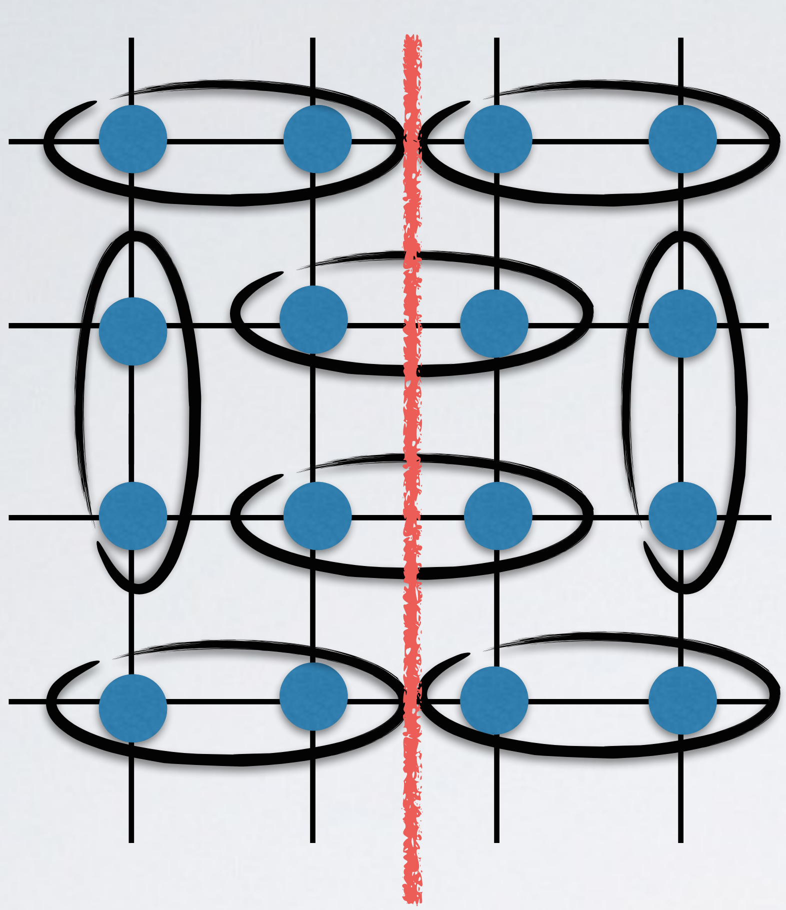
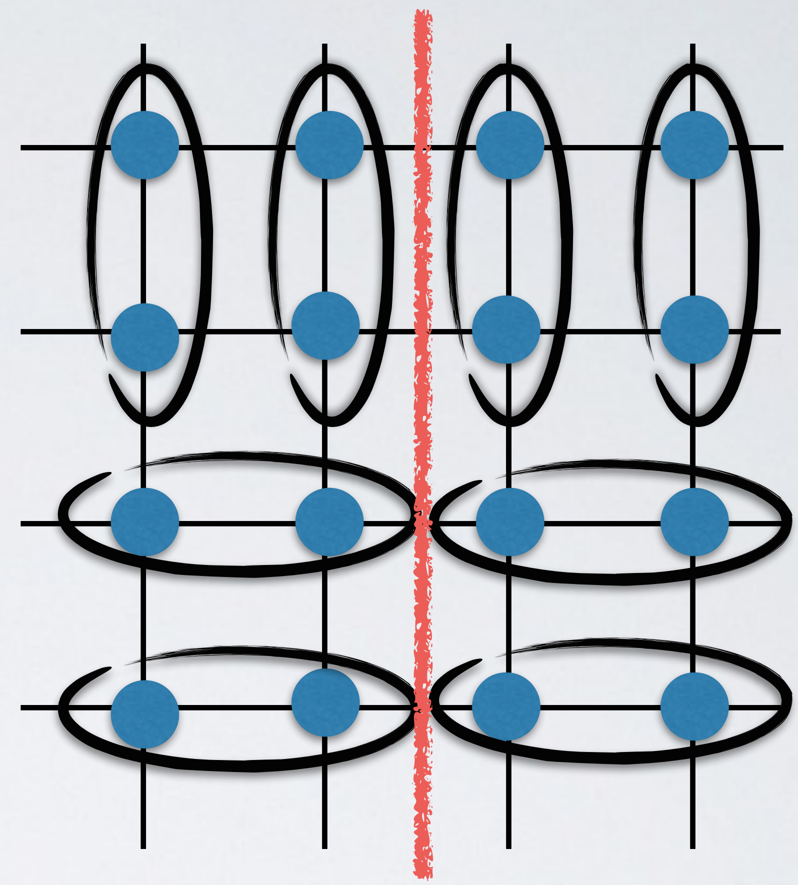
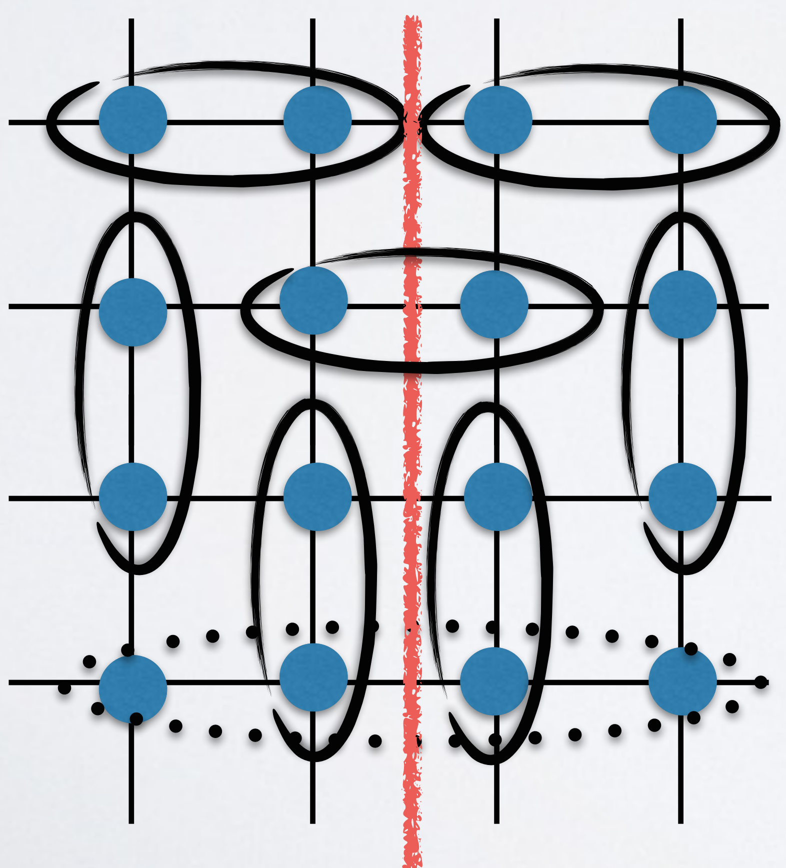
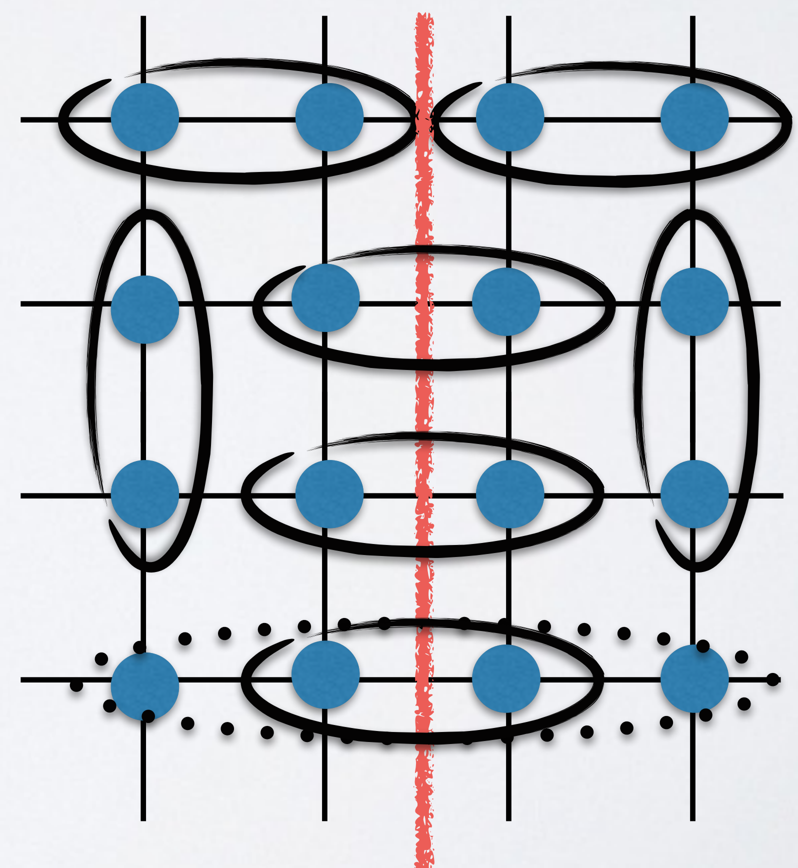
ALGORITHMIC PERSPECTIVE ON STRONGLY CORRELATED SYSTEMS

