# Quantum Ramps in the Transverse Field Ising Model

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CUNY Meeting on Non-Equilibrium

Scaling Collapse of Transverse Field Ising Model

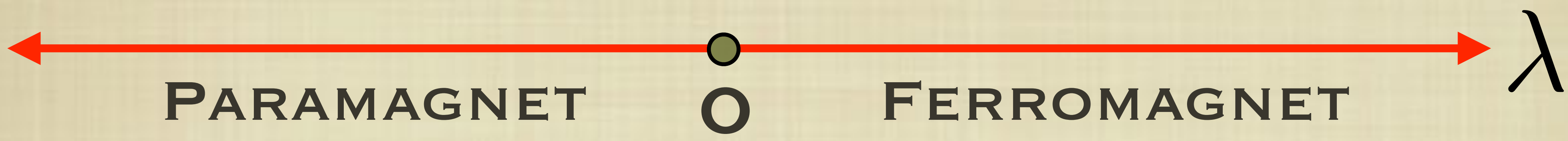
- Thermodynamic Limit
- Finite Size

Interesting States from quantum ramps

Do all things end in GGE or thermalization?

# Transverse Field Ising Model

$$H[\lambda] = -\frac{1}{2}\sum_j \left[ s_j^z s_{j+1}^z + (1-\lambda) s_j^x \right]$$




Wigner Fermionize

$$H[\lambda] = \sum_k H_k[\lambda]$$

$$H_k[\lambda] = (1 - \lambda - \cos k)\sigma_k^z + (\sin k)\sigma_k^x$$

Integrable system - Free Fermions

# Ramps

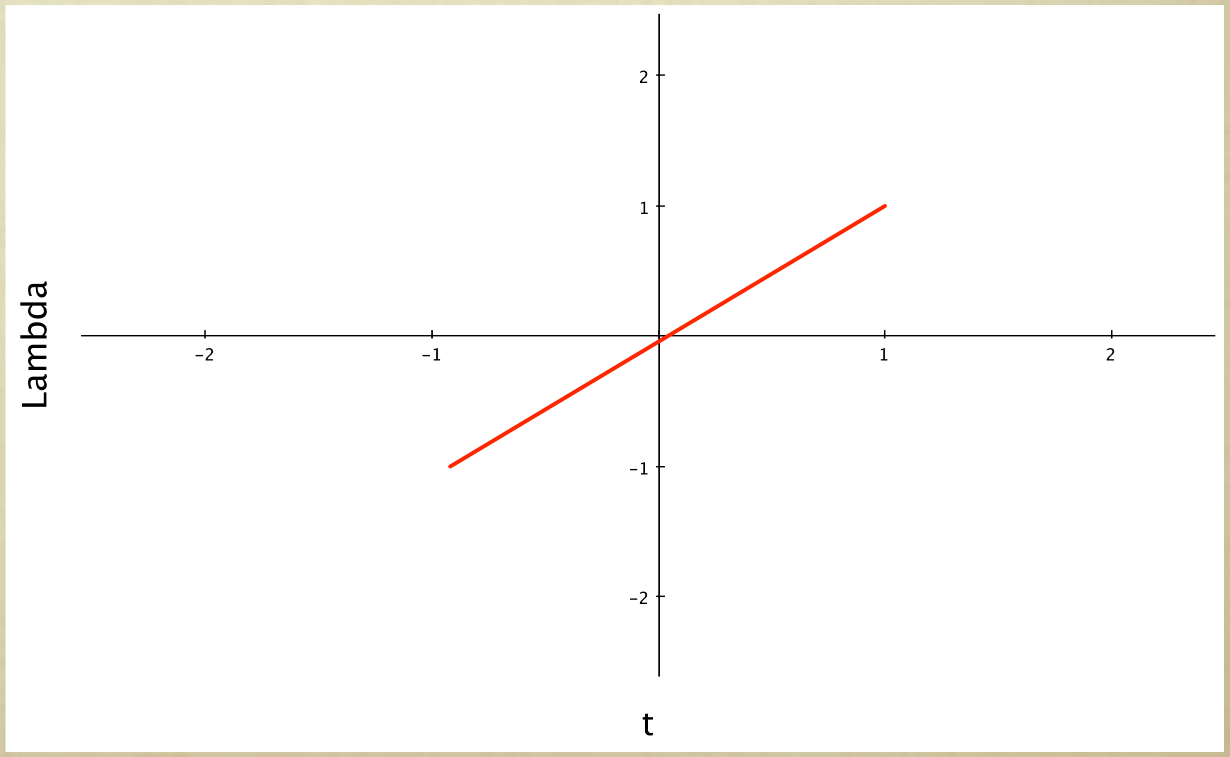
- **Ground States:** $H[\lambda]|\Psi_0[\lambda]\rangle = E_0[\lambda]|\Psi_0[\lambda]\rangle$

  Ground state at a given lambda!

- **Time Evolution:** $|\Psi(t)\rangle = \exp[iH[\lambda]t]|\Psi(0)\rangle$

- **For slow ramps, instantaneous ground state and time evolved ground state are the same!**

