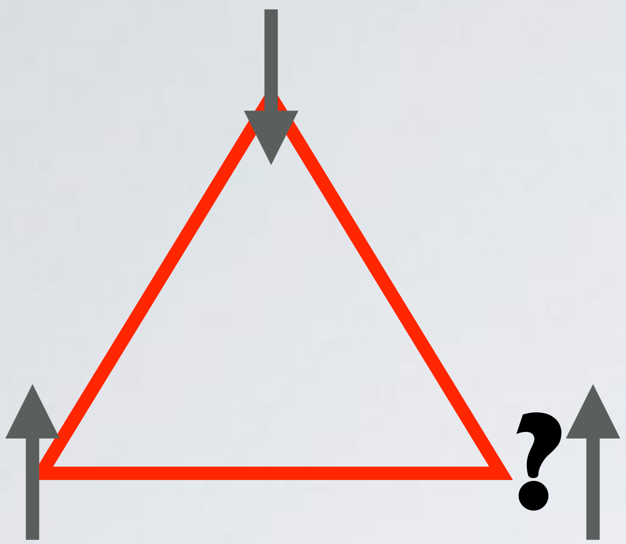
P. W. Anderson
Bell Laboratories, Murray Hill, New Jersey 07974
and
Cavendish Laboratory, Cambridge, England

(Received December 5, 1972; Invited**)

1973: Anderson predicts the Heisenberg model on the triangle lattice is a uniform RVB

frustration!

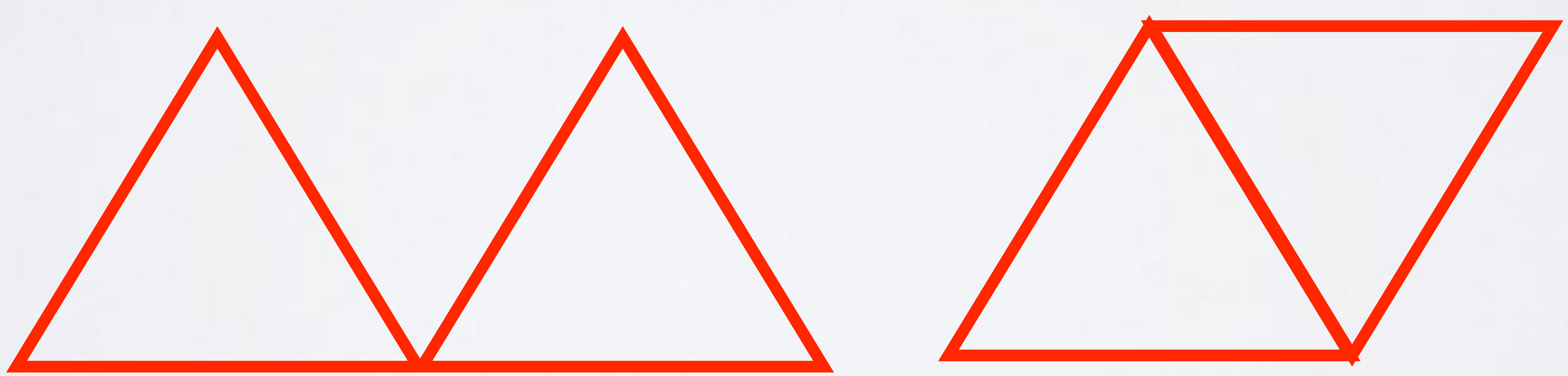




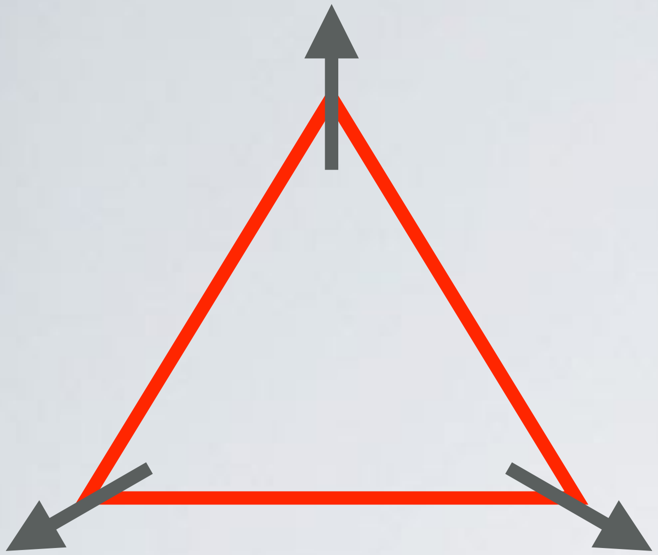
$$H_{xxz} = H_{xy} + J_z \sum_{ij} S_i^z S_j^z \quad \text{(ising limit } J_z \to \infty\text{)}$$

$$H_{\text{ising}} = \sum_{ij} S_i^z S_j^z$$

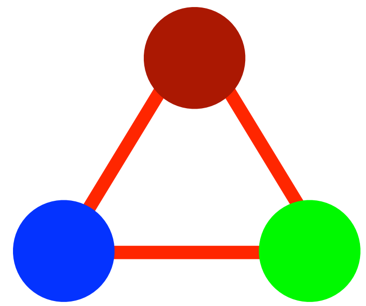
When you paste together many triangles, there are many degenerate states



But it wasn't....instead it was a 120 degree ordere



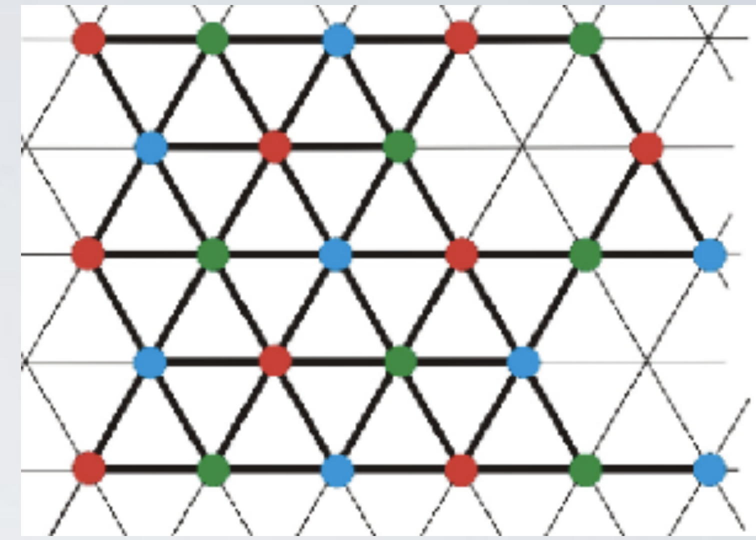
Define
3 "colors"



$$|a\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

$$|b\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + \omega|\downarrow\rangle\right)$$

$$|c\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + \omega^2|\downarrow\rangle\right)$$

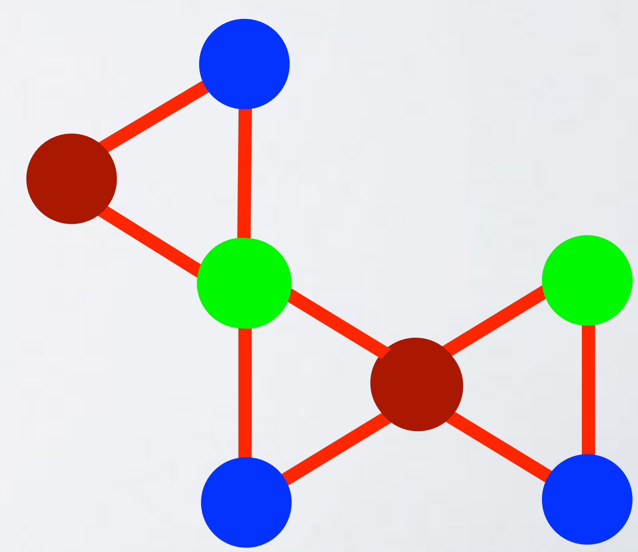


"Morally" this state but not exactly this state.

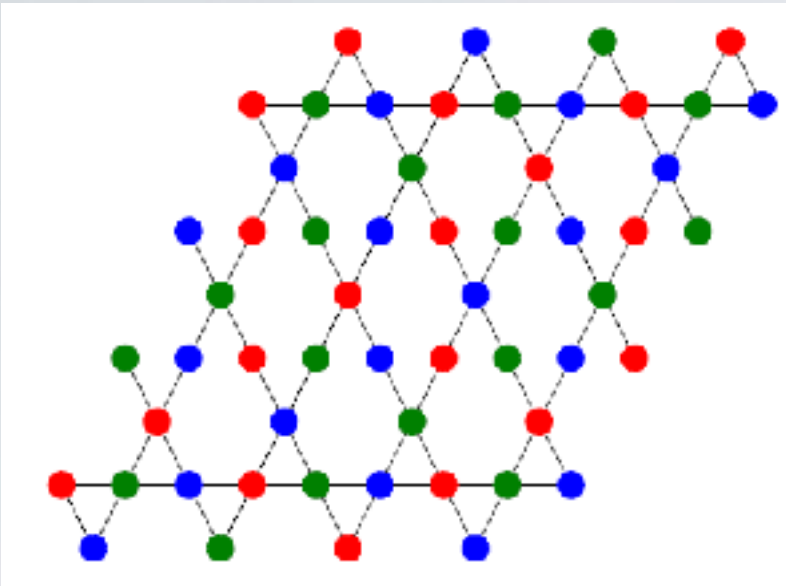
$$(|0\rangle + |1\rangle) \otimes (|0\rangle + \omega|1\rangle) \otimes (|0\rangle + \omega^2|1\rangle)$$

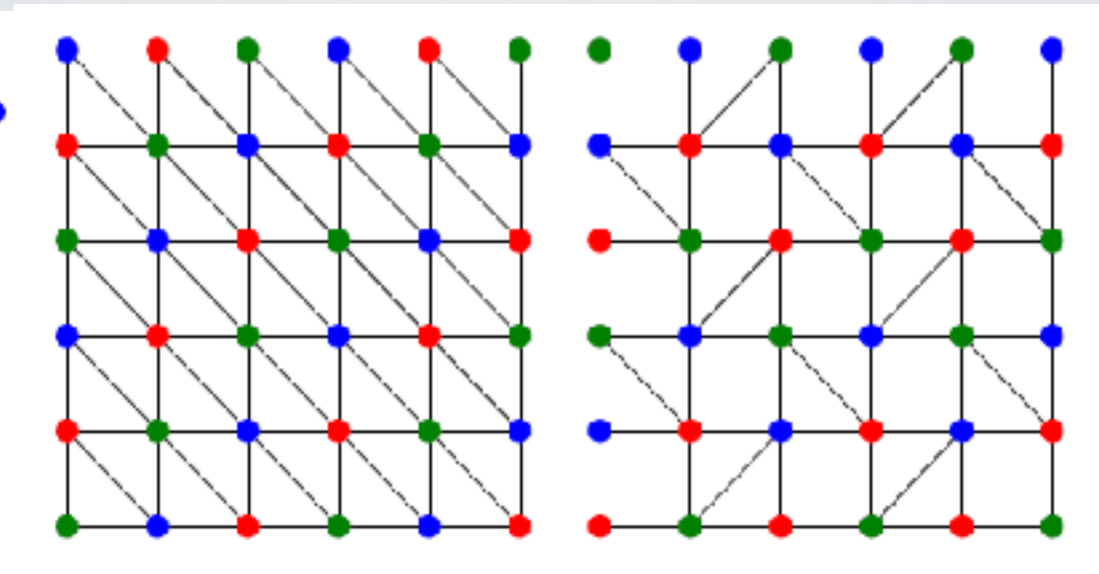
By projection ~~$|000\rangle + |111\rangle$~~ $+ |100\rangle + \omega|010\rangle + \omega^2|001\rangle + \ldots$

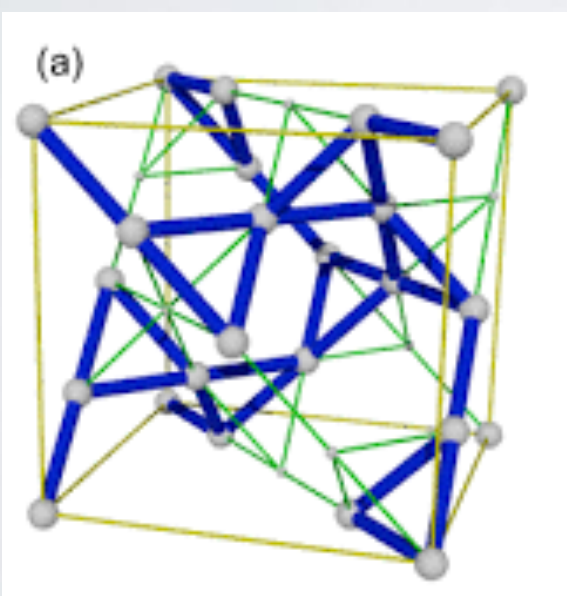
This is a high-energy eigenstate but projection removed it for us



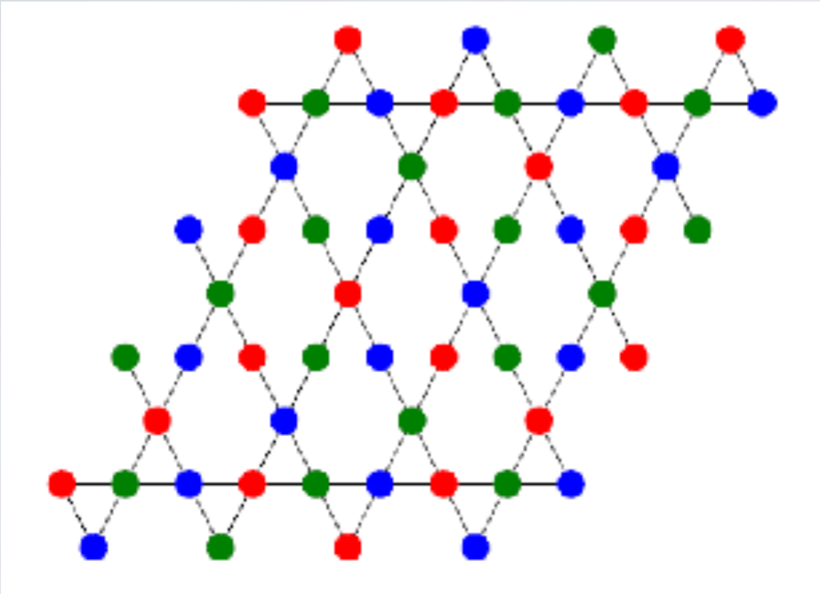
But there are other lattices of pasted-together triangles (shastry-sutherland, kagome, hyperkagome) (also all frustrated!)