Lecture: RVB and Spin Liquids

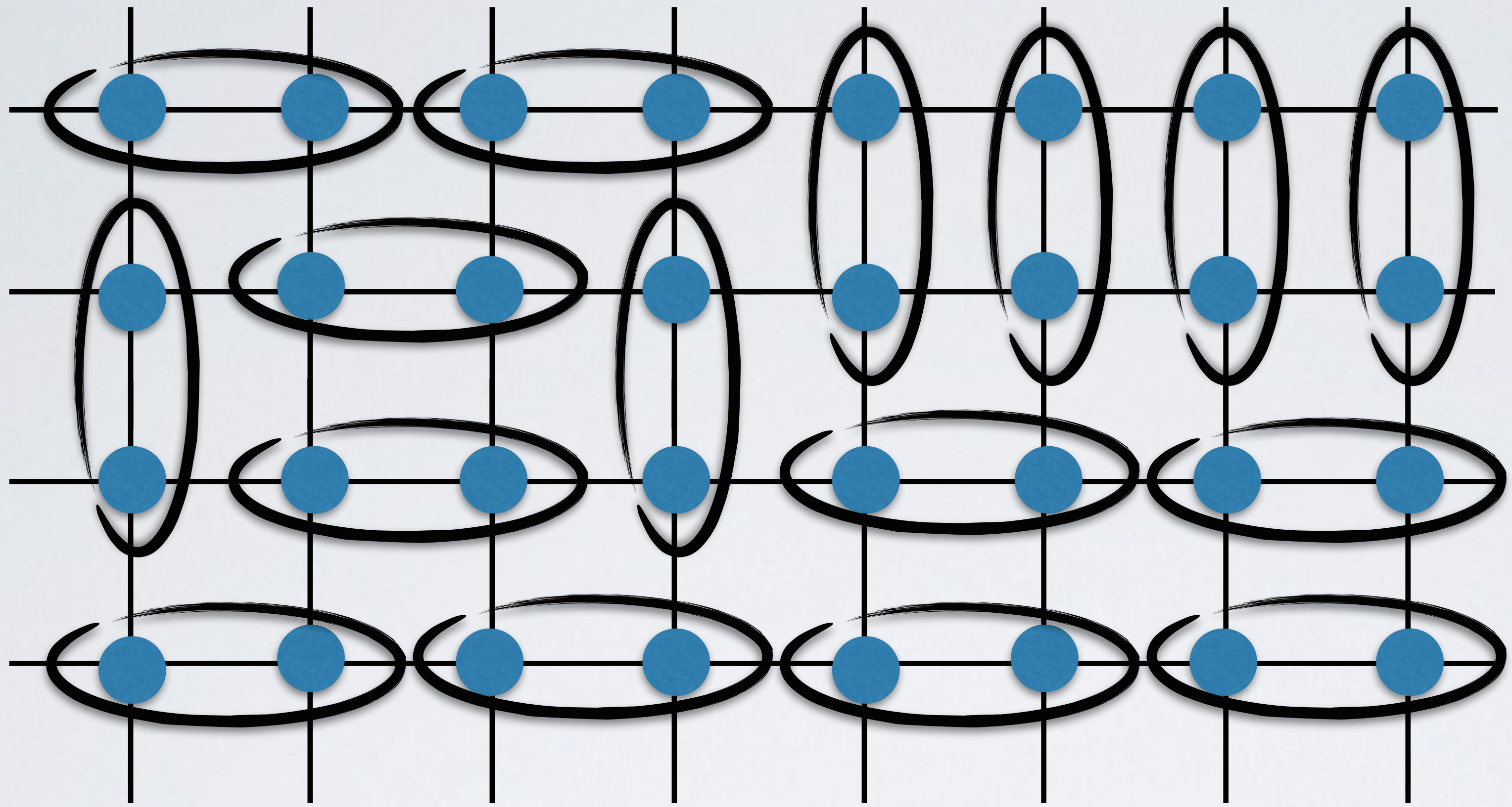
Bryan Clark

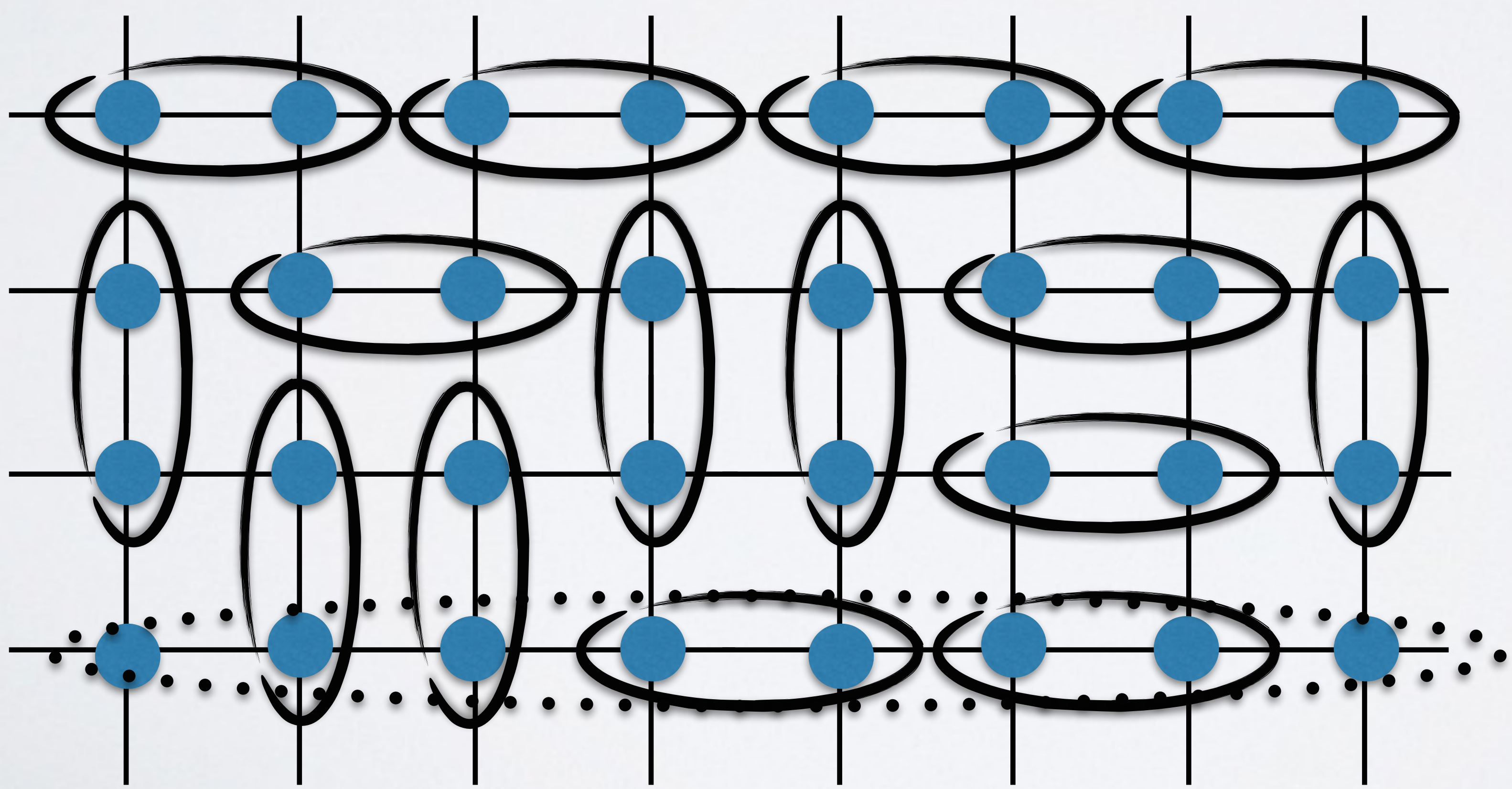
A special state: The uniform nearest-neighbor RVB state