**Adiabatic Theorem**



# Fast vs. Slow

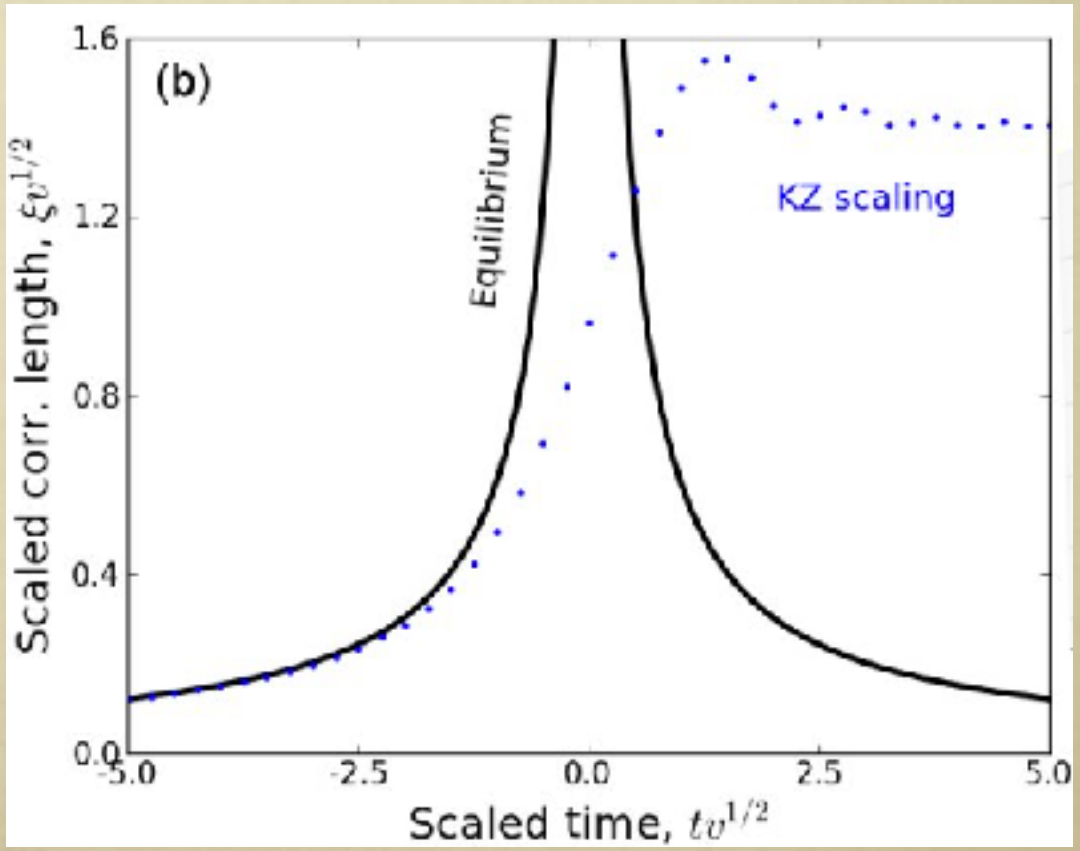
For linear ramps, adiabatic if $T \gtrsim \frac{||H_{\text{final}} - H_{\text{init}}||}{\Delta^2}$

Gap

At a QCP, $\Delta \to 0$

All ramps eventually fall out of the ground state.

Physically: Correlation time grows faster then time spent in region.



# Approach:

- TFIM is Exactly solvable.
- Two Steps:
  - Rewrite things in scaled units
  - Choose the correct regime

# Scaled Units

$t_k \equiv v^{-1/2}$ ⟷ $l_k \equiv v^{-1/2}$

Ising Universality Class: $\nu = z = 1$

$\tau = t/t_k$

$\Lambda = L/l_k$

$\kappa = kl_k$

$$H_k(\lambda) = (1 - \lambda - \cos k)\sigma_k^z + (\sin k)\sigma_k^x$$

↓ Long wavelength

$$H_k = \left(-\lambda + \frac{k^2}{2} + \ldots\right)\sigma_k^z + \left(k - \frac{k^3}{6} + \ldots\right)\sigma_k^x$$

$$i\frac{d\Psi_\kappa}{d\tau} = \left(-\tau + \frac{\kappa^2}{2}v^{1/2} - \ldots\right)\sigma_\kappa^z + \left(\kappa - \frac{\kappa^3}{6}v + \ldots\right)\sigma_\kappa^x$$

# Scaling Limit

$$i\frac{d\Psi_\kappa}{d\tau} = \left(-\tau + \frac{\kappa^2}{2}v^{1/2} - \dots\right)\sigma^z_\kappa + \left(\kappa - \frac{\kappa^3}{6}v + \dots\right)\sigma^x_\kappa$$

$$\Big\downarrow \lim_{v\to 0}$$

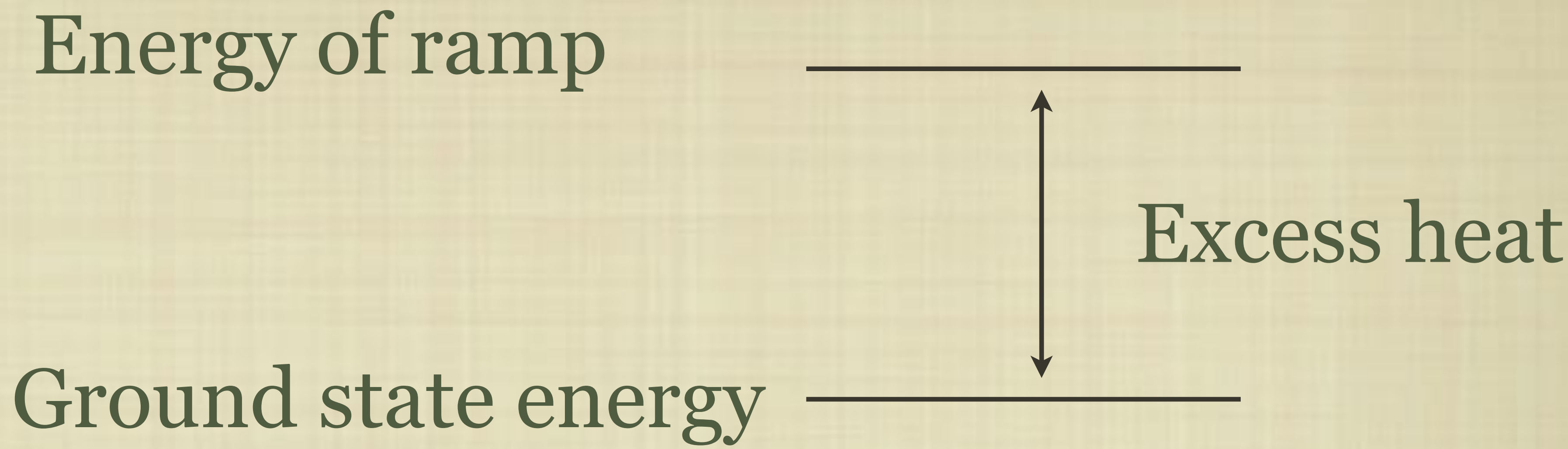
$$i\frac{d\Psi_\kappa}{d\tau} = \left(-\tau\sigma^z_\kappa + \kappa\sigma^x_\kappa\right)\Psi_\kappa$$