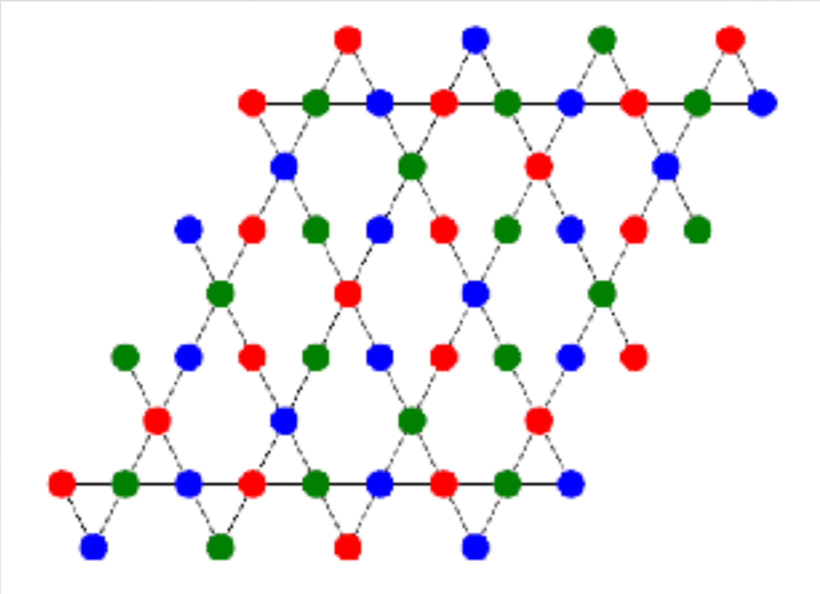


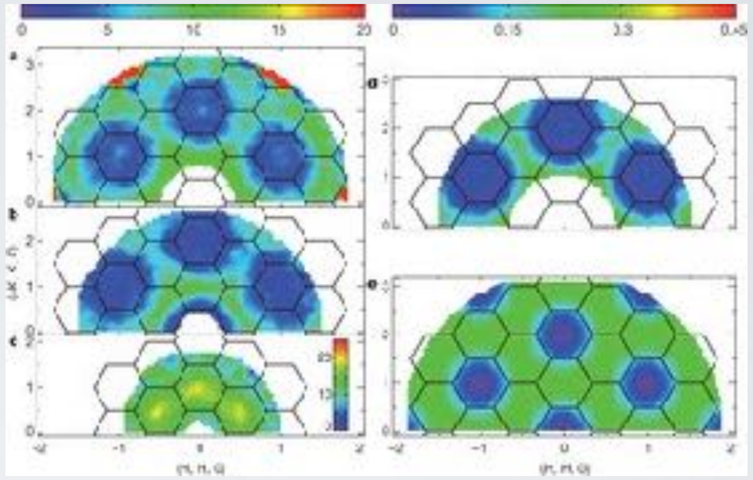


Among these, kagome stands out both experimentally and theoretically



Among these, kagome stands out both experimentally and theoretically

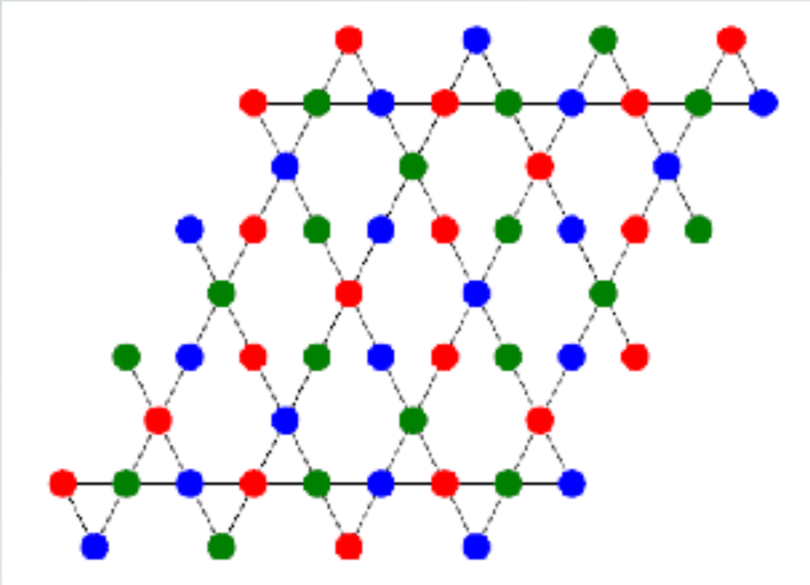


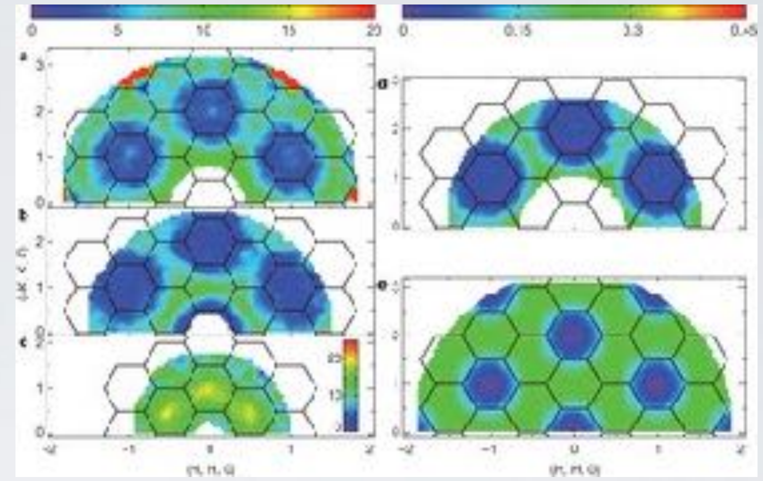


Nature 492, 406–41

Herbertsmithite

Among these, kagome stands out both experimentally and theoretically





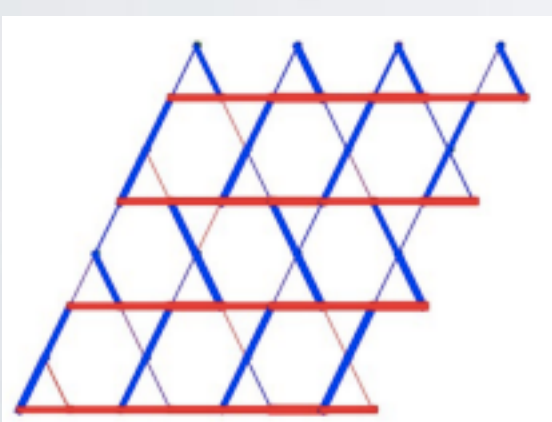
Nature 492, 406–41

Herbertsmithite

Volborthite

Kapellasite

Vesigniette



Phys. Rev. Lett. 111, 187205

Among these, kagome stands out both experimentally and theoretically

Z2 spin liquid: Heisenberg (White/Huse)

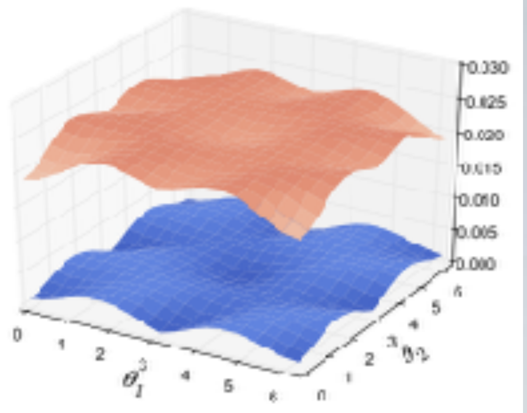
Chiral spin liquid: 1/3 plateau (this work)

1/3 plateau + J2-J3 (Donna Sheng)

Sz=0 chiral (Bela Bauer, Andreas Ludwig)

Sz=0 J1,J2,J3 (Donna Sheng)

Kagame spin liquids everywhere....



Heisenberg $S_z = 0$ (KAHF)	[Gapped or gapless spin-liquid]
XY, $S_z = 2/3$	[Chiral Spin Liquid]
Uniform Chirality	[Chiral Spin Liquid]
Non-uniform Chirality	[Gapless Spin Liquid]
J1-J2-J3	[Chiral Spin Liquid]

+ many kagome ordered states.

$q = 0$ state

$\sqrt{3} \times \sqrt{3}$ state

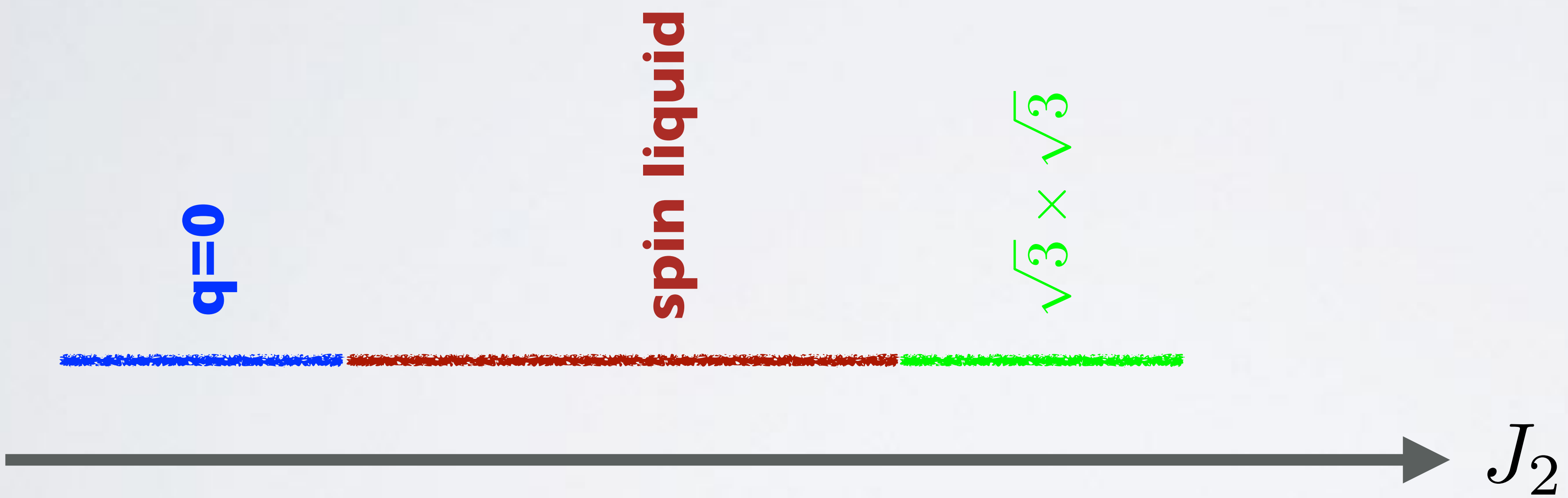
ferromagnetic state

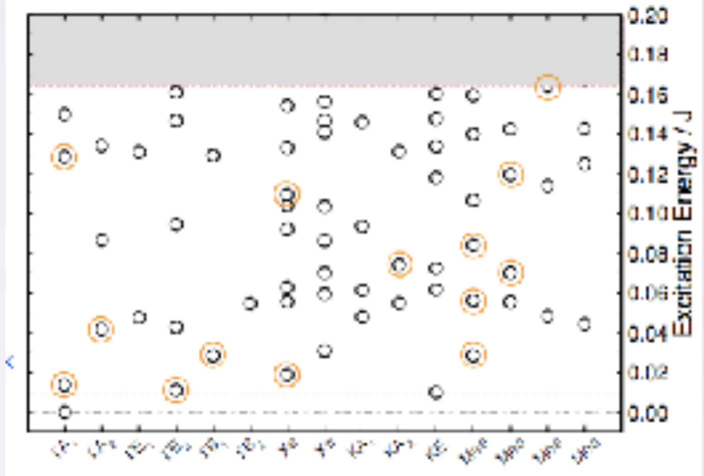
The ising frustration doesn't seem to be a good explanation for the panalopy of spin-liquids.

(1) Why kagome and not triangular?

Both are equally frustrated in the ising limit.

(2) Ising seems to have little to do with competing phases around the spin liquid.





(3) Mainly classical degeneracy….maybe quantum fluctuations resolve into spin-liquid but why?

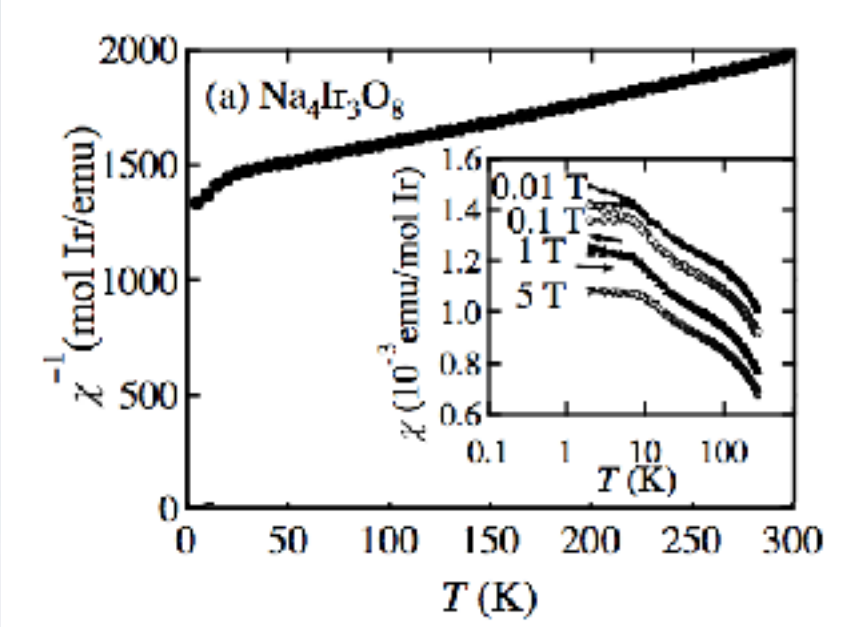
In addition there is some experimental evidence for hyperkagome $Na_4Ir_3O_8$

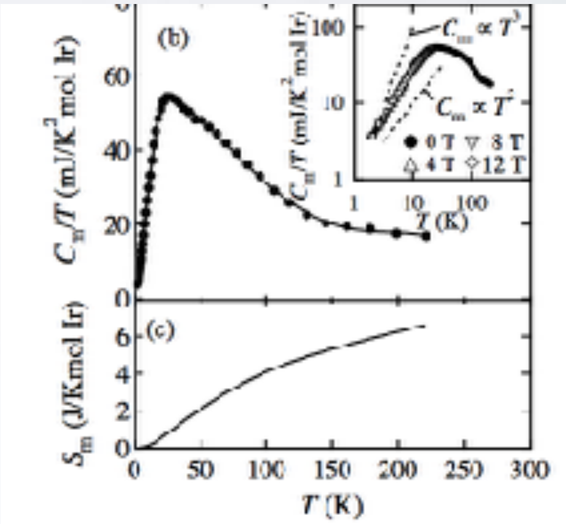
(depleted pyrochlore)

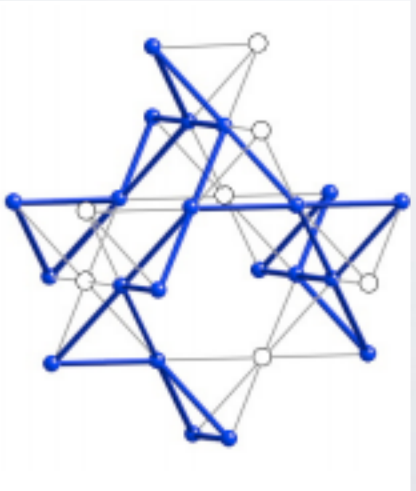
No sign of magnetic ordering down to a few Kelvin

Curie-Weiss temperature of 650K

Gapless excitations



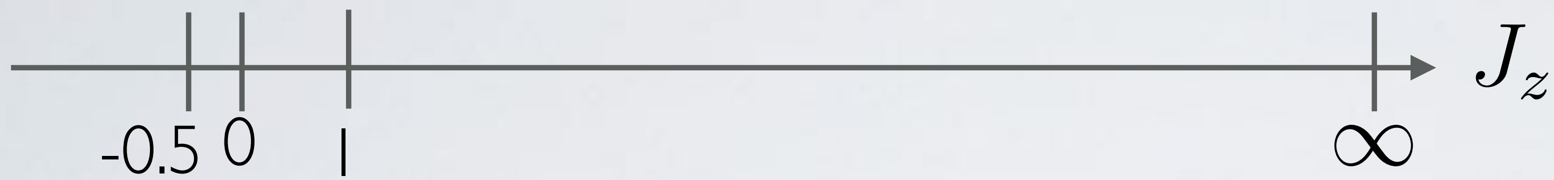




Y. Okamoto, M. Nohara, H. Aruga-Katori, and H. Takagi (2006), arXiv:0705.2821

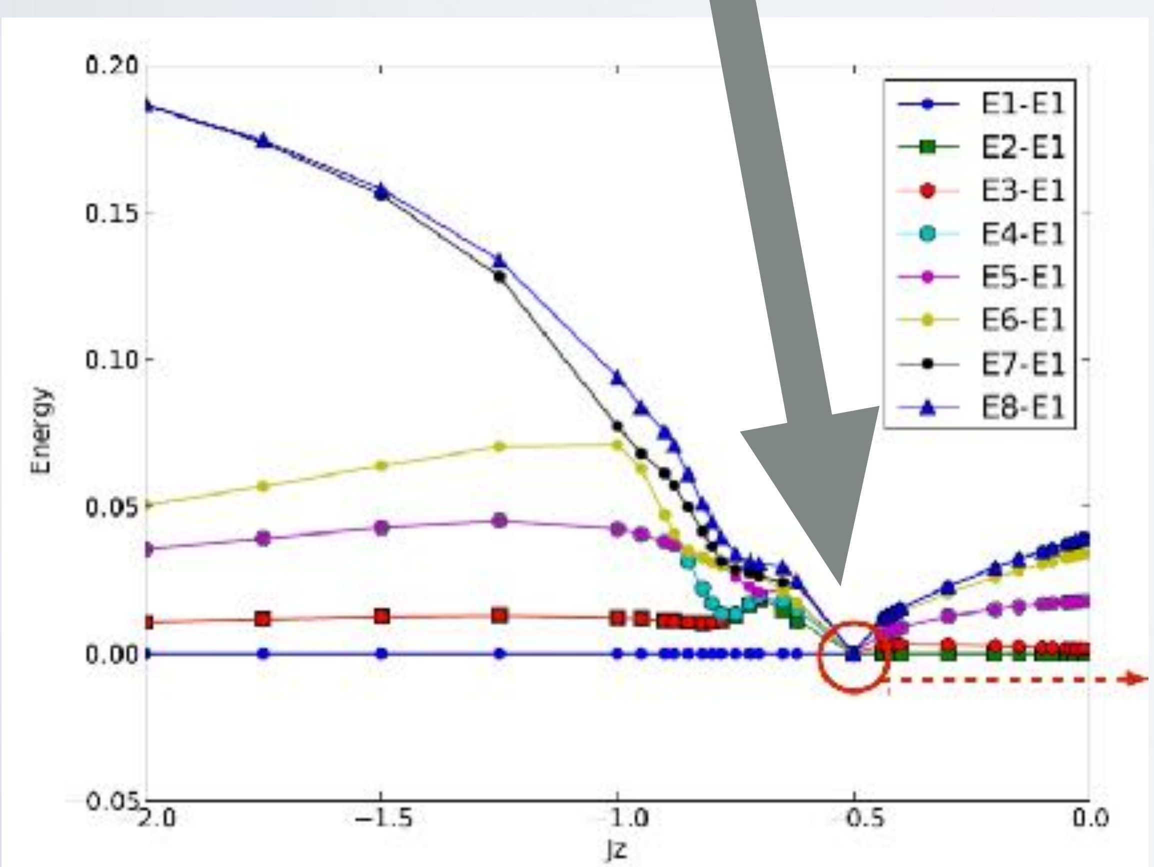
A new answer (amazing it hasn't been known for 30 years)

