THE WELLSPRING OF ALL PHASES ON THE KAGOME LATTICE

University of Illinois at Urbana Champaign
with Hitesh Changlani, Dmitrii Kochkov, Krishna Kumar, Eduardo Fradkin

Pittsburgh Quantum Institute: Wed. April 17, 2018

Other talks I didn't give that I'm happy to chat about.

An inverse approach to strongly correlated systems

Many-Body Localization and Holography

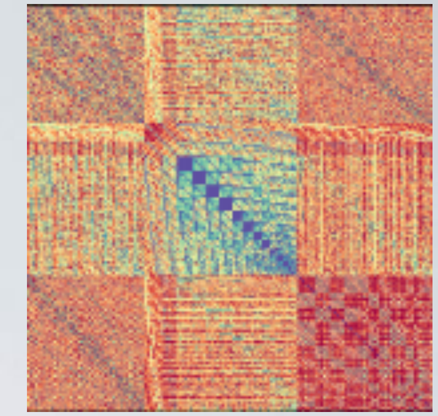
Beyond many-body localization: eigenstates with logarithmic entanglement.

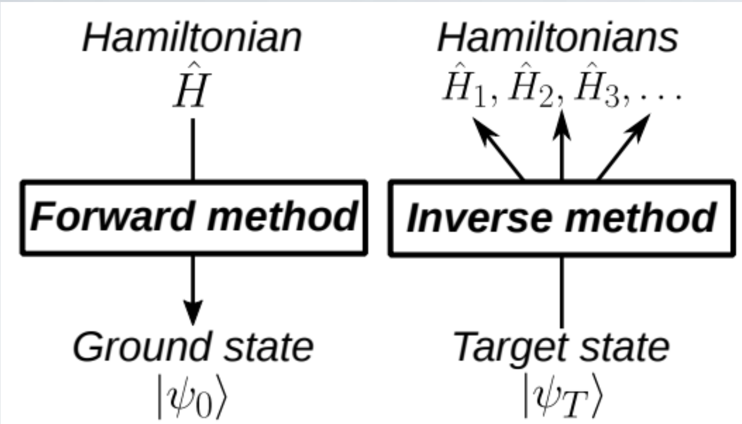
Finite Temperature Variational Monte Carlo

Wave-functions from Deep Neural Nets (without the hype)

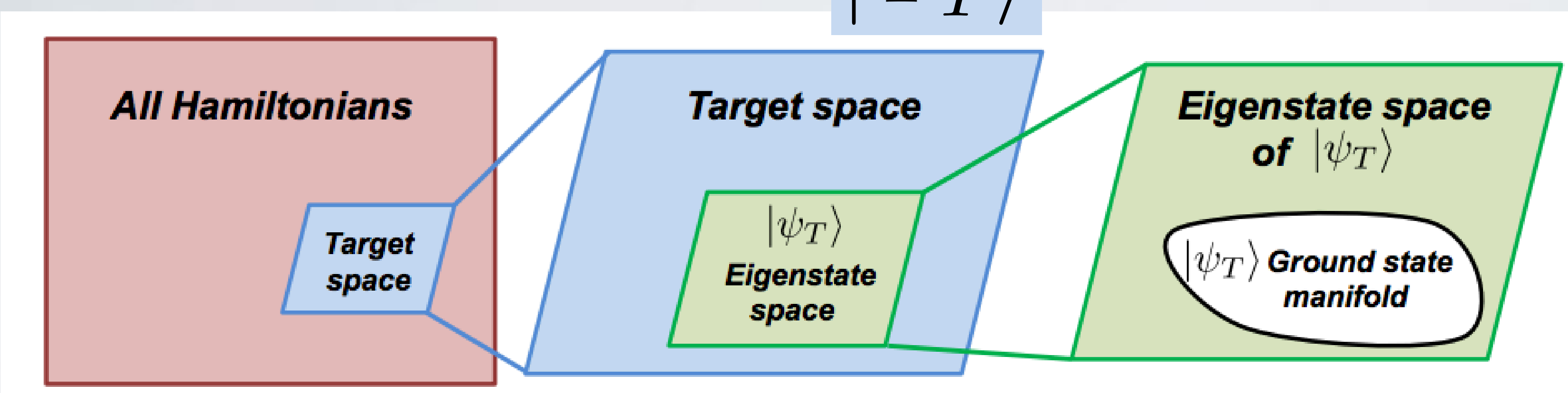
But first an aside…

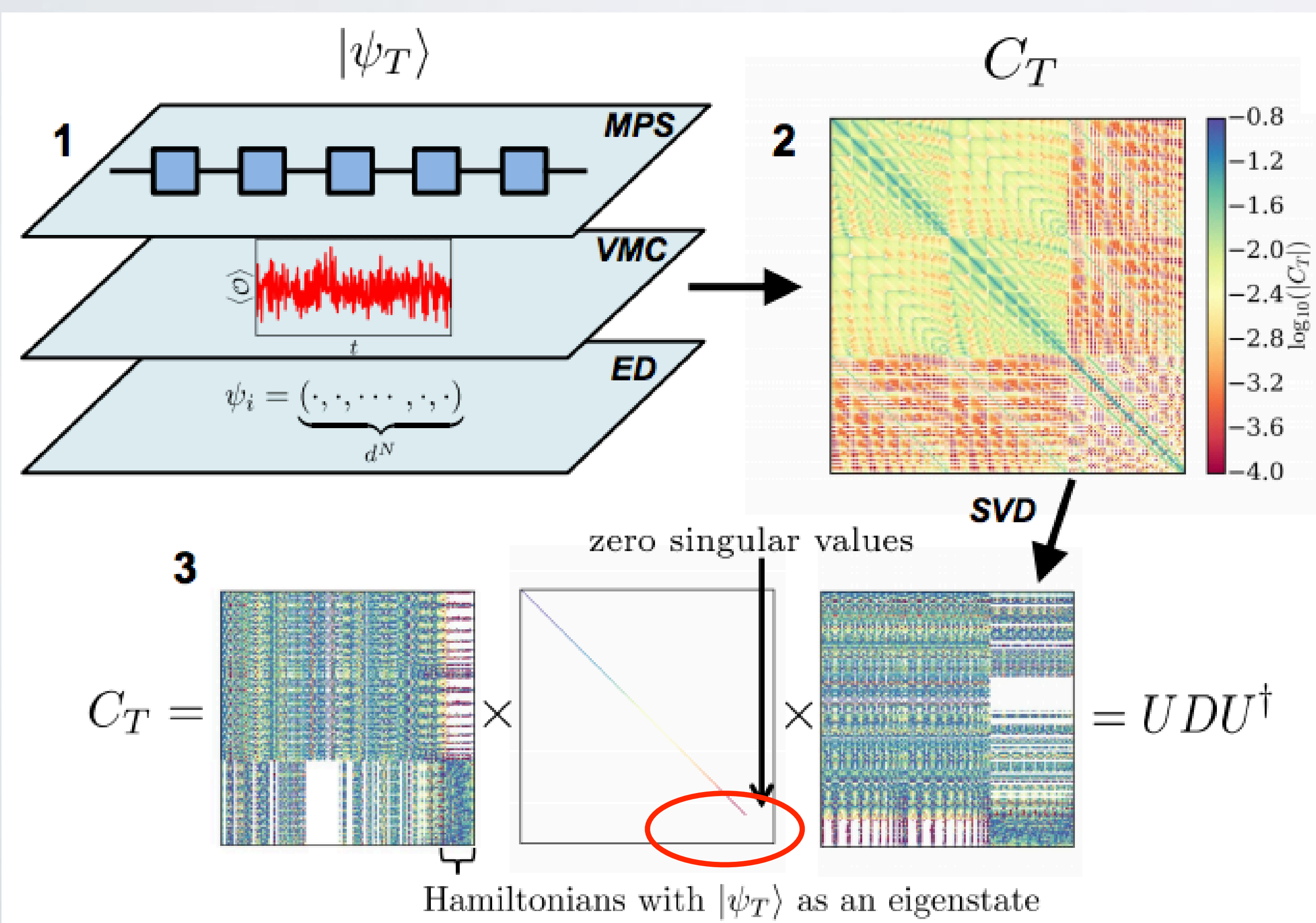
Inverse design in strongly correlated systems



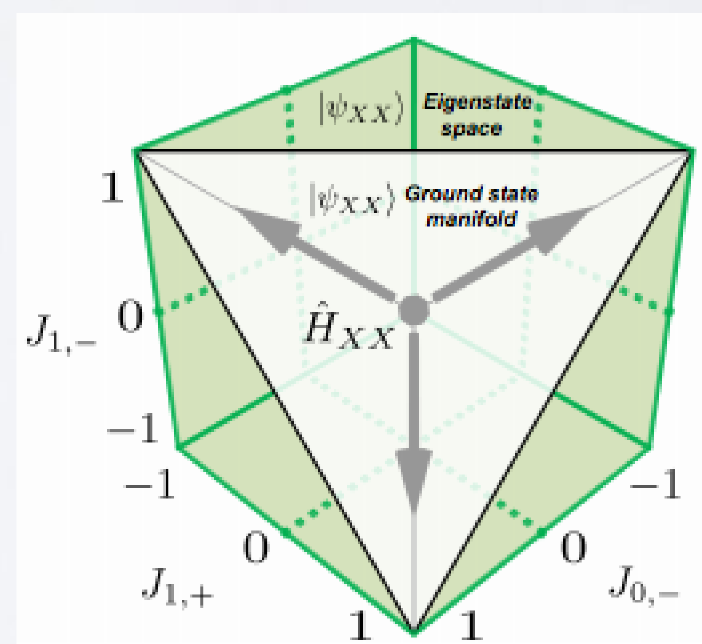


$|\Psi_T\rangle$





$$(C_T)_{ab} = \langle \hat{h}_a \hat{h}_b \rangle_T - \langle \hat{h}_a \rangle_T \langle \hat{h}_b \rangle_T$$



Eli Chertkov

Quantum Materials.....

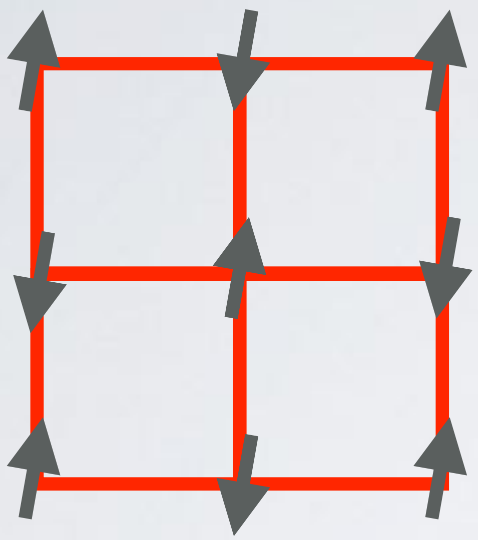
- Insulator - Electrons don't move
- Interaction between electron spins

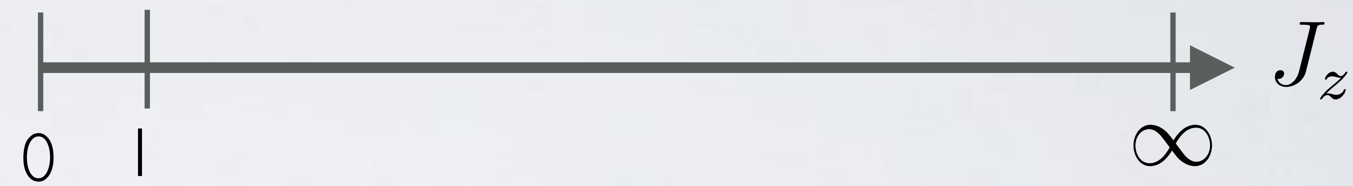
$$H_{xy} = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y$$

$$H_{xxz} = H_{xy} + J_z \sum_{ij} S_i^z S_j^z$$

$$H_{\text{ising}} = \sum_{ij} S_i^z S_j^z \quad \text{(ising limit } J_z \to \infty\text{)}$$

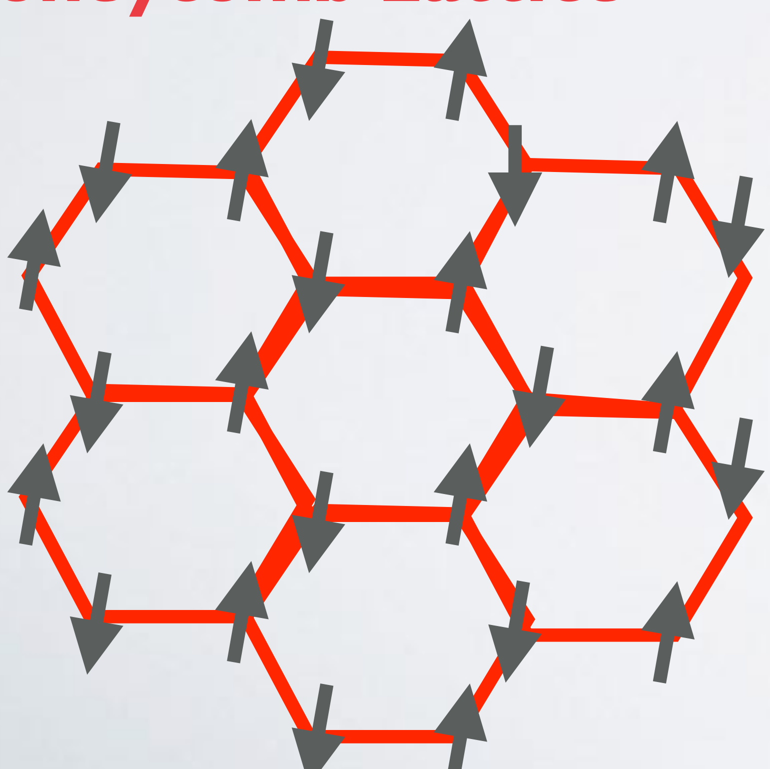
Square Lattice





Anti-ferromagnetic interaction: spins want to anti-align.

Honeycomb Lattice

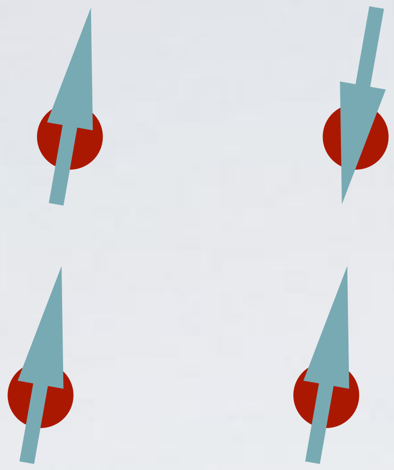


This is the exact wave-function in the Ising limit.

This phase survives down to $J_z = 1$

Bipartite lattices are all understood.

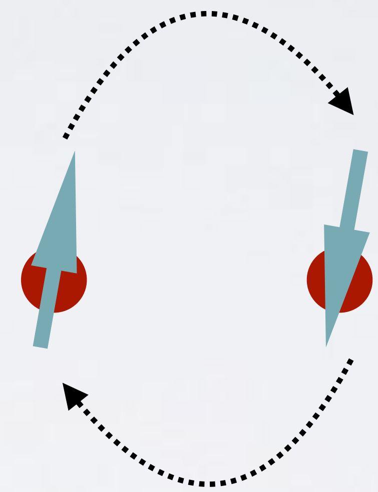
$$H_{ij}^{z} = \sigma_i^z \sigma_j^z$$



$-J_z$

$+J_z$

$$H_{ij}^{xy} = \left(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y\right)$$



$-J_{xy}$

The story of frustration.

$$H_{xxz} = H_{xy} + J_z \sum_{ij} S_i^z S_j^z \quad \text{(ising limit } J_z \to \infty\text{)}$$

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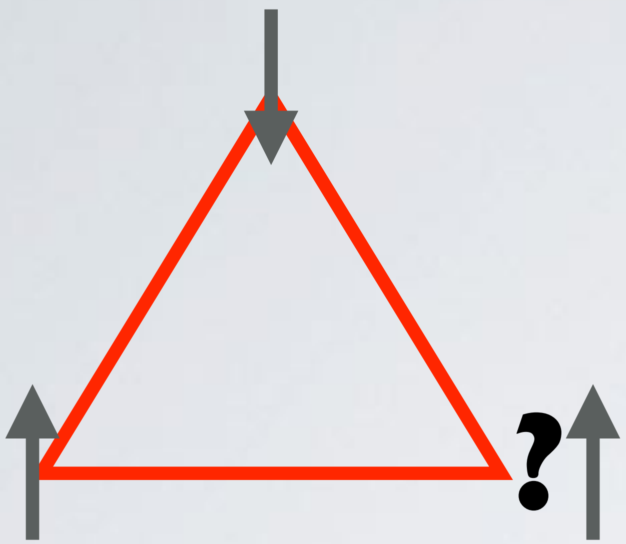
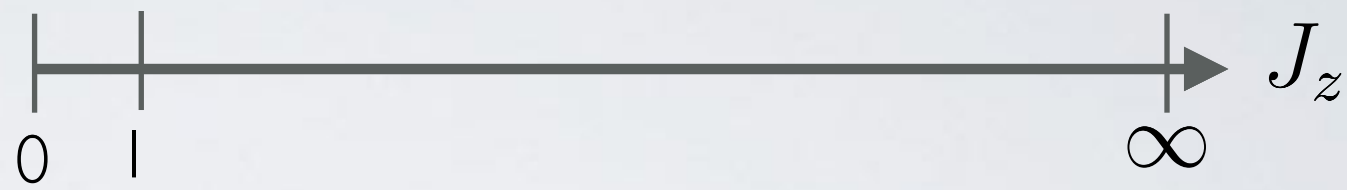
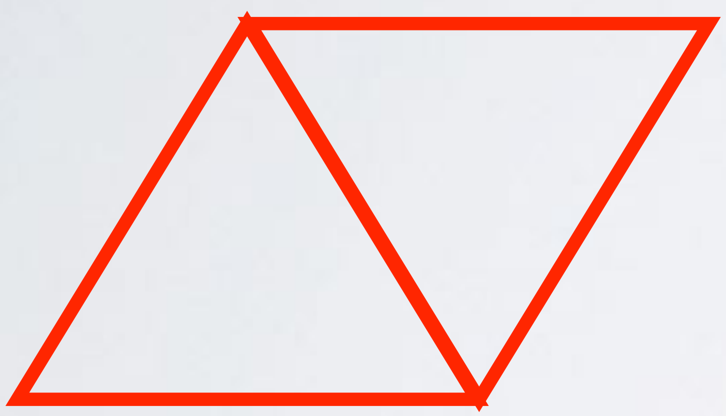
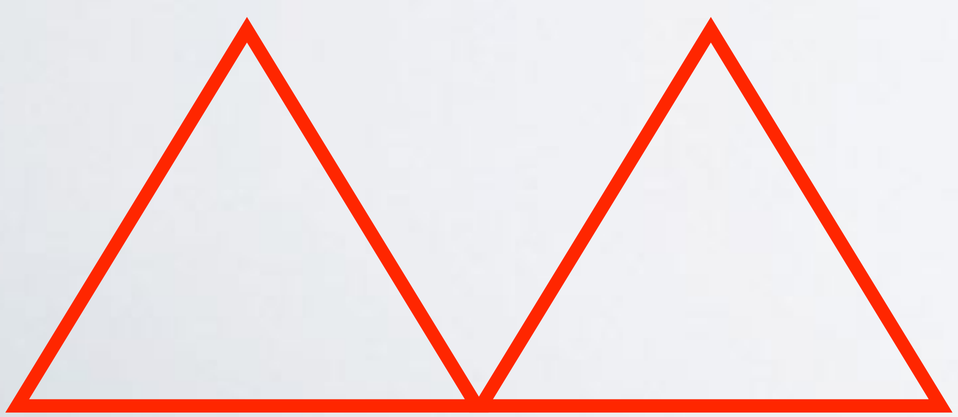




In this Ising limit, however you paste together triangles, there are many degenerate states





Frustrated magnets!