We've written $\Psi(t,k,v) = \Psi_\kappa(\tau)$

$$\lim_{v\to 0}$$

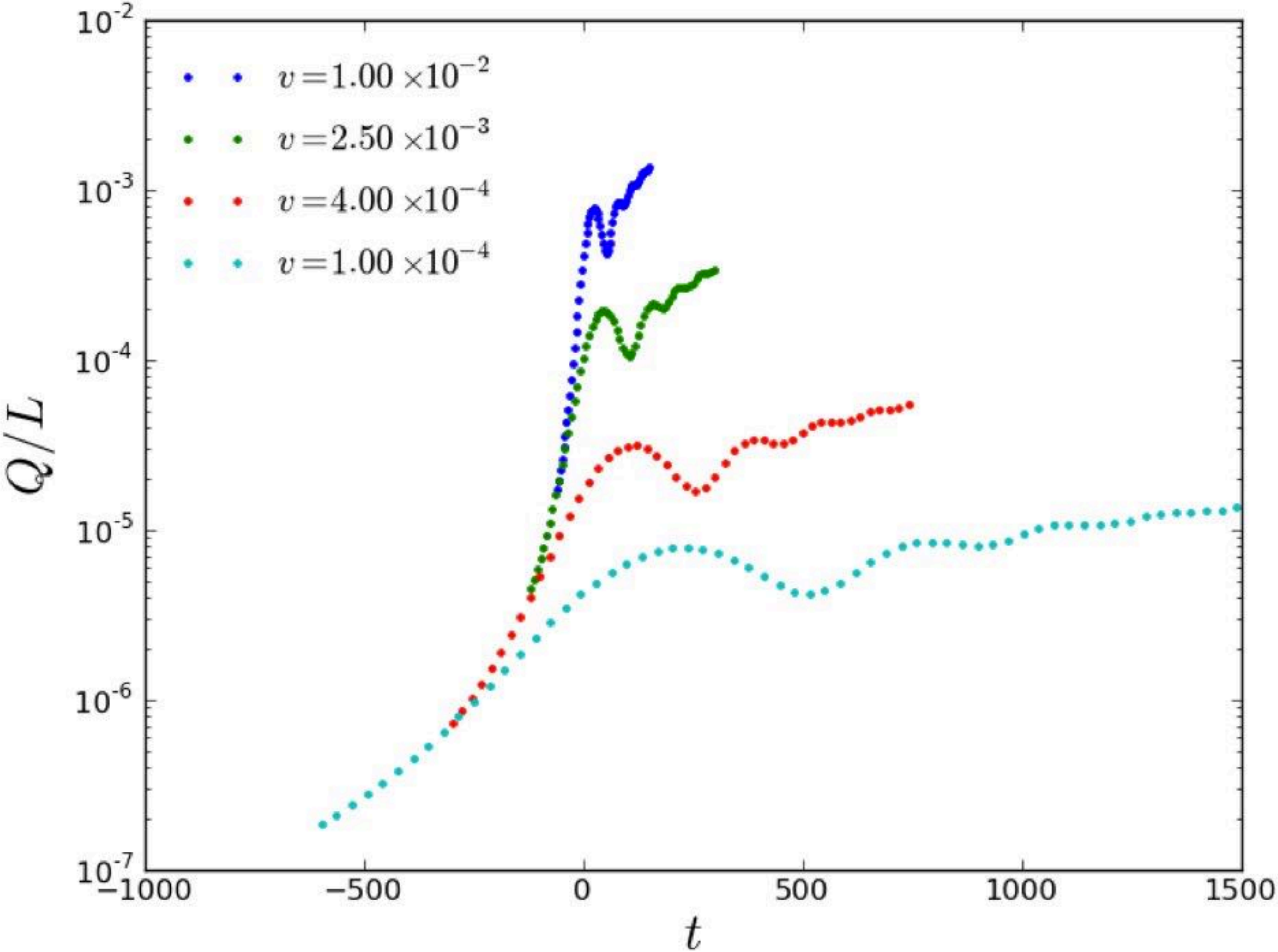
FIXED: $\tau$, $\kappa$, $\Lambda$

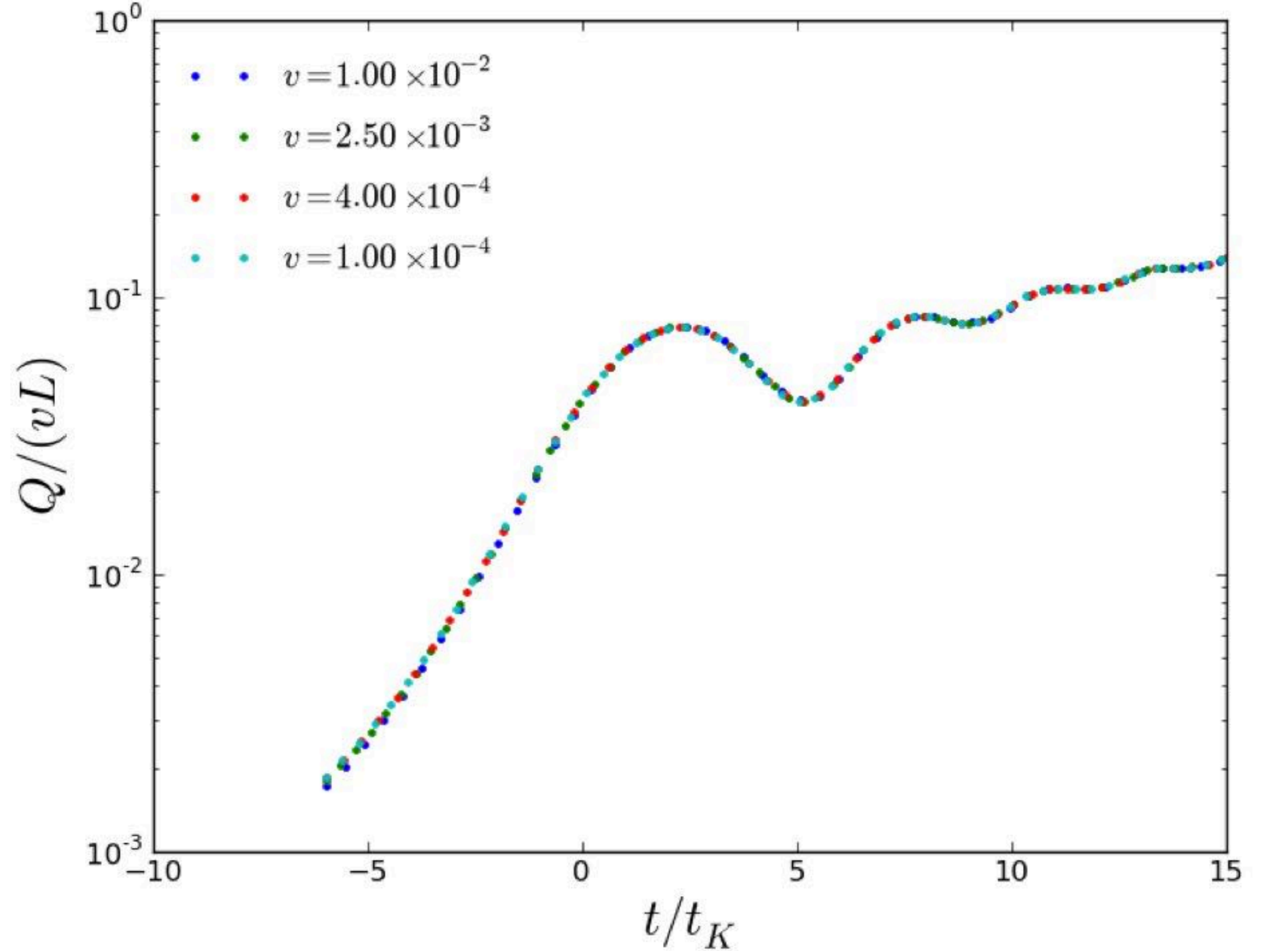
# What to measure




**Excess Heat:** $$Q = \langle \Psi(t) | H(t) | \Psi(t) \rangle - \langle \Psi_0 | H(t) | \Psi_0 \rangle$$

$$\frac{Q t_k}{L / l_k} = \frac{Q}{v L} = q(\tau, \Lambda)$$


$v = 1.00 \times 10^{-2}$
$v = 2.50 \times 10^{-3}$
$v = 4.00 \times 10^{-4}$
$v = 1.00 \times 10^{-4}$