$\Psi_{\mathrm{RVB}}^{\mathrm{even}} =$  $+$  $+ \ldots$

$\Psi_{\mathrm{RVB}}^{\mathrm{odd}} =$  $+$ 

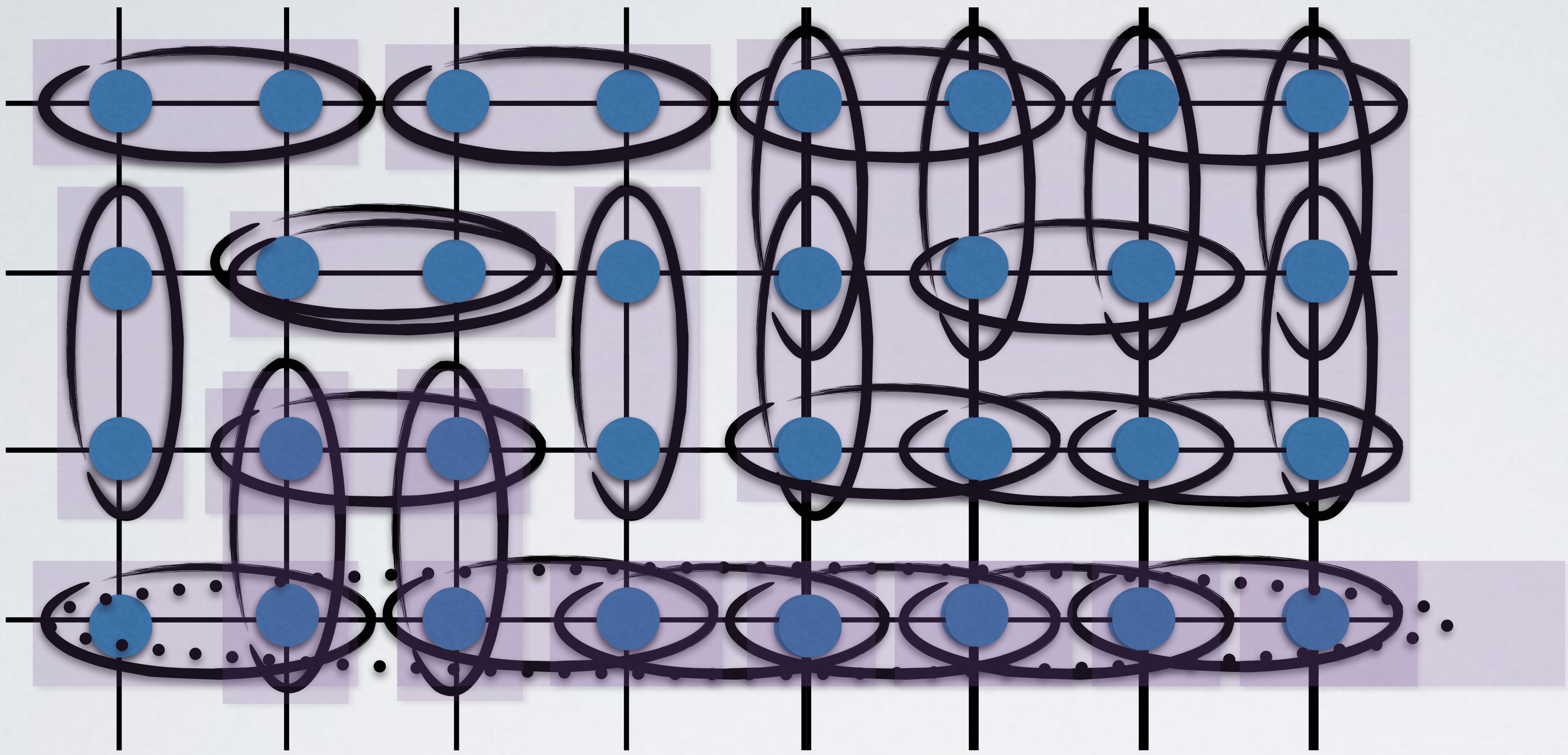
A special state: The uniform nearest-neighbor RVB state





$$2^{N_{\text{loop}}-N/2}$$

A special state: The uniform nearest-neighbor RVB state

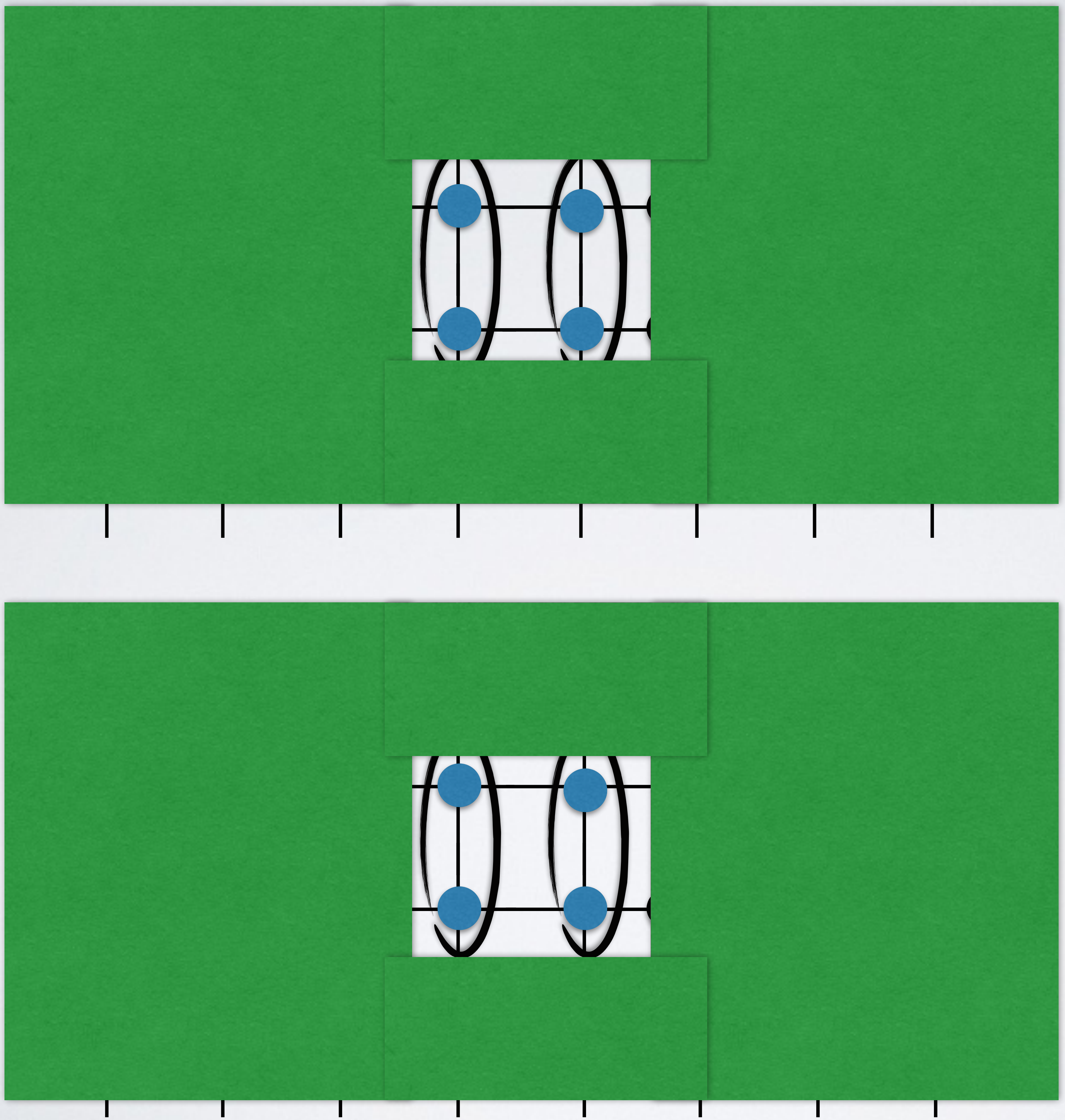


Orthogonal?

(homework)