The *old* story of frustration.

$$H_{xxz} = H_{xy} + J_z \sum_{ij} S_i^z S_j^z \quad \text{(ising limit } J_z \to \infty\text{)}$$

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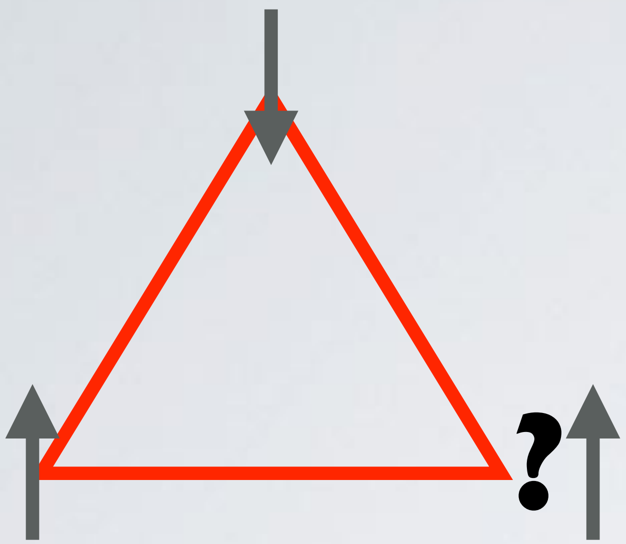

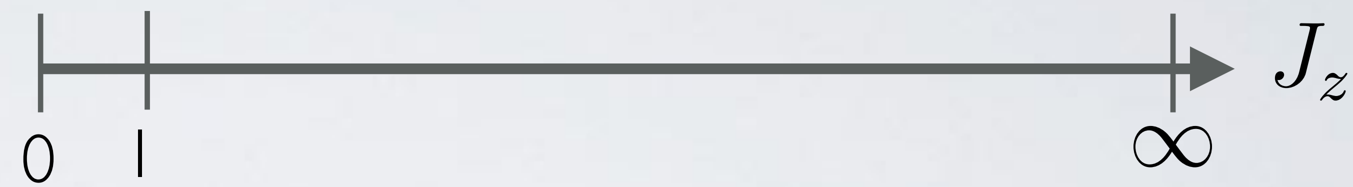

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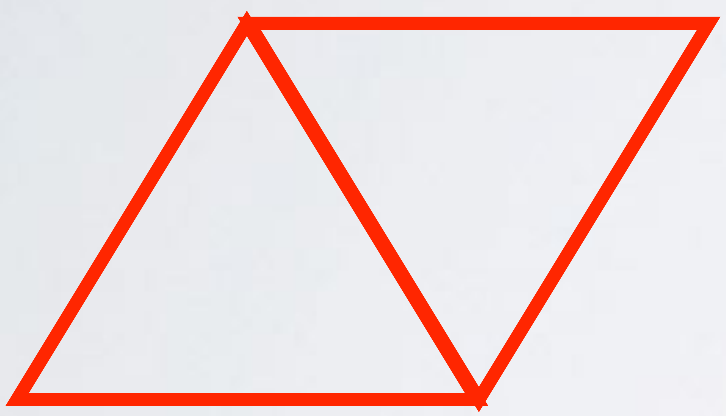
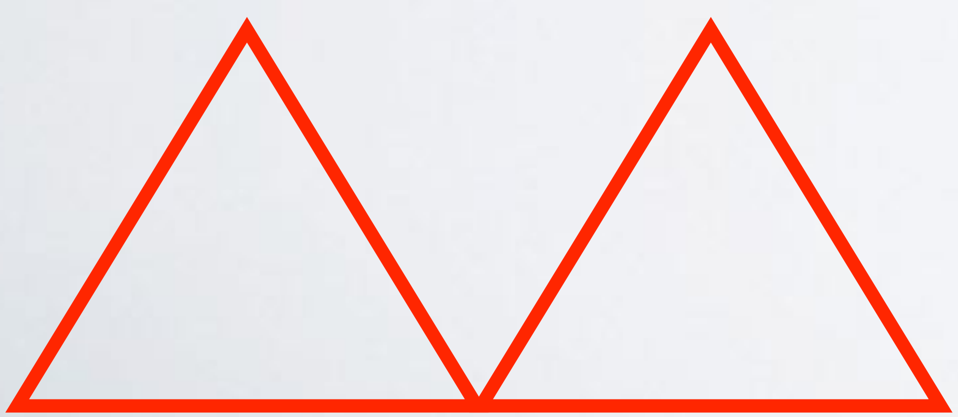
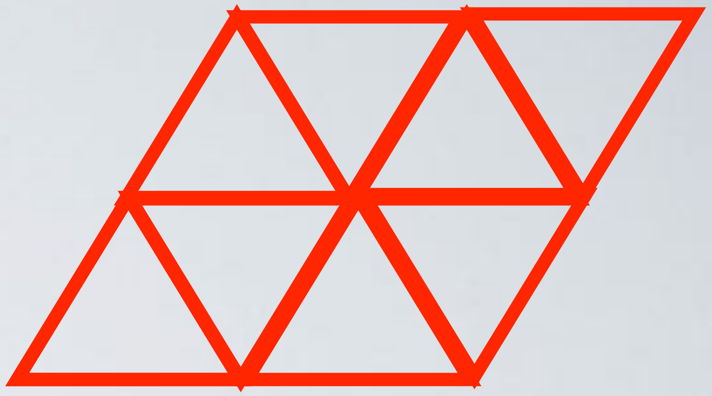




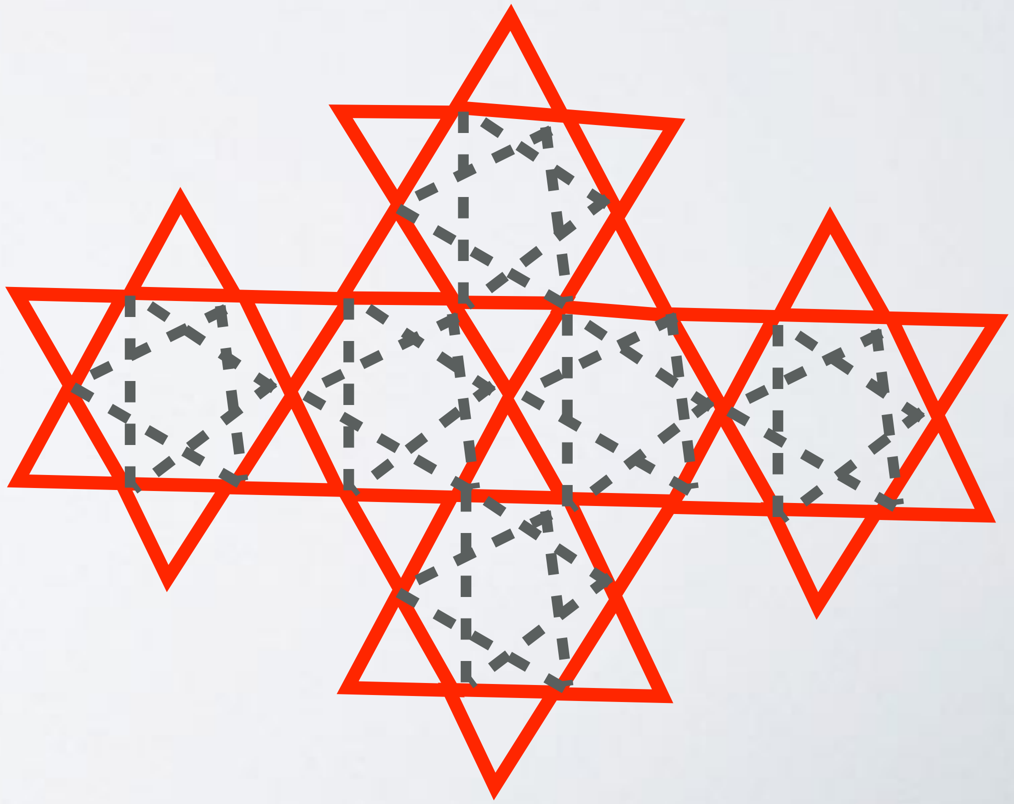
Frustrated magnets!

Triangle lattice



Kagome Lattice

J1-J2 Kagome Lattice



Kagome Materials

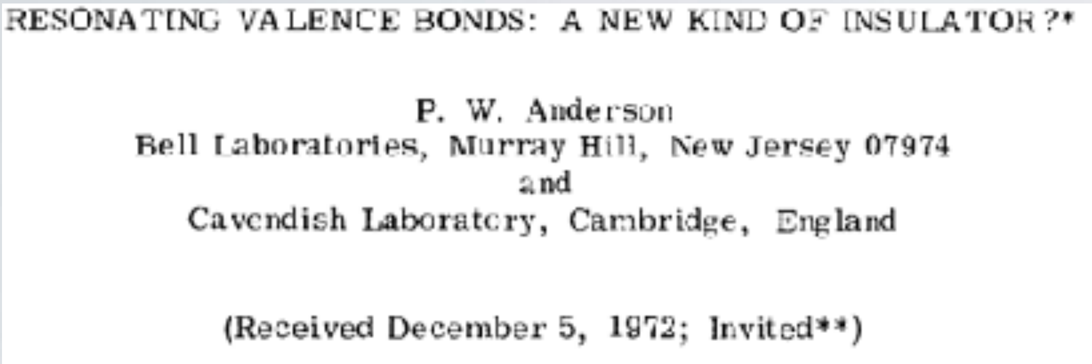
Herbertsmithite

Volborthite

Kapellasite

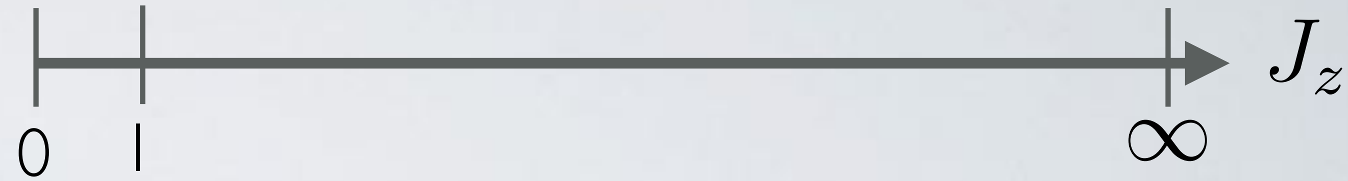
Vesigniette

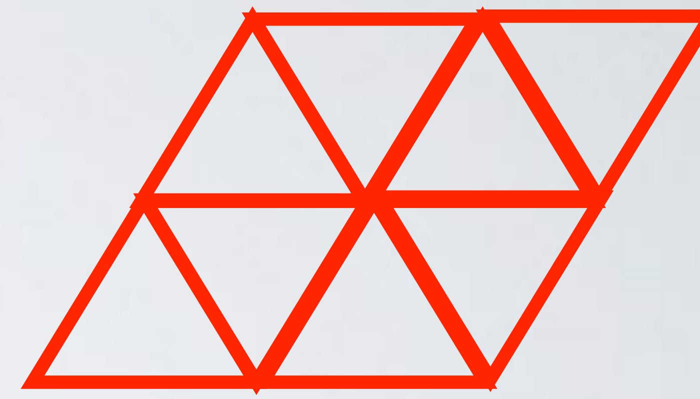
Phil Anderson suggested that the n.n. Heisenberg model on the triangular lattice wasn't a neel state (frustration!)


RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR ?*

P. W. Anderson
Bell Laboratories, Murray Hill, New Jersey 07974
and
Cavendish Laboratory, Cambridge, England

(Received December 5, 1972; Invited**)





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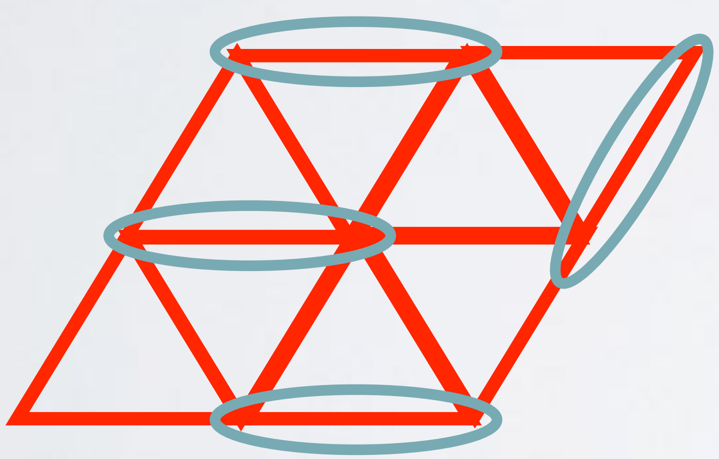
(today we would call such a thing a spin-liquid).

$$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



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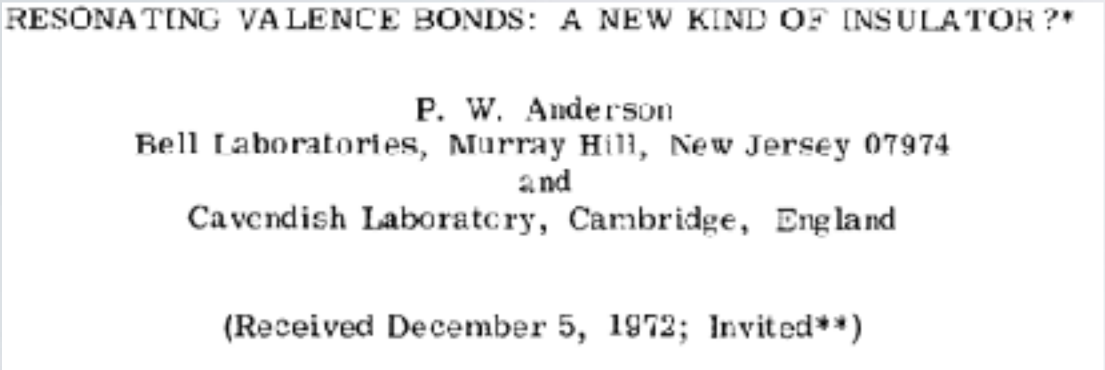

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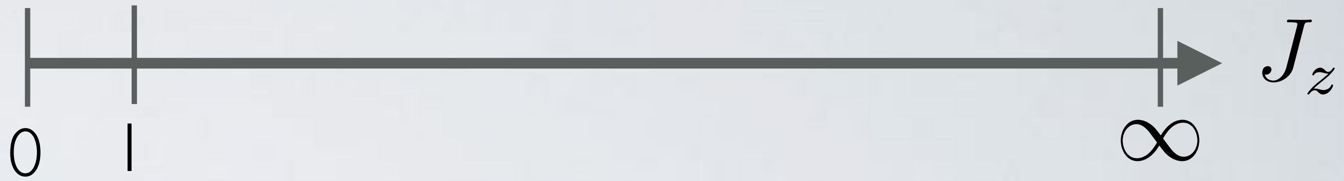

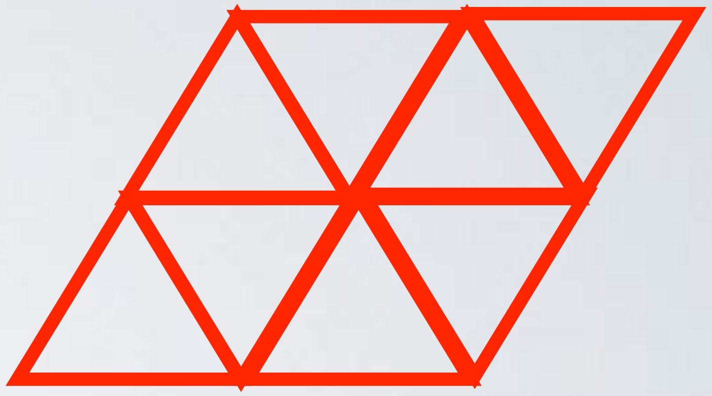

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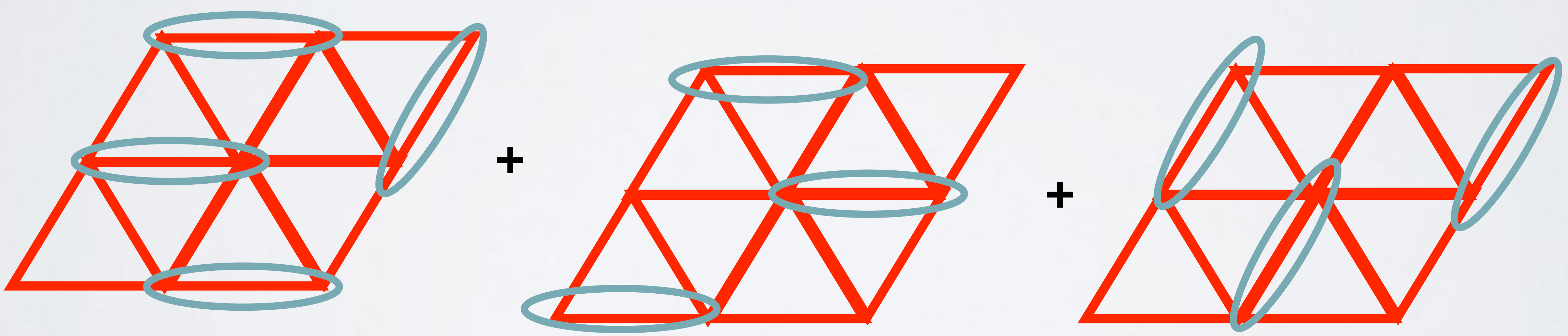
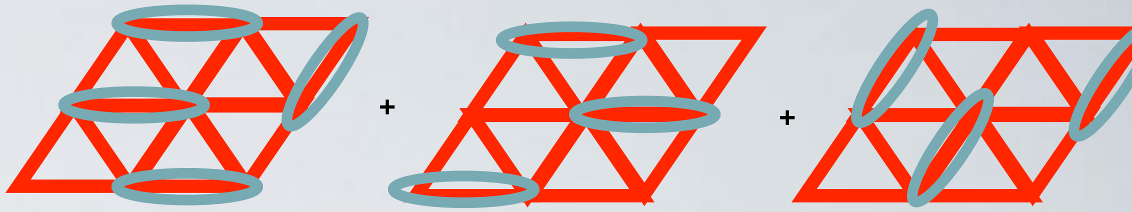


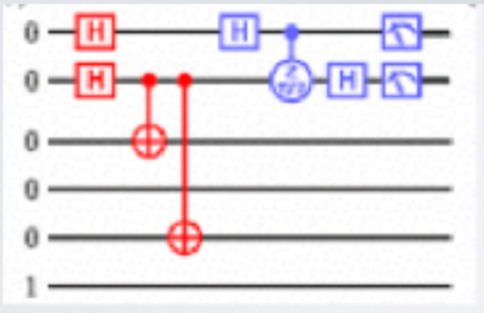
Like Benzene

The hunt for **spin liquids** is one of the forefront areas of condensed matter research!

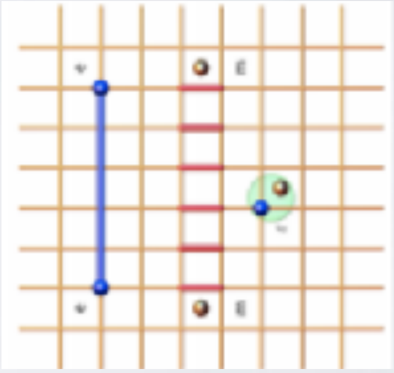


Beyond the Landau theory of phases - no broken symmetries!

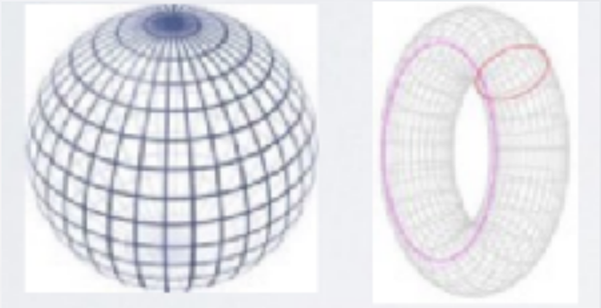
Long Range Entanglement - Can't be produced from a product state via a short quantum circuit



Fractionalized Excitations - Electron breaks into multiple emergent pieces.



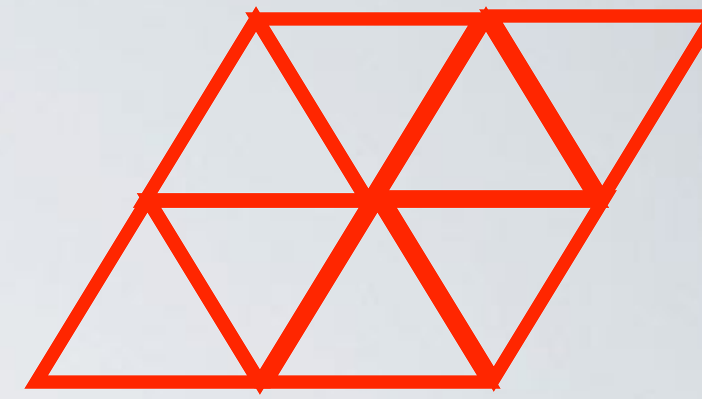
Topological Degeneracy - Manifold dependent geometry



The search for spin liquids is truly a hunt. We haven't had any good story for what sort of lattices should support spin liquids.

Was Phil Anderson right? Was the triangular lattice at the Heisenberg point a spin-liquid?

No!

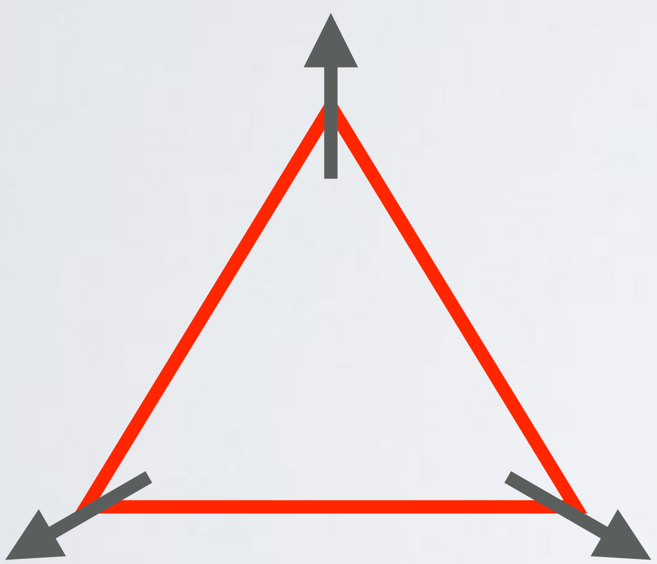


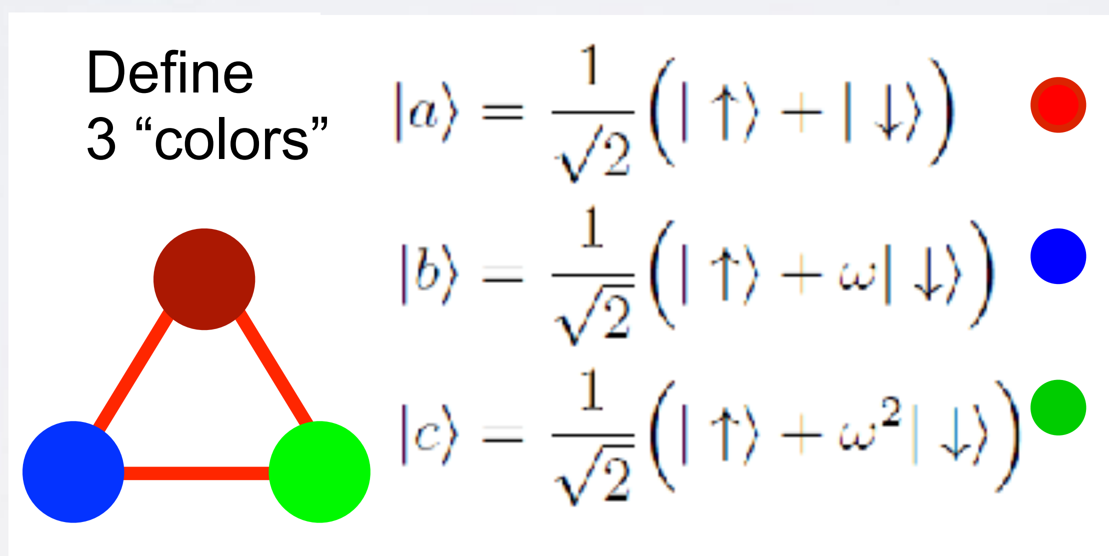
How do we know?

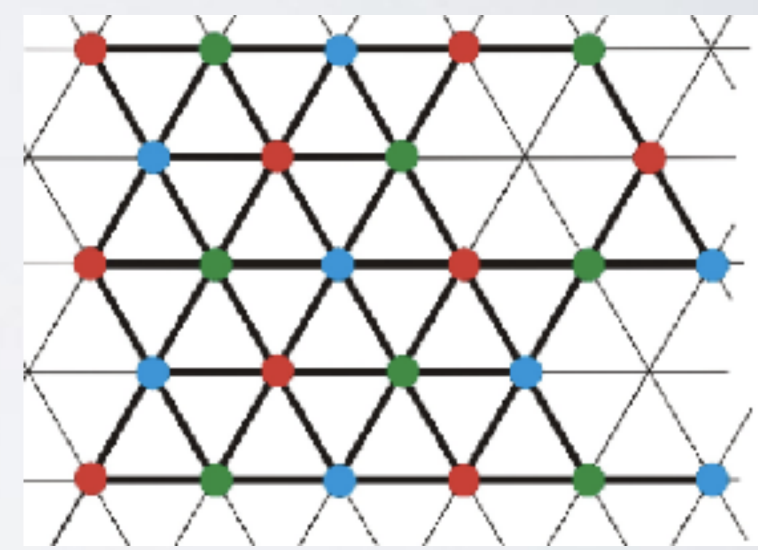
Years and years of numerics…

- Huse-Elser
- Green's Function Monte Carlo

The triangular lattice is 120 degree ordered.
No hint from the frustrated Ising limit.







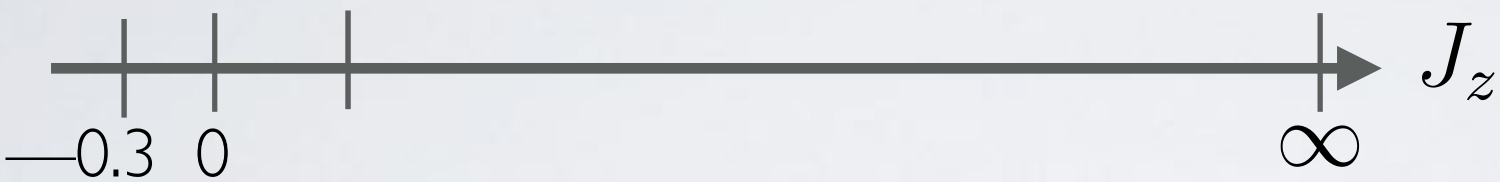
Morally, but not actually the wave-function. The actual wave-function is a highly complicated dressed version of this (that's why it took forever to verify this).