$Q/L$
$t$
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
$10^{-6}$
$10^{-7}$
−1000
−500
0
500
1000
1500

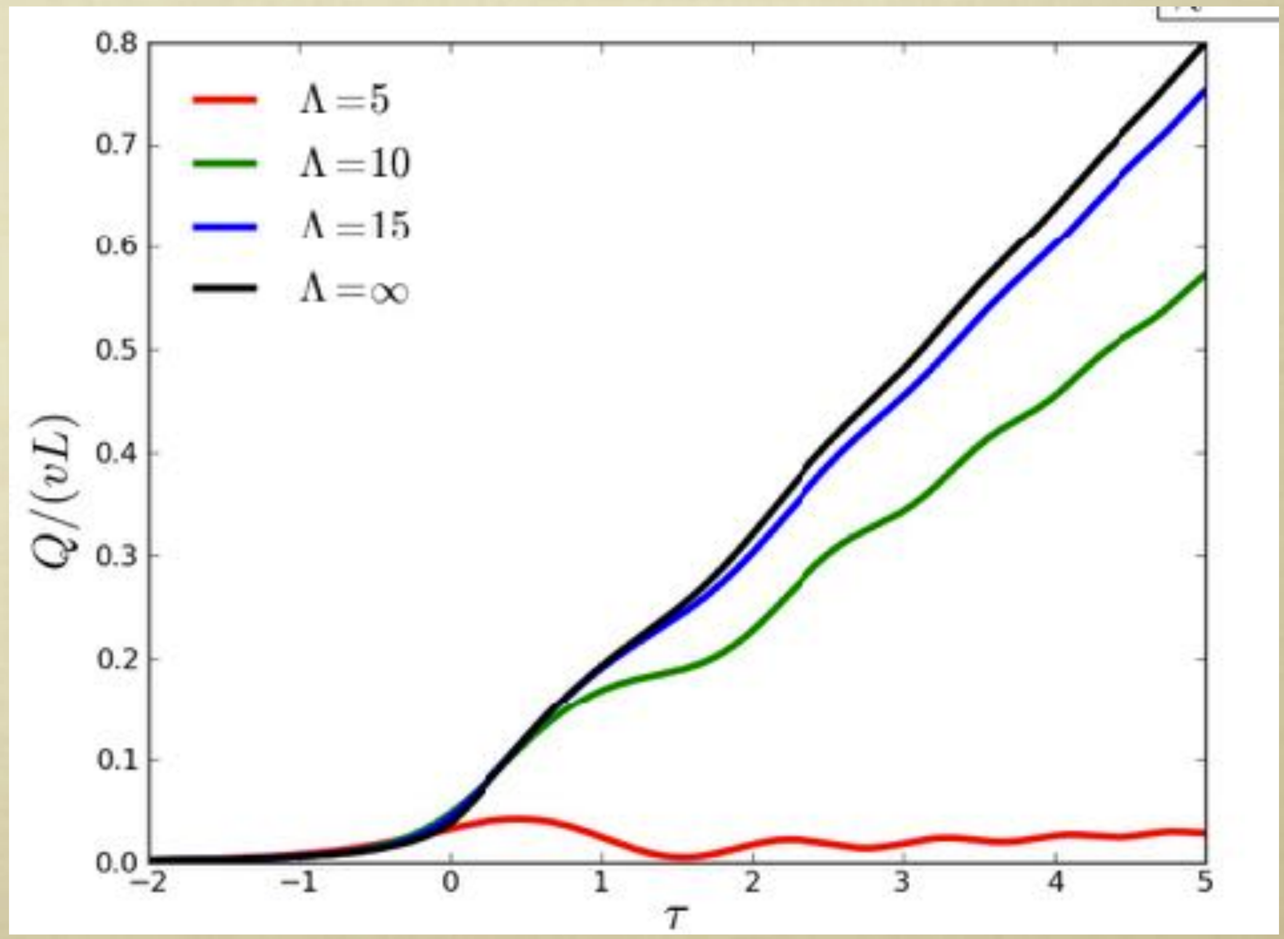

$v = 1.00 \times 10^{-2}$
$v = 2.50 \times 10^{-3}$
$v = 4.00 \times 10^{-4}$
$v = 1.00 \times 10^{-4}$
$Q/(vL)$
$t/t_K$
$10^0$
$10^{-1}$
$10^{-2}$
$10^{-3}$
−10
−5
0
5
10
15

# FINITE SIZE SYSTEMS

There is always a gap!

Can work out expected curves for different “scaled” system sizes.

These expected to be universal.