$$2^{N_{\mathrm{loop}}-N/2}$$

A special state: The uniform nearest-neighbor RVB state

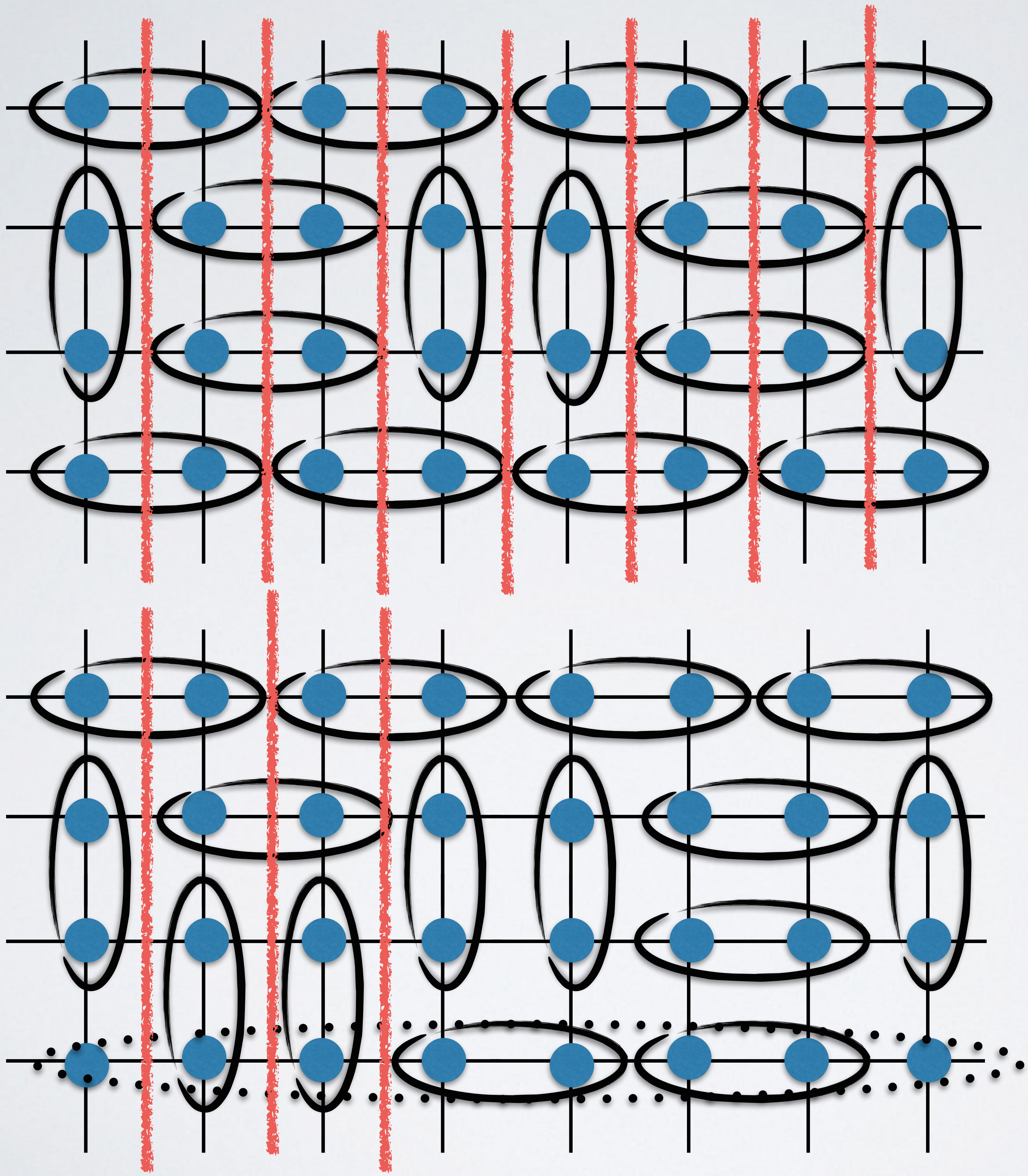


Q: Can the energy of these states be different?

Degenerate!

Locally indistinguishable!

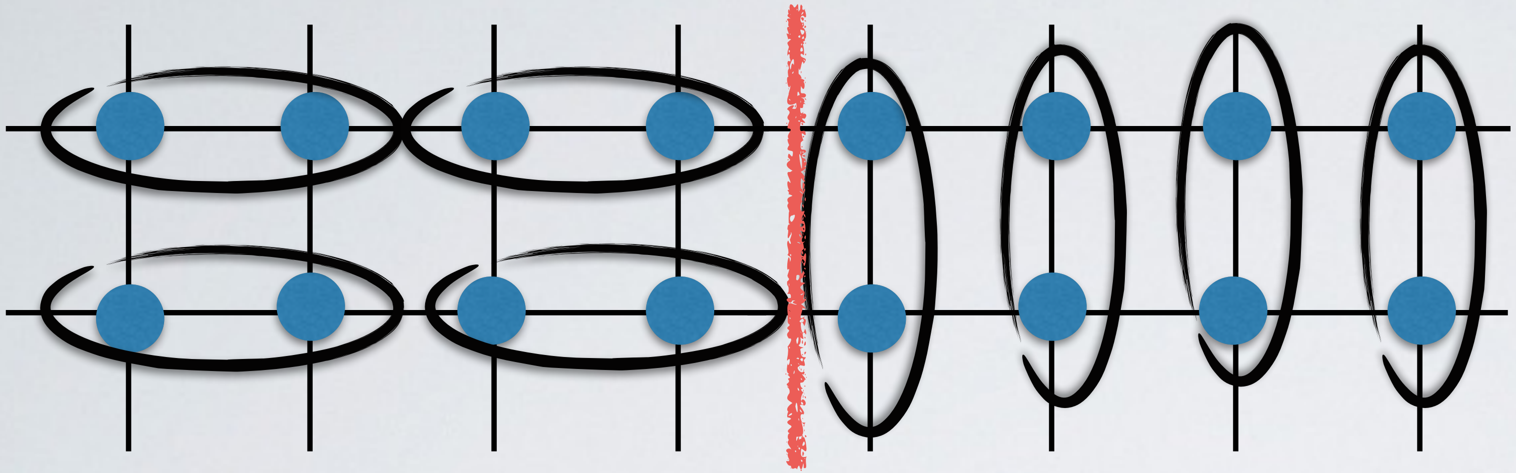
A special state: The uniform nearest-neighbor RVB state



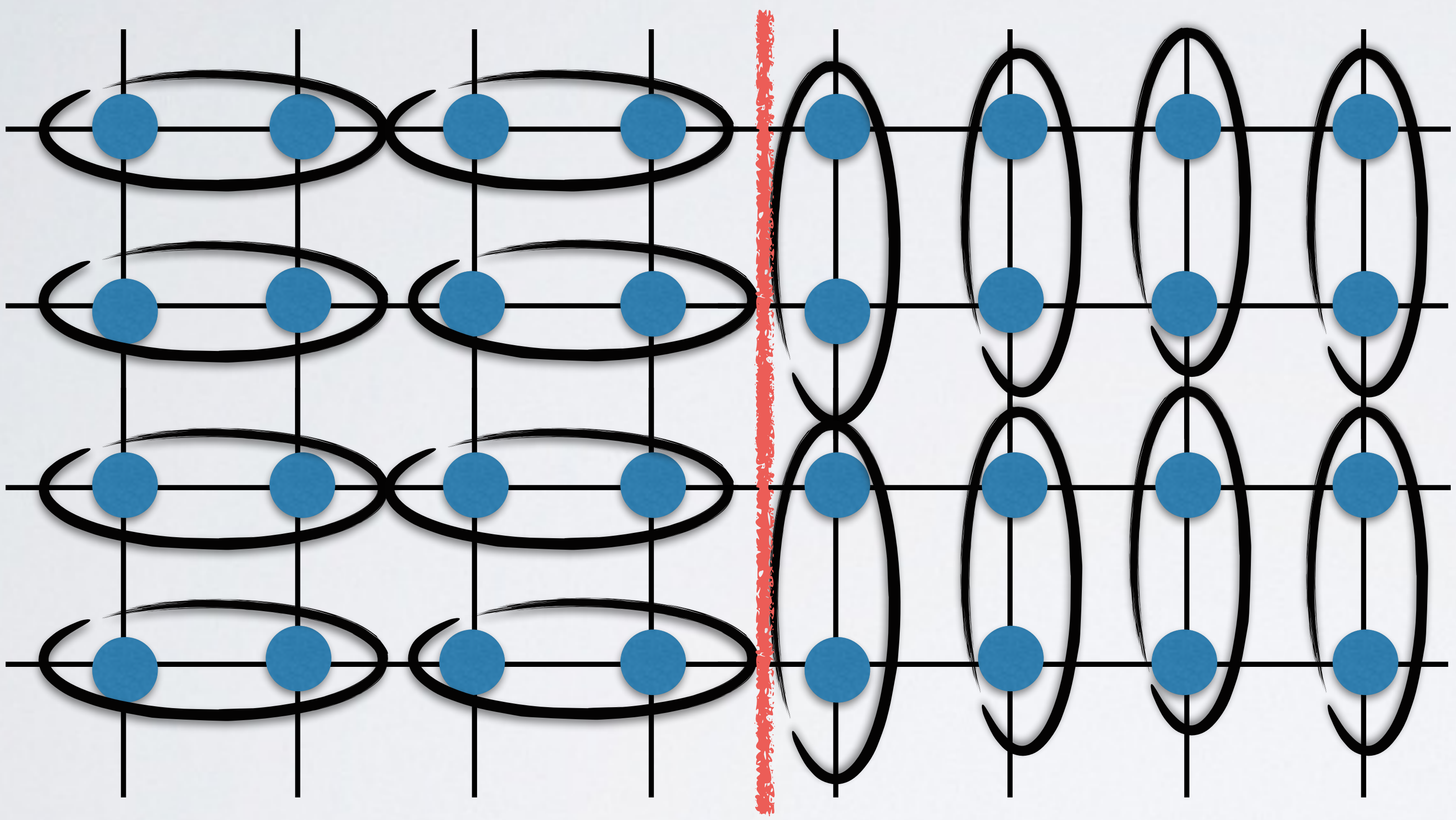
Q: How do you change from one state to the other?