Why? Because numerics says so….

How about if we have negative Jz?

$$H_{xxz} = H_{xy} + J_z \sum_{ij} S_i^z S_j^z \quad \text{(ising limit } J_z \to \infty\text{)}$$

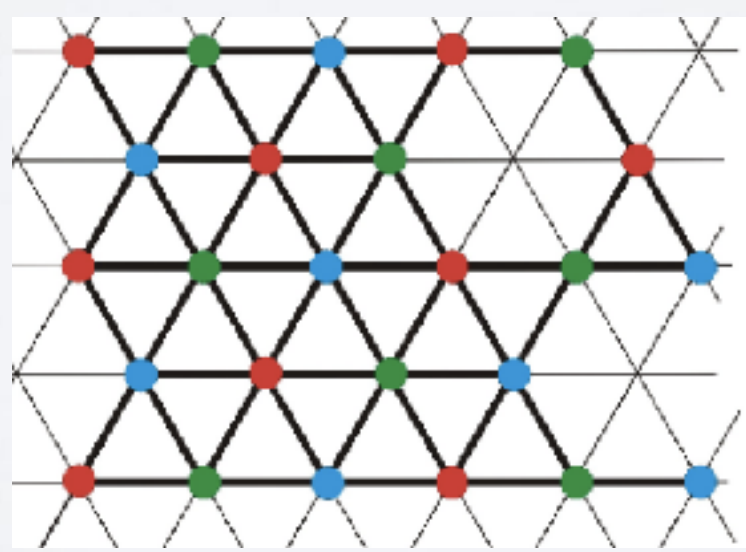
$$H_{\text{ising}} = \sum_{ij} S_i^z S_j^z$$



Wouldn't expect a spin-liquid.
But at least Ising-like?

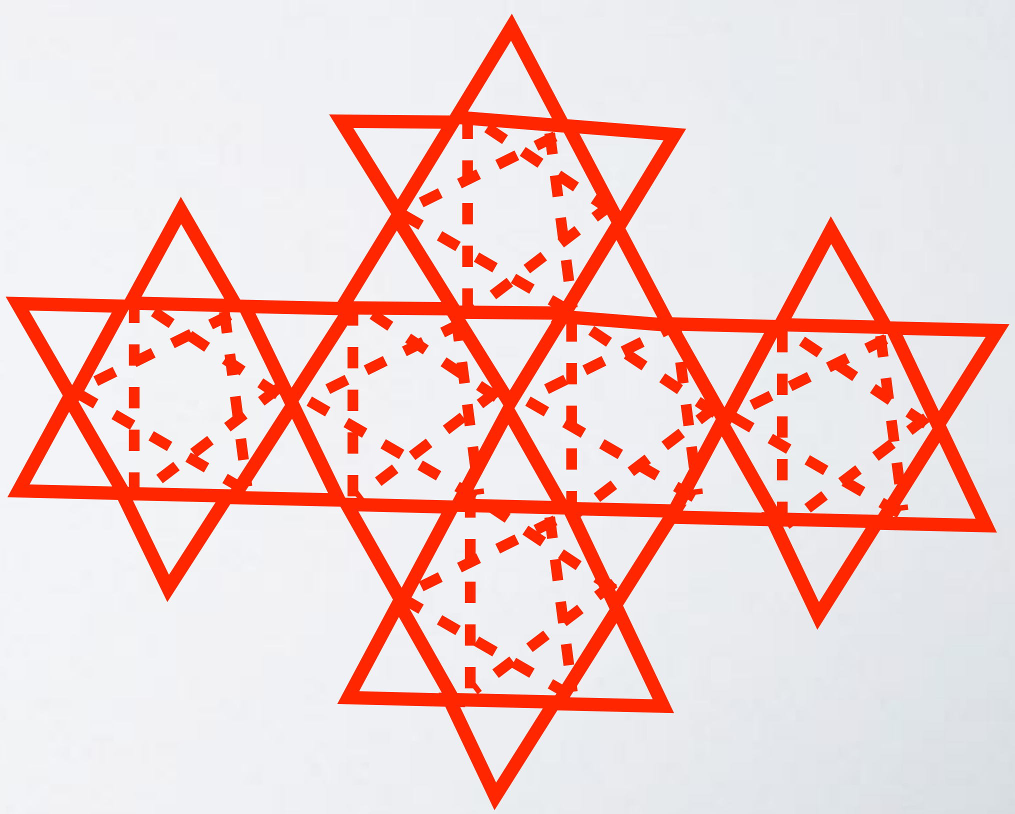
Nope...

coplanar



What about the J1-J2 kagome lattice?

$$J1 = 1$$
$$J2 = 1$$
$$J_z = 1$$



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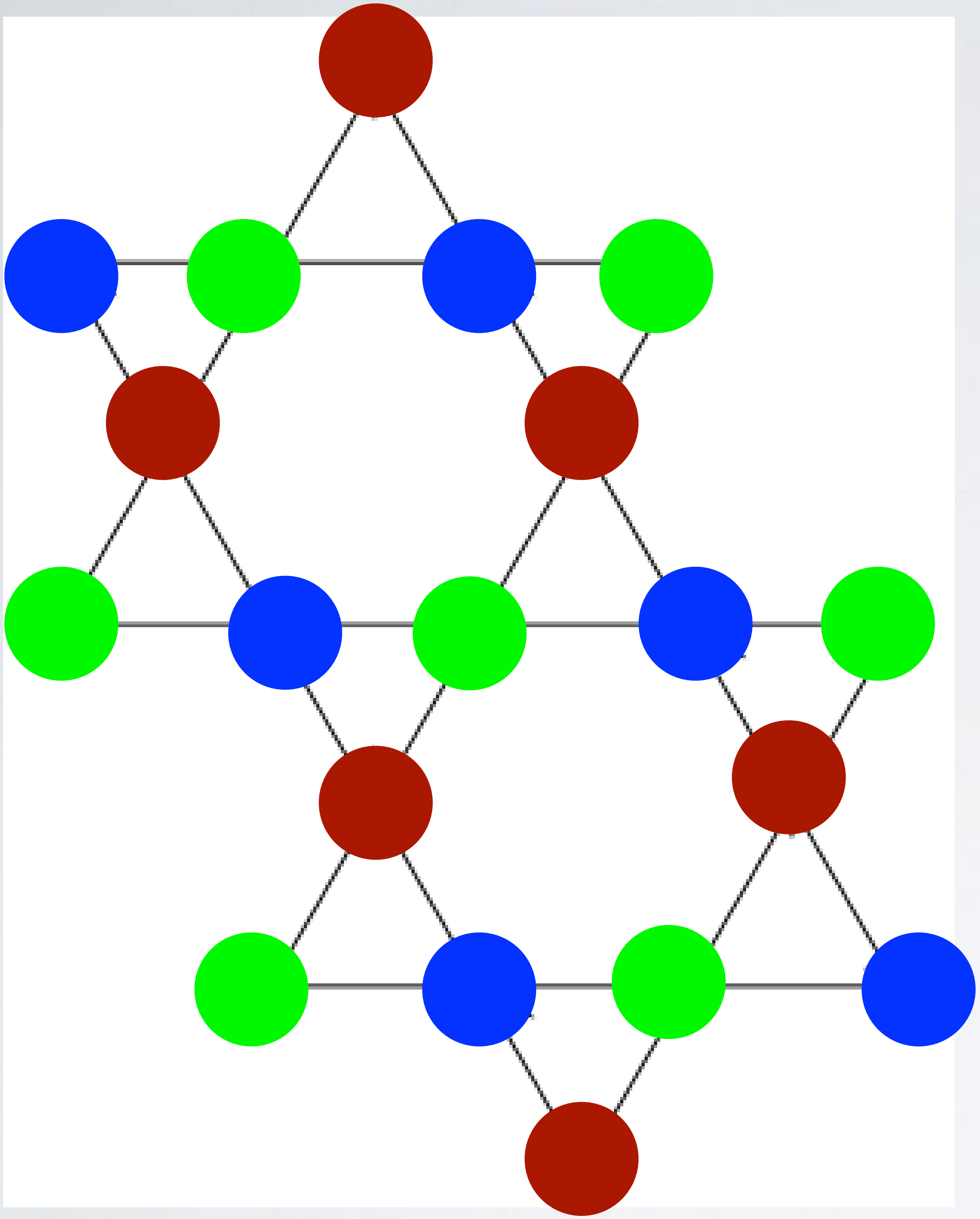
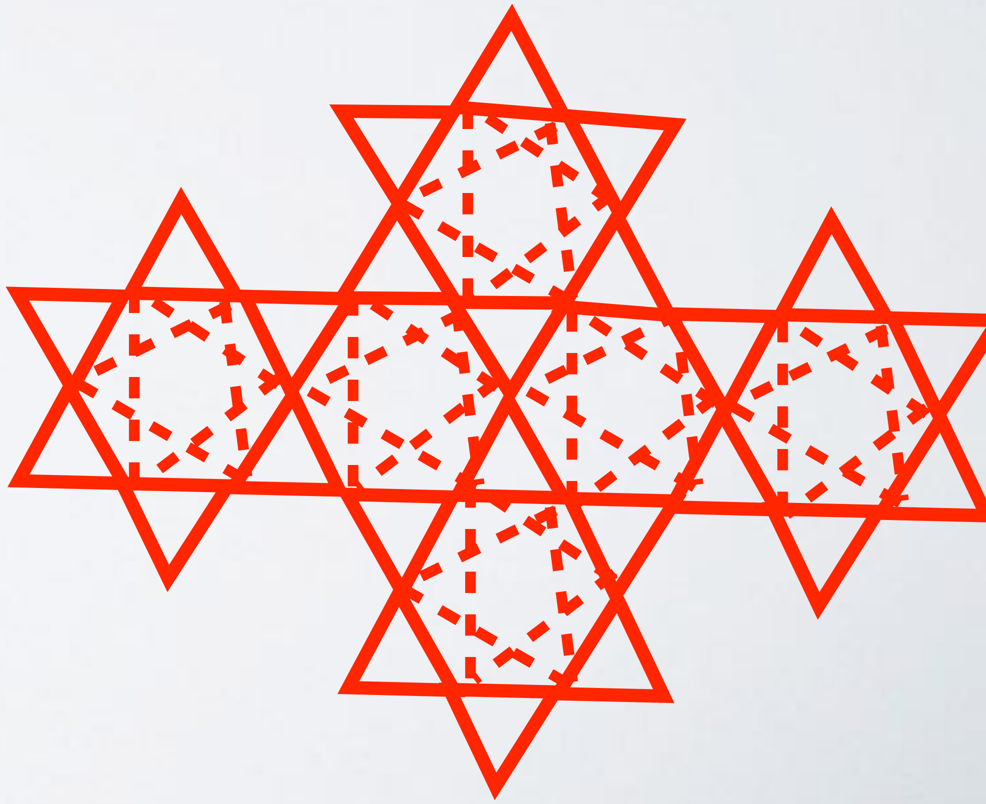


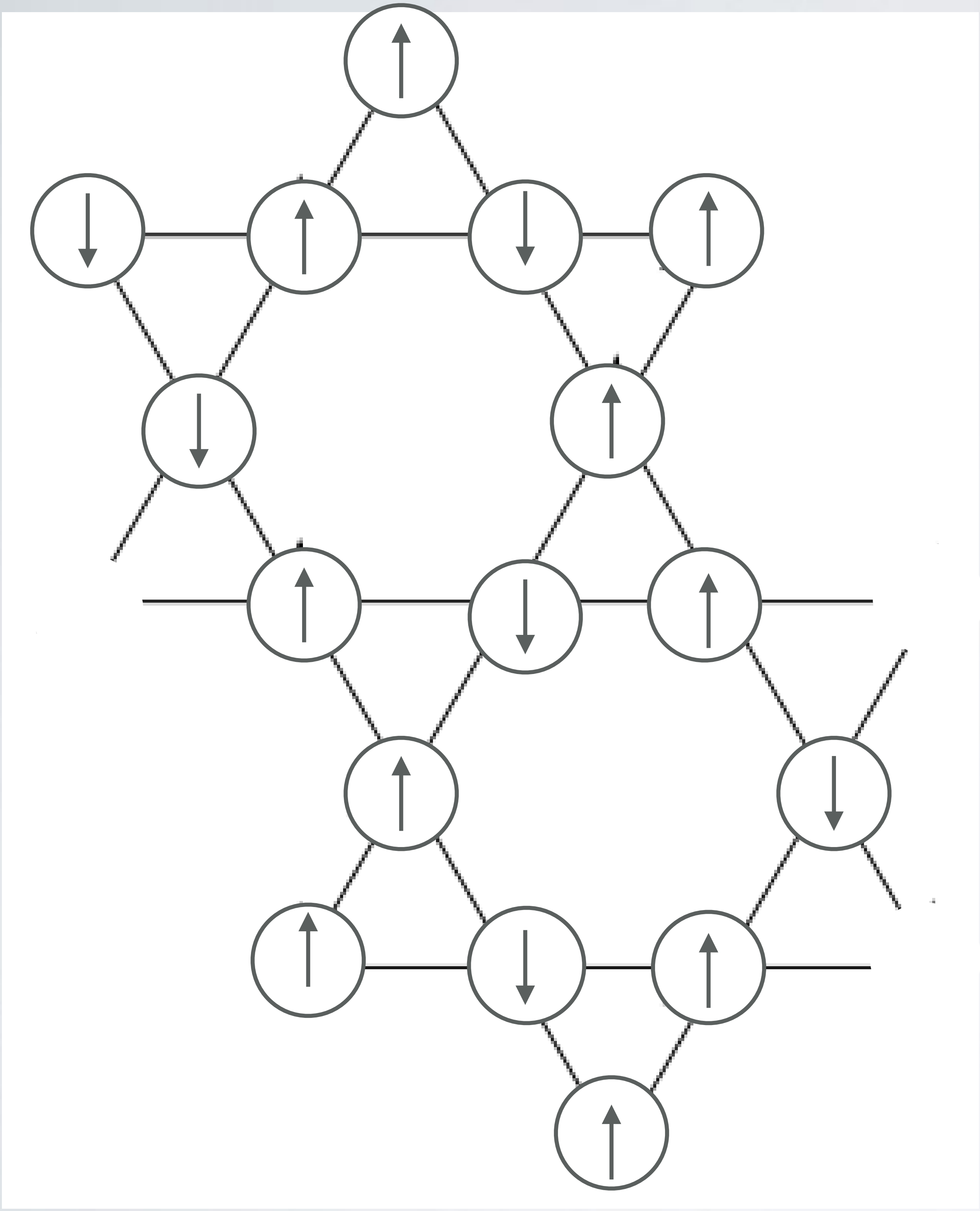
coplanar (q=0)



Why? Because numerics says so….

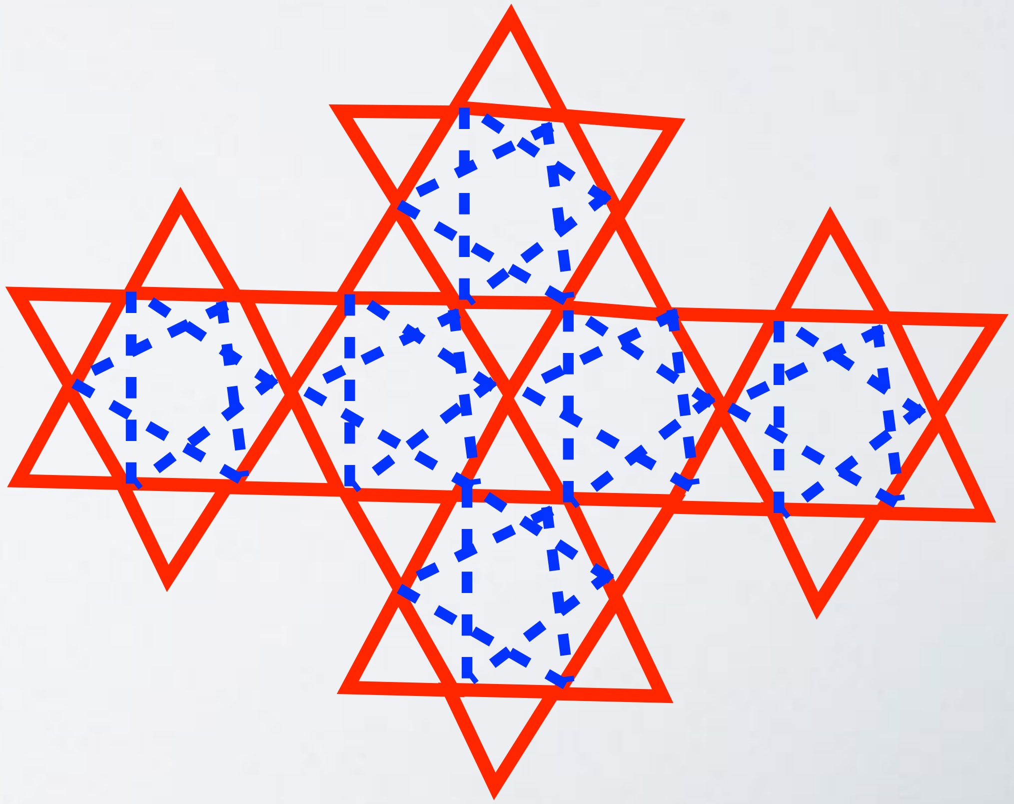
What about the J1-J2 kagome lattice?

$$J1 = 1$$
$$J2 = -1$$
$$J_z = \infty$$



This is the answer in the Ising Limit

Surely, it must be this at $J_z = 1$



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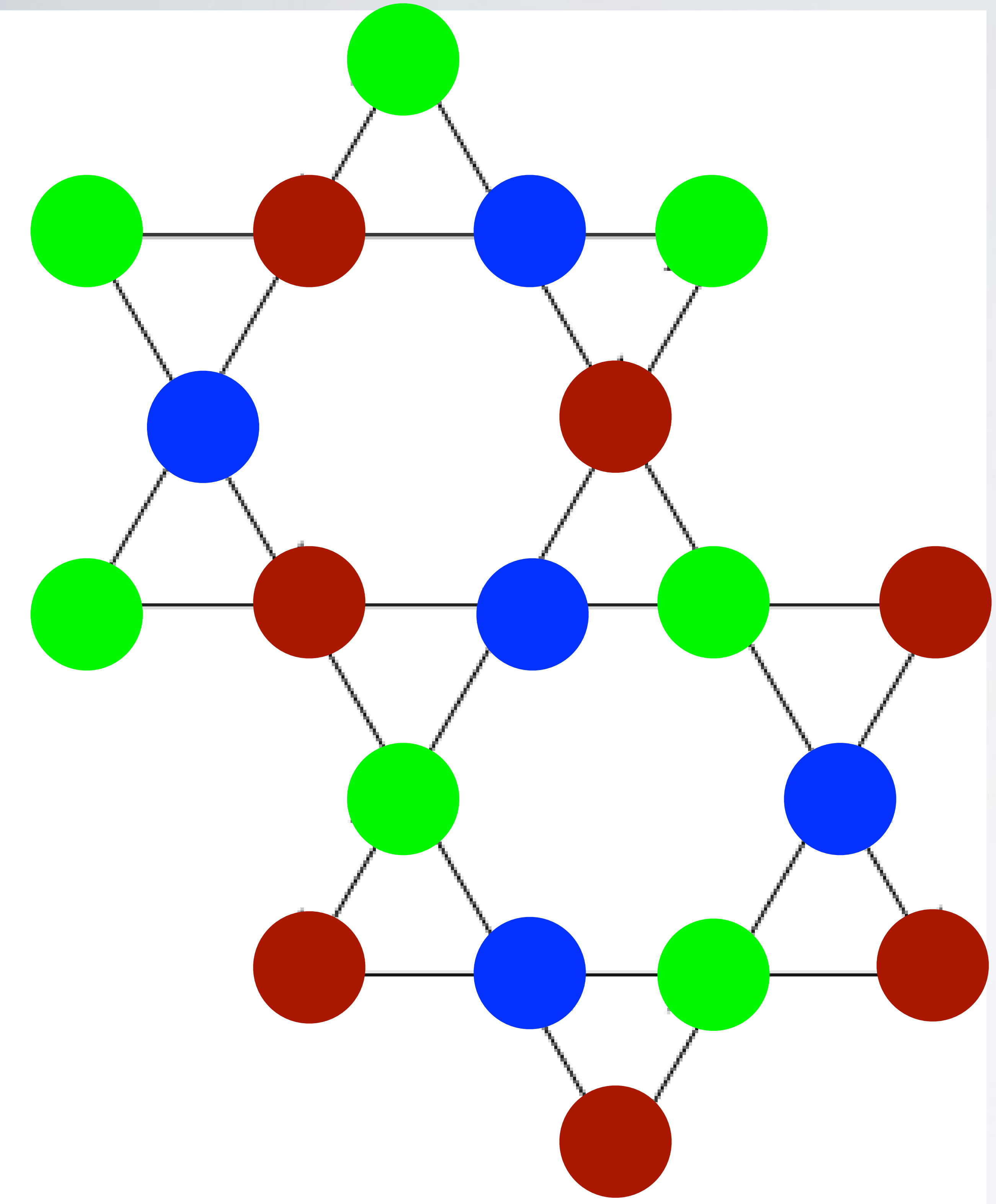
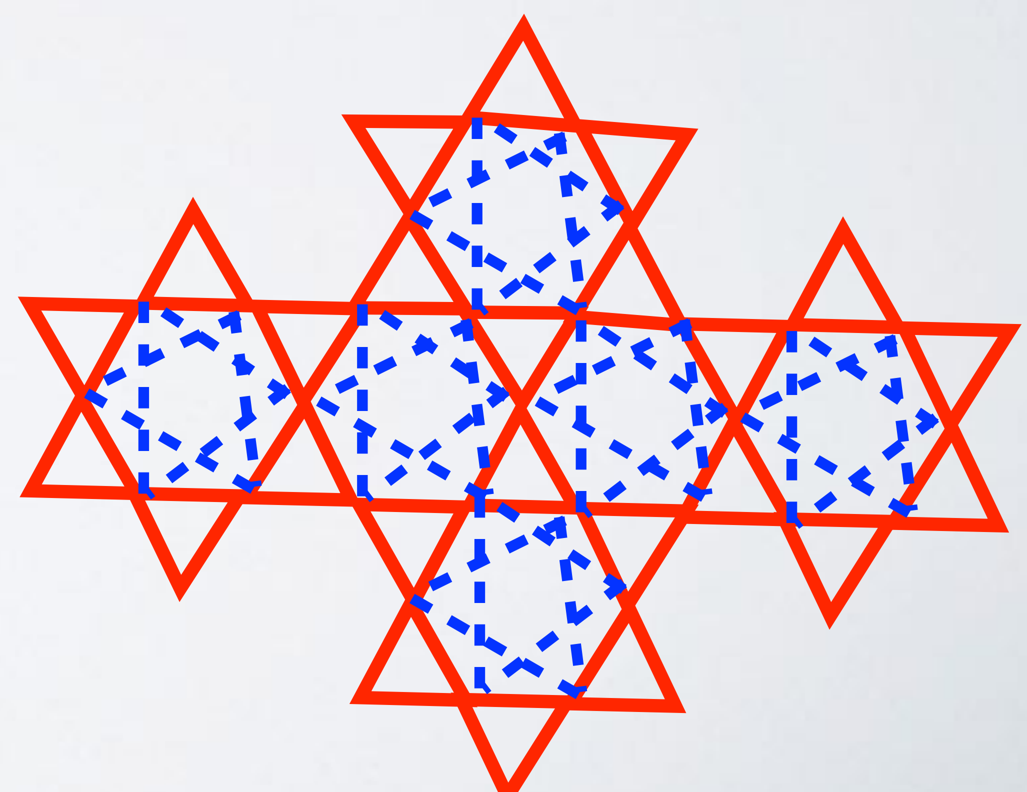


coplanar $\sqrt{3} \times \sqrt{3}$



Why? Because numerics says so….