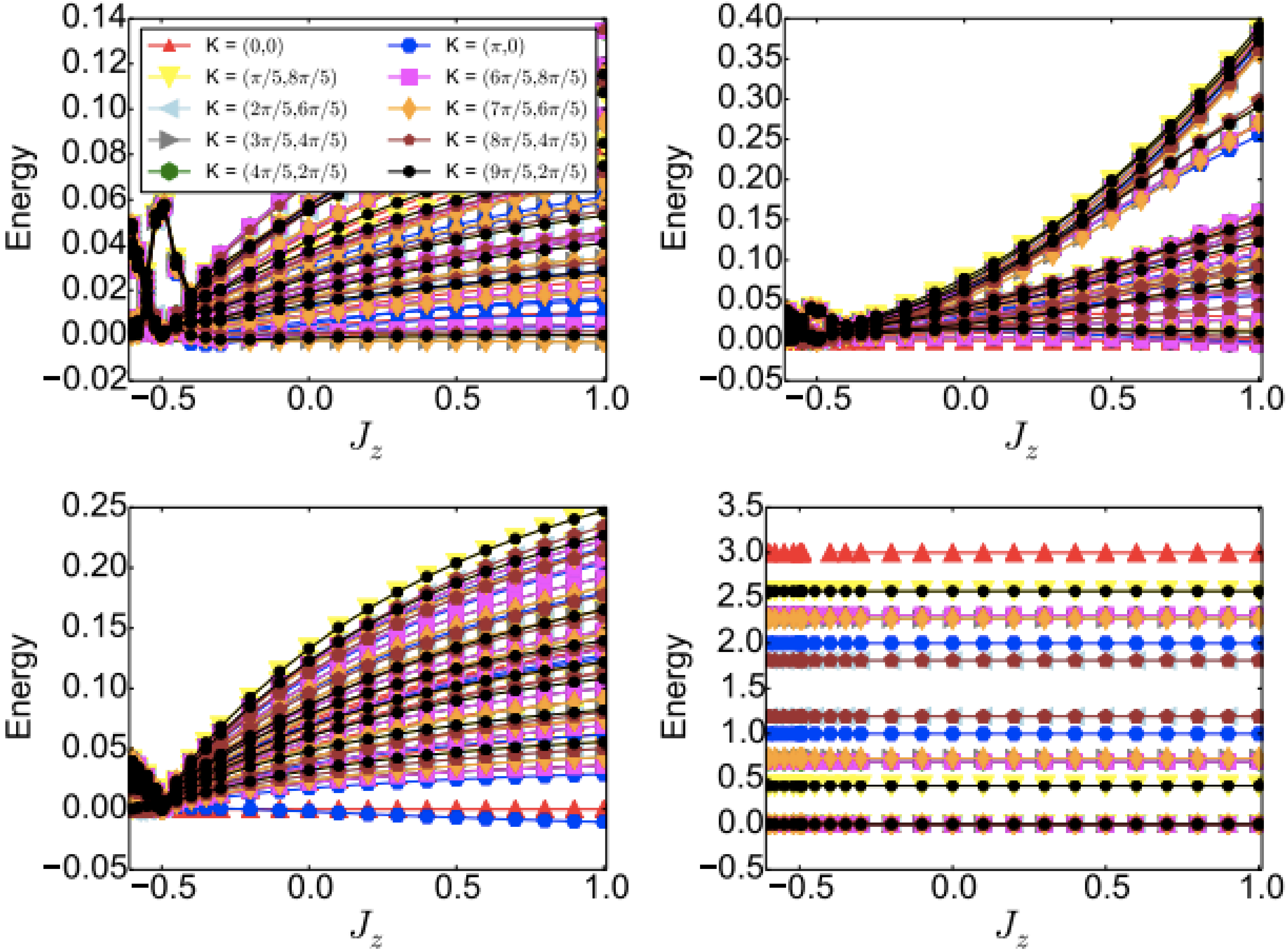
$$H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$



massive exact degeneracy in the XXZ model!

exactly -J/4

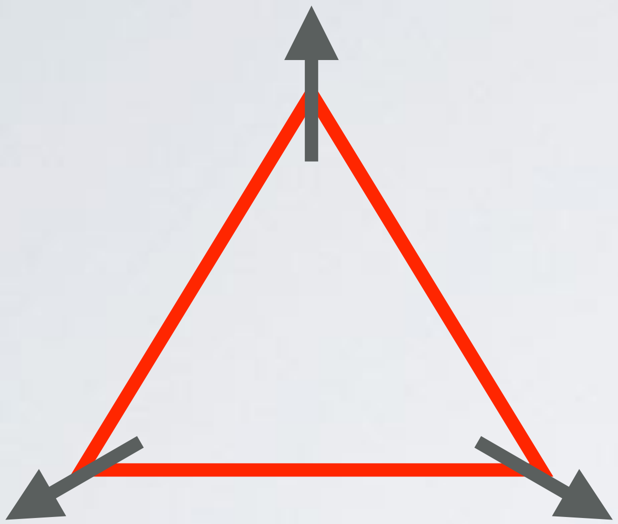




K = (0,0)
K = (π,0)
K = (π/5,8π/5)
K = (6π/5,8π/5)
K = (2π/5,6π/5)
K = (7π/5,6π/5)
K = (3π/5,4π/5)
K = (8π/5,4π/5)
K = (4π/5,2π/5)
K = (9π/5,2π/5)
Energy
J_z

Who ordered that?

For a single triangle at the XY point, we can ***relieve frustration***.

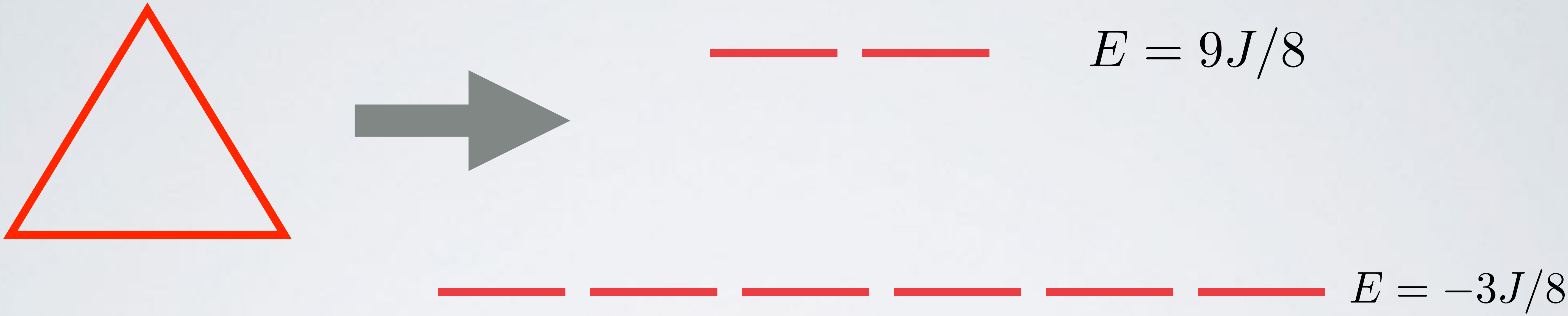


$$H = J_{xy} \sum_{ij} S_i^x S_j^x + S_i^y S_j^y$$

This is the exact ground state for (Sz=1/2) and everyone is happy

Who ordered that?

$$H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$



Who ordered that?

$$H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$

$$E = 9J/8$$

$$E = -3J/8$$

$$|1\rangle \equiv |\uparrow\uparrow\uparrow\rangle$$

$$|2\rangle \equiv \frac{1}{\sqrt{3}}\Big(|\uparrow\uparrow\downarrow\rangle + \omega|\uparrow\downarrow\uparrow\rangle + \omega^2|\downarrow\uparrow\uparrow\rangle\Big)$$

$$|3\rangle \equiv \frac{1}{\sqrt{3}}\Big(|\uparrow\uparrow\downarrow\rangle + \omega^2|\uparrow\downarrow\uparrow\rangle + \omega|\downarrow\uparrow\uparrow\rangle\Big)$$

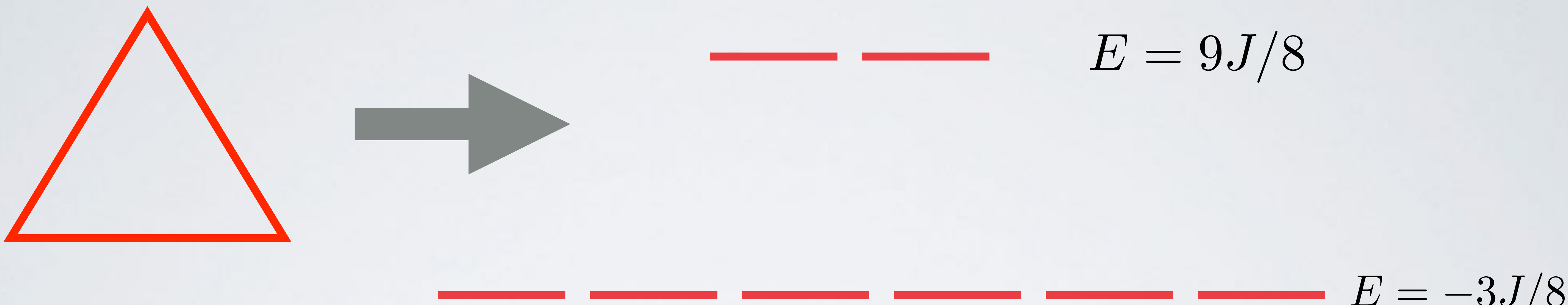
$$|4\rangle \equiv \frac{1}{\sqrt{3}}\Big(|\downarrow\downarrow\uparrow\rangle + \omega|\downarrow\uparrow\downarrow\rangle + \omega^2|\uparrow\downarrow\downarrow\rangle\Big)$$

$$|5\rangle \equiv \frac{1}{\sqrt{3}}\Big(|\downarrow\downarrow\uparrow\rangle + \omega^2|\downarrow\uparrow\downarrow\rangle + \omega|\uparrow\downarrow\downarrow\rangle\Big)$$

$$|6\rangle \equiv |\downarrow\downarrow\downarrow\rangle$$

Who ordered that?

$$H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$



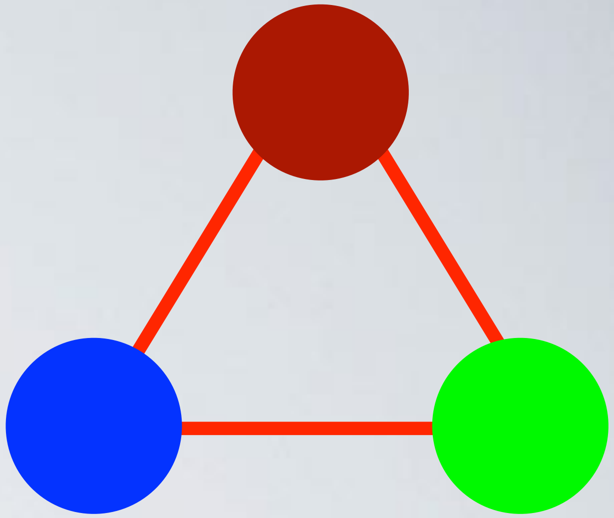
$$|+\rangle \equiv \frac{1}{\sqrt{3}}\Big(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle\Big)$$

$$|-\rangle \equiv \frac{1}{\sqrt{3}}\Big(|\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\rangle\Big)$$

$$H_{\rm tri} = -\frac{3J}{8}\sum_{i=1}^{6}|i\rangle\langle i| + \frac{9J}{8}(|+\rangle\langle +| + |-\rangle\langle -|)$$

$$-\frac{3J}{8}(1 - |+\rangle\langle +| - |-\rangle\langle -|)$$

Who ordered that?



$$|+\rangle \equiv \frac{1}{\sqrt{3}}\left(|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle\right)$$

$$|-\rangle \equiv \frac{1}{\sqrt{3}}\left(|\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\rangle\right)$$

$$-\frac{3J}{8} + \frac{3J}{2}\left(|+\rangle\langle+| + |-\rangle\langle-|\right)$$

Projectors

Constant

Positive coefficient

We want to **minimize the energy** by zeroing out the projectors

Frustration Free!

Many Triangles

$$H = \sum_{\text{tri}} H_{\text{tri}} = \frac{3}{2} \sum_{\text{tri}} P_{\text{tri}} - \frac{3}{8} N_{\text{tri}}$$

$$P_{\text{tri}} \equiv |+\rangle\langle+| + |-\rangle\langle-|$$

NOTE: projectors on triangles **DO NOT** commute with each other!!!

So an arbitrary eigenstate on one triangle need not be COMPATIBLE with other triangles

$$H = \sum_{\text{tri}} H_{\text{tri}} = \frac{3}{2}\sum_{\text{tri}} P_{\text{tri}} - \frac{3}{8}N_{\text{tri}}$$

$$P_{\text{tri}} \equiv |+\rangle\langle+| + |-\rangle\langle-|$$

$$|a\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

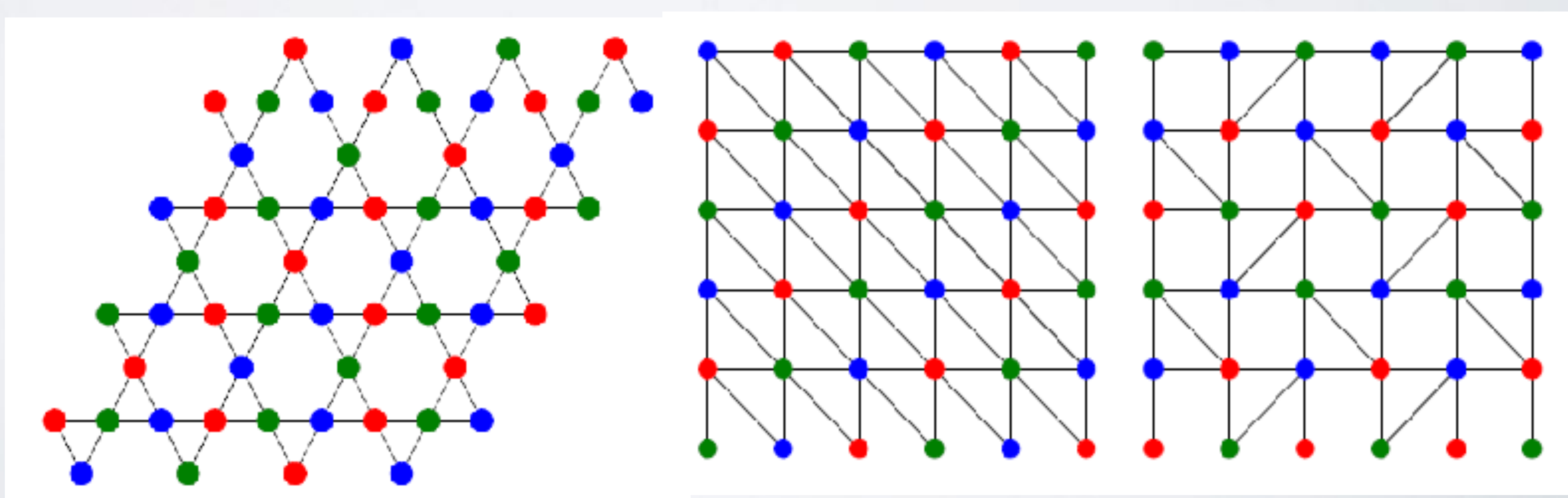
$$|b\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + \omega|\downarrow\rangle\right)$$

$$|c\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + \omega^2|\downarrow\rangle\right)$$

We want projector to annihilate our proposed solution

$$|\psi\rangle \equiv \prod_s \otimes |C_s\rangle_s$$

As long as we have ONLY one color per triangle the tensor product of all colors is an EXACT eigenstate



But there are more ground states….

$$|\psi\rangle \equiv \prod_s \otimes |C_s\rangle_s$$

This mixes Sz sectors

But the Hamiltonian doesn't.

$$|\psi^C\rangle \equiv P_{S_z}\Big(\prod_{\text{valid}} \otimes |C_s\rangle\Big)$$

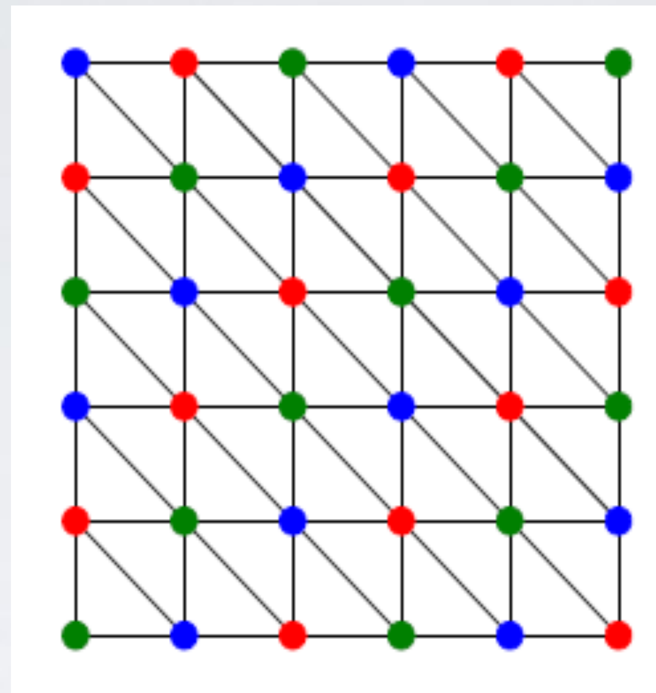
So projecting to Sz sectors are ground states.