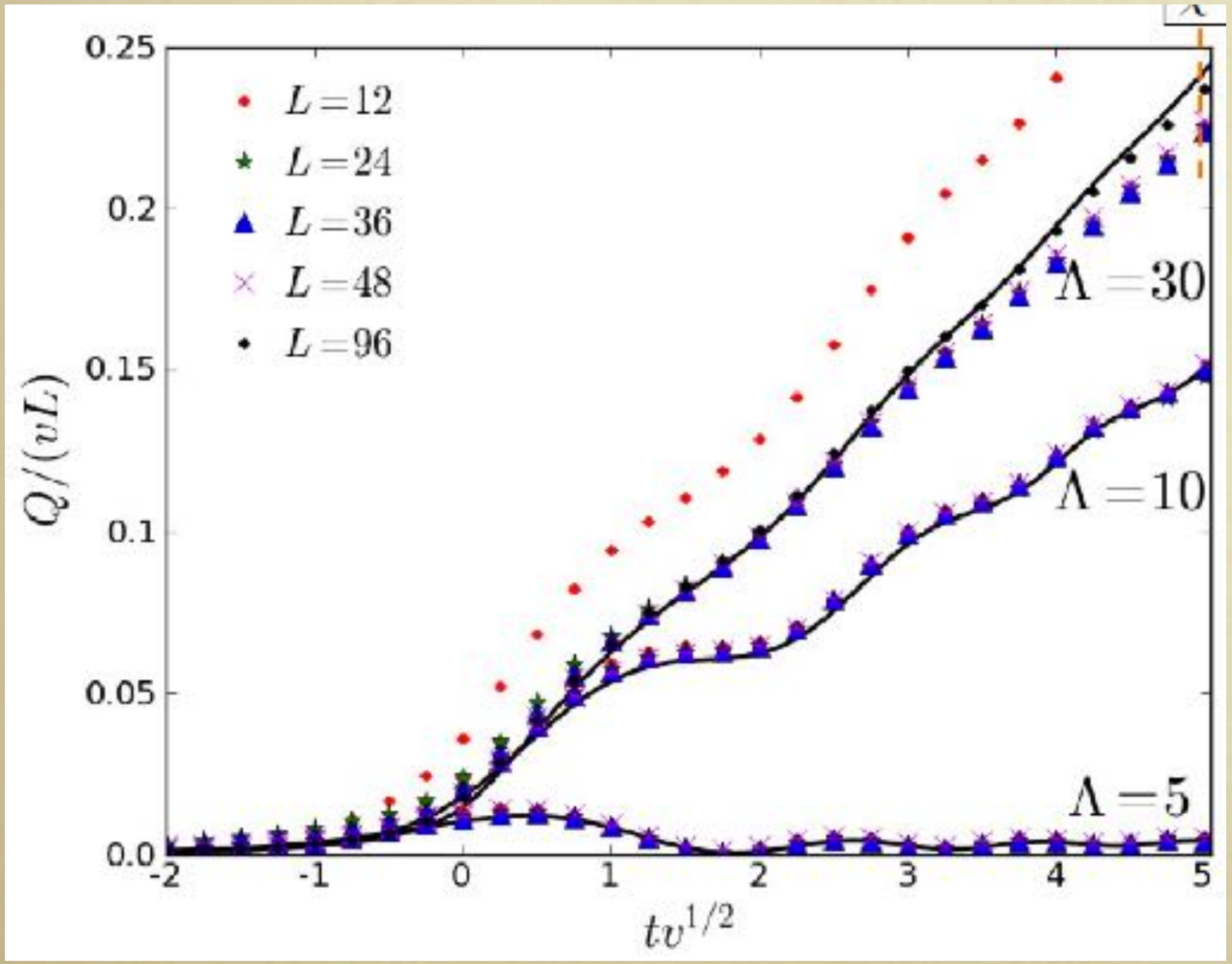


# Mott Insulating Bosons in Tilted Lattice

$$P\left[-\sum_i \sigma_i^x + \delta(t)\sum_i \frac{1}{2}(\sigma_i^z + 1)\right] P$$

Projects out $\uparrow\uparrow$

Same universality class as transverse field ising model.



# Time Evolution

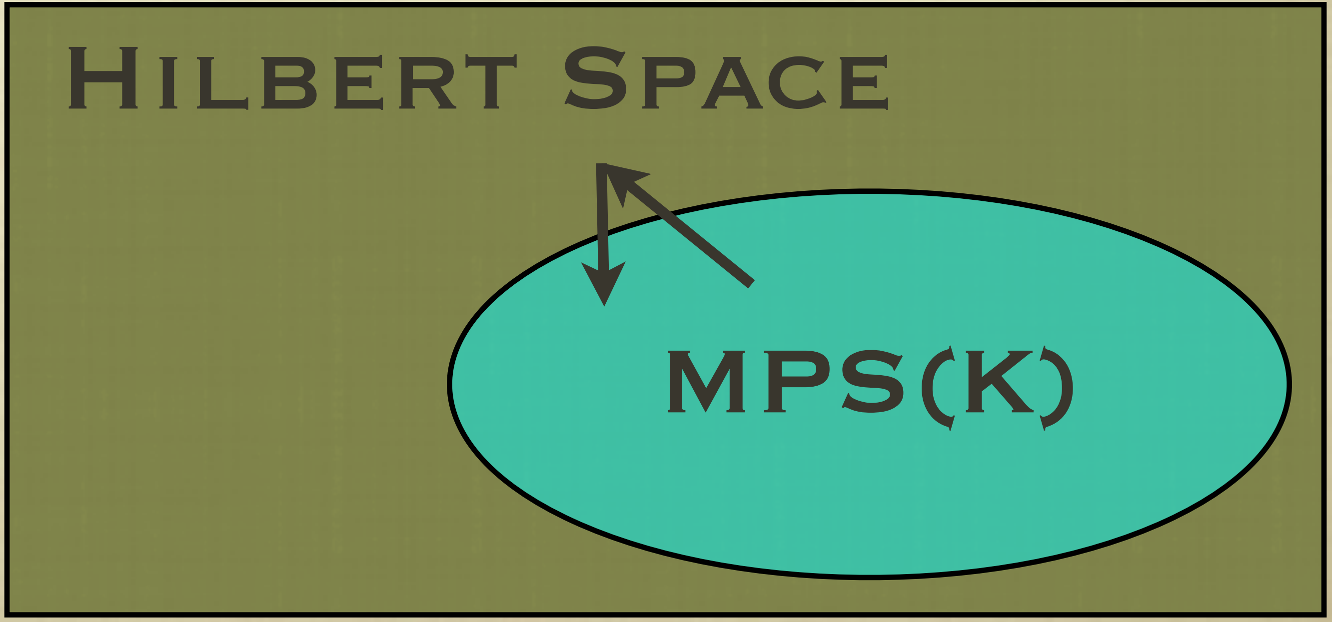
1. Start with $\Psi_{\text{MPS}} = \sum_{\sigma_i} Tr[M_1^{\sigma_1} M_2^{\sigma_2} \ldots M_k^{\sigma_k}] |\sigma_1 \sigma_2 \ldots \sigma_k\rangle$ for ferromagnet ground state.

$\begin{pmatrix} & \end{pmatrix}$ k x k matrix.

2. Apply $\exp\left[-it\left(-\sum_i \sigma_i^x + \delta(t) \sum_i \frac{1}{2}(\sigma_i^z + 1)\right)\right]$ exactly. This increases the bond dimension to something too large.

$\begin{pmatrix} & \end{pmatrix}$ 4k x 4k matrix.