Is everything coplanar?

Triangular: $J_z = 1$ coplanar

$J_z = -0.3$ coplanar

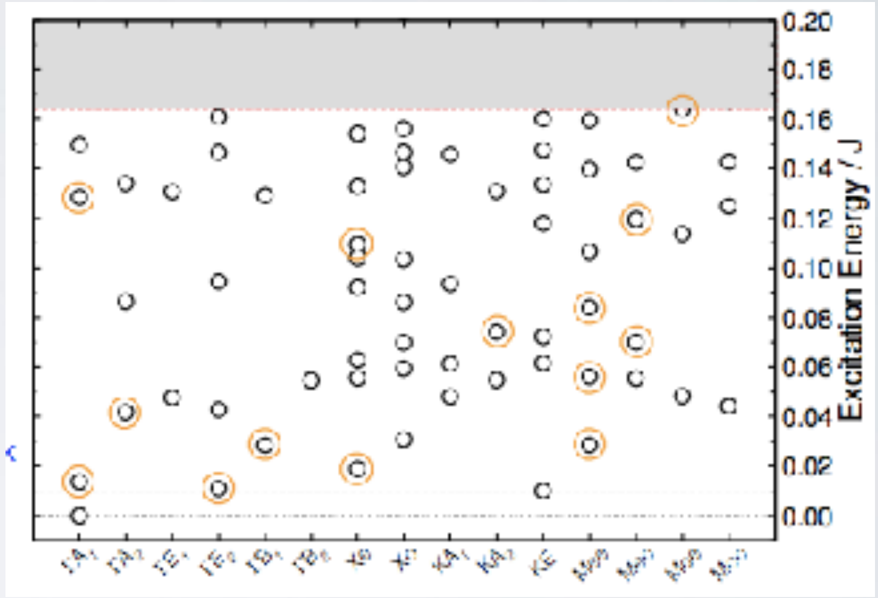
Kagome: $J_2 = 1 \quad J_z = 1$ coplanar

$J_2 = -1 \quad J_z = 1$ coplanar

$J_2 = 0.5 \quad J_z = 1$ Spin Liquid! (White/Huse)

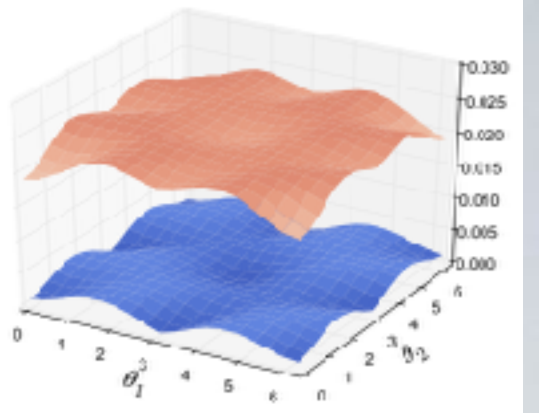
but a low energy mess

Why? years and years of numerics



Lauchli, et. Al

Kagame spin liquids everywhere....



Z2 (or Dirac) Spin Liquid — Heisenberg (White/Huse)

Chiral Spin Liquid — 2/3 Plateau (this work)

1/3 Plateau (Donna Sheng)

Chiral Term (Bela Bauer, Andreas Ludwig)

J1, J2, J3 (Donna Sheng)

Experimental evidence for spin-liquids in hyperkagome

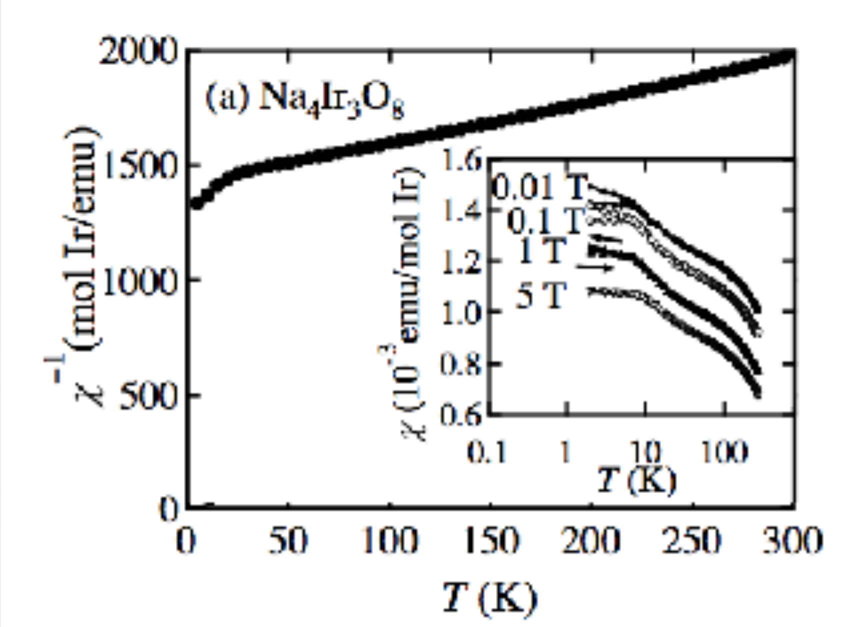
$Na_4Ir_3O_8$

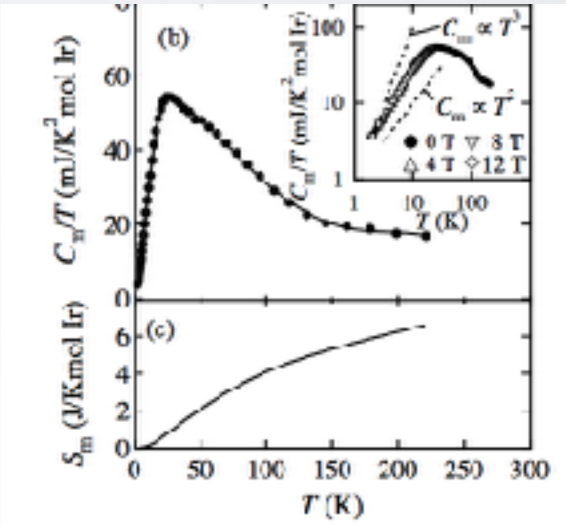
(depleted pyrochlore)

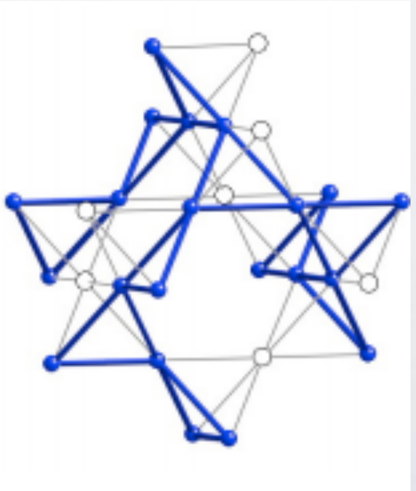
No sign of magnetic ordering down to a few Kelvin

Curie-Weiss temperature of 650K

Gapless excitations







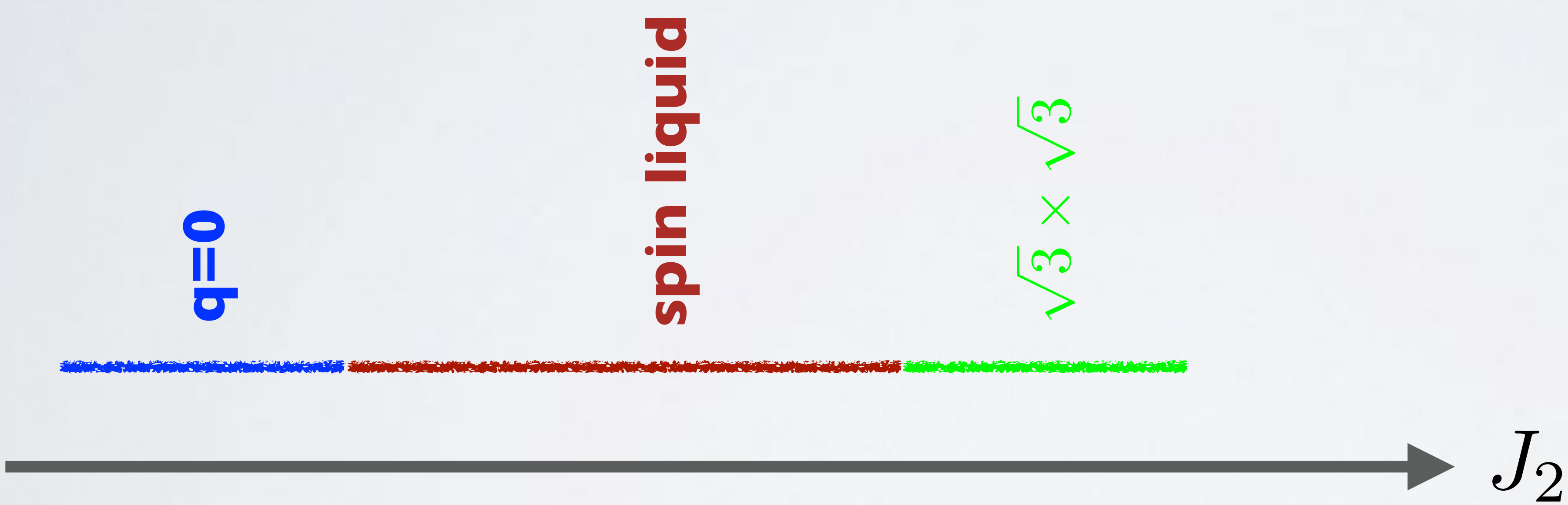
Y. Okamoto, M. Nohara, H. Aruga-Katori, and H. Takagi (2006), arXiv:0705.2821

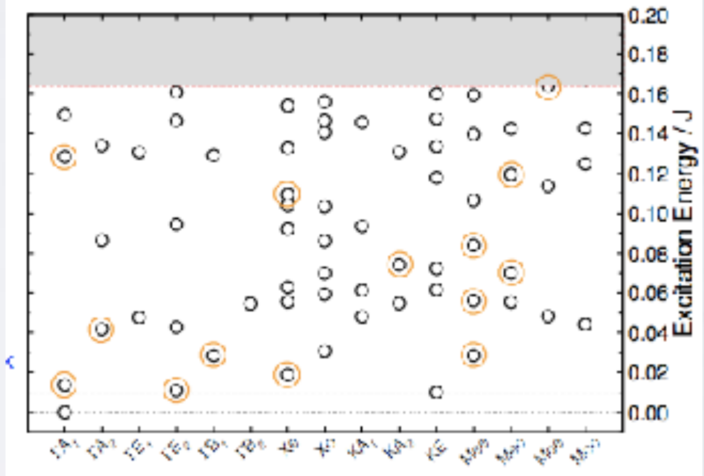
The Ising frustration doesn't seem to be a good explanation for anything.

(1) Why hyperkagome and kagome and not triangular?

Both are equally frustrated in the ising limit.

(2) Ising seems to have little to do with the coplanar phases





(3) Why does kagome have so many real phases and so many competing low-energy states?

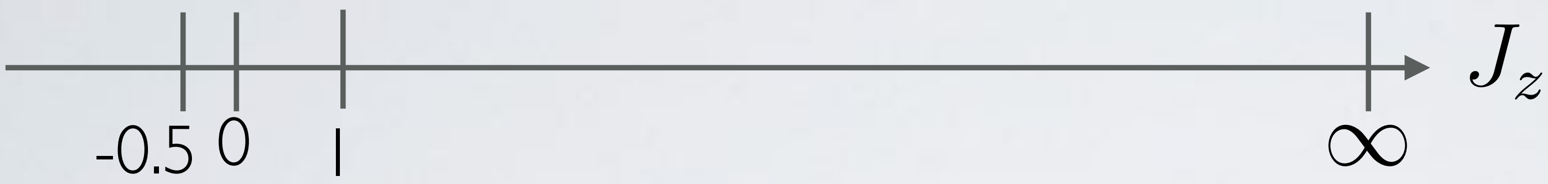
Is the take away point that strongly correlated systems is hard.

So… use numerics :)

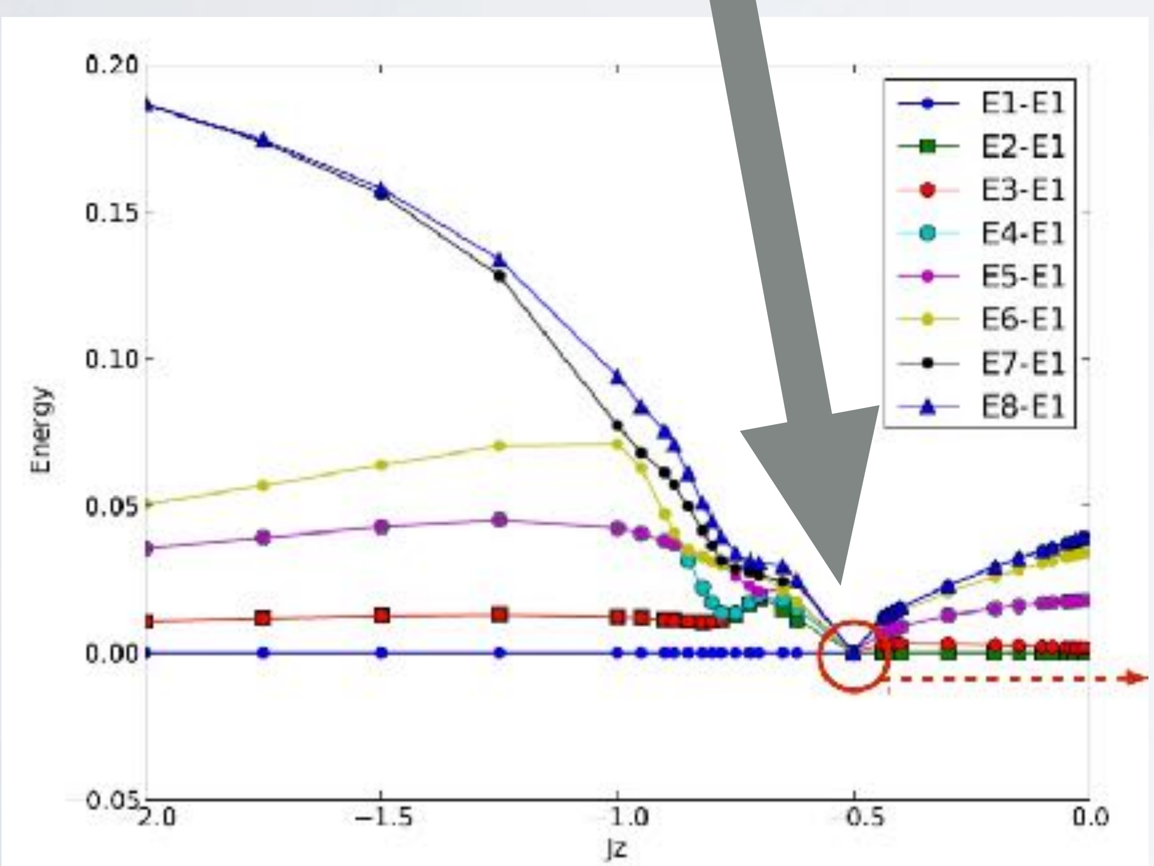
Q: Is there a better story?

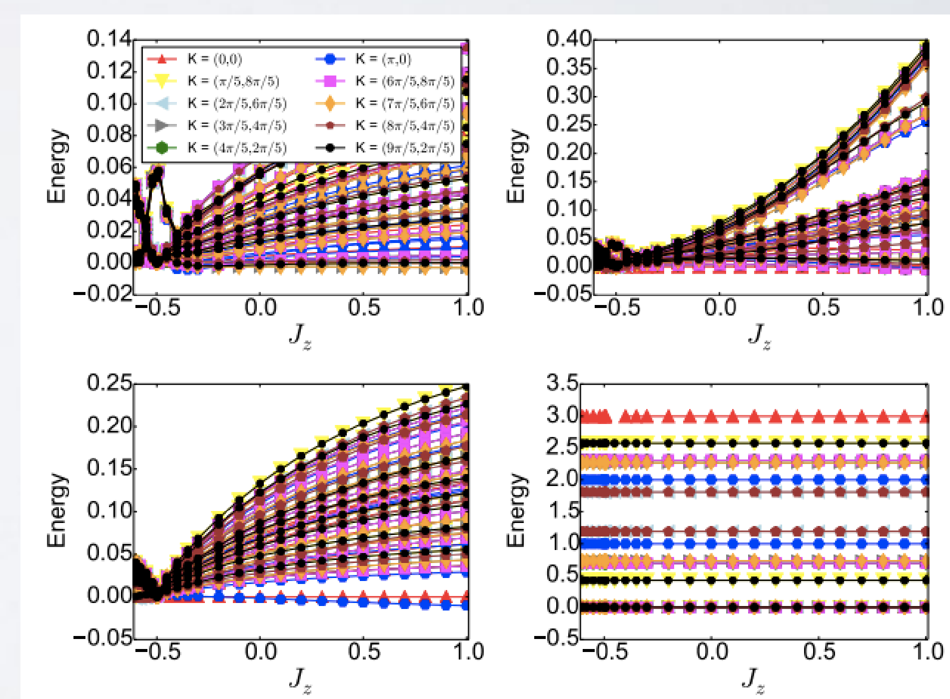
An interesting discovery.... (amazing it hasn't been known for 30 years)

$$H = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$



On the kagome:
massive exact degeneracy in the XXZ model!
exactly -J/4





What's going on? Who ordered this?

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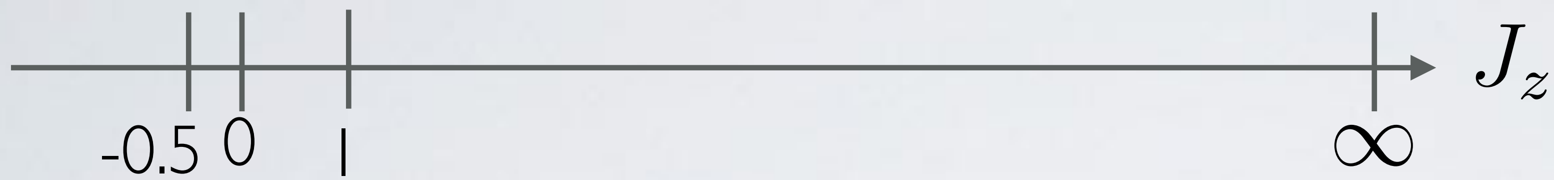
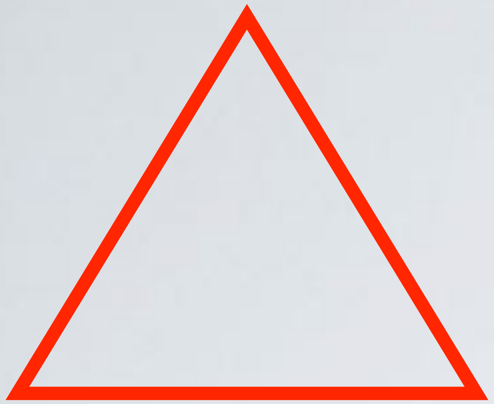


On the triangle:
no massive exact degeneracy.
exactly -J/2

What's going on? Who ordered this?