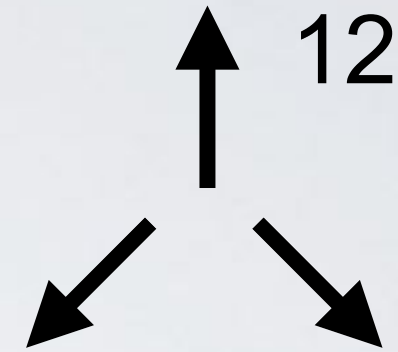
Roughly, each color gives N ground states (one per Sz sector)

(A bit of a lie because colors are non-orthogonal and may be more-so after projection)

An example: triangular lattice

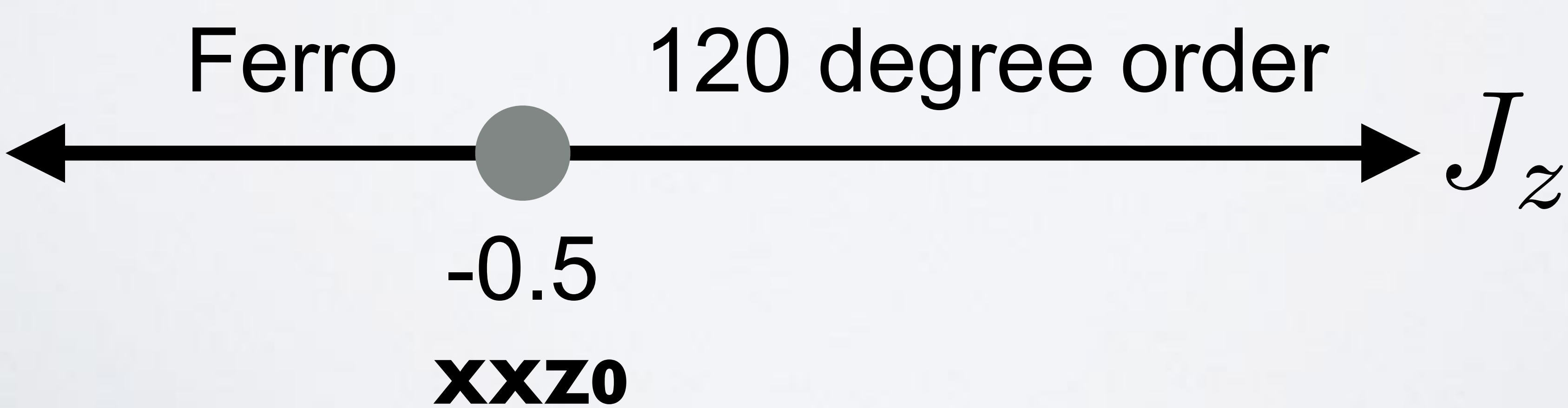
Ferro





120 degree order

Linear Degeneracy



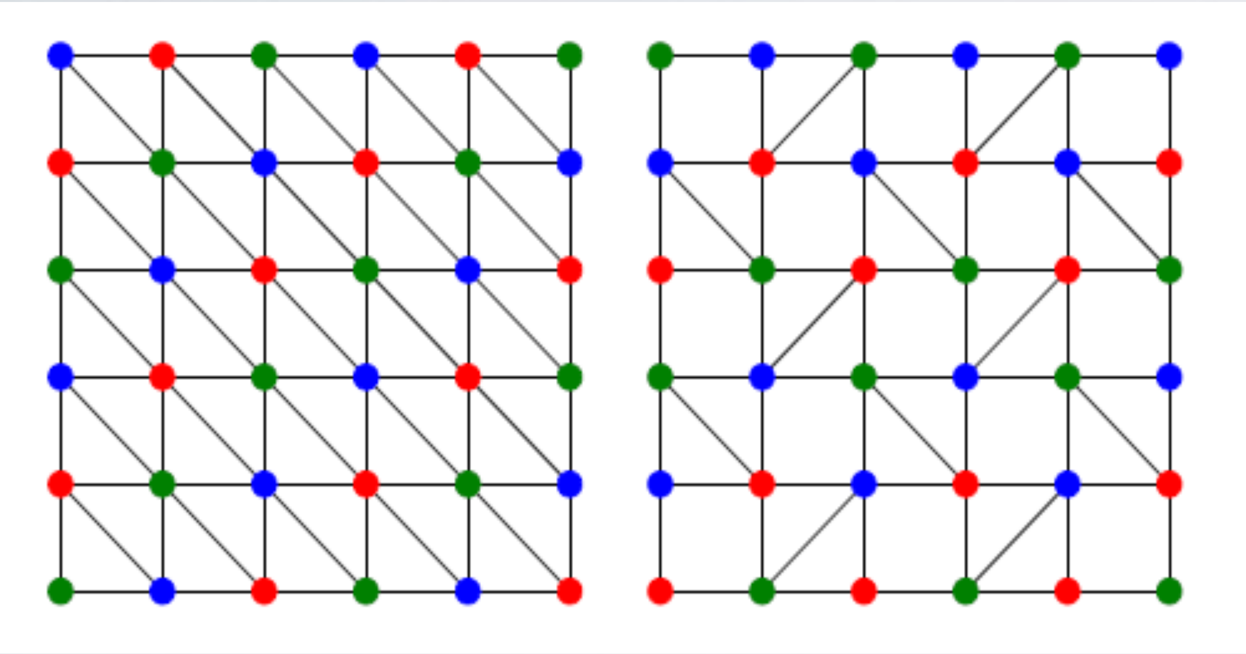
Question: How many colorings is this?

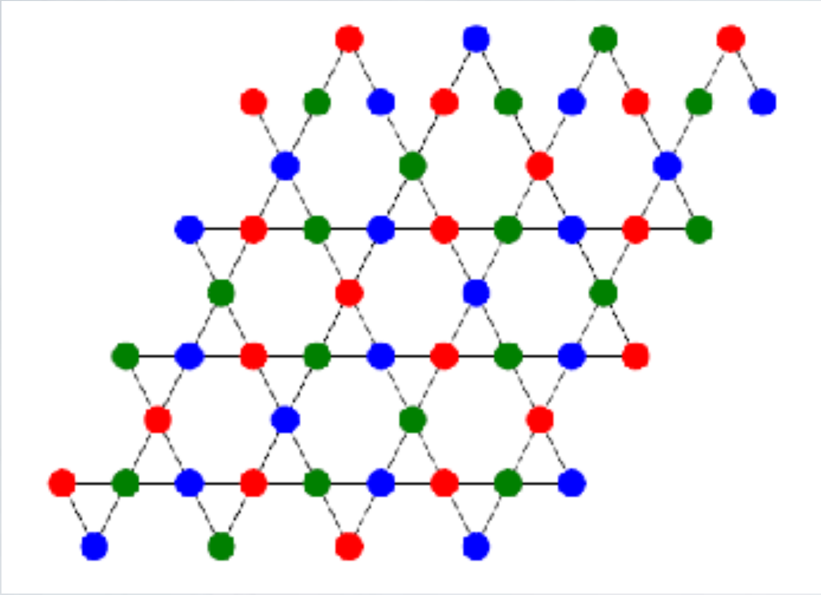
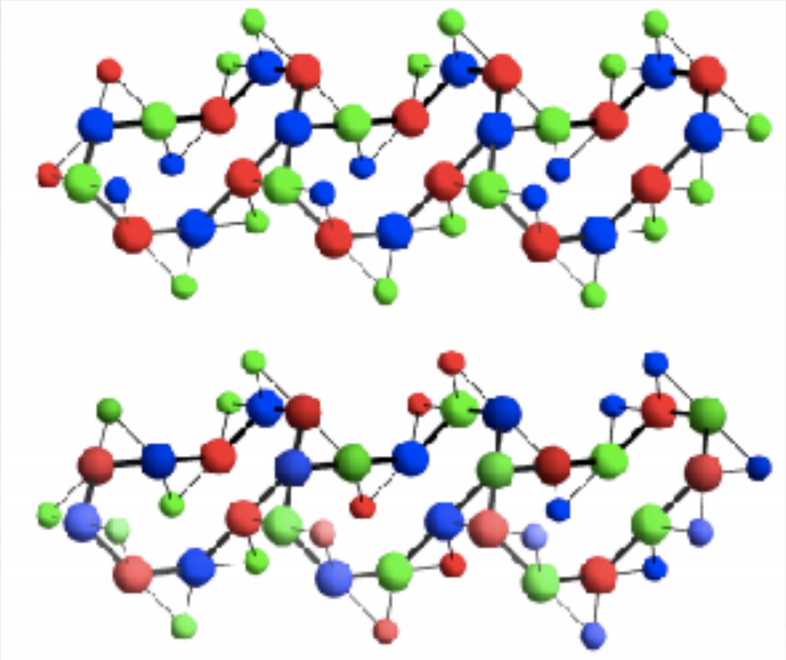
$$|a\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle + |\downarrow\rangle\Big)$$

$$|b\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle + \omega|\downarrow\rangle\Big)$$

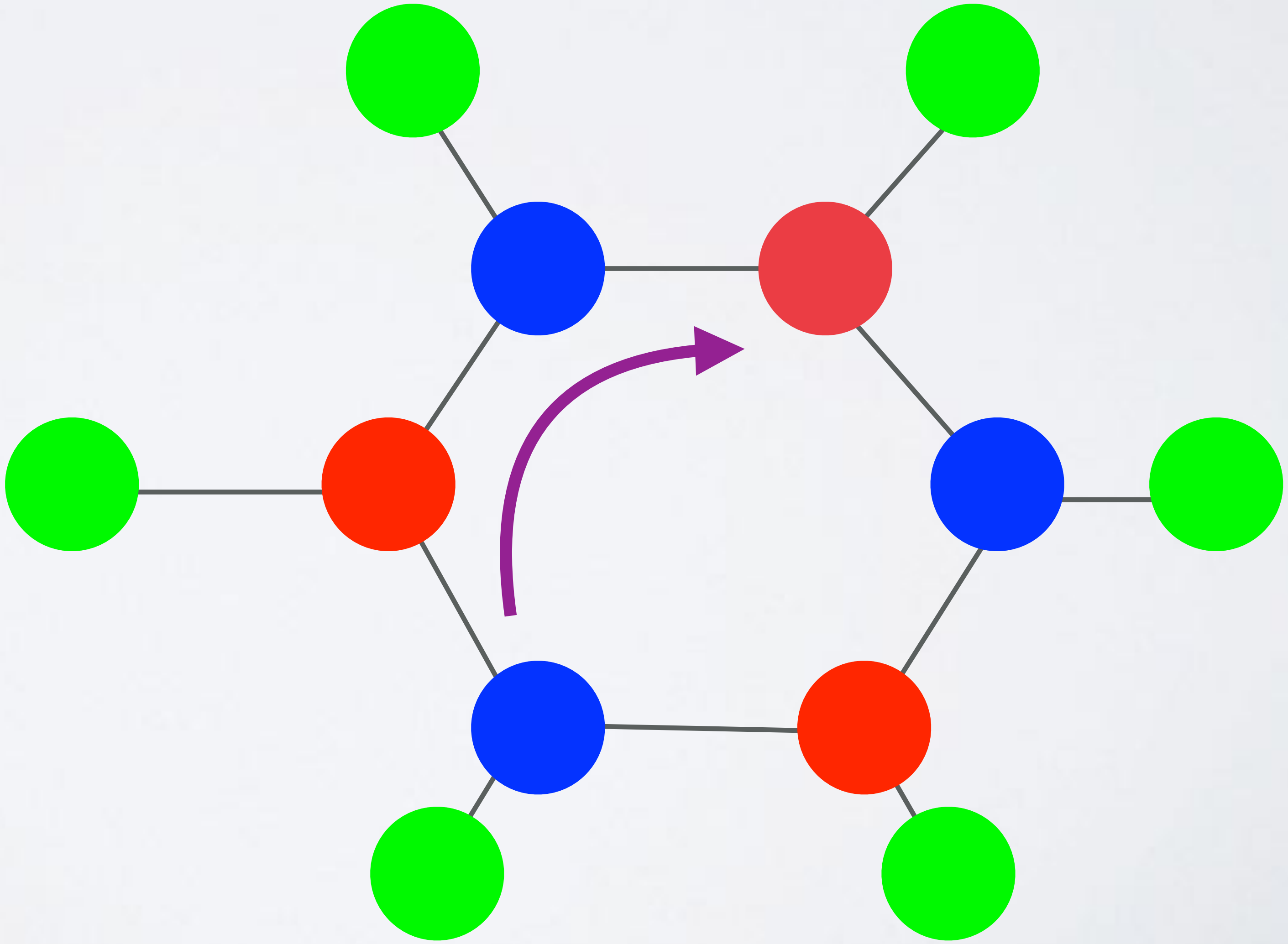
$$|c\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle + \omega^2|\downarrow\rangle\Big)$$

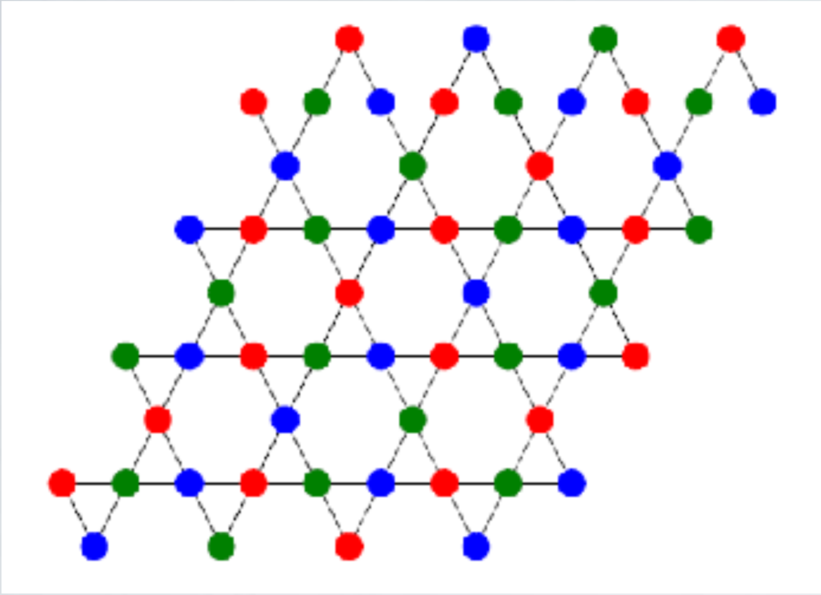
Only one (or two) colorings.