Topological Entanglement Entropy

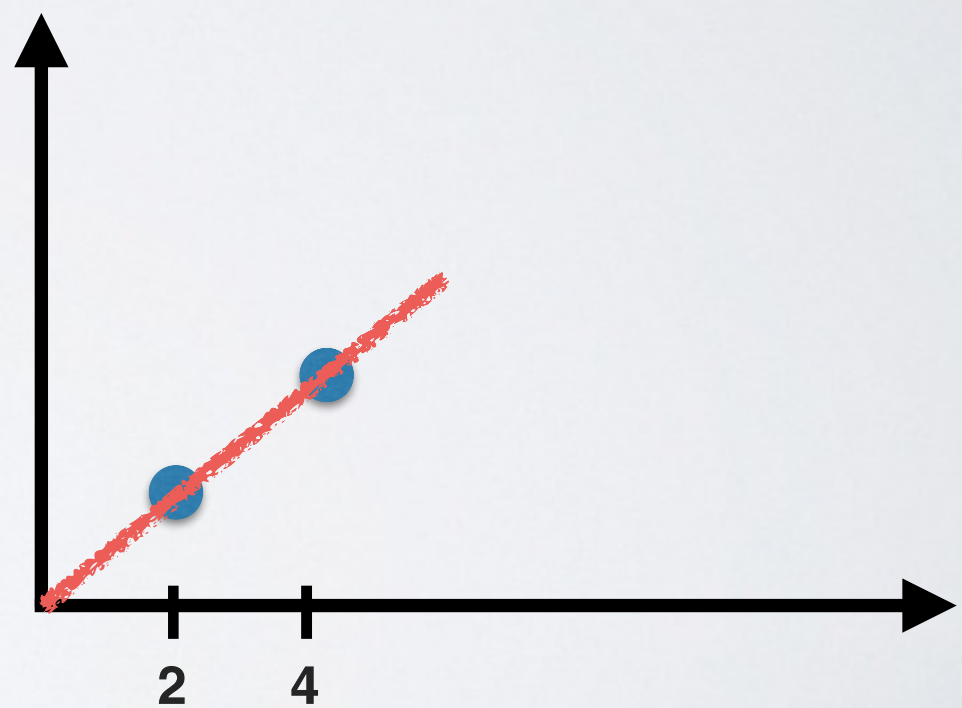
$$S(W) = \alpha W - \gamma$$



W=2; S= [(0+2)/2] ln(2) = ln(2)

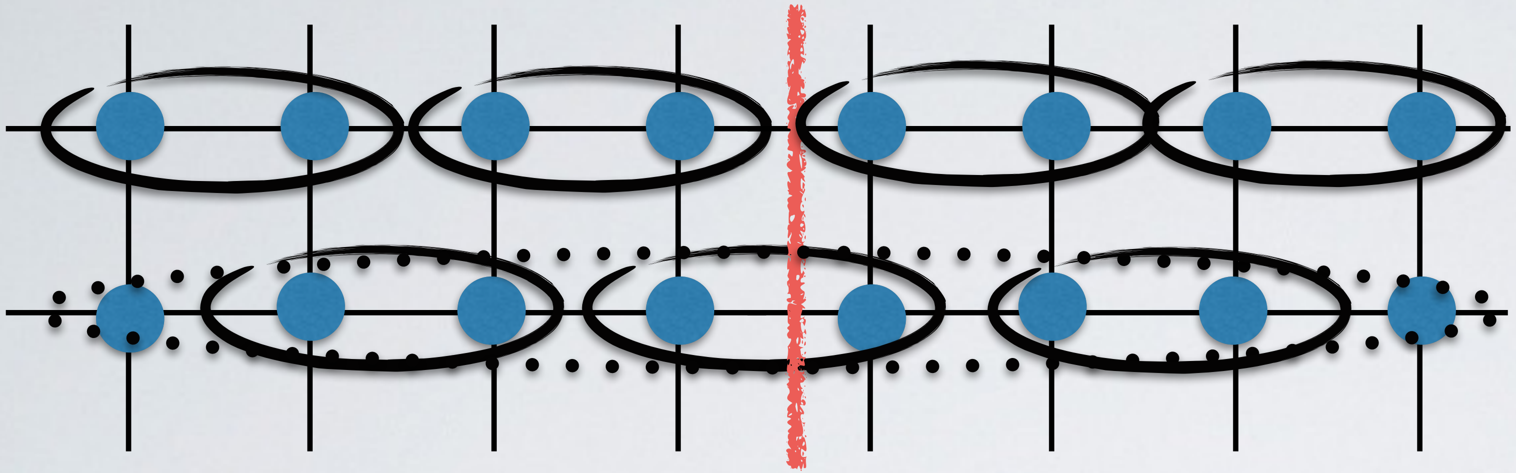


W=4; S= [(0+2+4)/2] ln(2) = 3 ln(2)