Who ordered that?

$$H_{XXZ0} = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$



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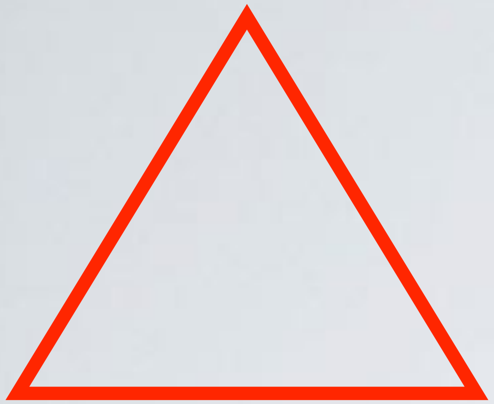
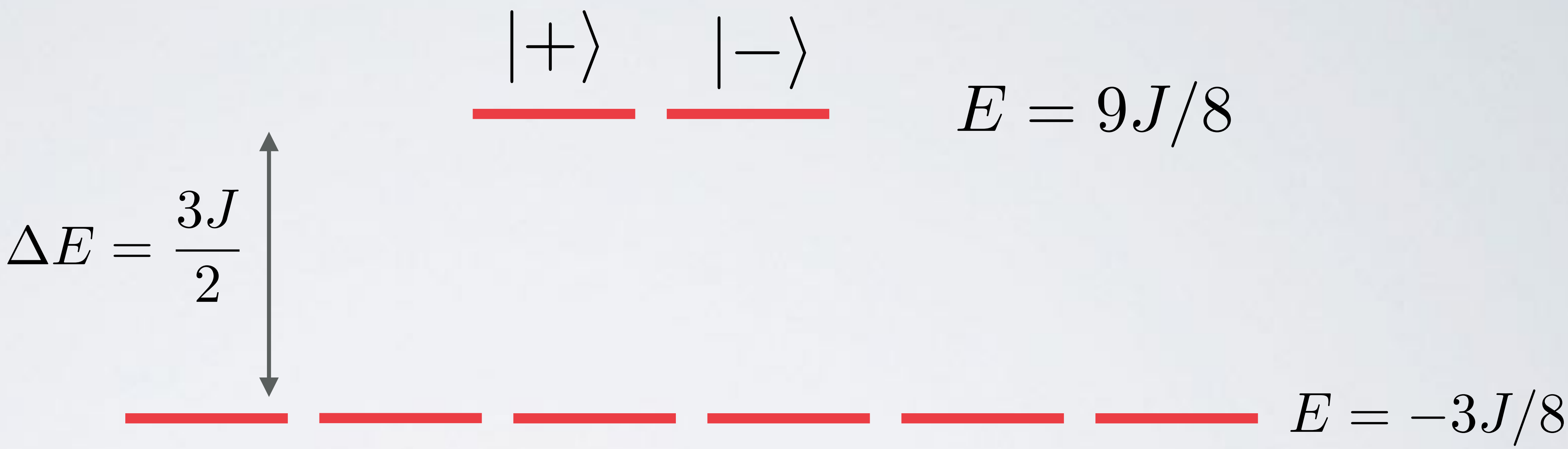




$$-\frac{3J}{8} + \frac{3J}{2}\left(|+\rangle\langle+| + |-\rangle\langle-|\right)$$

Projectors

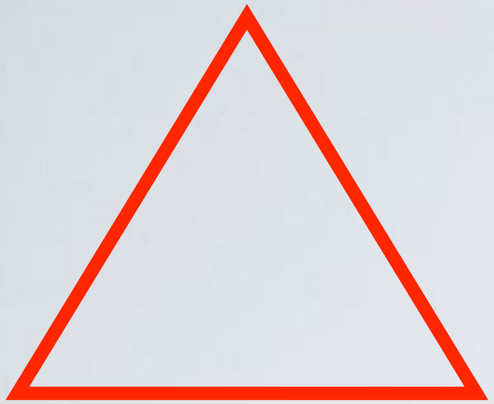
Constant

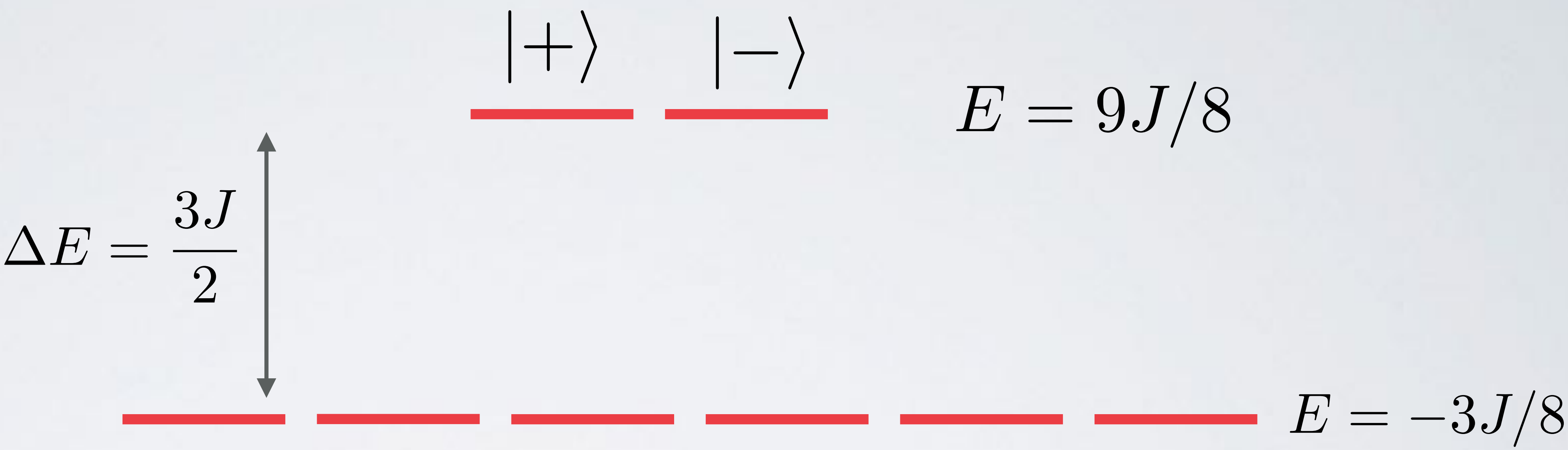
Positive coefficient

We want to **minimize the energy** by zeroing out the projectors

Who ordered that?

$$H_{XXZ0} = \sum_{ij} S_i^x S_j^x + S_i^y S_j^y - 0.5 \sum_{ij} S_i^z S_j^z$$





$$-\frac{3J}{8} + \frac{3J}{2}(|+\rangle\langle+| + |-\rangle\langle-|)$$

Projectors

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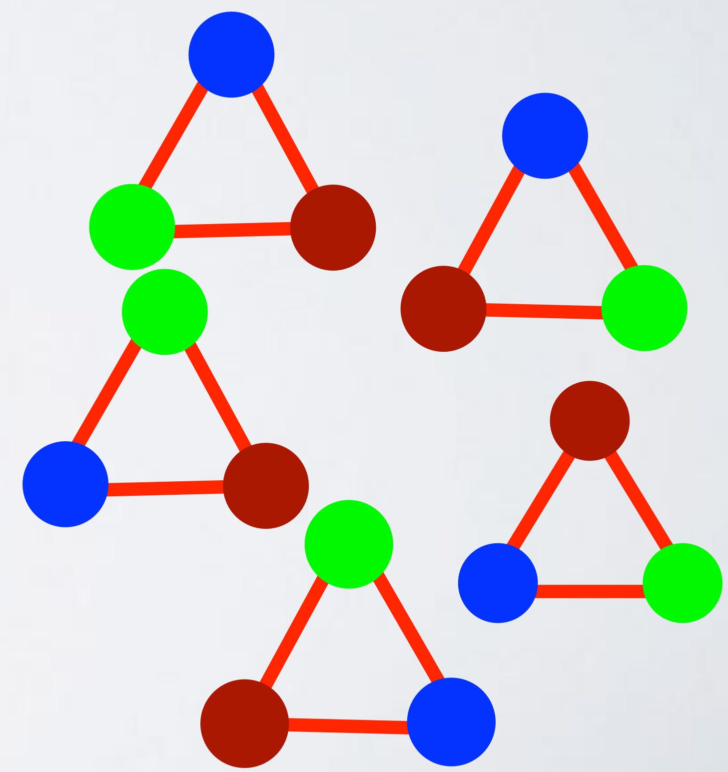

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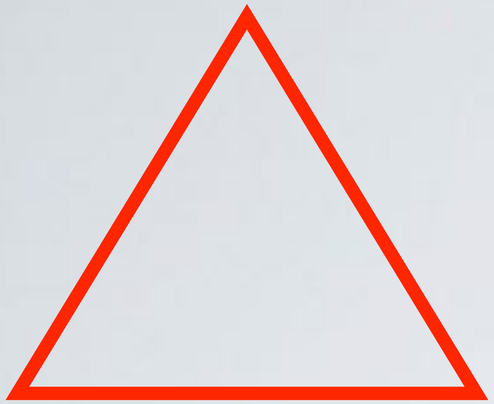
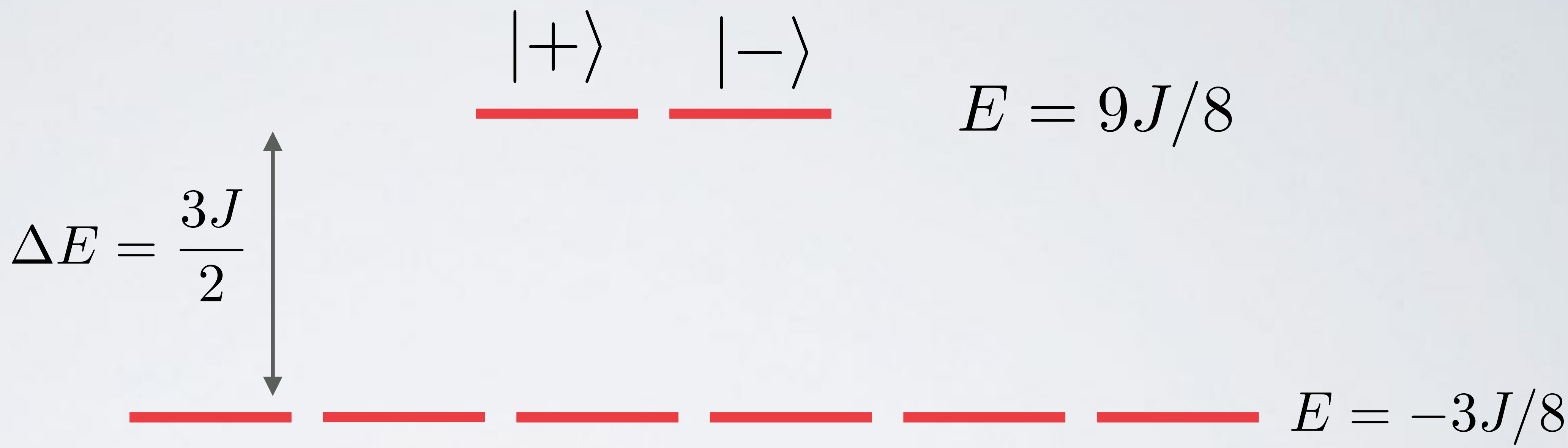




$$-\frac{3J}{8} + \frac{3J}{2}\left(|+\rangle\langle+| + |-\rangle\langle-|\right)$$

Projectors

Constant

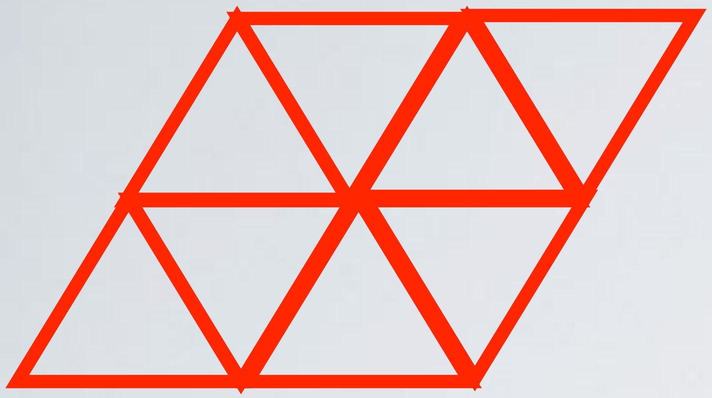
Positive coefficient

We want to **minimize the energy** by zeroing out the projectors

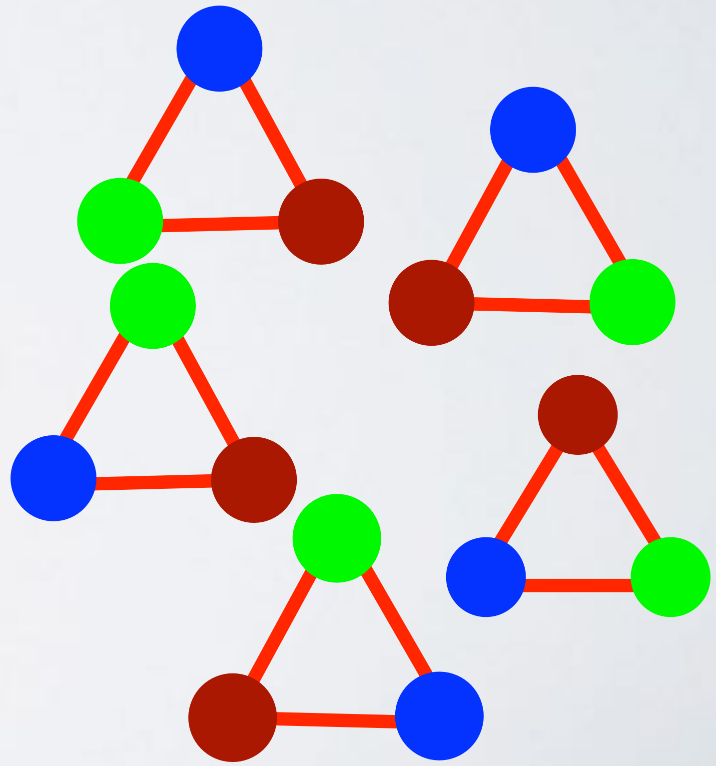
$$|1\rangle \equiv |\uparrow\uparrow\uparrow\rangle$$
$$|2\rangle \equiv \frac{1}{\sqrt{3}}\left(|\uparrow\uparrow\downarrow\rangle + \omega|\uparrow\downarrow\uparrow\rangle + \omega^2|\downarrow\uparrow\uparrow\rangle\right)$$
$$|3\rangle \equiv \frac{1}{\sqrt{3}}\left(|\uparrow\uparrow\downarrow\rangle + \omega^2|\uparrow\downarrow\uparrow\rangle + \omega|\downarrow\uparrow\uparrow\rangle\right)$$
$$|4\rangle \equiv \frac{1}{\sqrt{3}}\left(|\downarrow\downarrow\uparrow\rangle + \omega|\downarrow\uparrow\downarrow\rangle + \omega^2|\uparrow\downarrow\downarrow\rangle\right)$$
$$|5\rangle \equiv \frac{1}{\sqrt{3}}\left(|\downarrow\downarrow\uparrow\rangle + \omega^2|\downarrow\uparrow\downarrow\rangle + \omega|\uparrow\downarrow\downarrow\rangle\right)$$
$$|6\rangle \equiv |\downarrow\downarrow\downarrow\rangle$$

What about many triangles?

$$H = \sum_{\Delta} H_{XXZ0}$$



Paste together ground states over individual triangles



What about many triangles?

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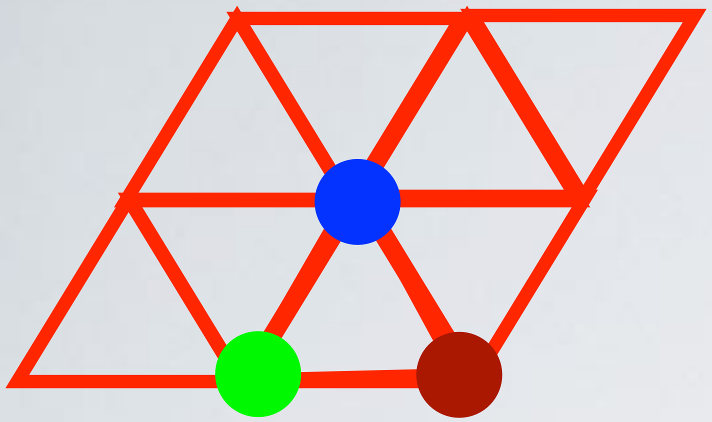
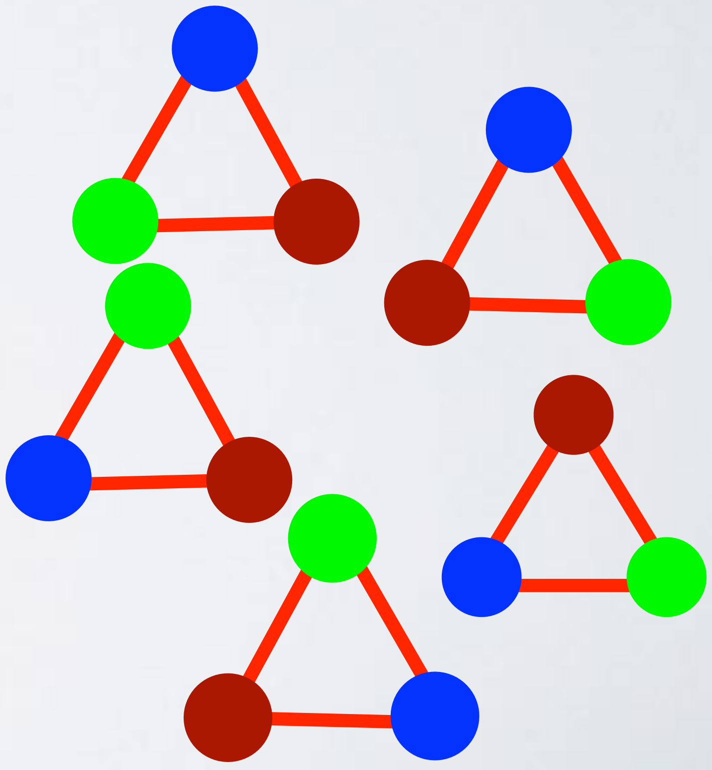


Paste together ground states over individual triangles



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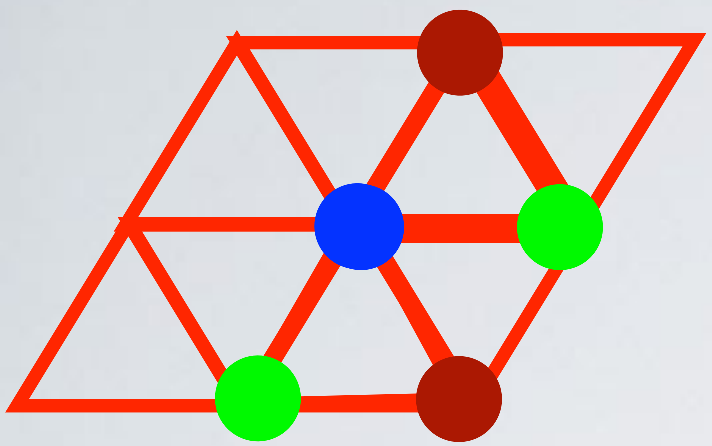
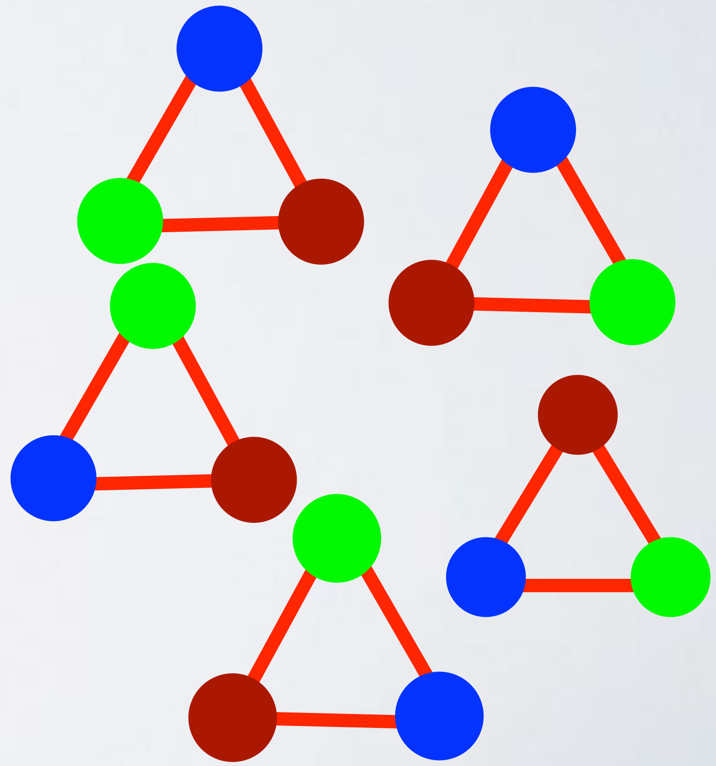


Paste together ground states over individual triangles



What about many triangles?

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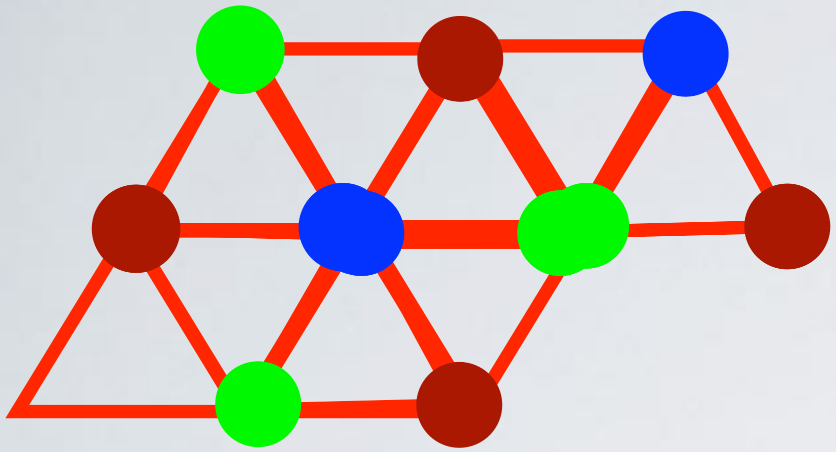
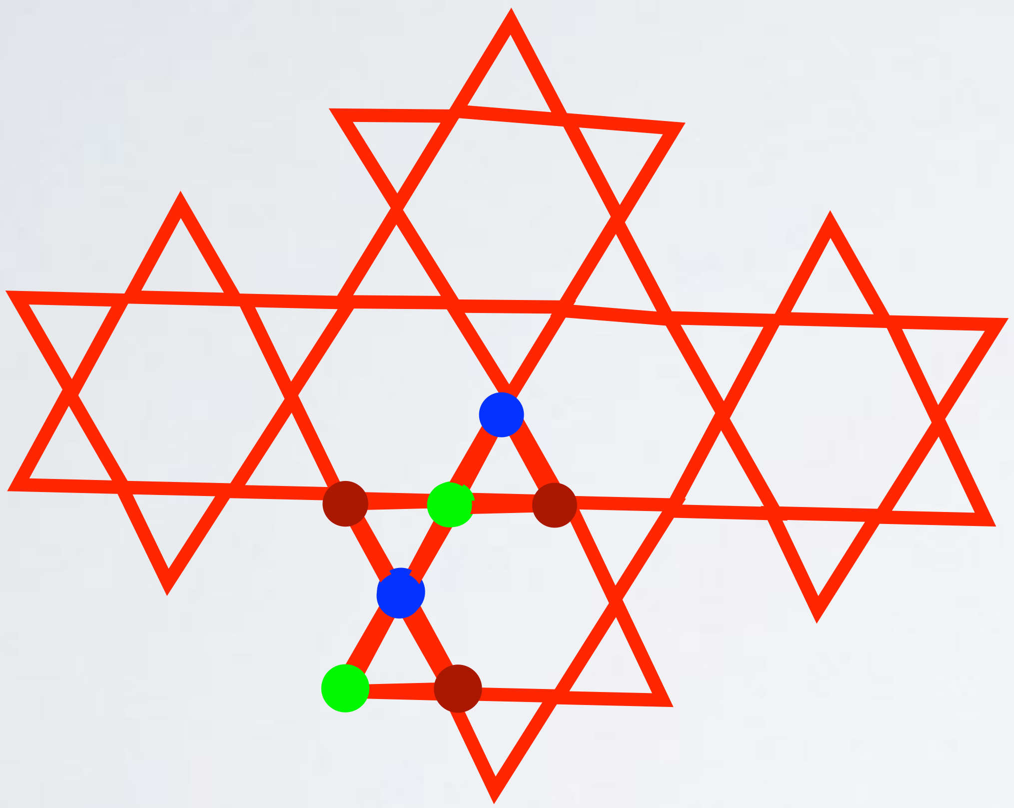
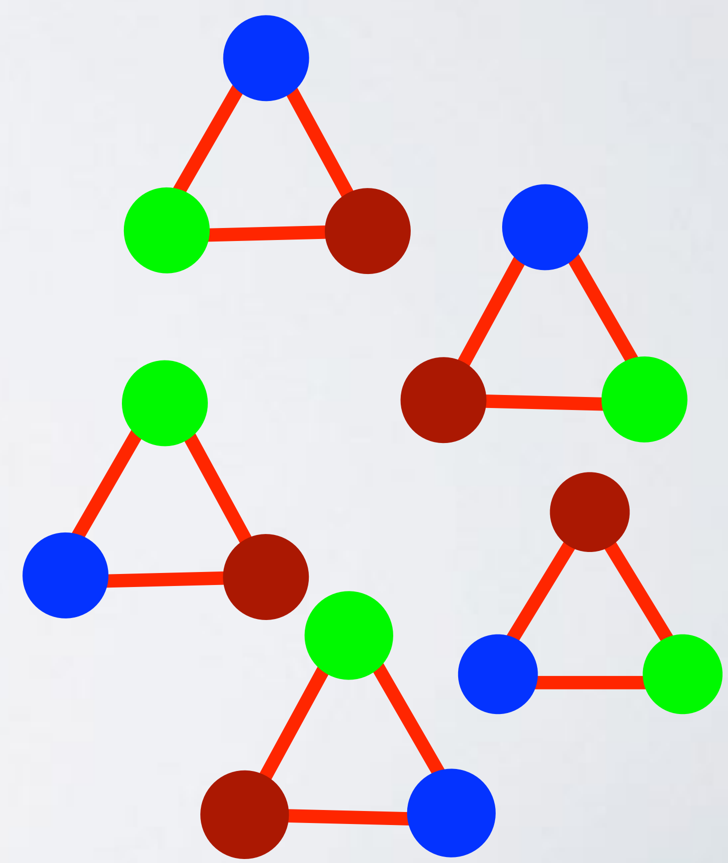