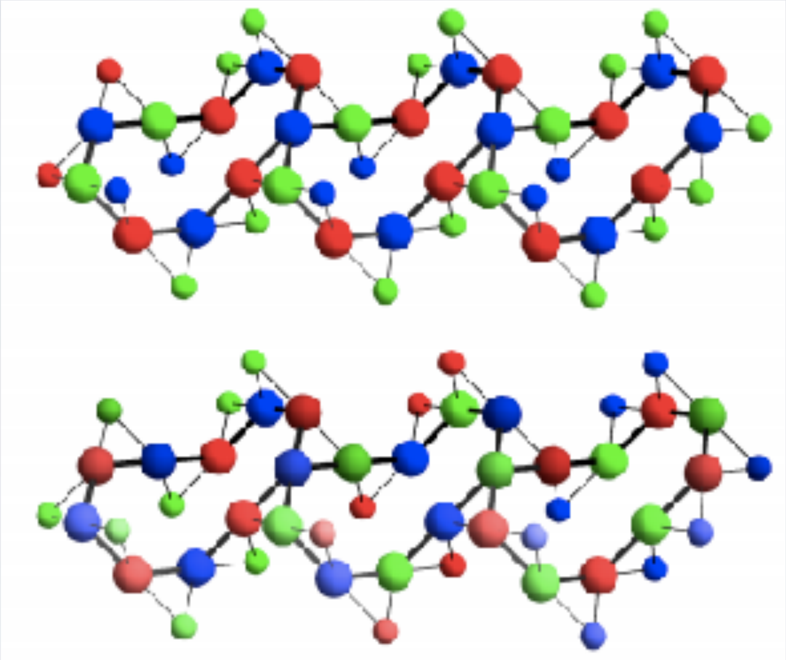


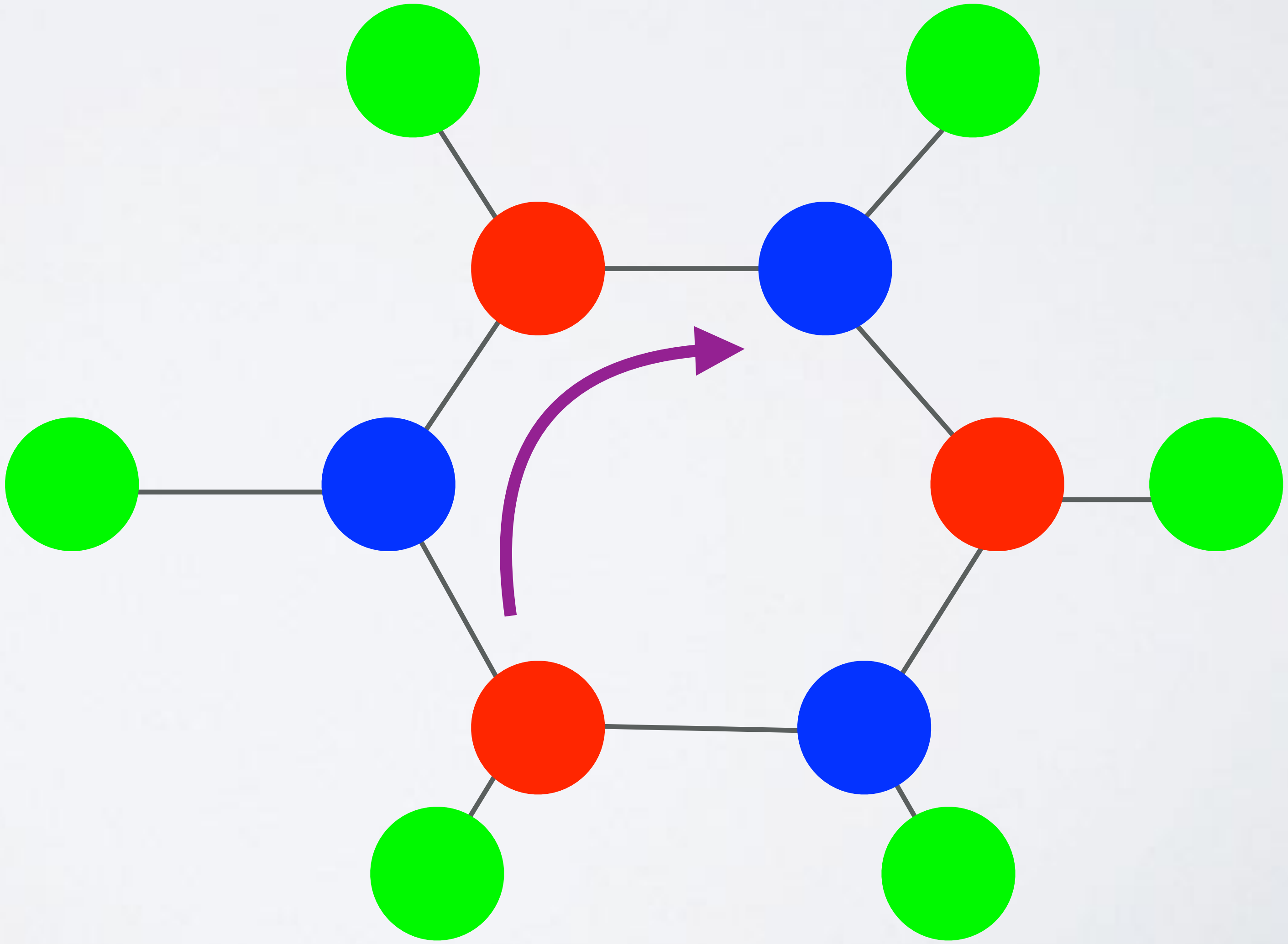
Consider kagome…

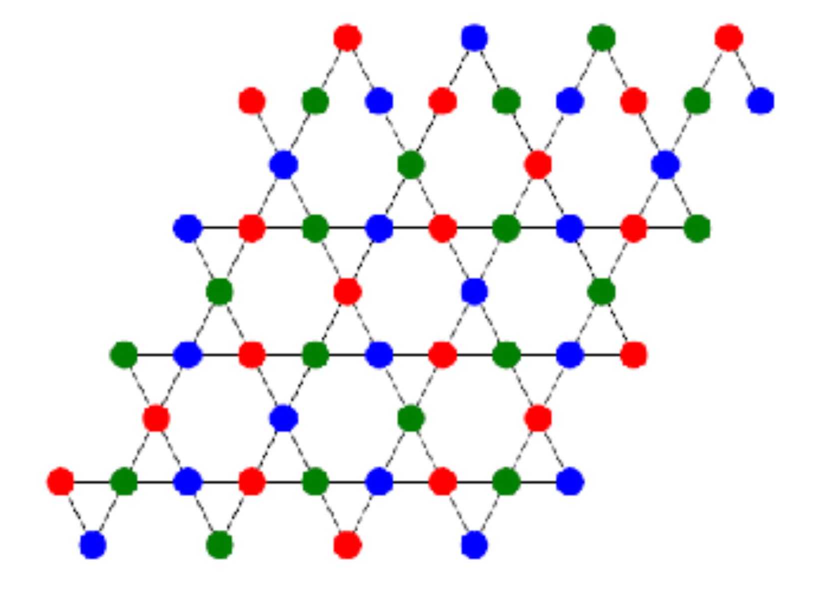
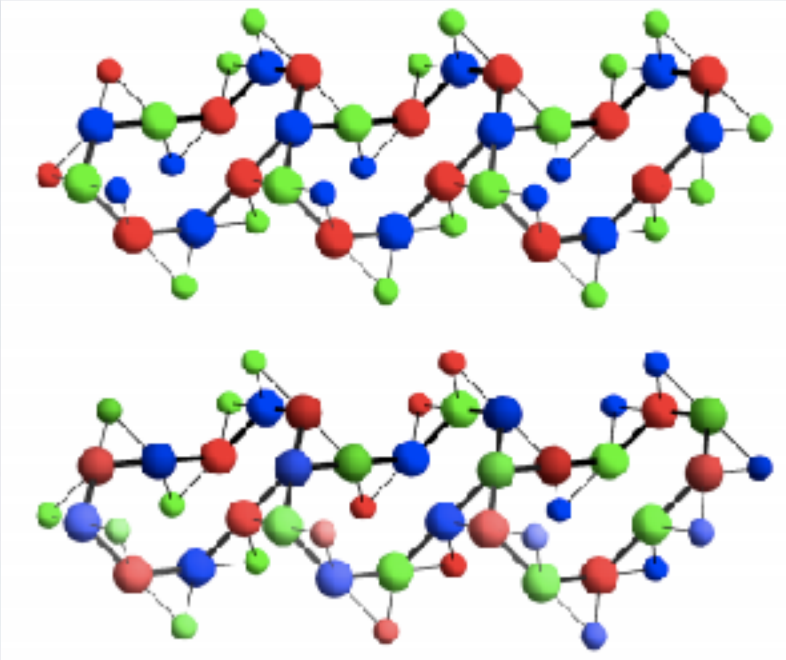






Consider kagome…



An exponential number of colorings!

1.208^N (from Baxter)

But much fewer then Ising configurations....

Lattice	Ising configs	Colorings
2x2x3	924	8
3x2x3	48620	16
4x2x3	2.7 million	32

This looks like a many-body flat band…does it have anything to do with the known one-body flat band in kagome?

Yes…it's a superset of it.

Take the product states and project to one spin-up.

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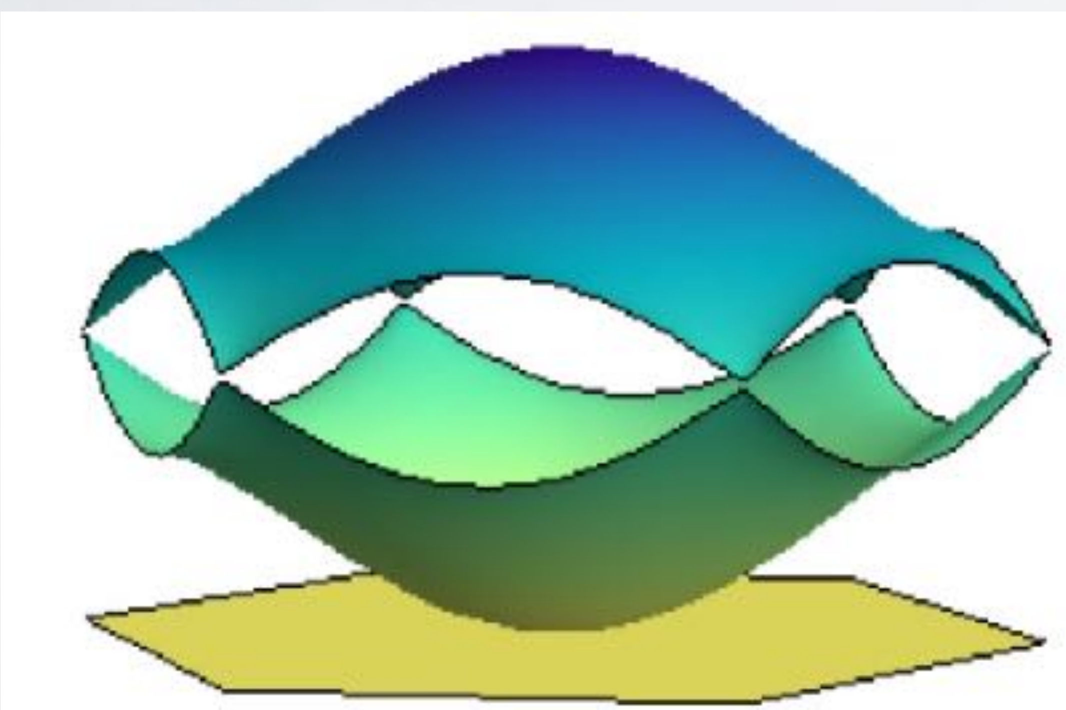
Band touching from real-space topology in frustrated hopping models

Doron L. Bergman,[1] Congjun Wu,[2] and Leon Balents[3]

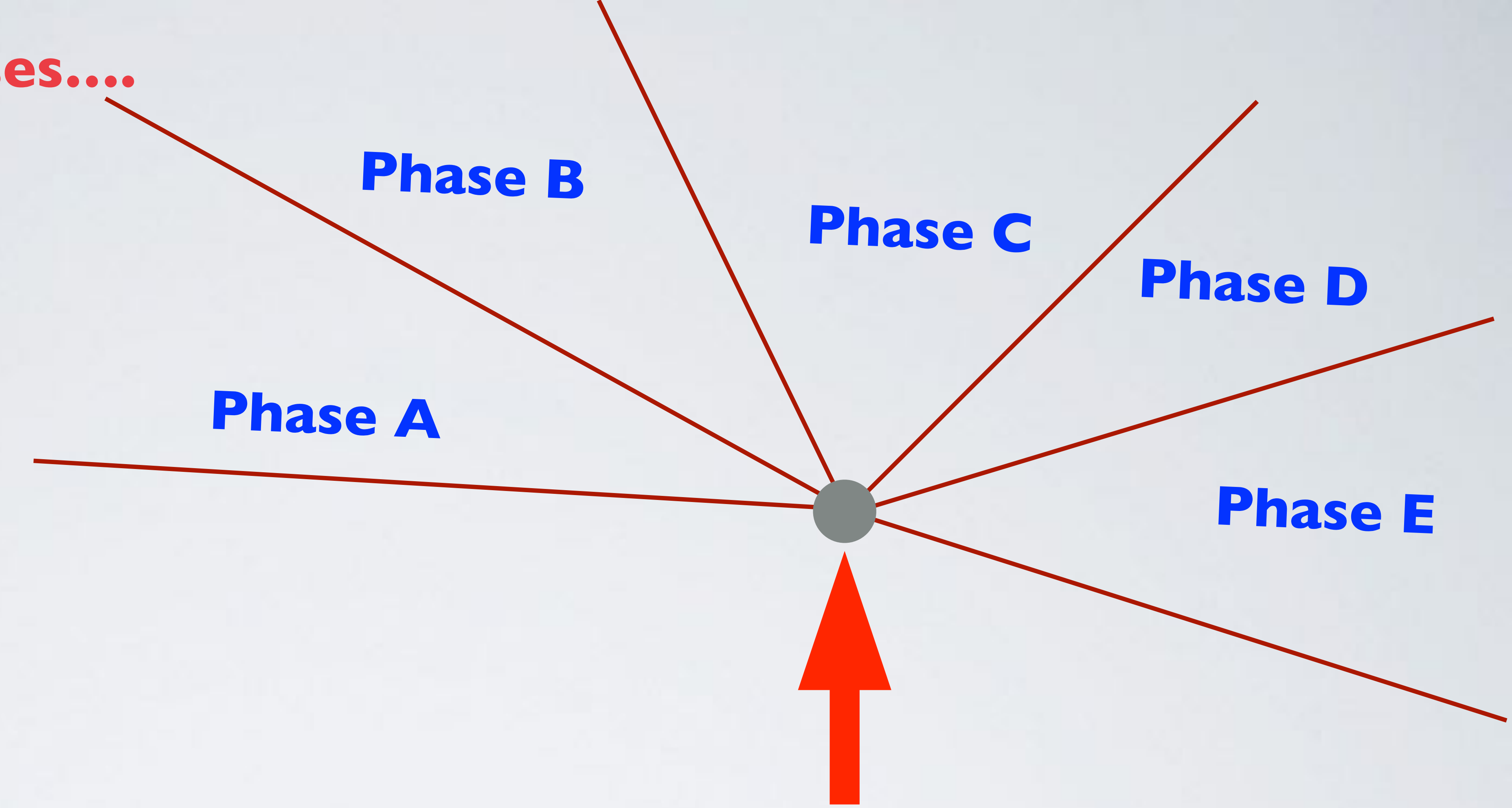
$$\mathcal{H}_t = -t \sum_{\langle ij \rangle} (c_i^\dagger c_j + \mathrm{H.c.}),$$



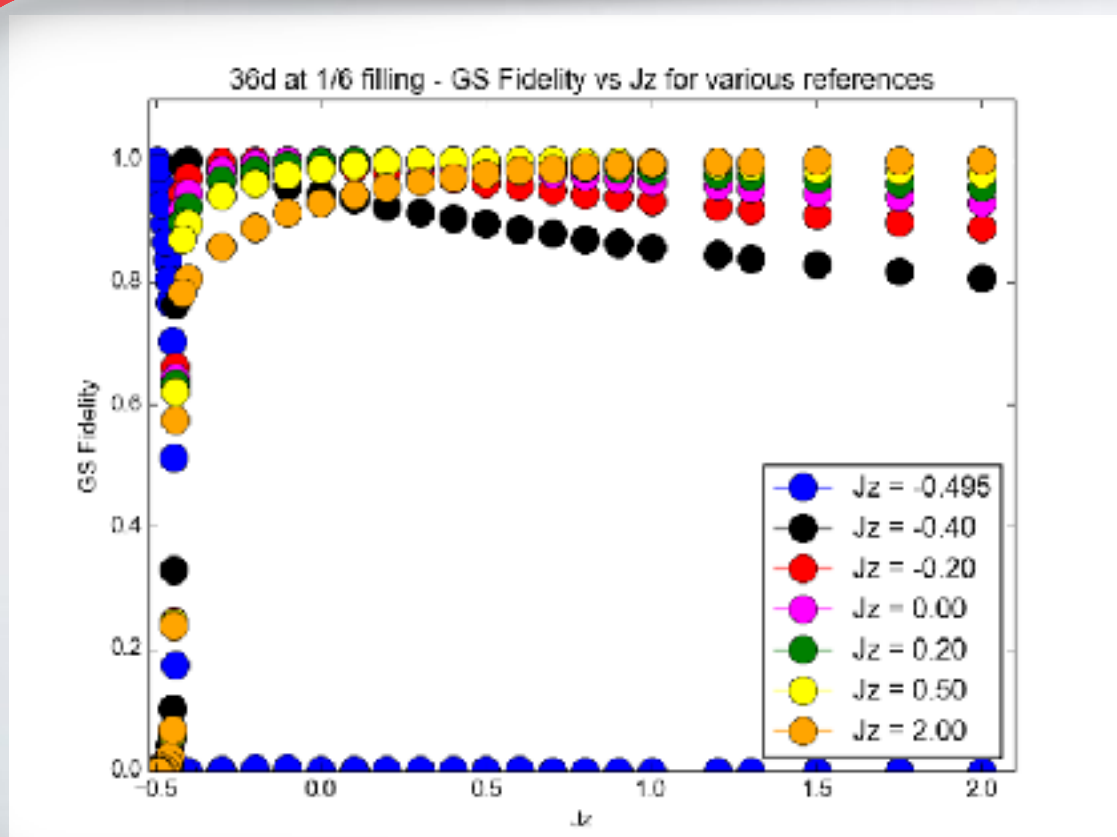
But there are even more ground states….

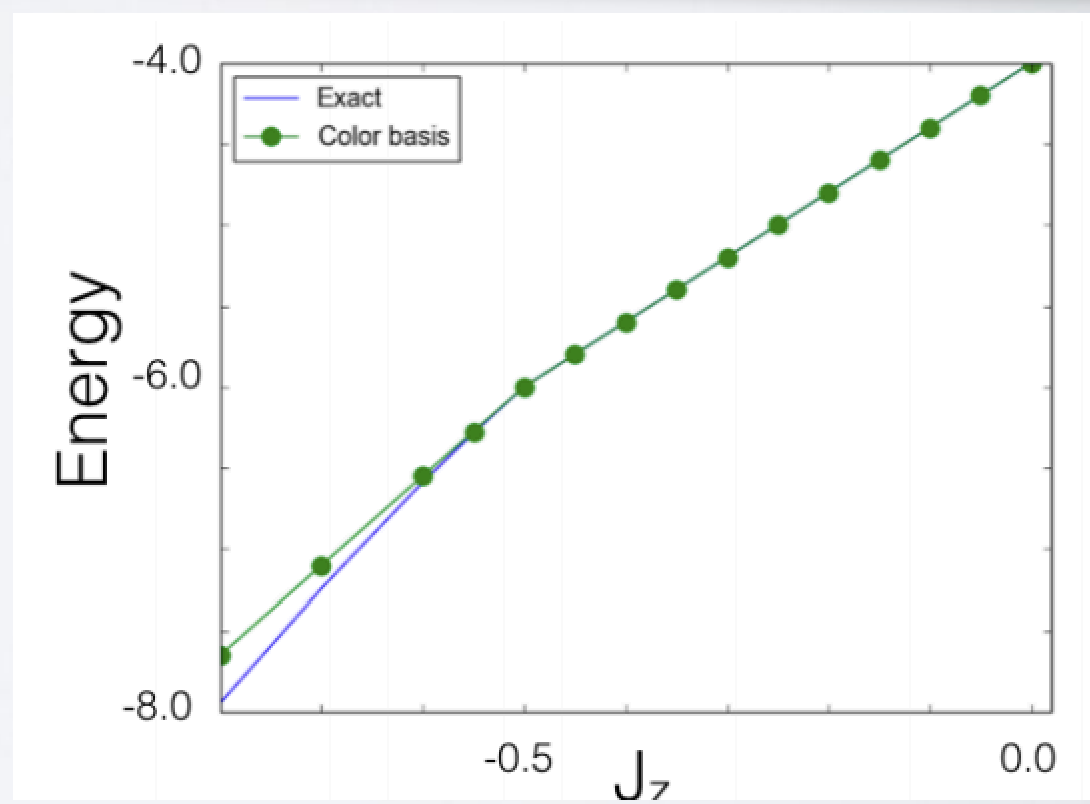
Lattice	Method	$n_b = 1$	$n_b = 2$	$n_b = 3$	$n_b = 4$	$n_b = 5$	$n_b = 6$	# 3-colorings
3×3 kagome obc	ED	15	102	414	1117			3808
(33 sites)	$R(S)$	15	102	414	1117	2136	3078	
3×3 kagome pbc	ED	10	38	60	41	40	40	40
	$R(S)$	10	34	40	40	40	40	
5×2 kagome pbc	ED	11	47	92	83	65	64	64
	$R(S)$	11	42	58	63	64	64	
4×3 kagome pbc	ED	13	68	169	172	137	136	136
	$R(S)$	13	68	134	136	136	136	

Connect to known phases....