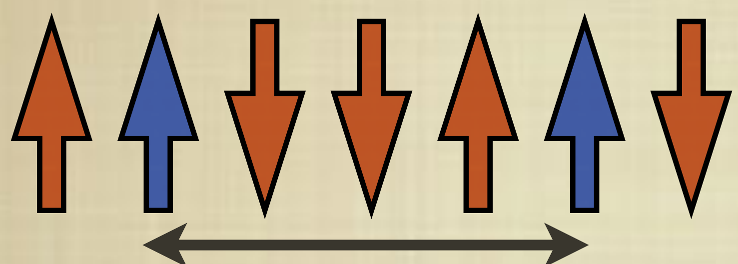
3. Project back to bond dimension of size k.



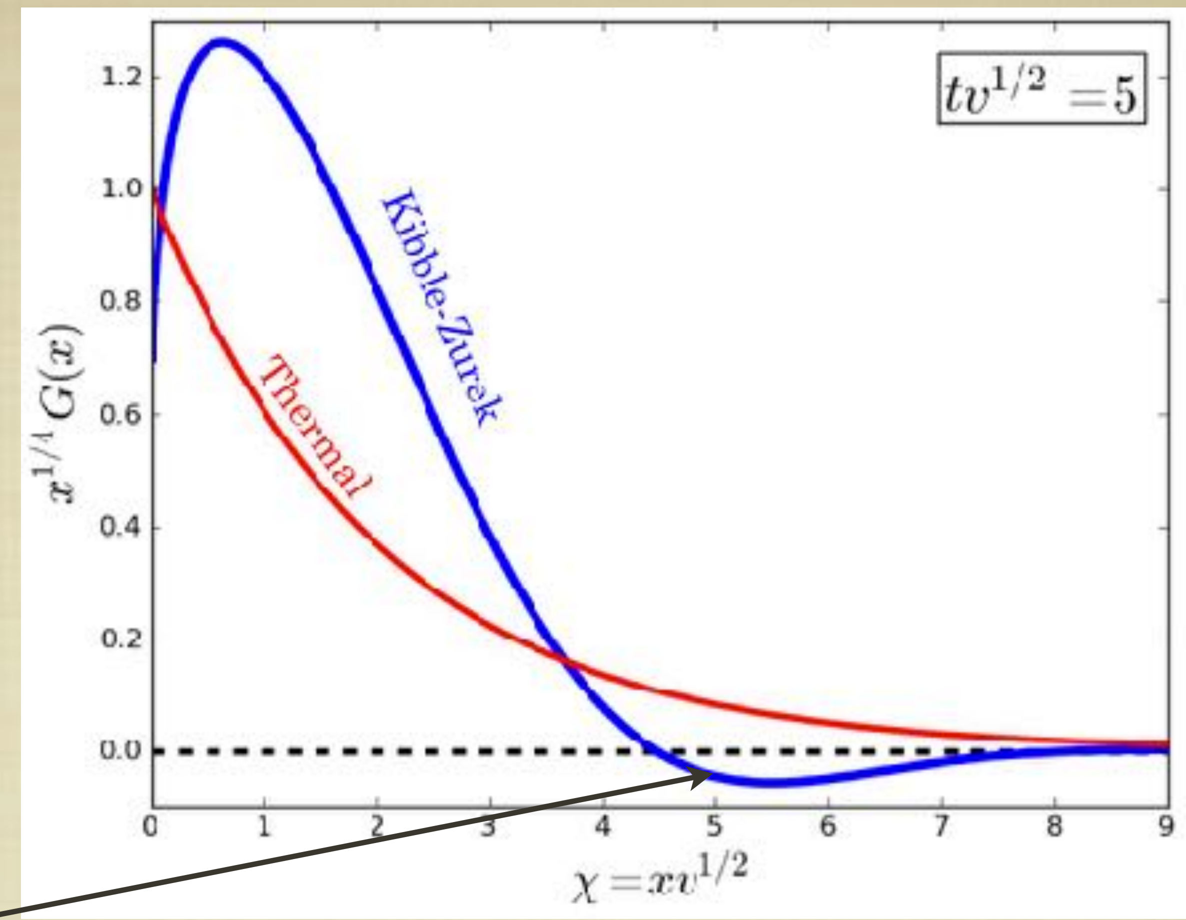
# Interesting States from Ramps

# Q: Can Ramps Produce interesting states?

## Thermal States

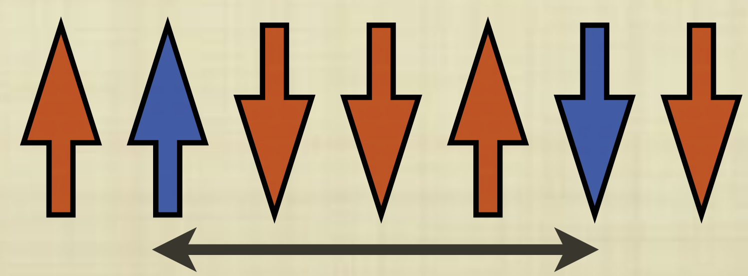


If I'm up, then you're probably up as well.




Robust even for experimentally accessible systems!