Paste together ground states over individual triangles

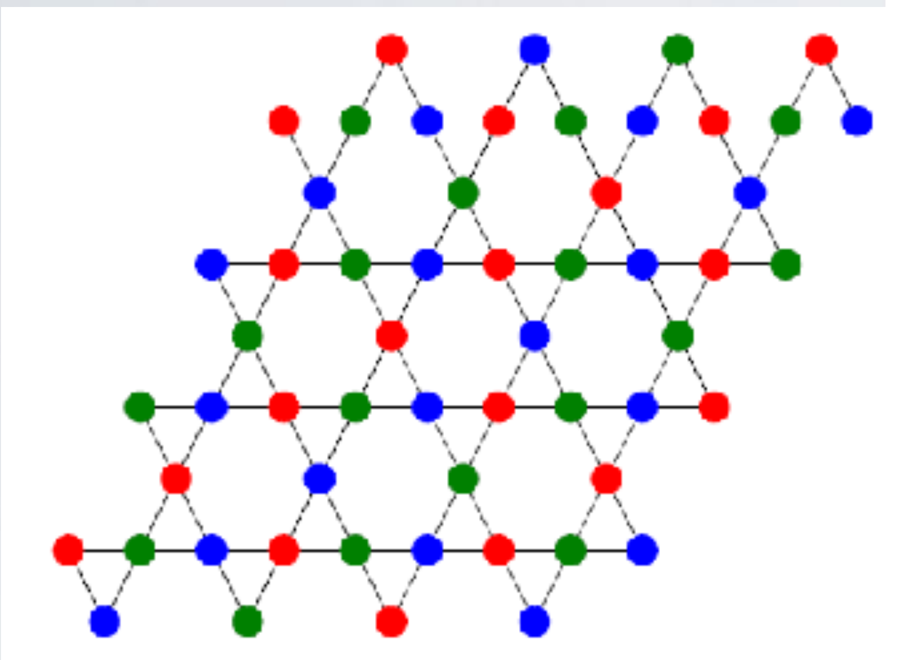


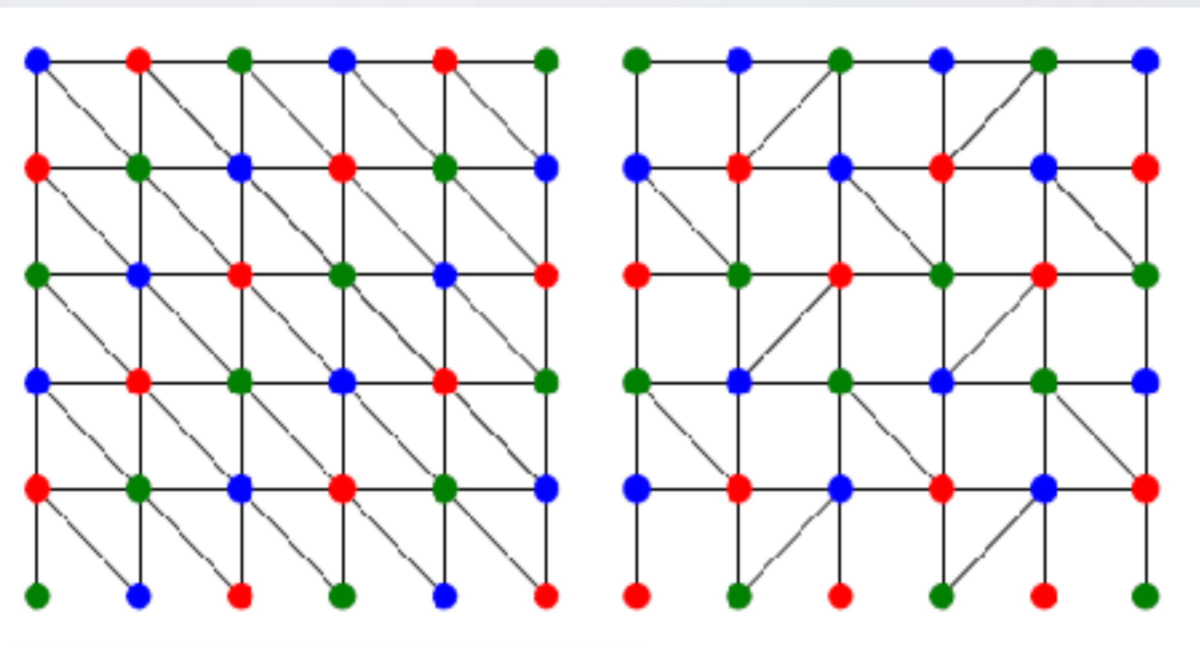


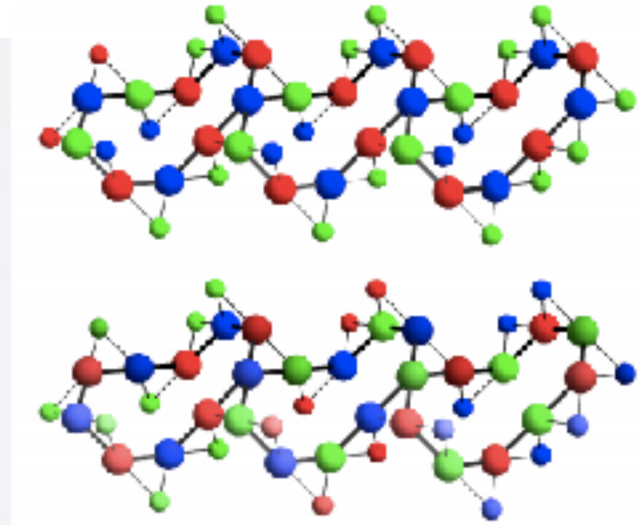
Frustration Free!
(but not commuting)

$$H = \sum_{\Delta} H_{XXZ0}$$

$$|\psi\rangle \equiv \prod_{s} \otimes |C_s\rangle_s$$







Actually there are more ground states….

$$|\psi\rangle \equiv \prod_s \otimes |C_s\rangle_s$$

This mixes Sz sectors

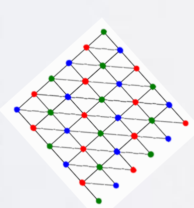
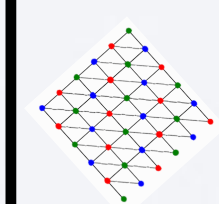
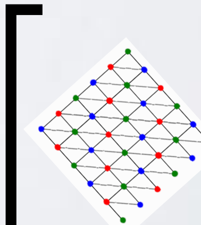
But the Hamiltonian doesn't.

$$|\psi^C\rangle \equiv P_{S_z}\left(\prod_{\text{valid}} \otimes |C_s\rangle\right)$$

So projecting to Sz sectors are ground states.

Roughly, each color gives N ground states (one per Sz sector)

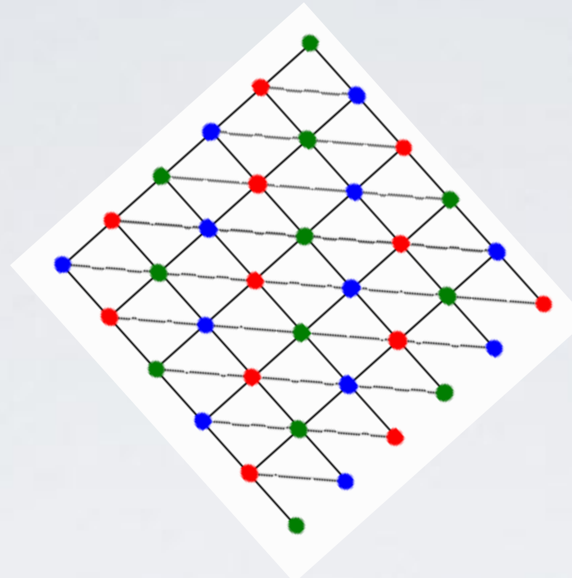
(A bit of a lie because colors are non-orthogonal and may be more-so after projection)

$\{ P_{S_z=0}\Big[$

$\Big], P_{S_z=1}\Big[$

$\Big], \ldots\ldots, P_{S_z=N}\Big[$

$\Big] \}$

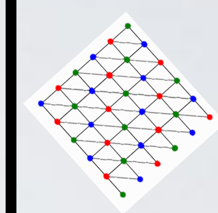
Can we use this to understand the triangular lattice?

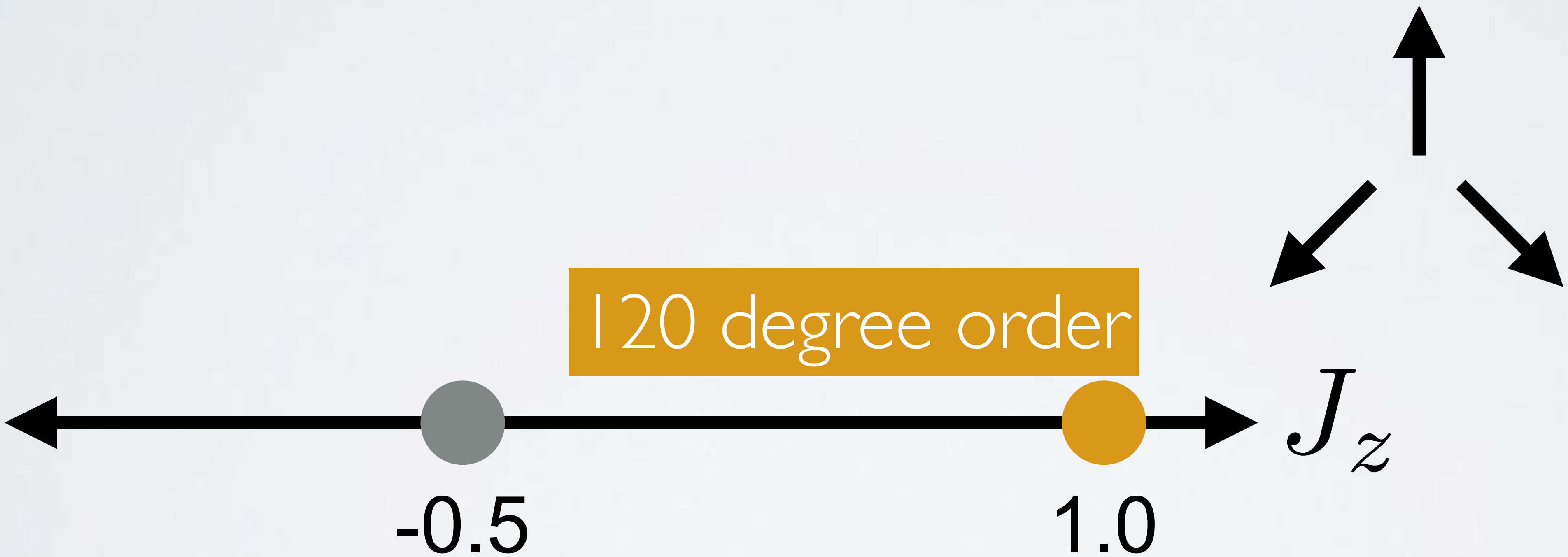
How many colorings?

Only one (or two) colorings.



$$P_{S_z=0}\left[\quad\right]$$





XXZ0

Heisenberg

$$H = H_{\mathrm{XXZ0}} + 1.5 \sum_{ij} S_i^z S_j^z$$

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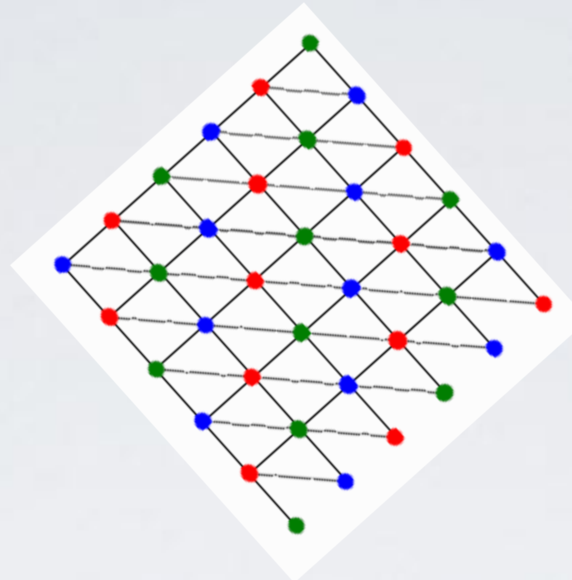
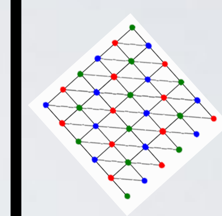
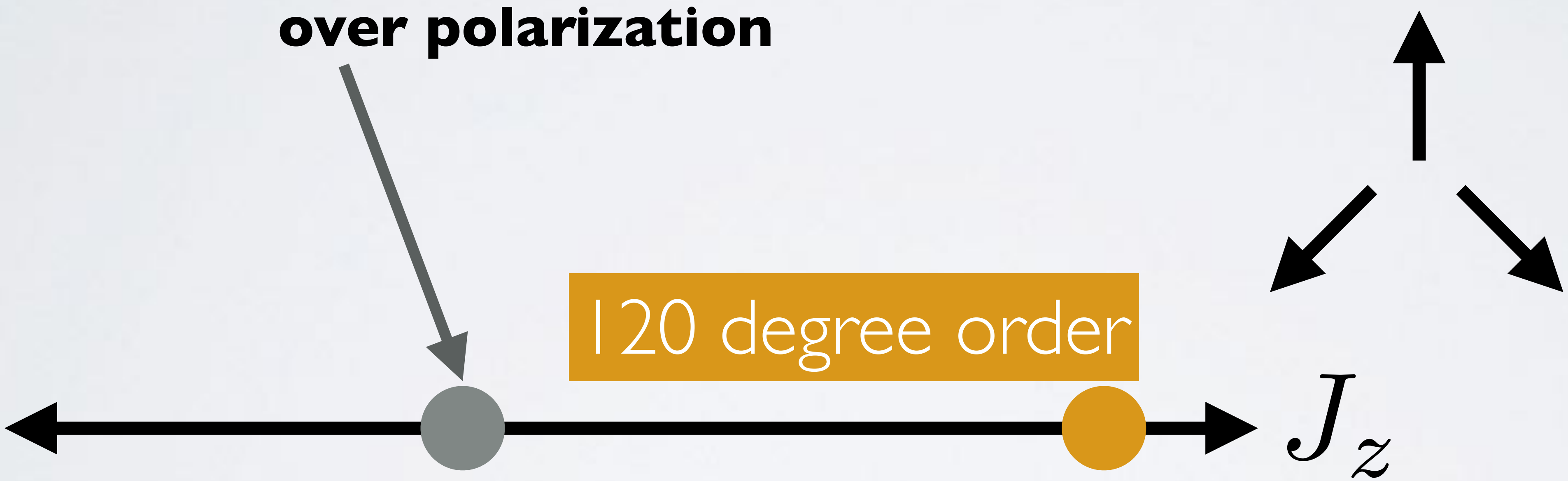


$$P_{S_z=0}\Big[$$  $$\Big]$$

`Linear Degenerate' over polarization



XXZ0

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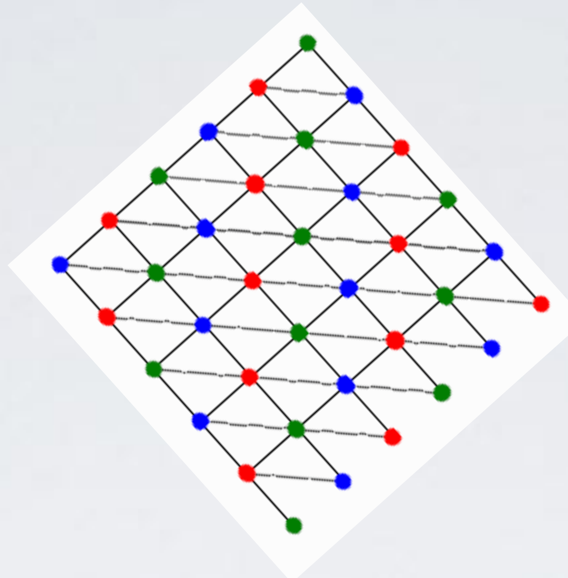
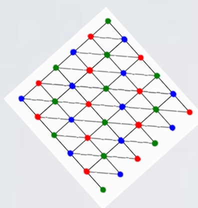
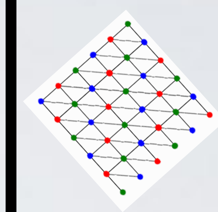
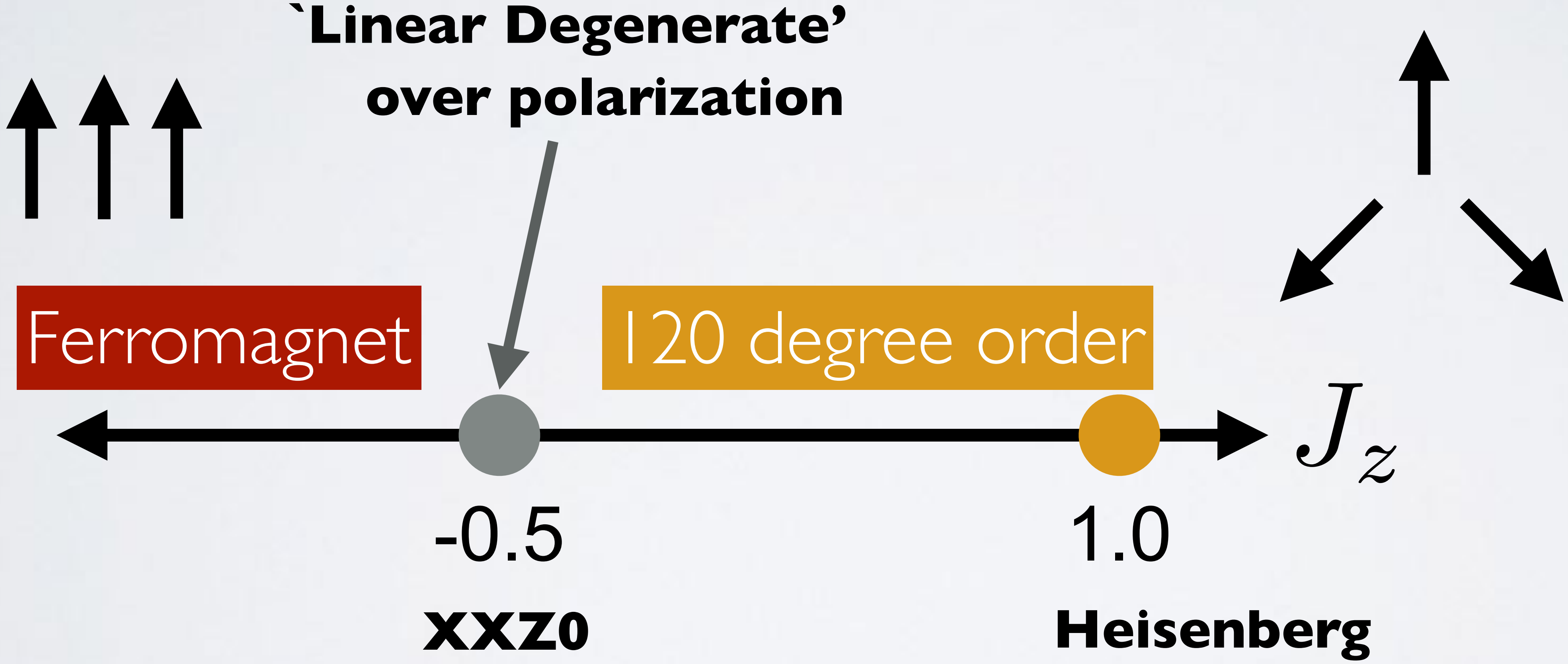


$P_{S_z=N}\big[$  $\big]$

$P_{S_z=0}\big[$  $\big]$

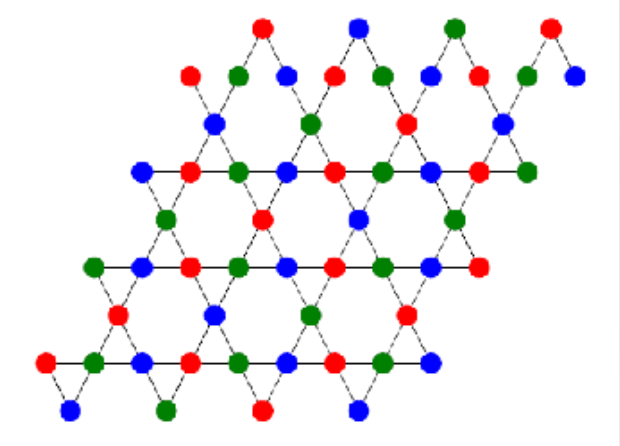


$$H = H_{\mathrm{XXZ0}} - 0.1 \sum_{ij} S_i^z S_j^z$$

$$H = H_{\mathrm{XXZ0}} + 1.5 \sum_{ij} S_i^z S_j^z$$

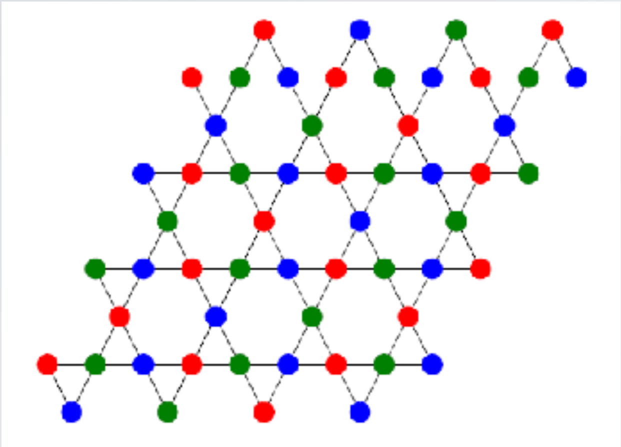
Can we use this to understand the kagome lattice?