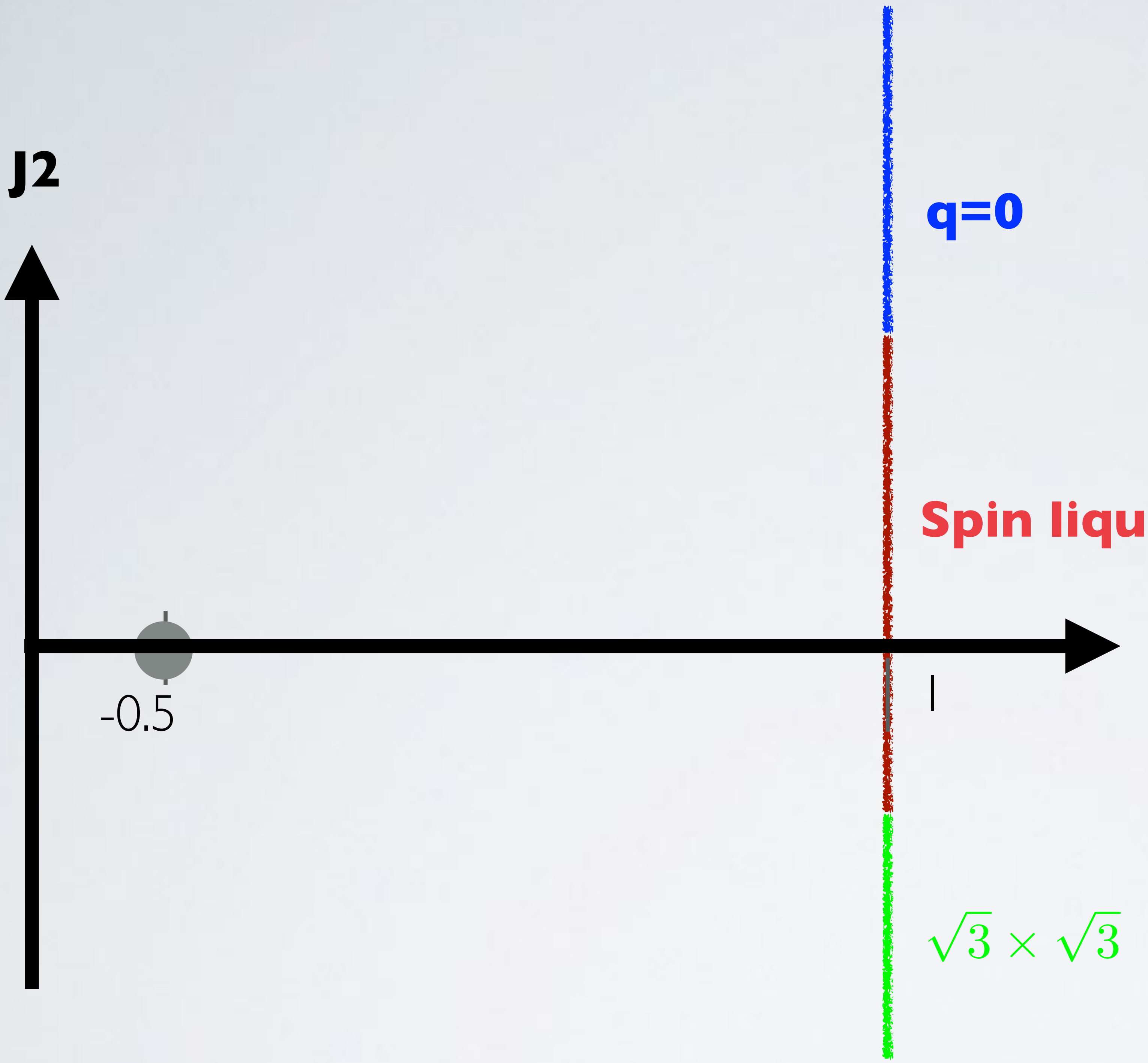


The mother of all phases?

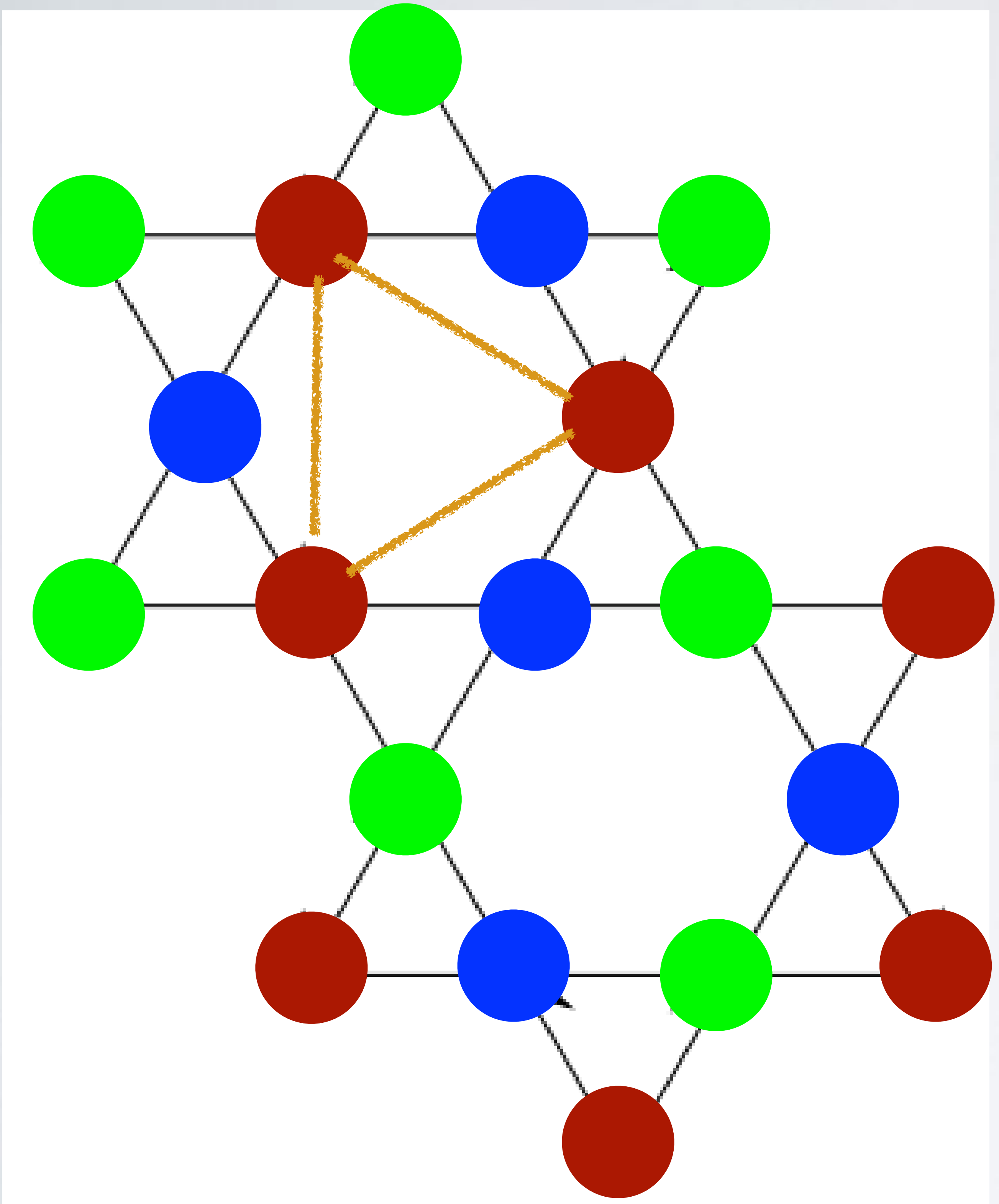




$S_z = 0$

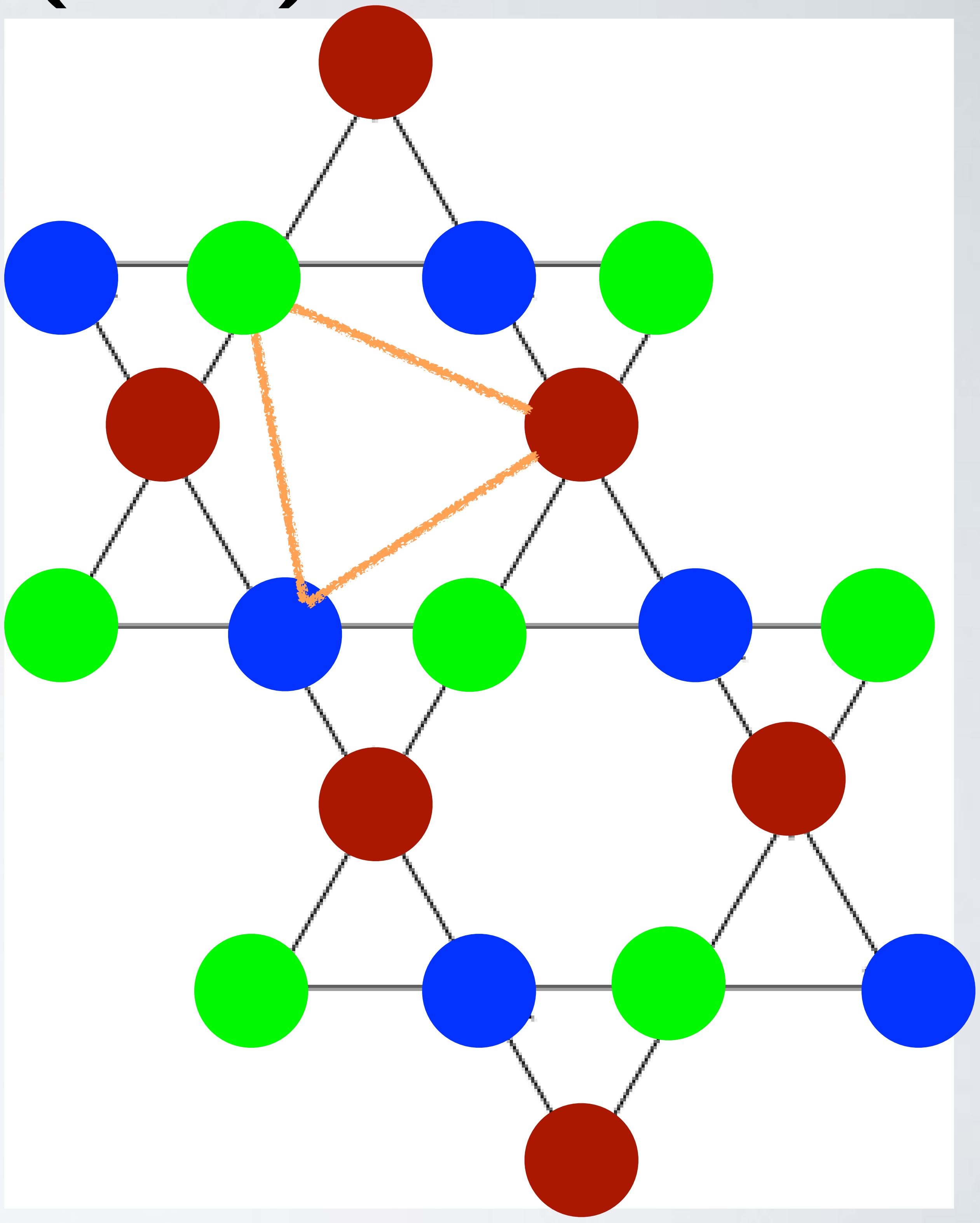


Tune J1-J2

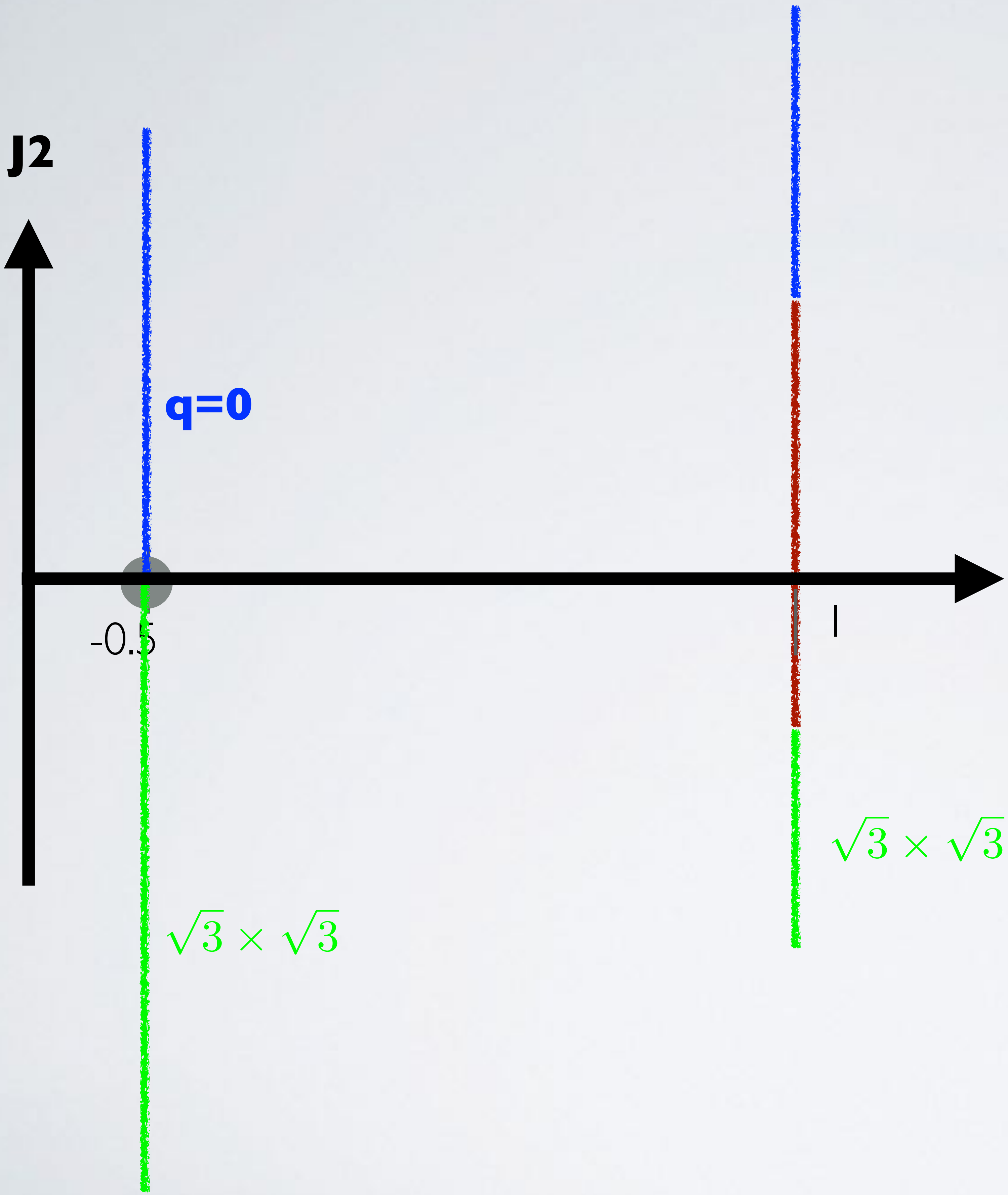


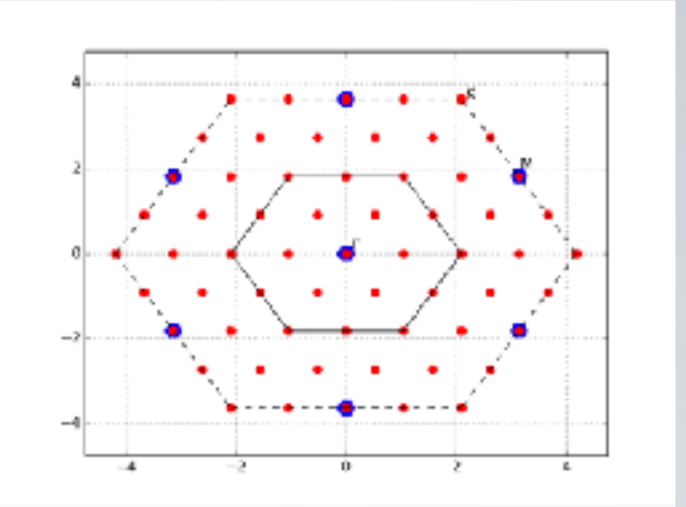
$\sqrt{3} \times \sqrt{3}$

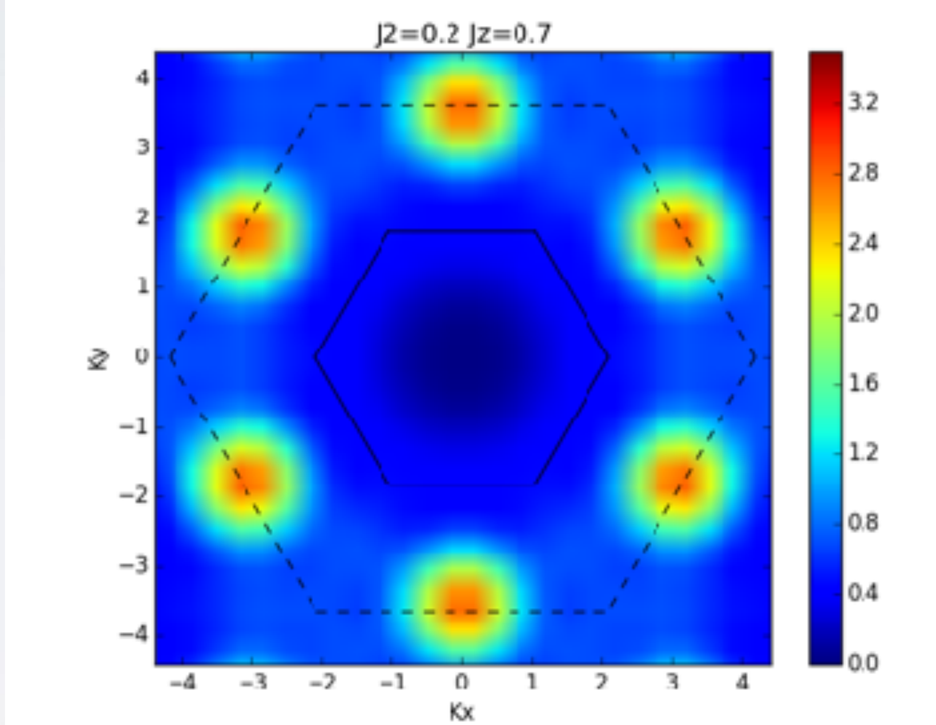
(exact)