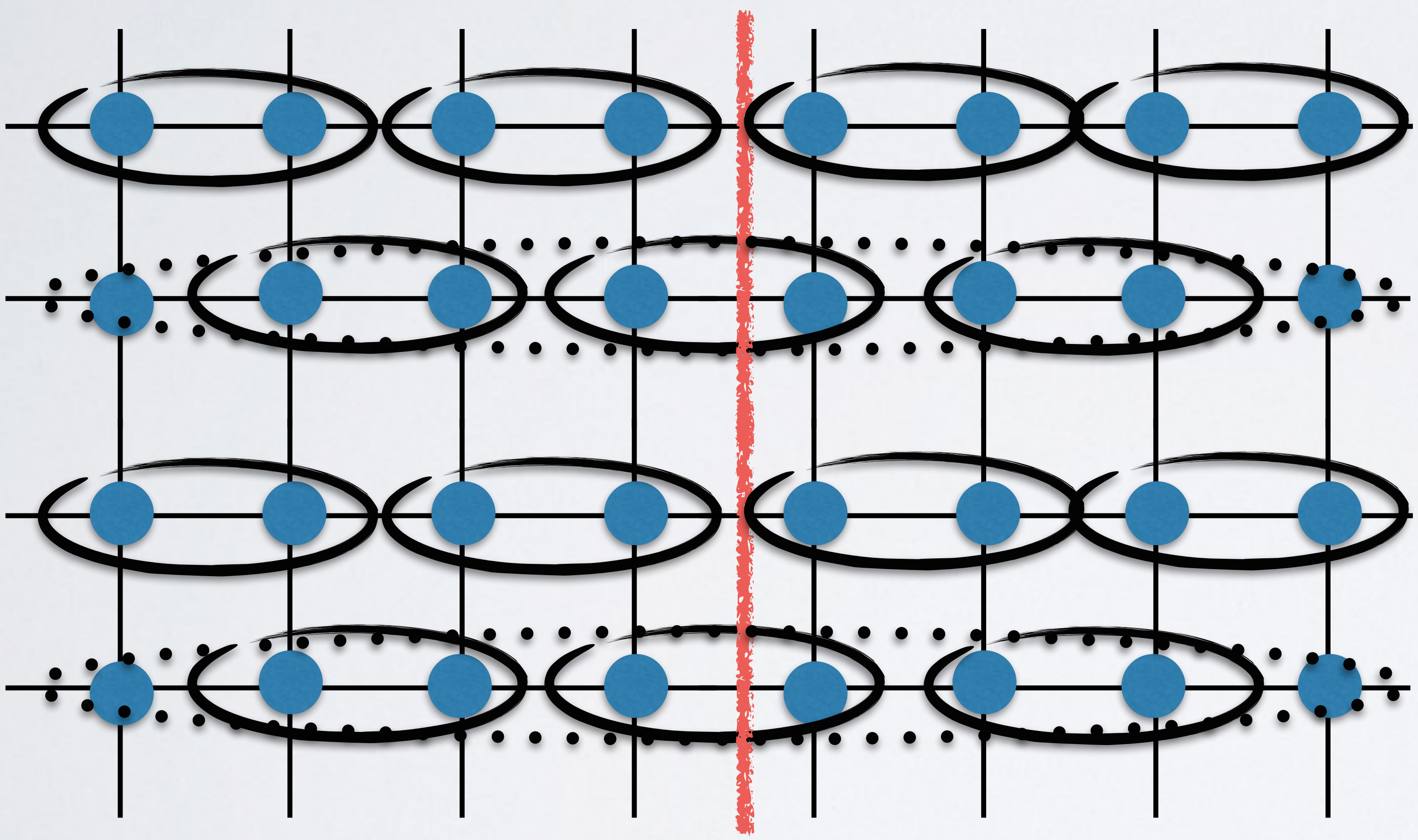


Topological Entanglement Entropy

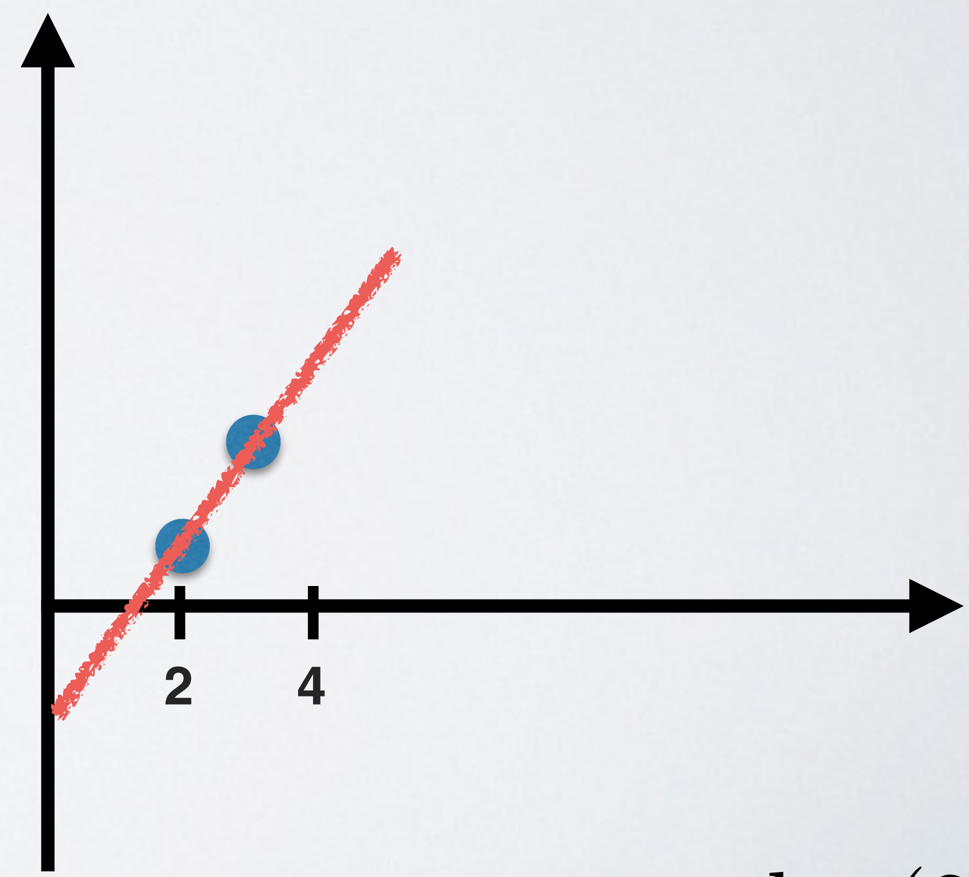
$$S(W) = \alpha W - \gamma$$



W=2; S= [1/2] ln(2) = 0.5 ln(2)



W=4; S= [(1+3)/2] ln(2) = 2 ln(2)



$$\gamma = \log(2)$$

We will consider two classes of wave-functions

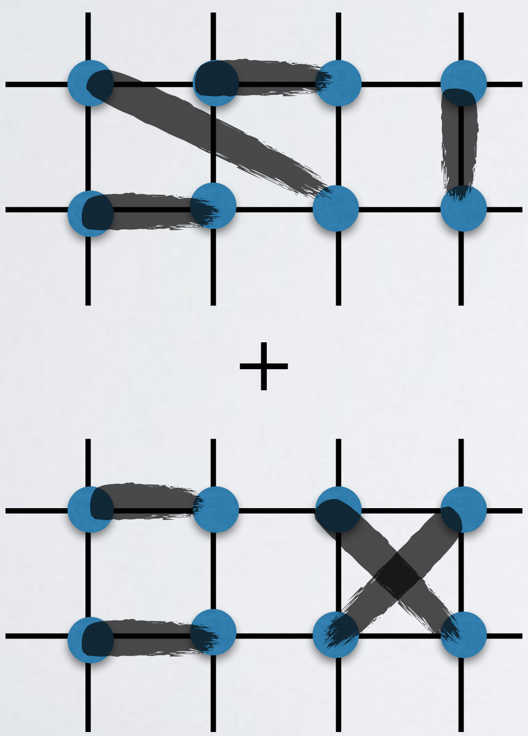
Fermionic RVB

Singlet: $(c^\dagger_{1\uparrow}c^\dagger_{2\downarrow} + c^\dagger_{1\downarrow}c^\dagger_{2\uparrow})|0\rangle$

$$\Theta_{12}|0\rangle$$

Notation: canonical ordering - all up and then all down

$$\sum_{[\text{dimer coverings A}]} \prod_{ij\in A} f_{ij}\Theta_{ij}|0\rangle$$

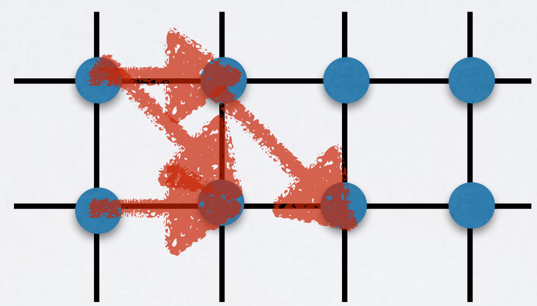


Bosonic RVB

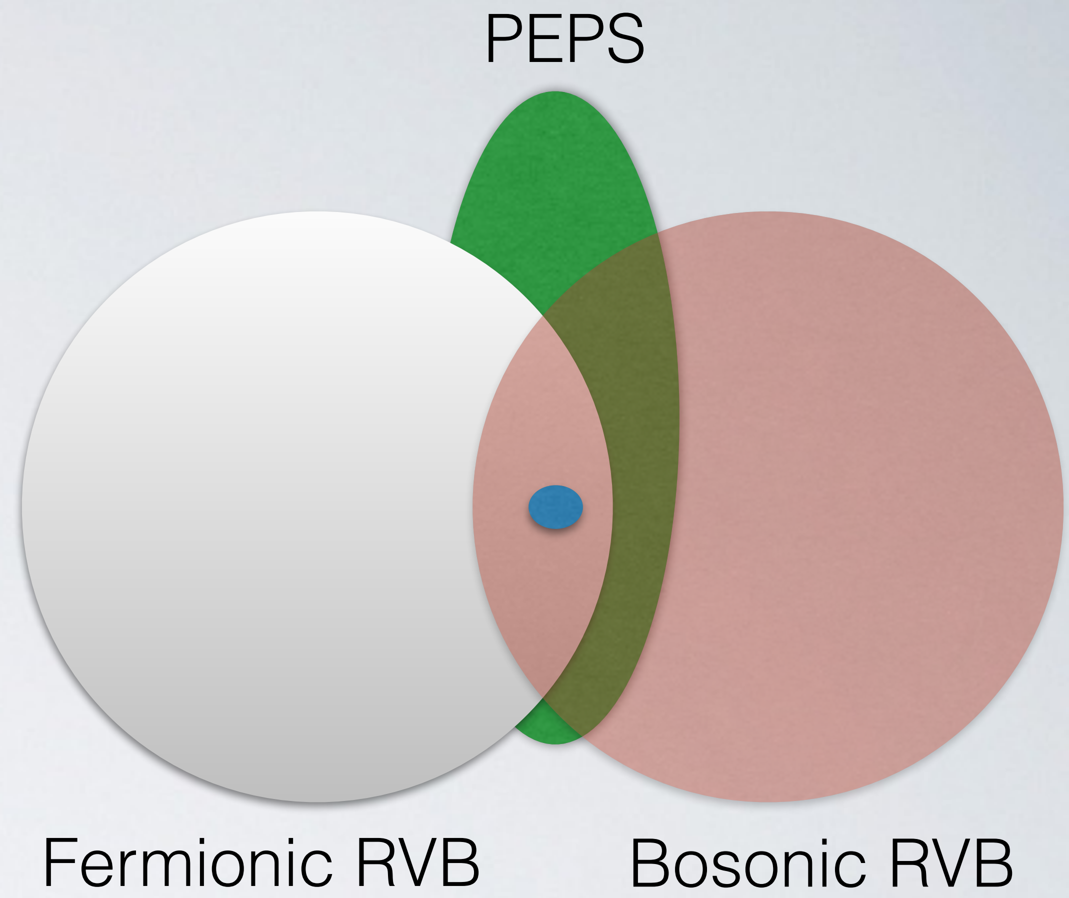
$$|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle$$

Notation: for each (i,j) you need to have an orientation

('gauge out this choice by changing f)



$$\sum_{[\text{dimer coverings A}]} \prod_{ij\in A} f^{\text{boson}}_{ij}(|\uparrow_i\downarrow_j\rangle - \downarrow_i\uparrow_j\rangle)$$



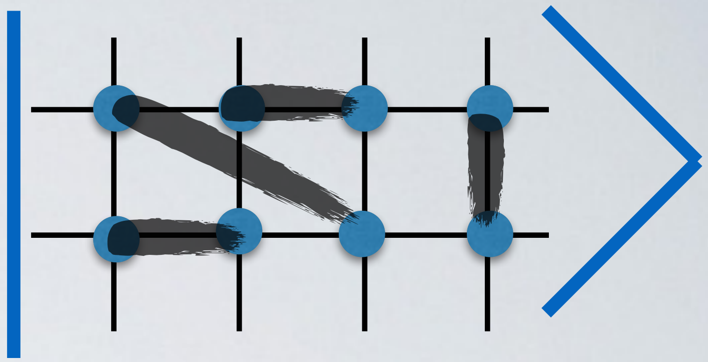
Important aside: this can be a basis….