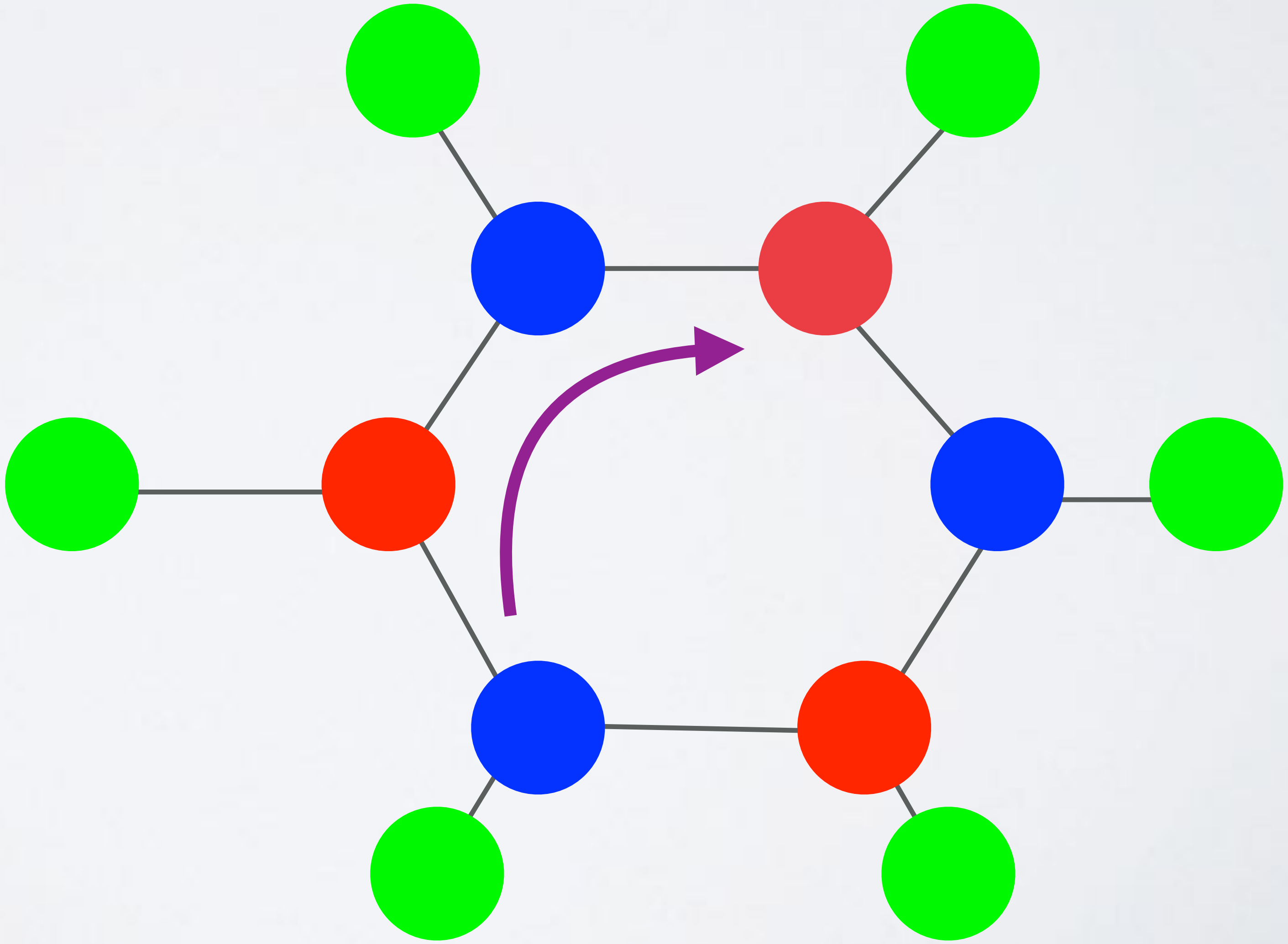
How many colorings?



Can we use this to understand the kagome lattice?

How many colorings?





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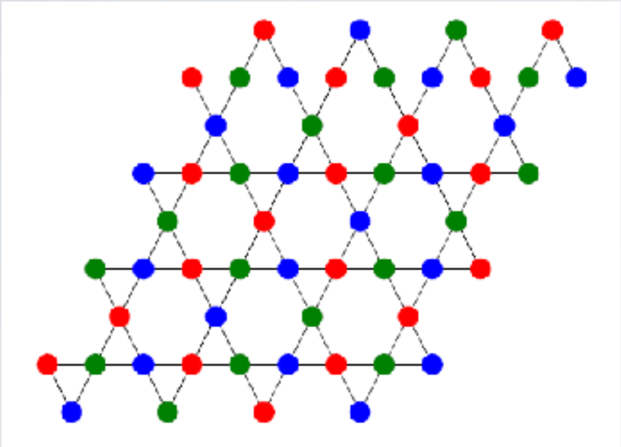

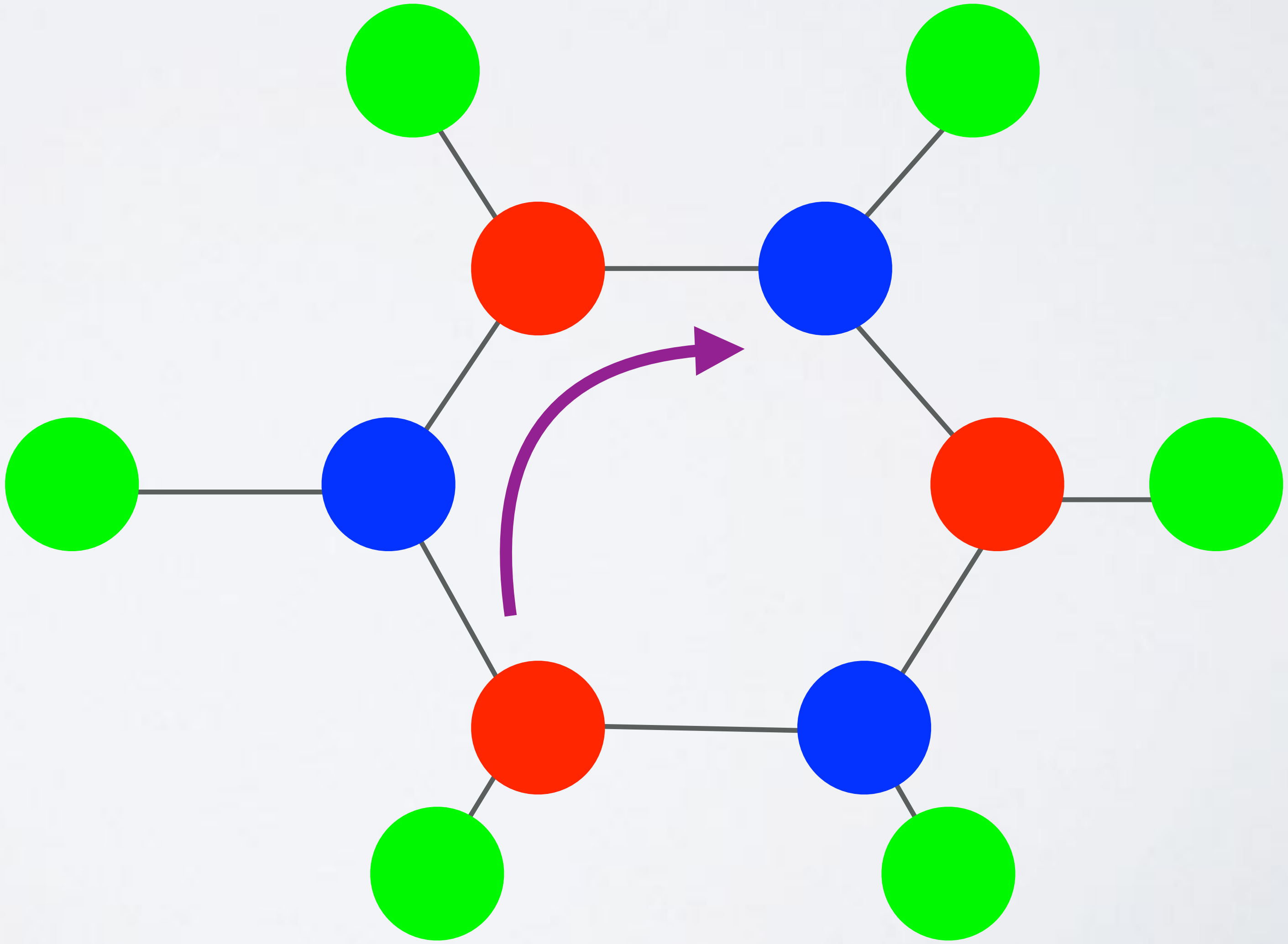

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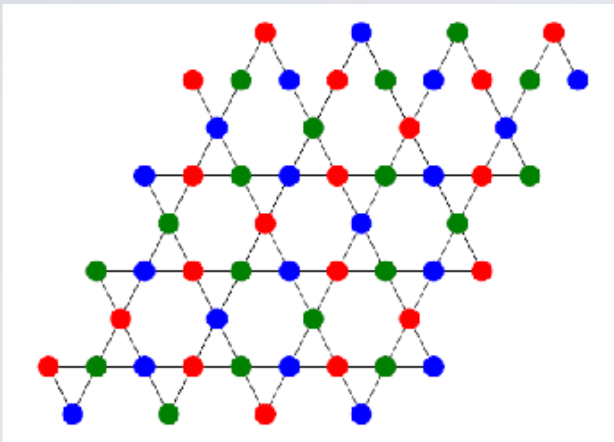


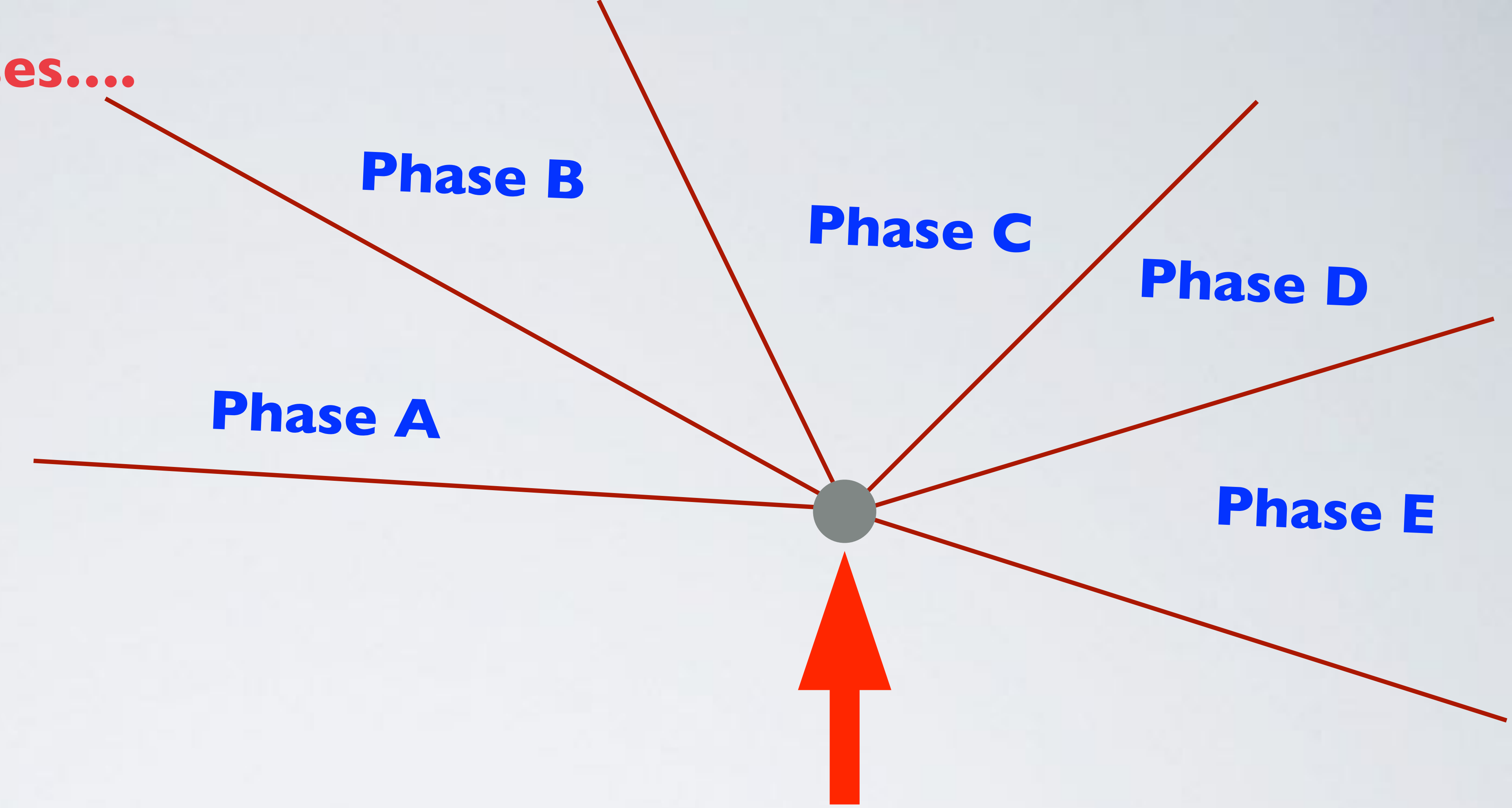
An exponential number of colorings!

1.208^N (from Baxter)

But much fewer then Ising configurations....

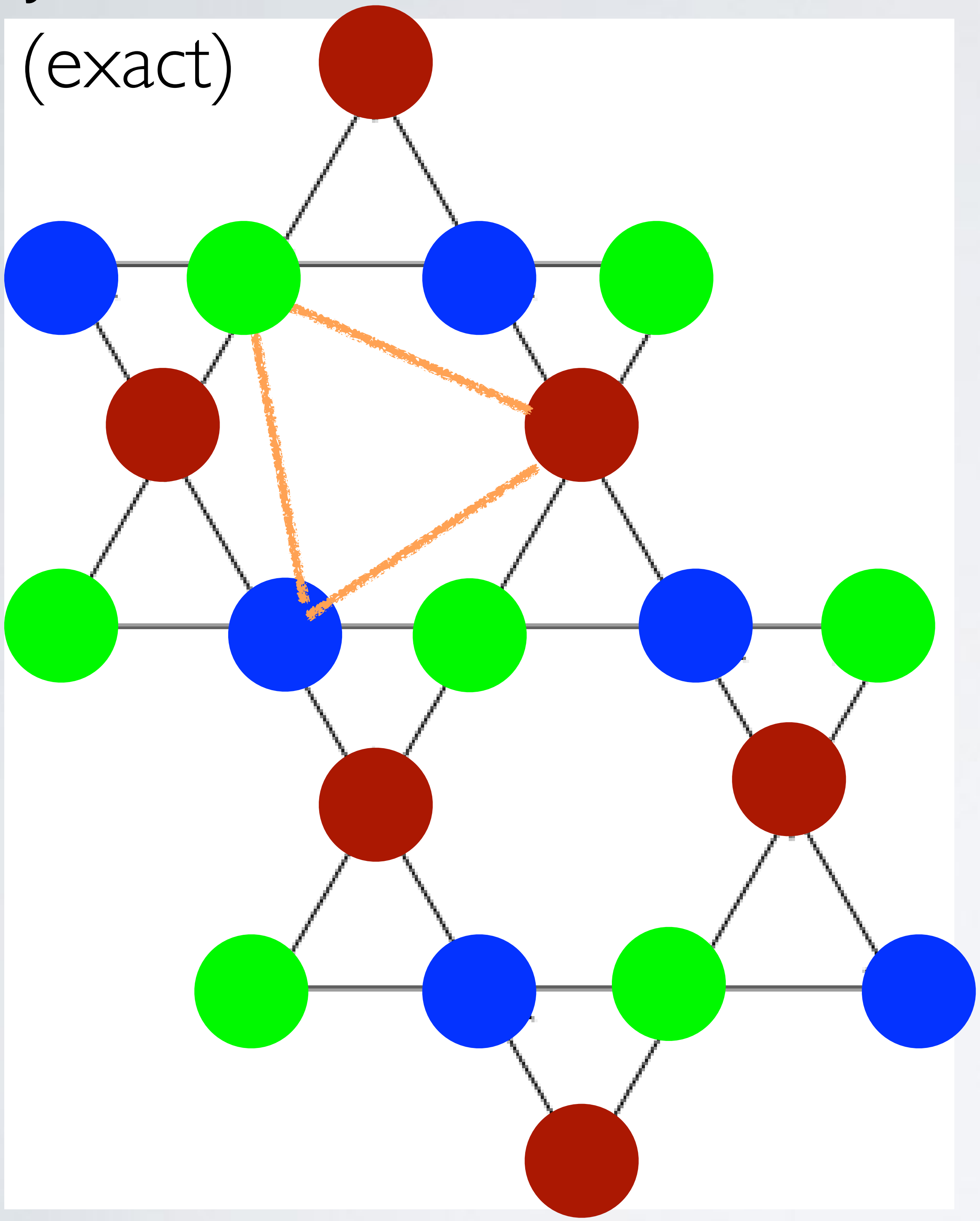
Lattice	Ising configs	Colorings
2x2x3	924	8
3x2x3	48620	16
4x2x3	2.7 million	32

Connect to known phases....



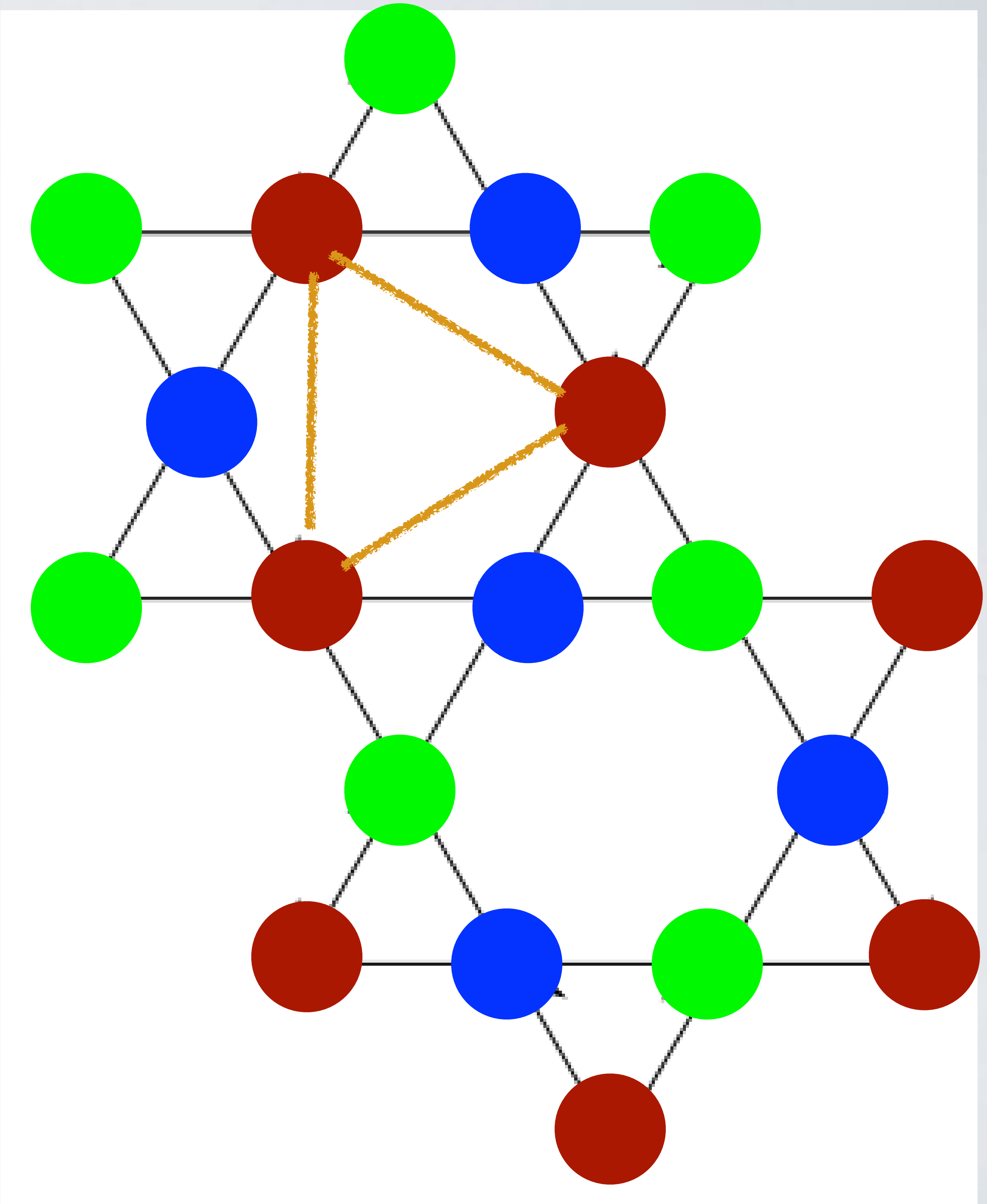
The mother of all phases?