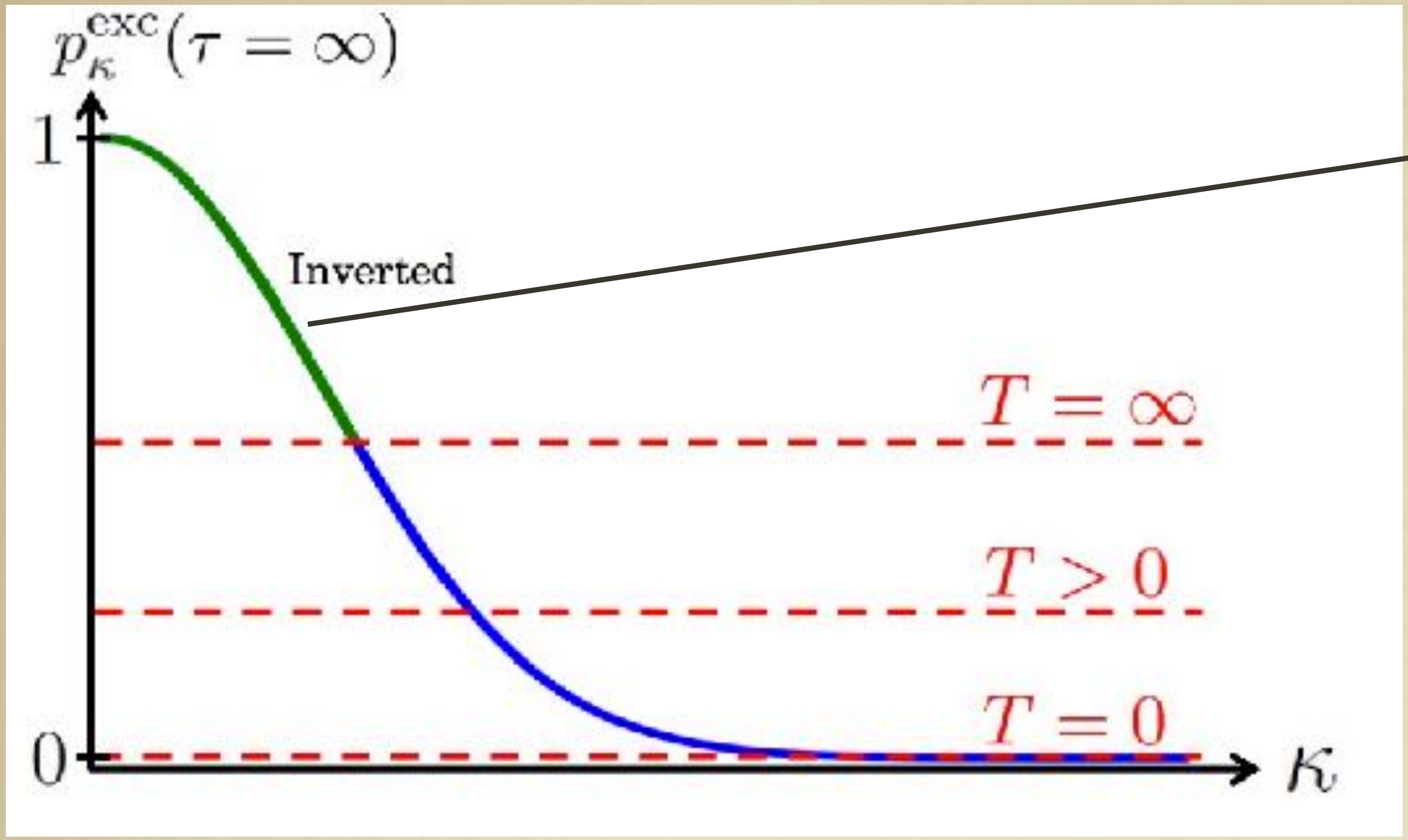
## Athermal States



If I'm up, then you're probably down.

!!!

# Why Athermal?




In a thermal state, this should be concave not convex!

# Generalized Gibbs Ensembles,
# Thermalization,
# and
# Steady States

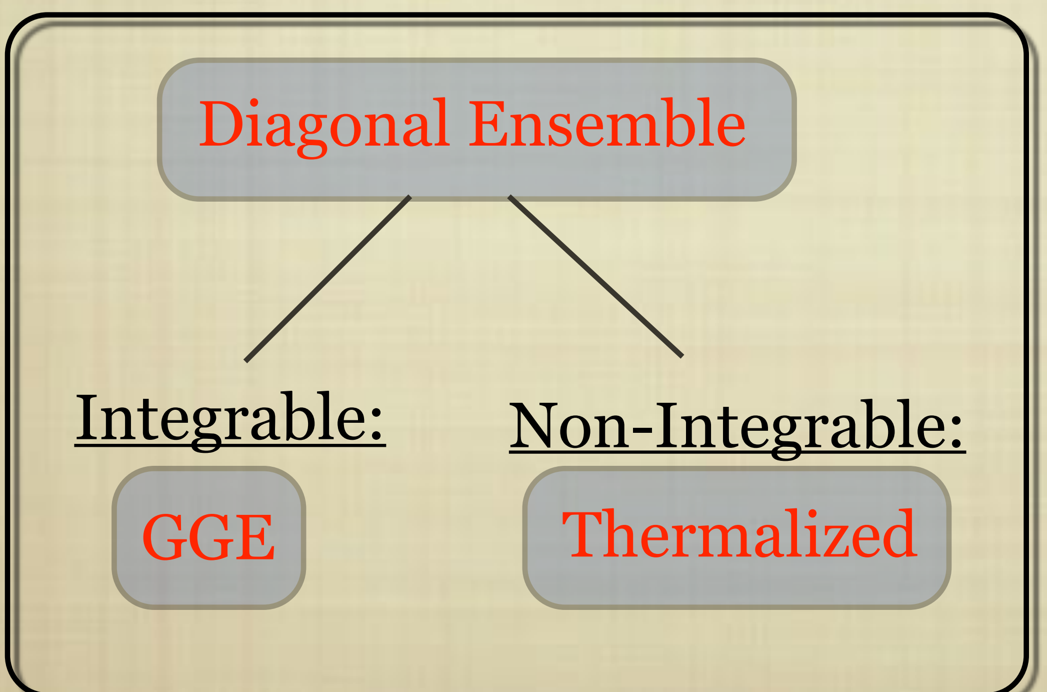
# Q: What do closed quantum systems evolve to?

Sit and wait under some Hamiltonian H:

$$\Psi(t) = \alpha_0 \exp[itE_0]|\Psi_0\rangle + \alpha_1 \exp[itE_1]|\Psi_1\rangle + \ldots$$

These phases dephase with respect to each other!

Observables in diagonal ensemble.




Can we get something else?

# Q: What do closed quantum systems evolve to?

Sit and wait under some Hamiltonian H(t):

$$\lambda = vt$$

$$\Delta E(\tau) = \sqrt{\tau^2 + 1} \approx \frac{1}{2\tau}$$

$$\Delta\phi = \int_{\tau_i}^{\tau_f} \Delta E(\tau) d\tau \approx \frac{1}{2}\log(\tau_f/\tau_i)$$

As $\tau \to \infty, \quad \Delta\phi \to \infty$

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$$\lambda = vt^3$$

$$\Delta E(\tau) = \sqrt{\tau^6 + 1} - \sqrt{\tau^6} \approx \frac{1}{\tau^3}$$

$$\Delta\phi = \int_{\tau_i}^{\tau_f} \frac{d\tau}{\tau^3}$$

As $\tau \to \infty, \Delta\phi \to K$

**Steady State Neither GGE Nor Thermal!**

# Conclusions

- Transverse Field Ising Model scaling collapse
- Athermal States
- Ramp Protocols exist which end in phase locked states

$$\hat{O} = \alpha_0 \langle \Psi_0 | \hat{O} | \Psi_0 \rangle + \alpha_1 \langle \Psi_1 | \hat{O} | \Psi_1 \rangle + ...$$

$$\Delta E(\tau) = \sqrt{\tau^2 + 1} - \tau \approx \frac{1}{2\tau}$$

$$\Delta\varphi = \int_{\tau_i}^{\tau_f} \Delta E(\tau) d\tau \approx \frac{1}{2} \log(\tau_f / \tau_i) \qquad (13)$$

$$2J(g - \cos(k))$$

$$H[\lambda] = -\frac{1}{2} \sum_j \left[ s_j^z s_{j+1}^z + (1 - \lambda) s_j^x \right]$$

$$H_k = \left( -\lambda + \frac{k^2}{2} + \ldots )\sigma_k^z + (k - \frac{k^3}{6} + \ldots \right) \sigma_k^x$$

$$i\frac{d\Psi_k}{dt} = (-vt\sigma_k^z + k\sigma_k^x)\,\Psi_k$$

$$i\frac{d\Psi_\kappa}{d\tau} = (-\tau\sigma_\kappa^z + \kappa\sigma_\kappa^x)\,\Psi_\kappa$$