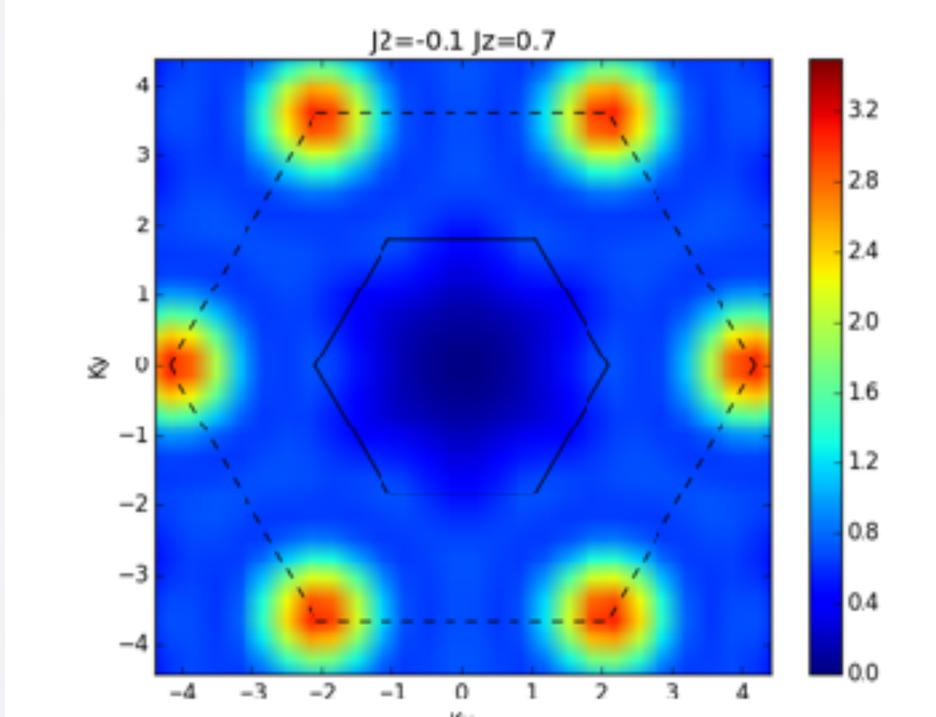


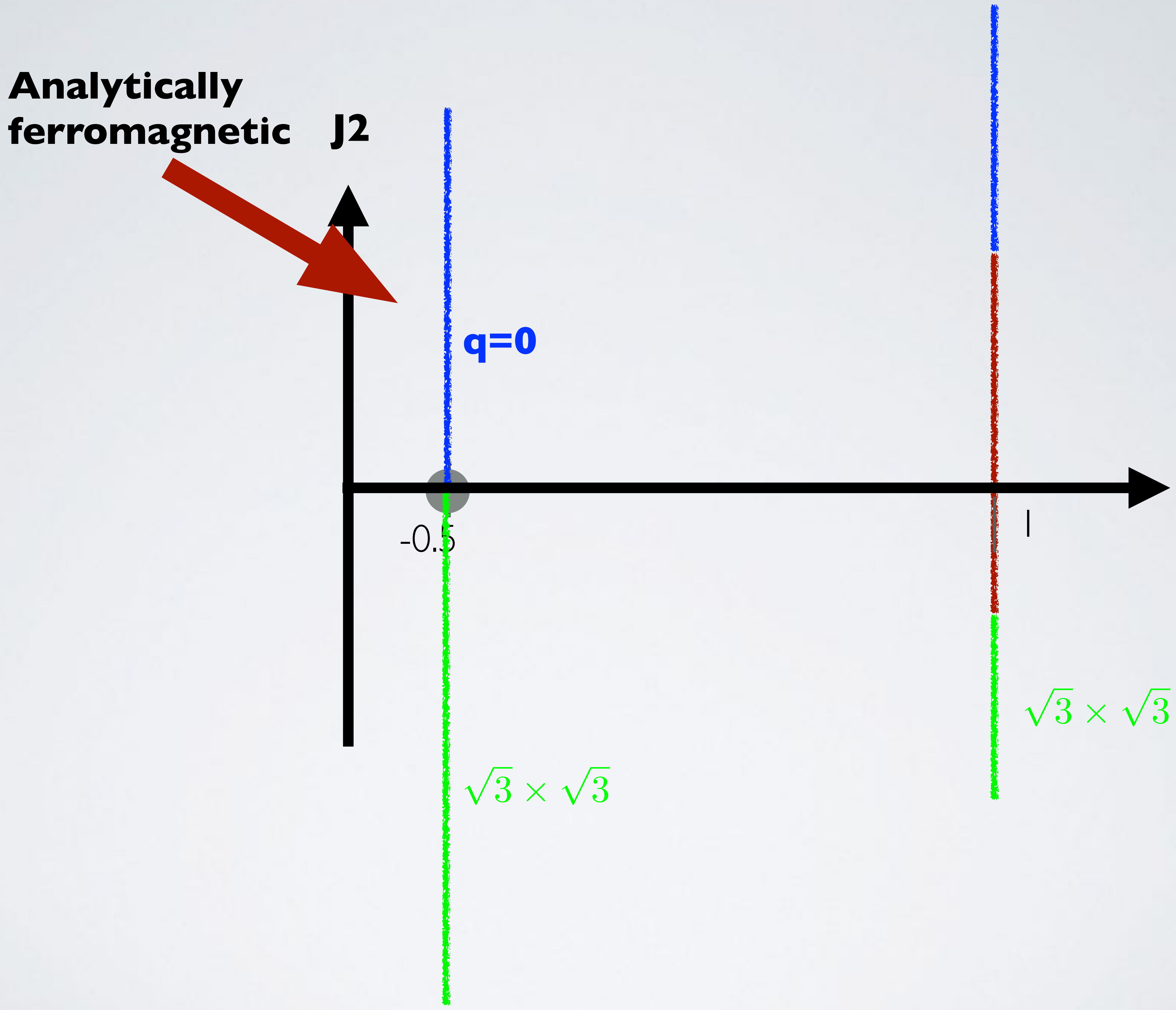
q=0


J2
q=0
-0.5
1
$\sqrt{3} \times \sqrt{3}$
$\sqrt{3} \times \sqrt{3}$

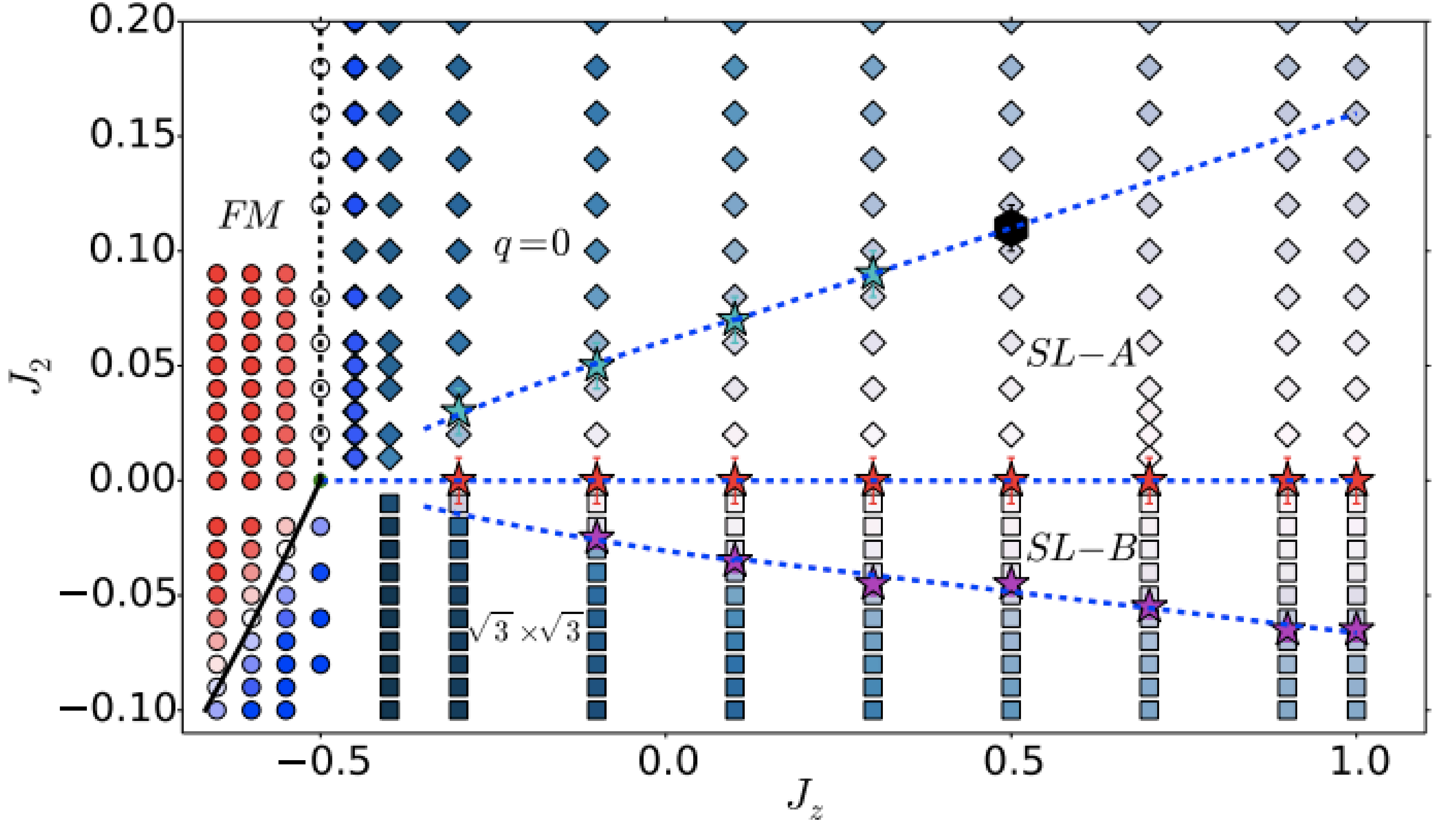



J2=0.2 Jz=0.7
Kx
Ky

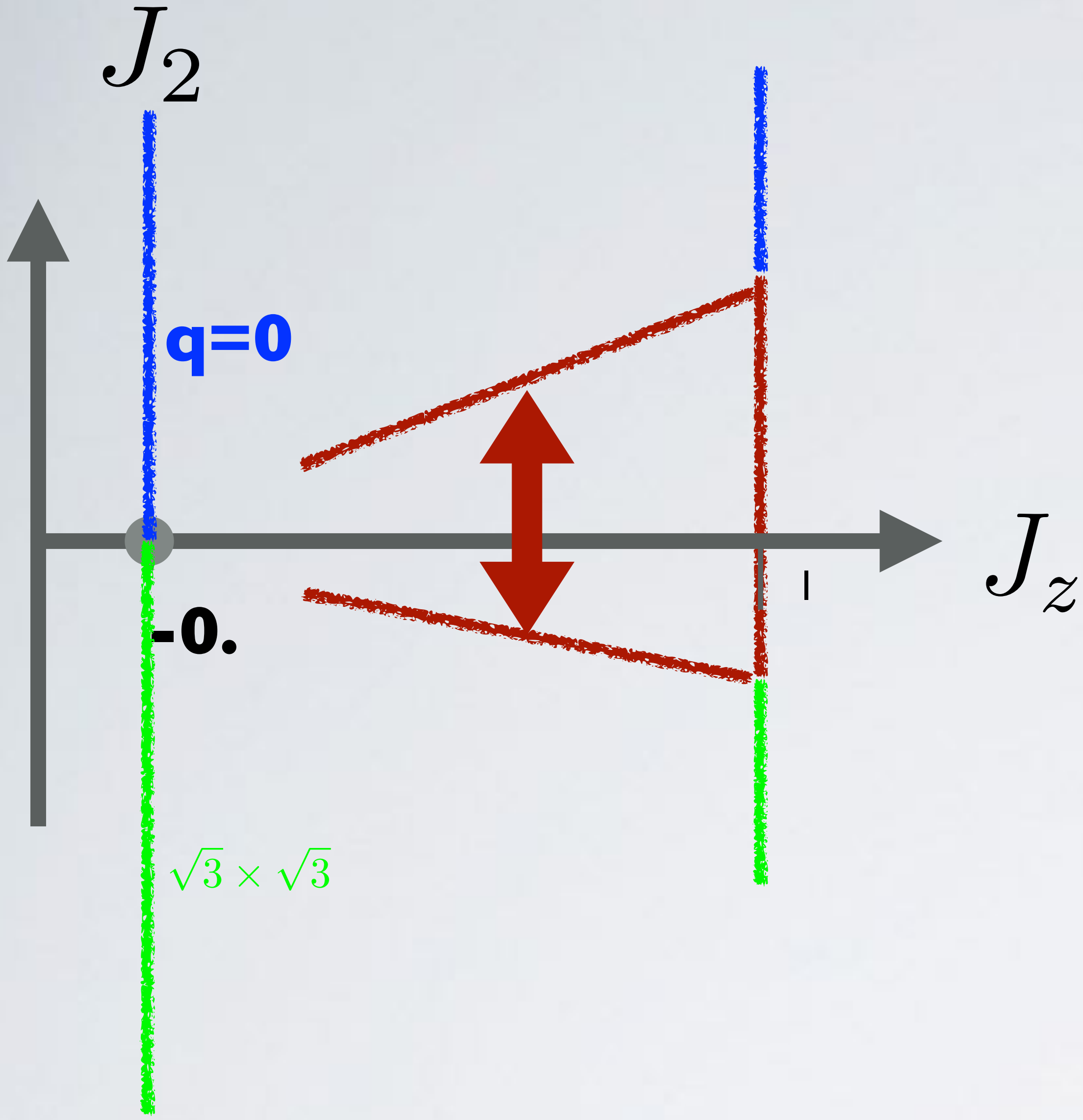

J2=-0.1 Jz=0.7
Ky

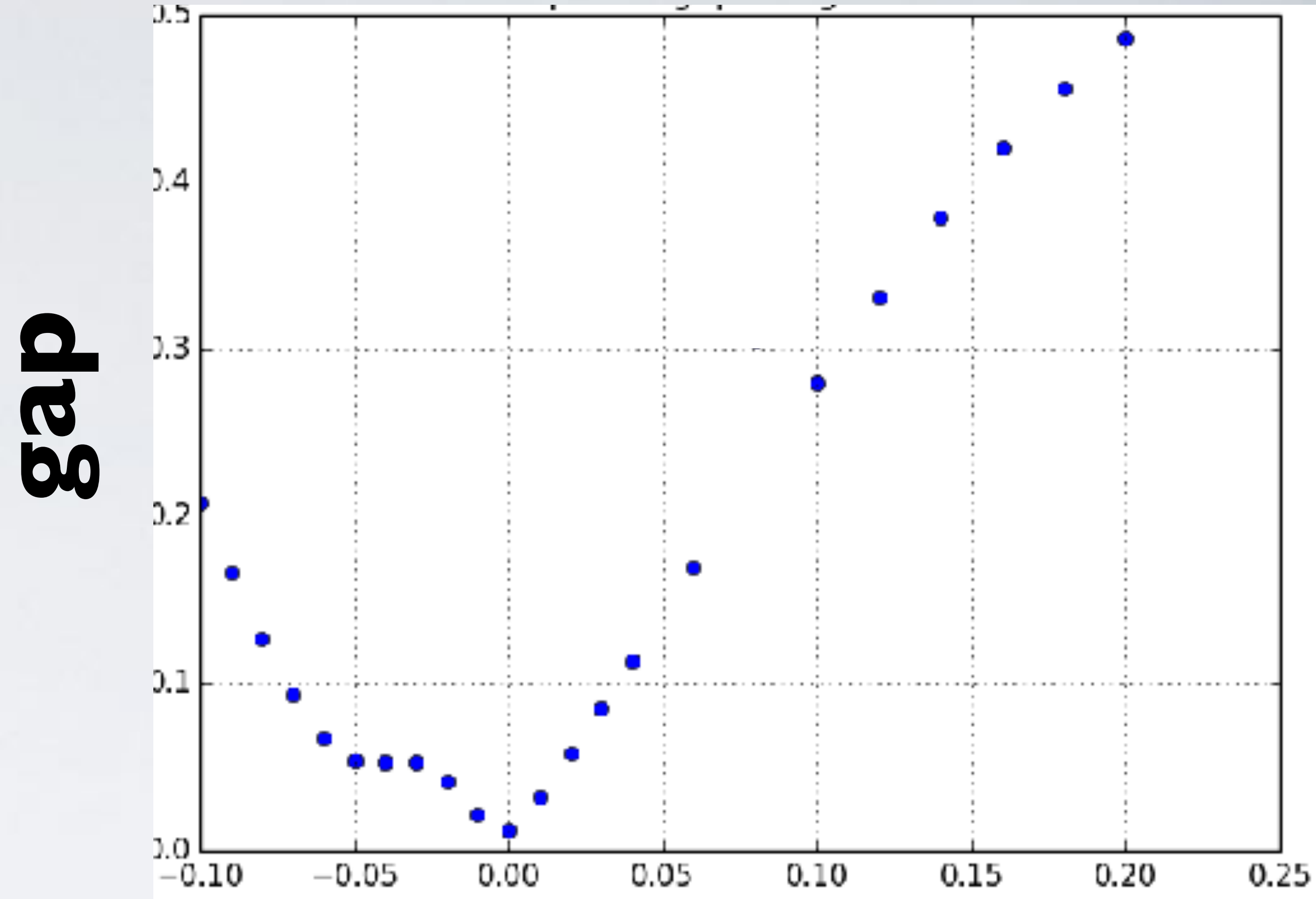


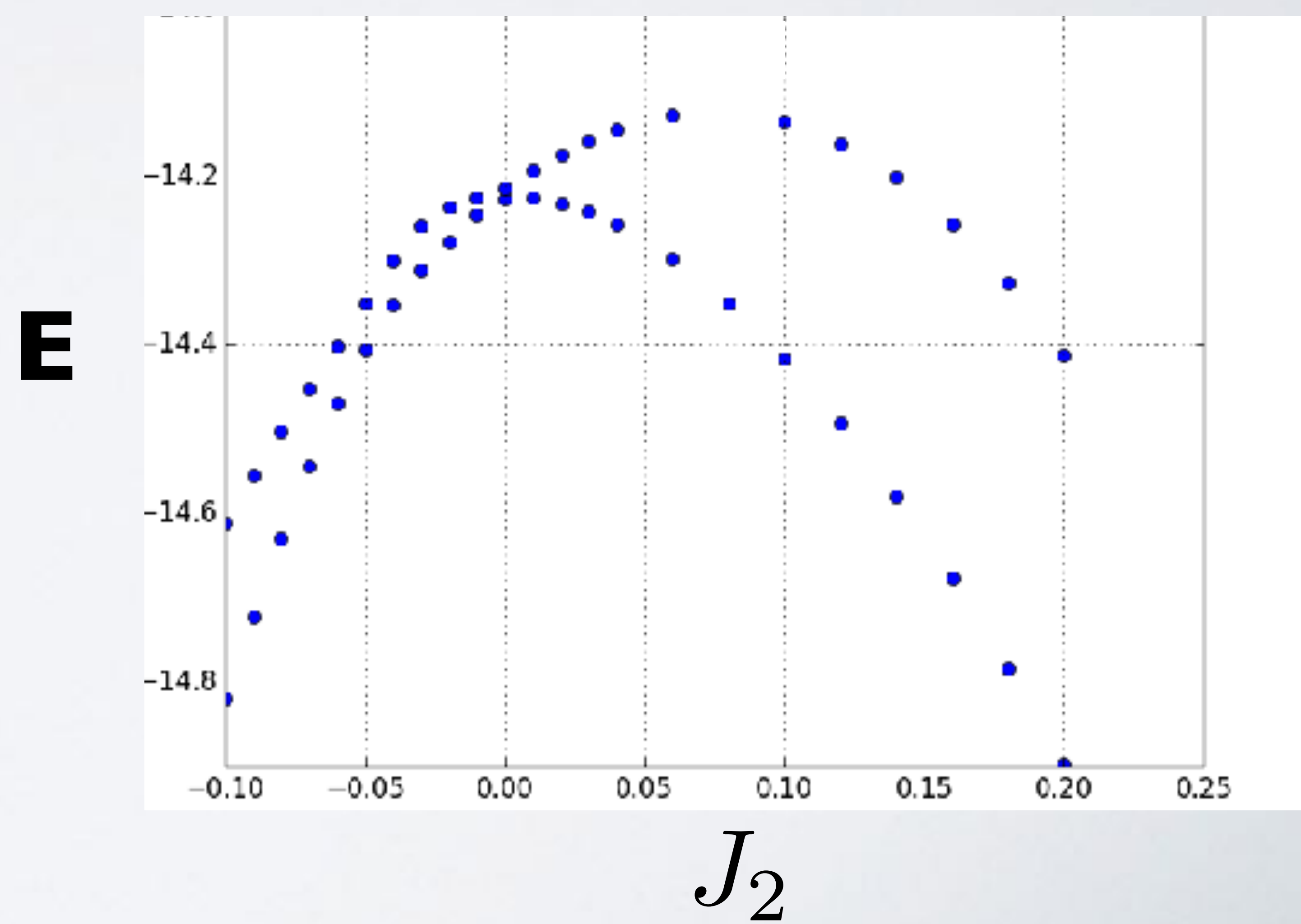
Analytically
ferromagnetic
J2
q=0
-0.5
1
$\sqrt{3}\times\sqrt{3}$
$\sqrt{3}\times\sqrt{3}$

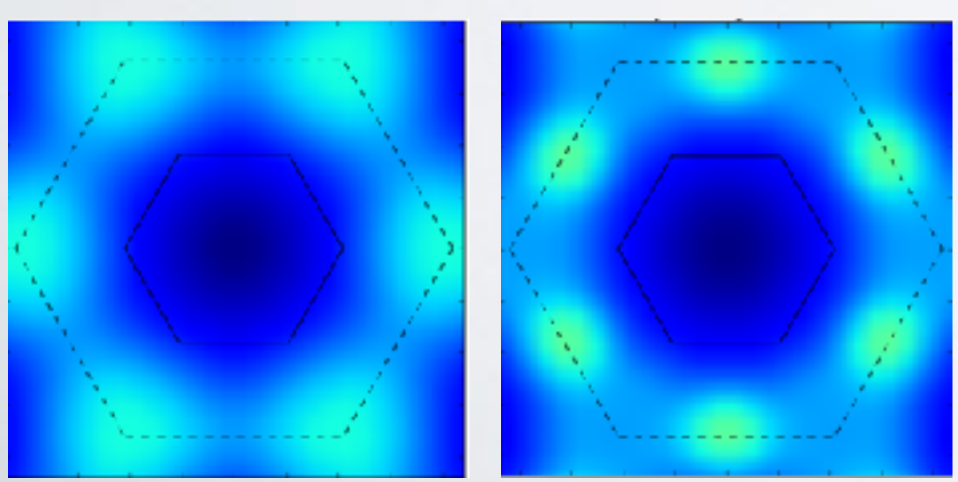


0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.5
0.0
0.5
1.0
J_2
J_z
FM
$q=0$
$SL-A$
$SL-B$
$\sqrt{3}\times\sqrt{3}$