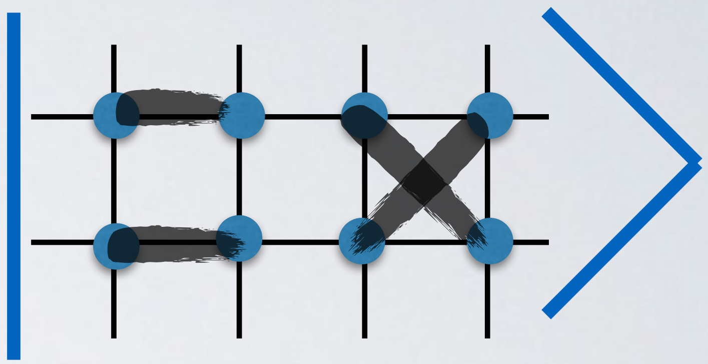
$$\sum_{\text{[dimer coverings A]}} \prod_{ij \in A} f_{ij}^{\text{boson}} (|\uparrow_i \downarrow_j\rangle - \downarrow_i \uparrow_j\rangle)$$

Q: Are these a complete basis? (multiplicity?)

Q: Are these orthogonal

Basis Elements:



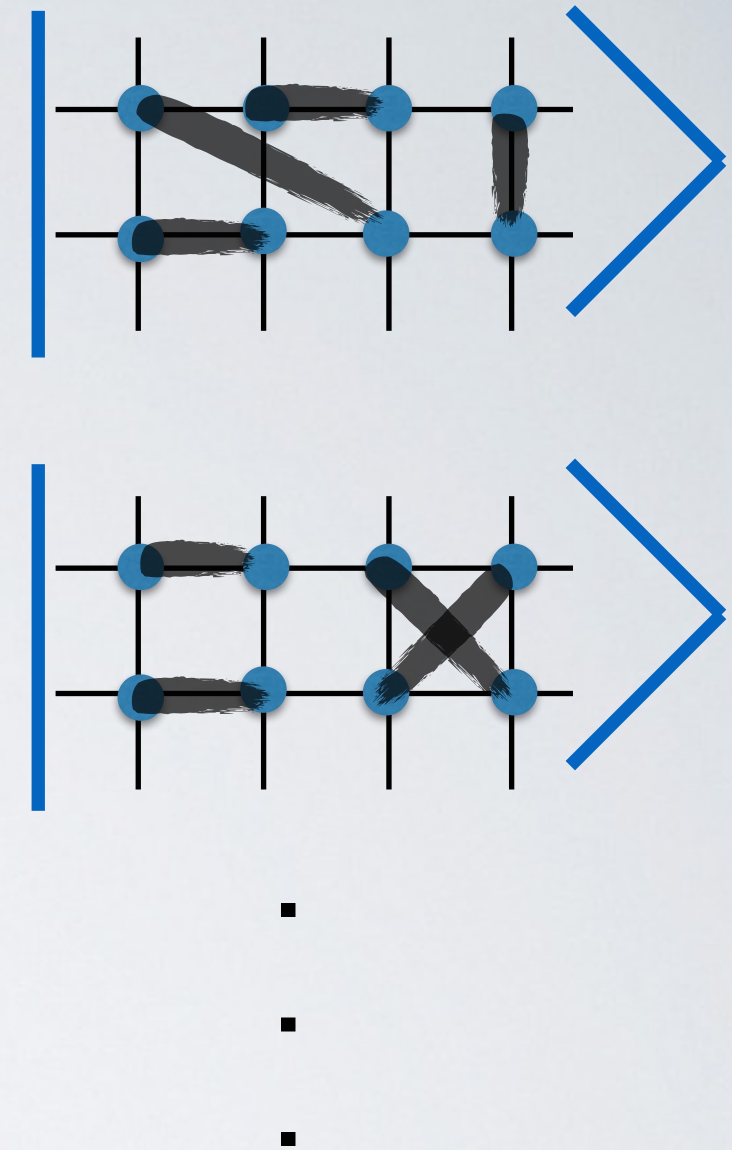


.
.
.

Note: could be fermionic too

Important aside: this can be a basis….

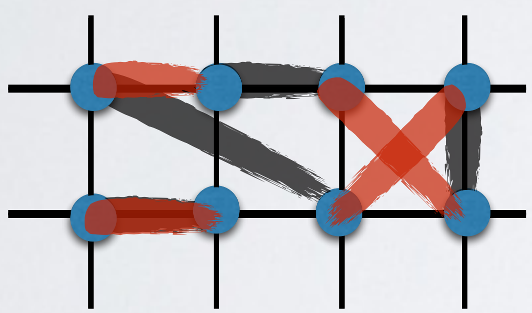
Basis Elements:



$$\sum_{[\text{dimer coverings A}]} \prod_{ij\in A} f_{ij}^{\text{boson}}(|\uparrow_i\downarrow_j\rangle - \downarrow_i\uparrow_j\rangle)$$

Q: Are these a complete basis?

Q: Are these orthogonal

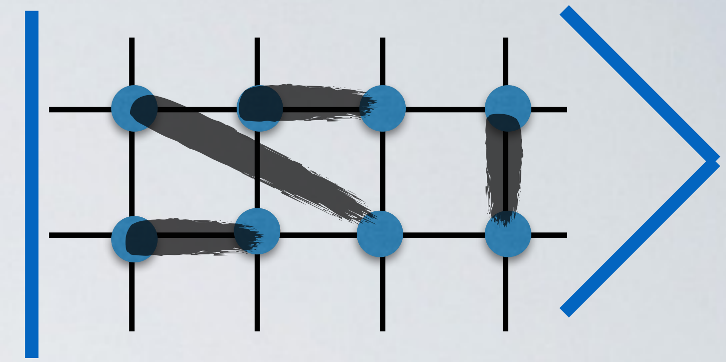


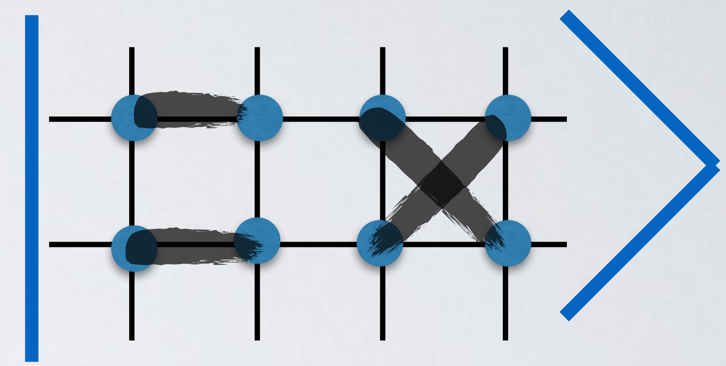
$$2^{N_{\text{loop}}-N/2}$$

Note: could be fermionic too

Important aside: this can be a basis….

Basis Elements:





⋮

$$\sum_{[\text{dimer coverings A}]} \prod_{ij\in A} f_{ij}^{\text{boson}}(|\uparrow_i\downarrow_j\rangle - \downarrow_i\uparrow_j\rangle)$$

Q: Are these a complete basis?

Q: Are these orthogonal

Doing exact diagonalization…

$$H_{ij} = \langle b_i|\hat{H}|b_j\rangle$$

$$S_{ij} = \langle b_i|b_j\rangle$$

$$H\Psi = ES\Psi$$

Q: Will this give you a ground state? excited states?

Q: Is this going to give you the values of f?