J2>0

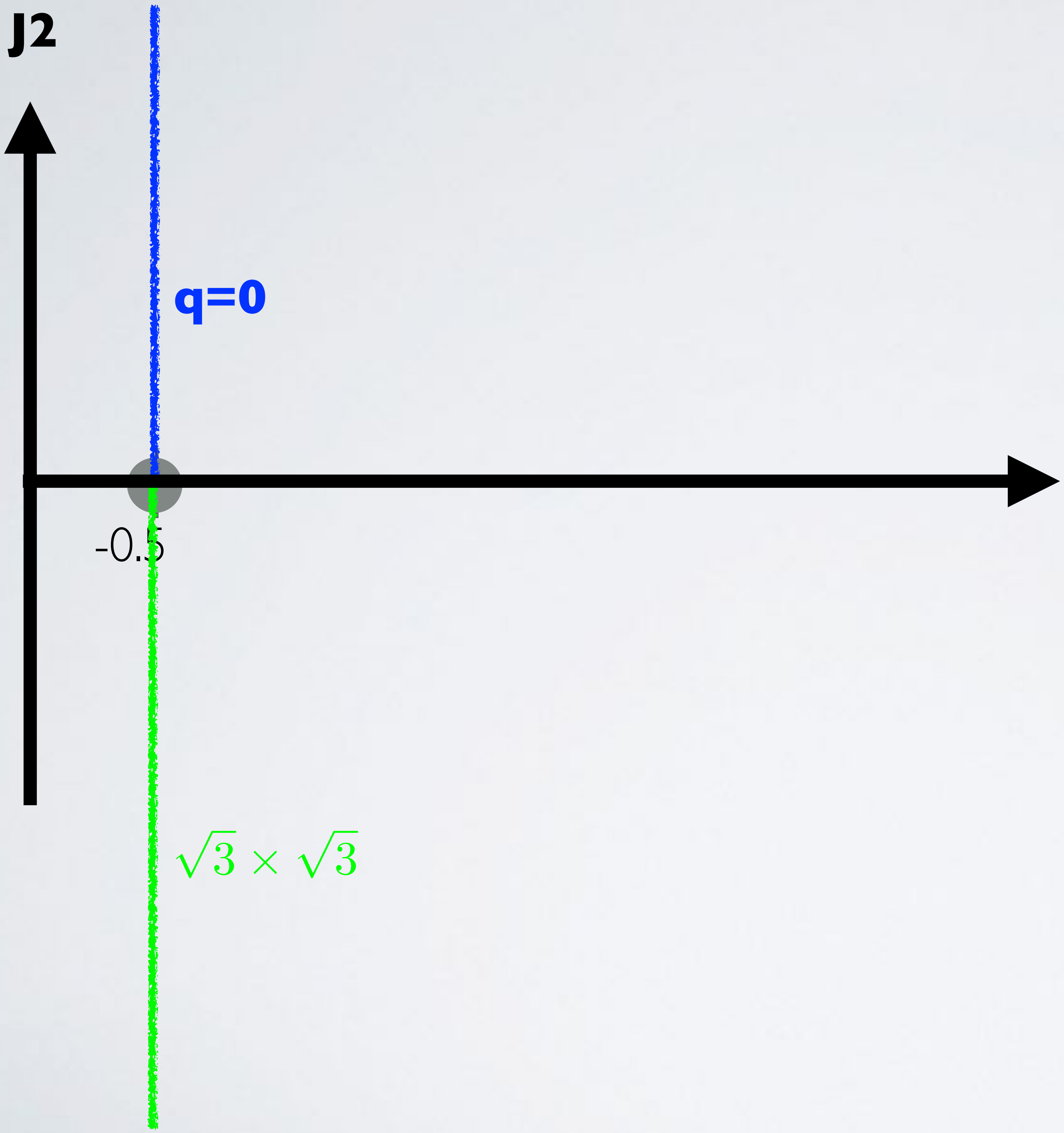


q=0

J2<0

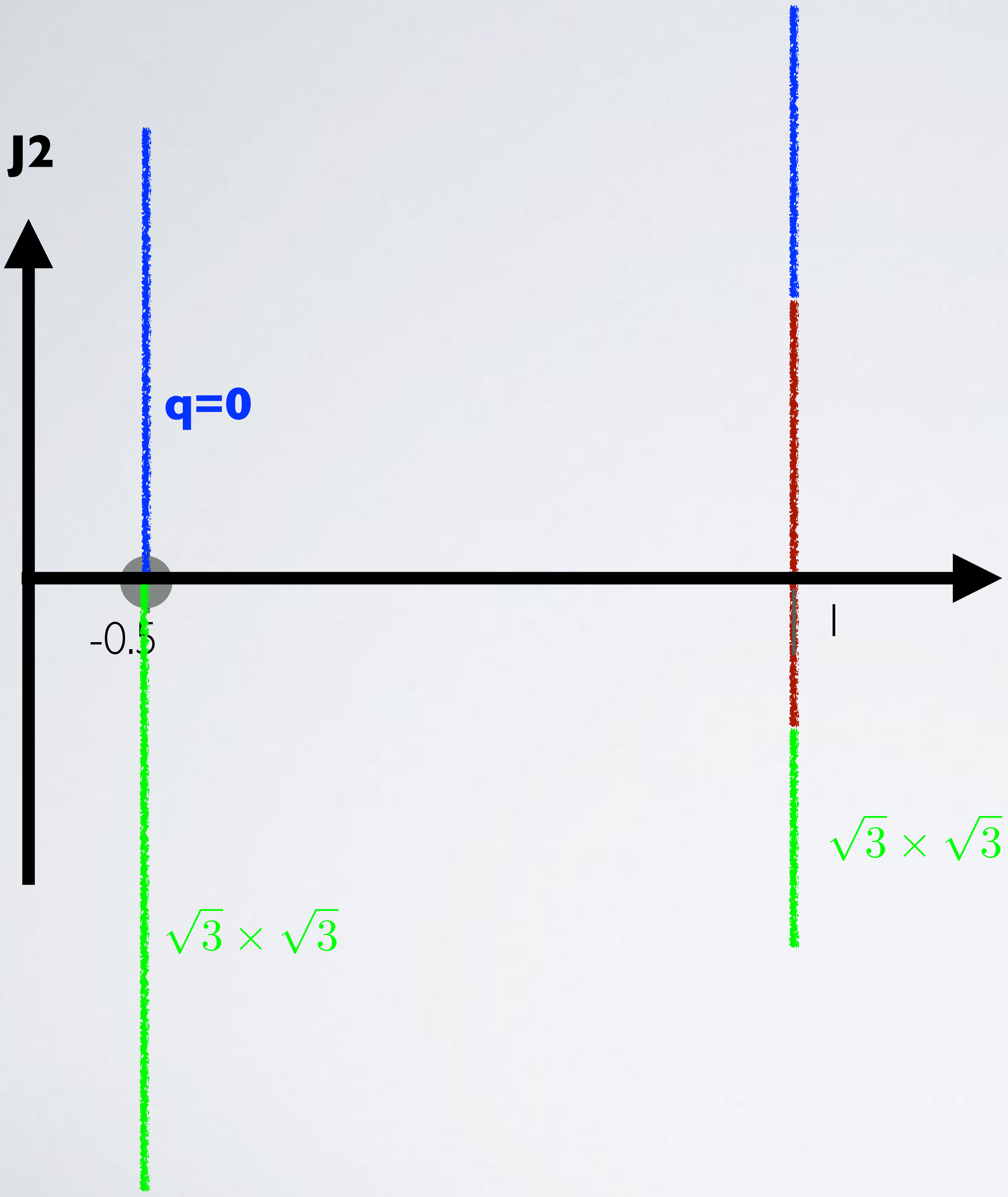


$\sqrt{3} \times \sqrt{3}$

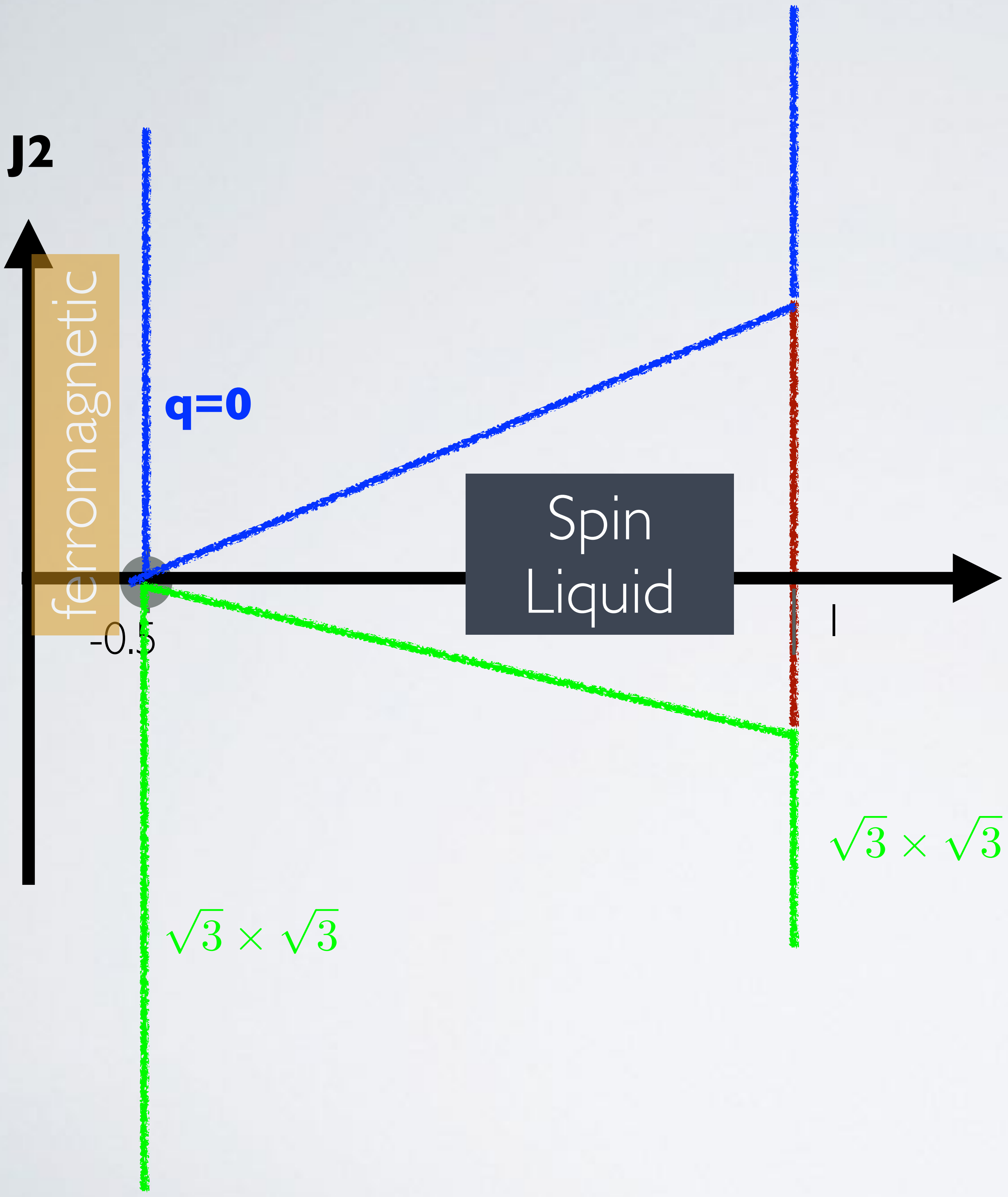


J2
q=0
-0.5
$\sqrt{3} \times \sqrt{3}$

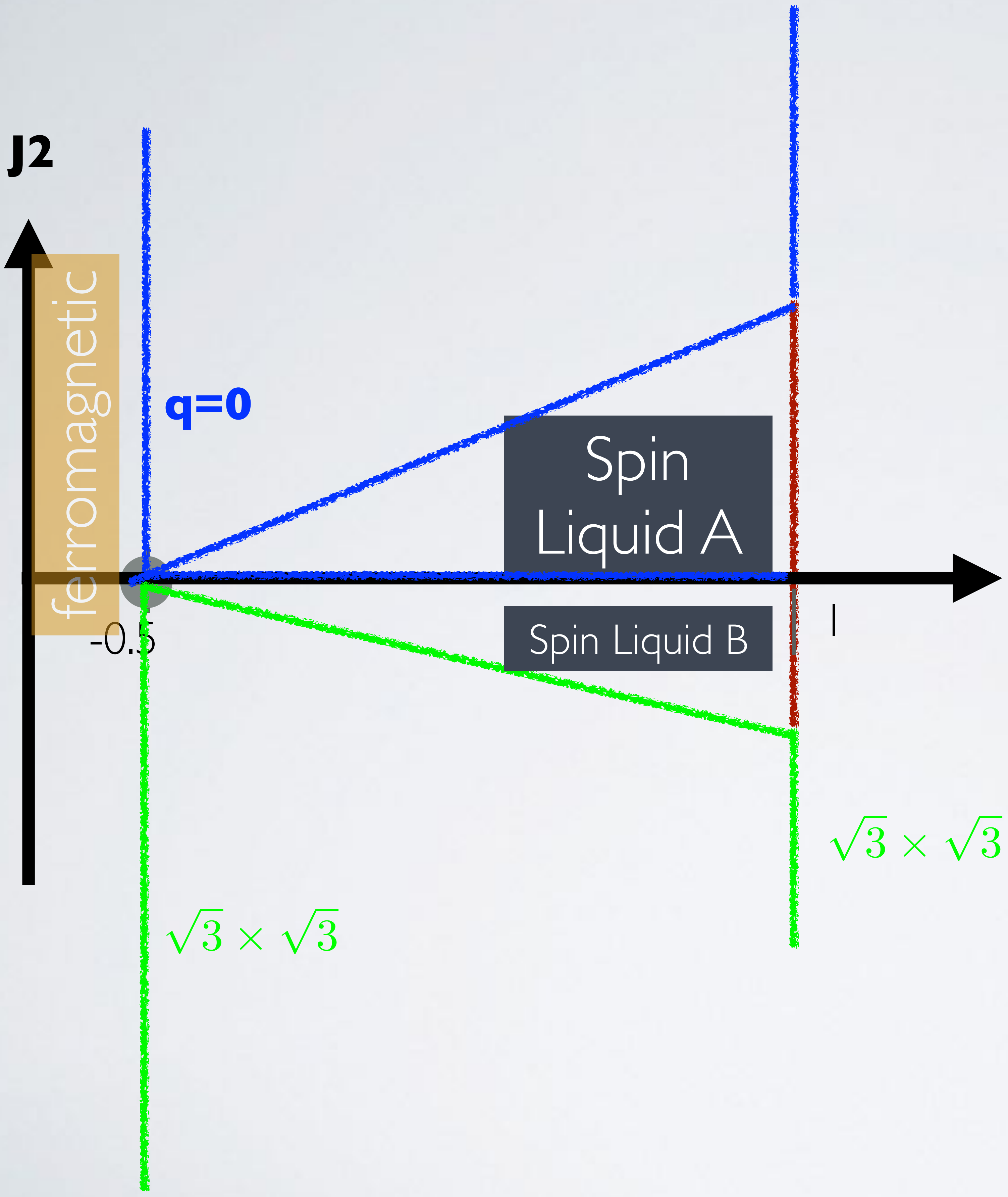
$S_z = 0$

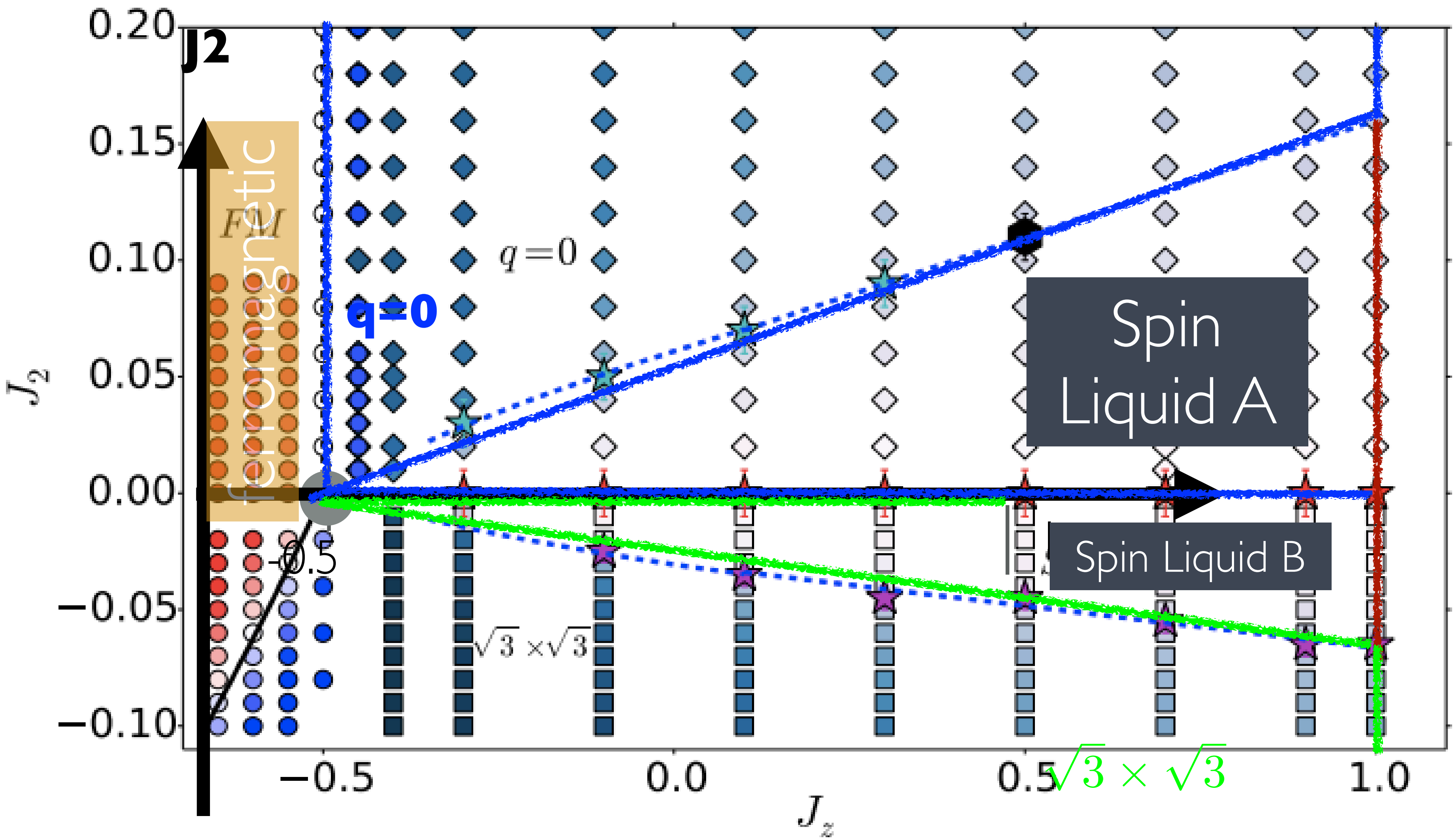


$S_z = 0$

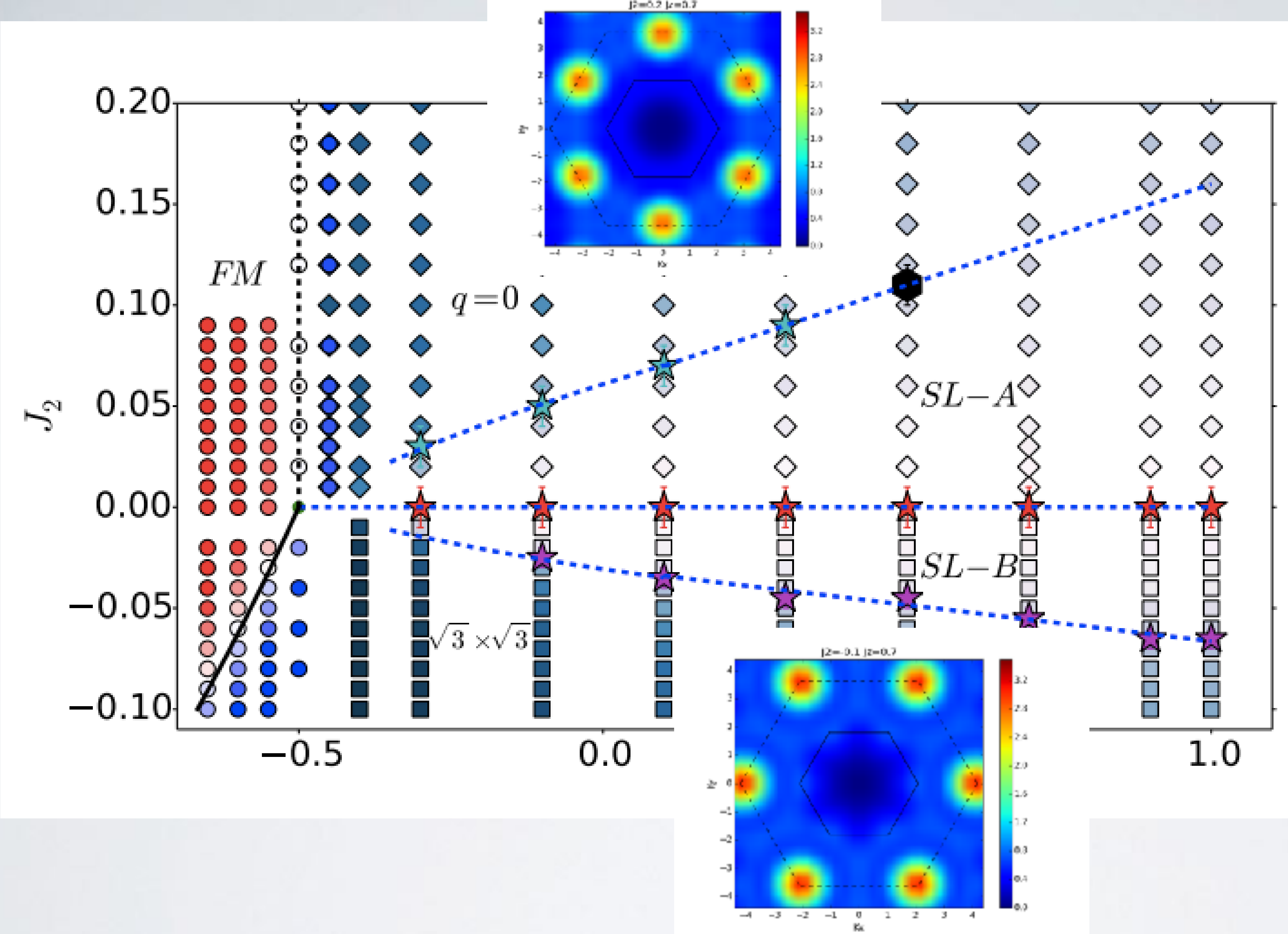

J2
ferromagnetic
q=0
Spin
Liquid
-0.5
1
$\sqrt{3} \times \sqrt{3}$
$\sqrt{3} \times \sqrt{3}$

$S_z = 0$


J2
ferromagnetic
q=0
Spin Liquid A
Spin Liquid B
-0.5
1
$\sqrt{3} \times \sqrt{3}$
$\sqrt{3} \times \sqrt{3}$



J2
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
J_2
FM
ferromagnetic
$q=0$
q=0
-0.5
$\sqrt{3}\times\sqrt{3}$
Spin Liquid A
Spin Liquid B
−0.5
0.0
0.5
$\sqrt{3}\times\sqrt{3}$
1.0
J_z



Q: Why co-planar states?

Colorings are all co-planar

Q: Why these co-planar states?

Fixed by colorings which satisfy J1-J2

Q: Why spin-liquids?

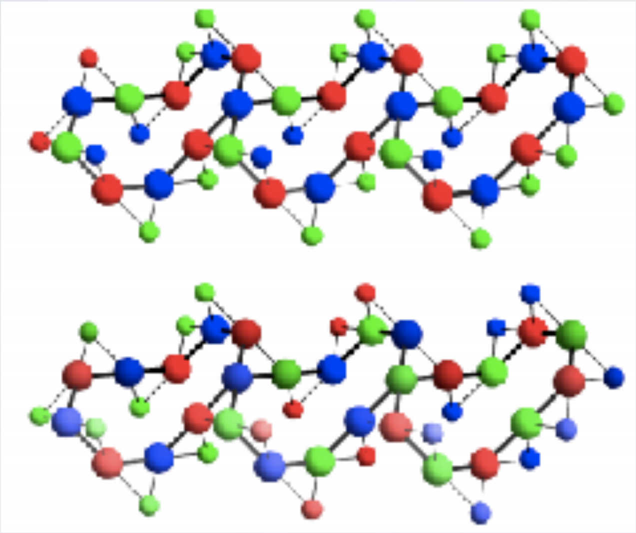
Q: Why so many competing phases?

Q: Why low-energy mess?

Exponential Degeneracy

Can we use this to understand the hyper-kagome lattice?

What's known: *experimental evidence of a spin-liquid.*

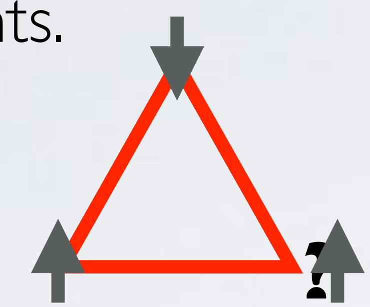


Also exponential degeneracy

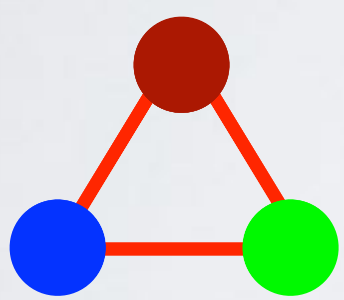
Conclusions

XXZ0 controls the physics of the Heisenberg point on lattices of pasted triangles in the way that the Ising limit doesn't.

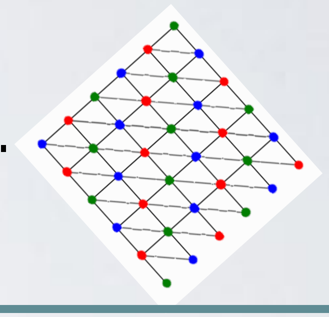
The story of frustration is not one of triangles which can't satisfy up-up-down constraints.



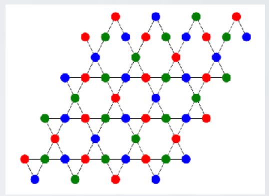
Instead, the story of frustrated magnetism is really one of coloring.



A single coloring which controls the triangular lattice.



And an exponential number of colorings which controls the kagome lattice.



From which all the known phases (and I conjecture arbitrarily many more) arise.