**We've written** $\Psi(t, k, v) = \Psi_\kappa(\tau)$

$$i\frac{d\Psi}{dt} = H\Psi$$

# Low Energy

$$H_k = \left( -\lambda + \xcancel{\frac{k^2}{2}} + \ldots \right)\sigma_k^z + \left( k - \xcancel{\frac{k^3}{6}} + \ldots \right) \sigma_k^x$$

$t_k \equiv v^{-1/2}$ $\longleftrightarrow$ $l_k \equiv v^{-1/2}$

Ising Universality Class: $\nu = z = 1$

$t/t_k$

$L/l_k$

$kl_k$

# Quantum ramps

$$H(\lambda) = (1 - \lambda - \cos k)\sigma_k^z + (\sin k)\sigma_k^x$$

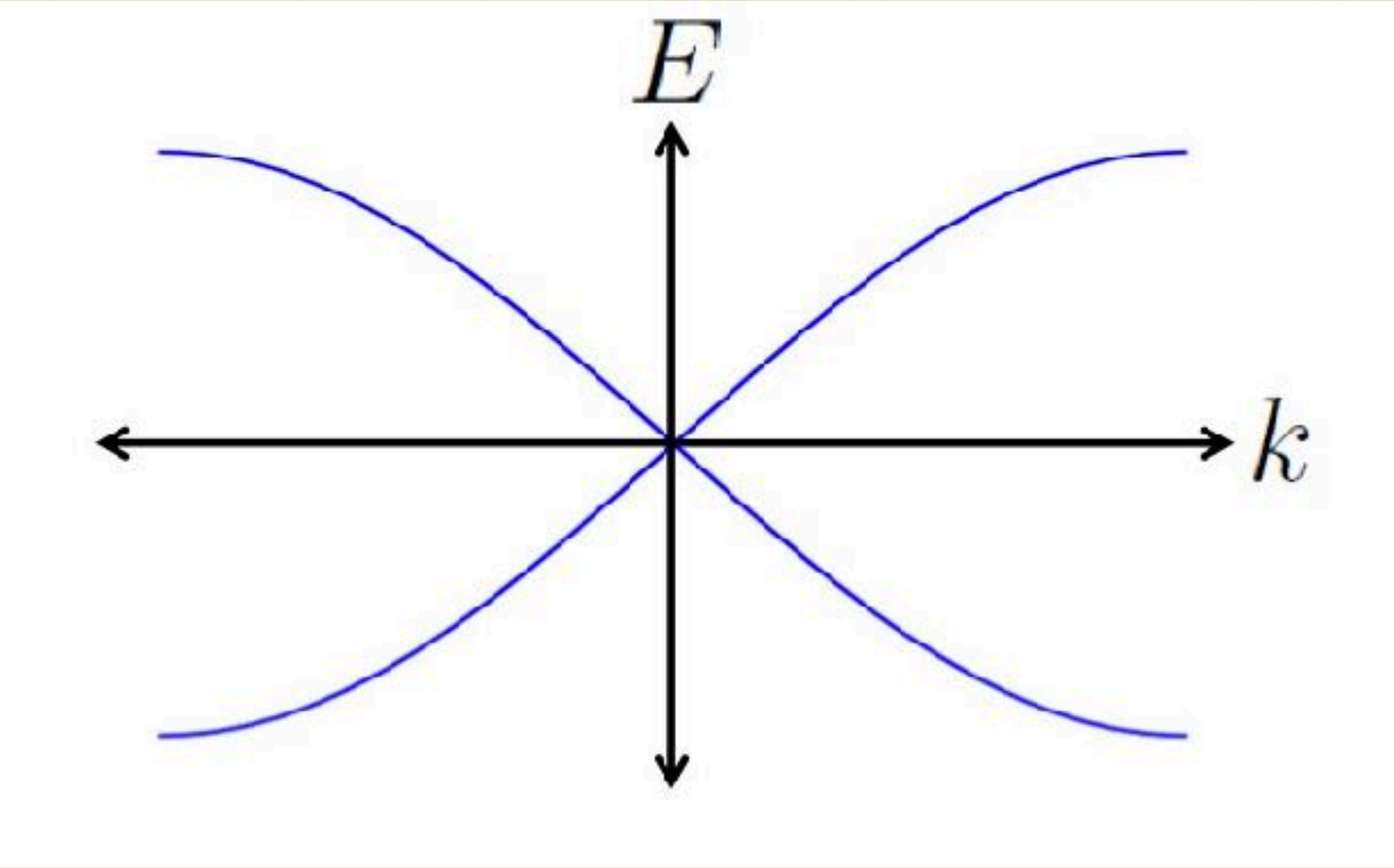
- Start Here
- End There

$$H[\lambda] = \sum_k H_k[\lambda]$$

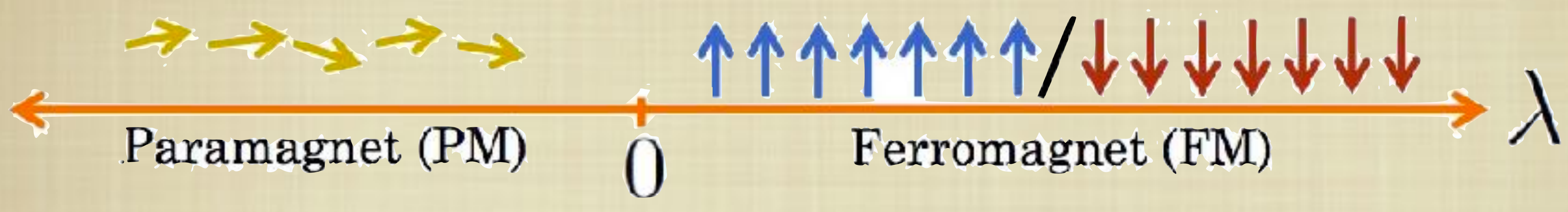
$$H_k[\lambda] = (1 - \lambda - \cos k)\sigma_k^z + (\sin k)\sigma_k^x$$

- Slow Ramps Track the Ground State

- You can't be Slow enough with a gapless system!



- Is their something universal here?



Paramagnet (PM)
0
Ferromagnet (FM)
λ

# Finite Size Effects