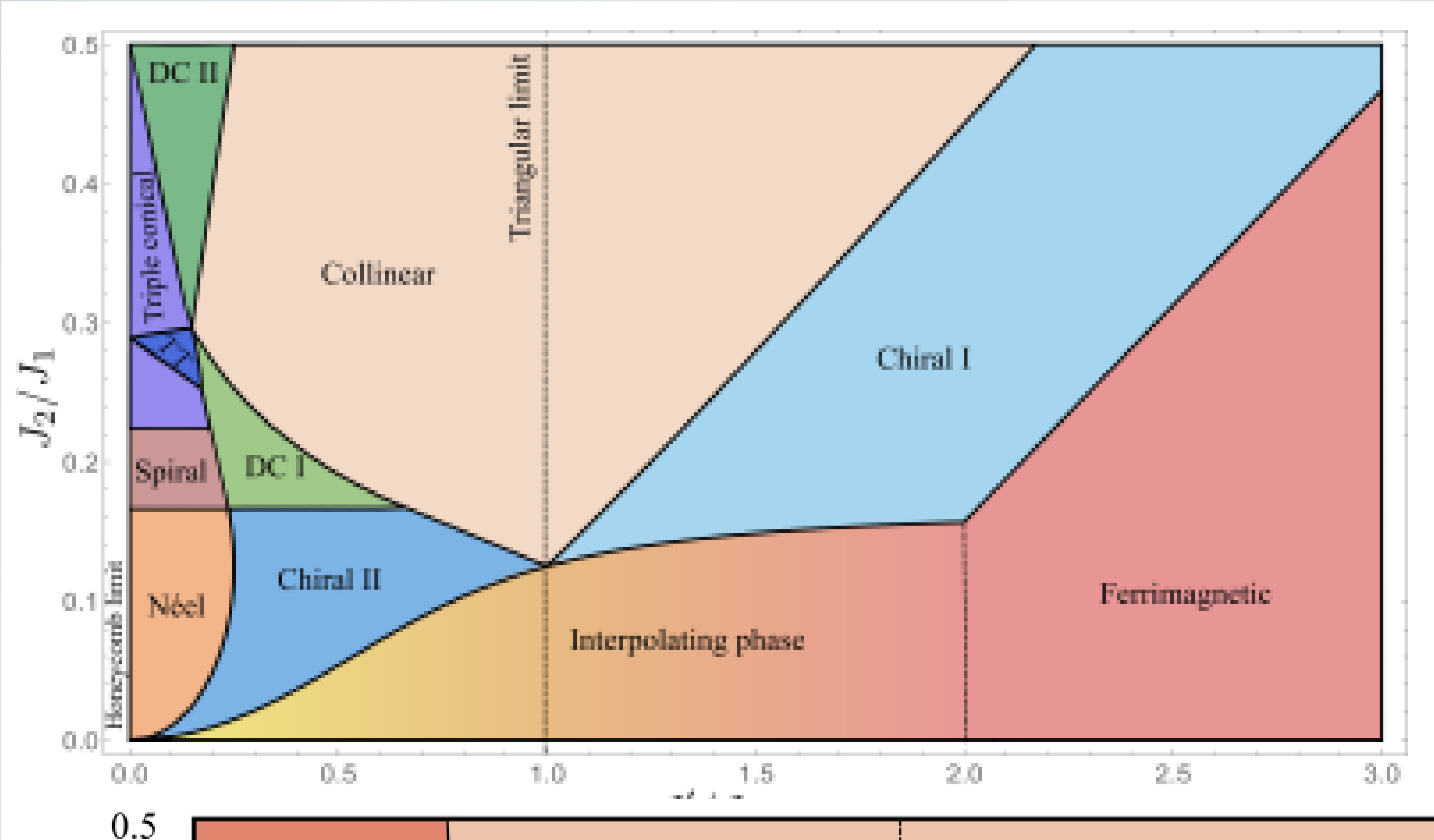

J2
q=0
Jz
-0.
√3 × √3

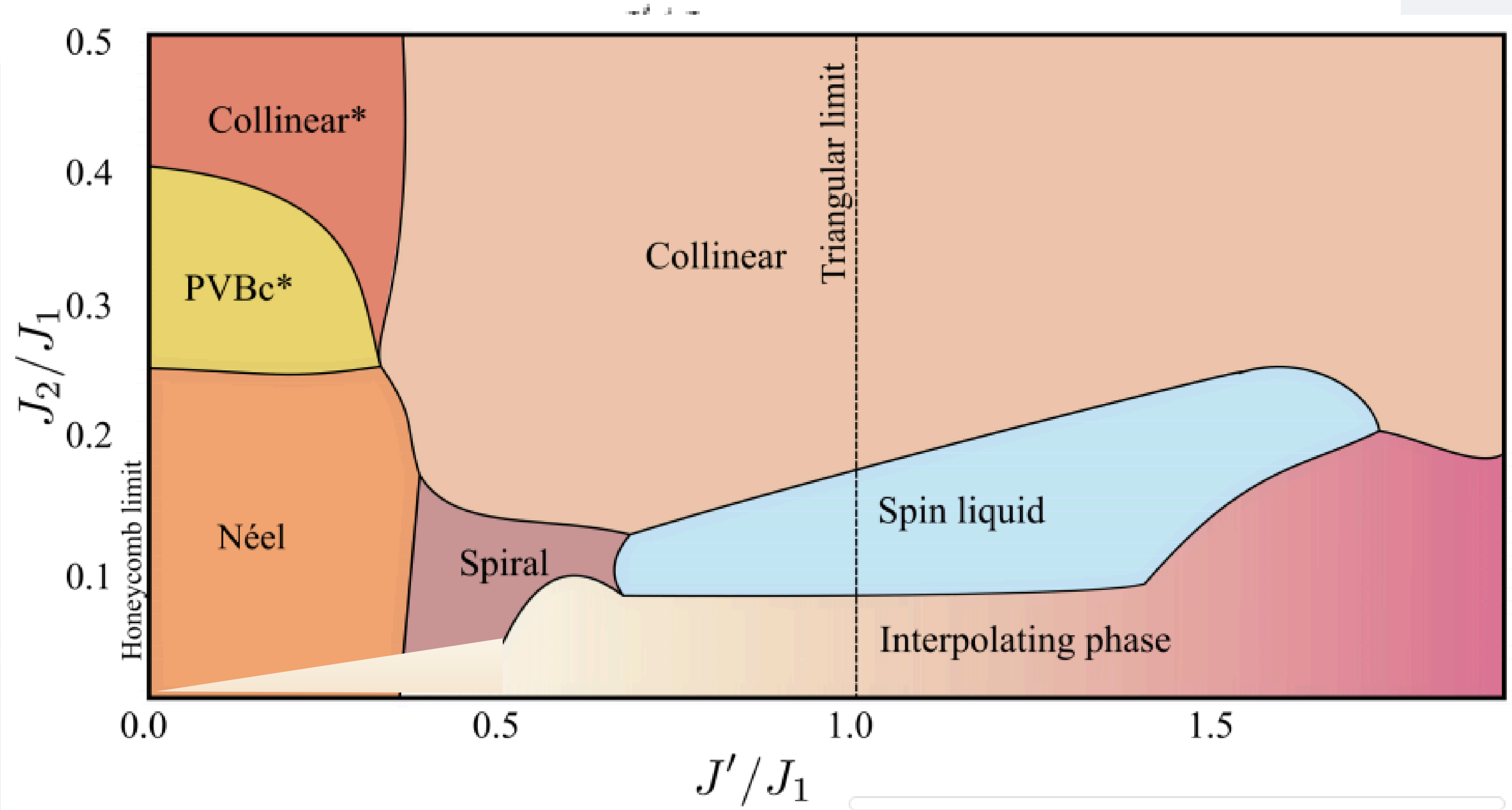

gap
0.5
0.4
0.3
0.2
0.1
0.0
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20
0.25

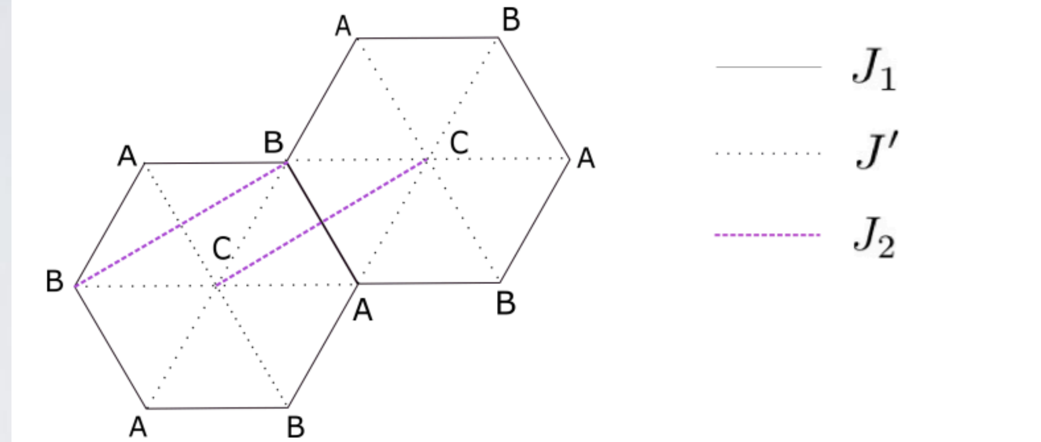

E
−14.2
−14.4
−14.6
−14.8
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20
0.25
J2



An aside on another model...







$$\hat{\mathcal{H}} = J_1 \sum_{\langle i,j \rangle} S_{iA} \cdot S_{jB} + J_2 \sum_{\langle\langle i,j \rangle\rangle \eta = A,B,C} S_{i\eta} \cdot S_{j\eta} + J' \sum_{\langle i,j \rangle \eta = A,B} S_{i\eta} \cdot S_{j\eta}$$

Stuffed Honeycomb

Conclusions