

Possibly useful formulae

$$\begin{aligned}\vec{F} &= -\vec{\nabla}U(\vec{r}) \\ U(x) &= -\int_{x_0}^x dx' F(x') \\ \frac{1}{2}m(v_2^2 - v_1^2) &= \int_{x_1}^{x_2} dx' F(x')\end{aligned}$$

Rocket with gravity

$$m\dot{v} + u\dot{m} = -mg$$

Cylindrical coordinates

$$\begin{aligned}\hat{r} &= \cos\phi\hat{i} + \sin\phi\hat{j} \\ \hat{\phi} &= -\sin\phi\hat{i} + \cos\phi\hat{j} \\ \vec{v} &= \dot{r}\hat{r} + r\dot{\phi}\hat{\phi} + \dot{z}\hat{z} \\ \vec{a} &= (\ddot{r} - r\dot{\phi}^2)\hat{r} + (2\dot{r}\dot{\phi} + r\ddot{\phi})\hat{\phi} + \ddot{z}\hat{z}\end{aligned}$$

Spherical coordinates

$$\begin{aligned}\hat{r} &= \sin\theta\cos\phi\hat{i} + \sin\theta\sin\phi\hat{j} + \cos\theta\hat{k} \\ \hat{\theta} &= \cos\theta\cos\phi\hat{i} + \cos\theta\sin\phi\hat{j} - \sin\theta\hat{k} \\ \hat{\phi} &= -\sin\phi\hat{i} + \cos\phi\hat{j} \\ \vec{v} &= \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\dot{\phi}\sin\theta\hat{\phi} \\ \vec{a} &= \hat{r}(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta) \\ &\quad + \hat{\theta}(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta) \\ &\quad + \hat{\phi}(r\ddot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta + 2\dot{r}\dot{\phi}\sin\theta) \\ \hat{r} \times \hat{\theta} &= \hat{\phi}; \quad \hat{\phi} \times \hat{r} = \hat{\theta}; \quad \hat{\theta} \times \hat{\phi} = \hat{r}\end{aligned}$$

Quadratic formula:

$$\begin{aligned}\text{if } ax^2 + bx + c &= 0, \\ \text{then } x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\end{aligned}$$

$$\begin{aligned}\cos(a+b) &= \cos a \cos b - \sin a \sin b \\ \sin(a+b) &= \sin a \cos b + \cos a \sin b\end{aligned}$$

Taylor series:

$$\begin{aligned}f(x) &= f(x_0) + \left(\frac{df}{dx}\right)_{x=x_0} (x-x_0) \\ &\quad + \frac{1}{2!} \left(\frac{d^2f}{dx^2}\right)_{x=x_0} (x-x_0)^2 + \mathcal{O}(x-x_0)^3\end{aligned}$$

$$\begin{aligned}\int \frac{dx}{x+a} &= \ln(x+a) + C, \quad \int e^x dx = e^x + C, \\ \int \ln x dx &= x \ln x - x + C, \quad \int \sin x dx = -\cos x + C, \\ \int \cos x dx &= \sin x + C, \quad \int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) \\ \int \frac{xdx}{\sqrt{x^2+a^2}} &= \sqrt{a^2+x^2}.\end{aligned}$$

$$r(\phi) = \frac{\alpha}{1 + \varepsilon \cos\phi} \quad \alpha = \frac{l^2}{GM} \quad \varepsilon = \sqrt{1 + \frac{2el^2}{G^2M^2}}$$

$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix} \quad \vec{\nabla} = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$$

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t) \exp(-i\omega t) dt,$$

$$f(t) = \int_{-\infty}^{\infty} \tilde{f}(\omega) \exp(i\omega t) \frac{d\omega}{2\pi}$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}, \quad \zeta = \frac{c}{2m\omega_n}$$

$$G(t) = \frac{\sin(\omega_d t)}{m\omega_d} \exp\left(-\frac{ct}{2m}\right),$$

$$|\tilde{G}(\omega)| = \frac{1}{k} \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + 4\left(\frac{\omega}{\omega_n}\right)^2 \zeta^2}}$$

$$\phi(\omega) = \tan^{-1} \left[2\zeta\omega\omega_n / (\omega_n^2 - \omega^2) \right]$$

$$\tilde{x}(\omega) = \tilde{G}(\omega) \tilde{F}(\omega)$$

$$\int_{-\infty}^{\infty} f(t) \delta(t-a) dt = f(a)$$

$$a_n = \frac{2}{T} \int_T F(t) \cos(2n\pi t/T) dt$$

$$b_n = \frac{2}{T} \int_T F(t) \sin(2n\pi t/T) dt$$

$$\begin{aligned}\vec{F}^{\text{fictitious}} &= -m\vec{a}_0 - 2m\vec{\omega} \times \vec{v}^{\text{relative}} \\ &\quad - m\vec{\omega} \times [\vec{\omega} \times \vec{r}] - m\dot{\vec{\omega}} \times \vec{r}\end{aligned}$$