Note: could be fermionic too

Suppose we want the amplitude of a bosonic RVB state…

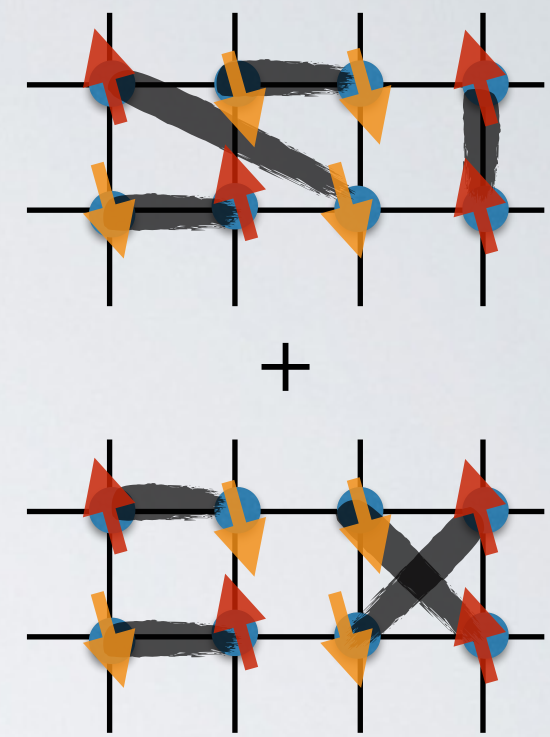
$$\sum_{\text{[dimer coverings A]}} \prod_{ij\in A} f_{ij}^{\text{boson}}(|\uparrow_i\downarrow_j\rangle - \downarrow_i\uparrow_j\rangle)$$

in the Sz basis

$$|\uparrow,\downarrow,\downarrow,\uparrow,\downarrow,\uparrow,\downarrow,\uparrow\rangle$$

Loop through all dimer configurations D consistent with pattern of valence bonds… (Q: how many?)

For each D, multiply the amplitudes and signs

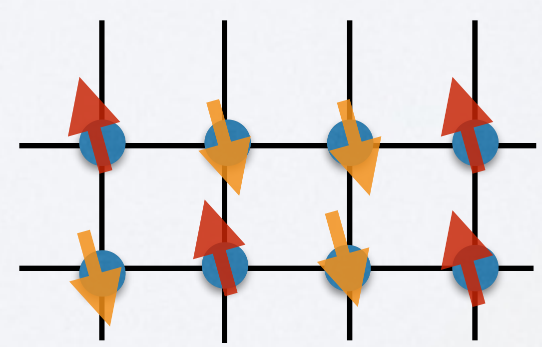


Q: Can we do better?

Permanent:

	1'st spin down (2)	2'nd spin down (3)	3'rd spin down (5)	4'th spin down (7)
1'st spin up (1)	f(1,2)	f(1,3)	f(1,5)	f(1,7)
2'nd spin up (4)	f(4,2)	f(4,3)	f(4,5)	f(4,7)
3'rd spin up (6)	f(6,2)	f(6,3)	f(6,5)	f(6,7)
4'th spin up (8)	f(8,2)	f(8,3)	f(8,5)	f(8,7)

Q: What are the "legal" valence bonds.
(1,2) (1,3) (1,5) (1,7) (4,2) (4,3) (4,5) …



Suppose we want the amplitude of a bosonic RVB state…

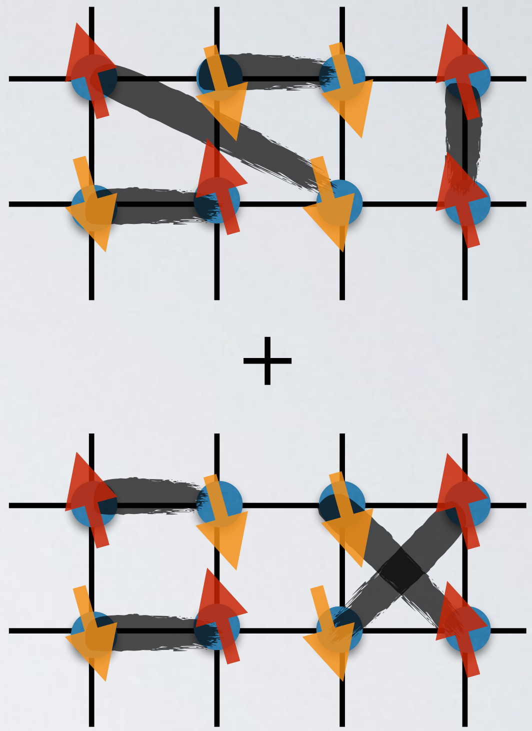
$$\sum_{[\text{dimer coverings A}]} \prod_{ij\in A} f_{ij}^{\text{boson}}(|\uparrow_i\downarrow_j\rangle - |\downarrow_i\uparrow_j\rangle)$$

in the Sz basis

$$|\uparrow,\downarrow,\downarrow,\uparrow,\downarrow,\uparrow,\downarrow,\uparrow\rangle$$

Loop through all dimer configurations D consistent with pattern of valence bonds… (Q: how many?)

For each D, multiply the amplitudes and signs



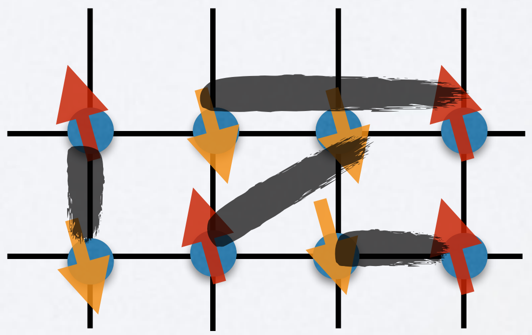
Q: Can we do better?

<u>Permanent:</u>

	1'st spin down (2)	2'nd spin down (3)	3'rd spin down (5)	4'th spin down (7)
1'st spin up (1)	f(1,2)	f(1,3)	f(1,5)	f(1,7)
2'nd spin up (4)	f(4,2)	f(4,3)	f(4,5)	f(4,7)
3'rd spin up (6)	f(6,2)	f(6,3)	f(6,5)	f(6,7)
4'th spin up (8)	f(8,2)	f(8,3)	f(8,5)	f(8,7)

Q: What are the "legal" valence bonds.
(1,2) (1,3) (1,5) (1,7) (4,2) (4,3) (4,5) …

Q: What is the amplitude of the one below?



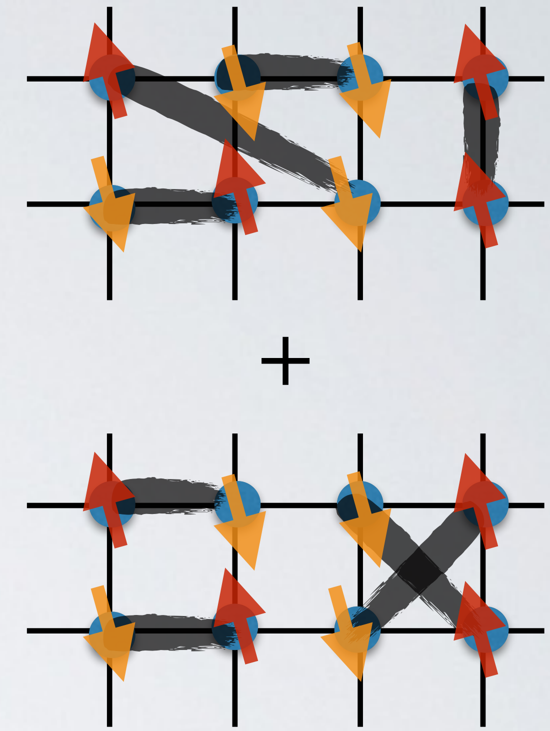
Suppose we want the amplitude of a bosonic RVB state… $\sum_{[\text{dimer coverings } A]} \prod_{ij\in A} f_{ij}^{\text{boson}}(|\uparrow_i\downarrow_j\rangle - \downarrow_i\uparrow_j\rangle)$

in the Sz basis

$|\uparrow,\downarrow,\downarrow,\uparrow,\downarrow,\uparrow,\downarrow,\uparrow\rangle$

Loop through all dimer configurations D consistent with pattern of valence bonds… (Q: how many?)

For each D, multiply the amplitudes and signs



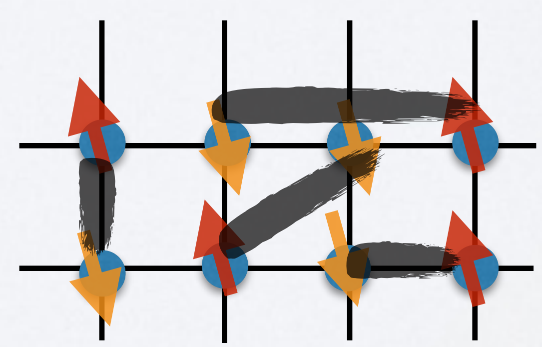
Q: Can we do better?

<u>Permanent:</u>

	1'st spin down (2)	2'nd spin down (3)	3'rd spin down (5)	4'th spin down (7)
1'st spin up (1)	f(1,2)	f(1,3)	f(1,5)	f(1,7)
2'nd spin up (4)	f(4,2)	f(4,3)	f(4,5)	f(4,7)
3'rd spin up (6)	f(6,2)	f(6,3)	f(6,5)	f(6,7)
4'th spin up (8)	f(8,2)	f(8,3)	f(8,5)	f(8,7)

Q: What are the "legal" valence bonds.
(1,2) (1,3) (1,5) (1,7) (4,2) (4,3) (4,5) …

Q: What is the amplitude of the one below?



Suppose we want the amplitude of a bosonic RVB state…

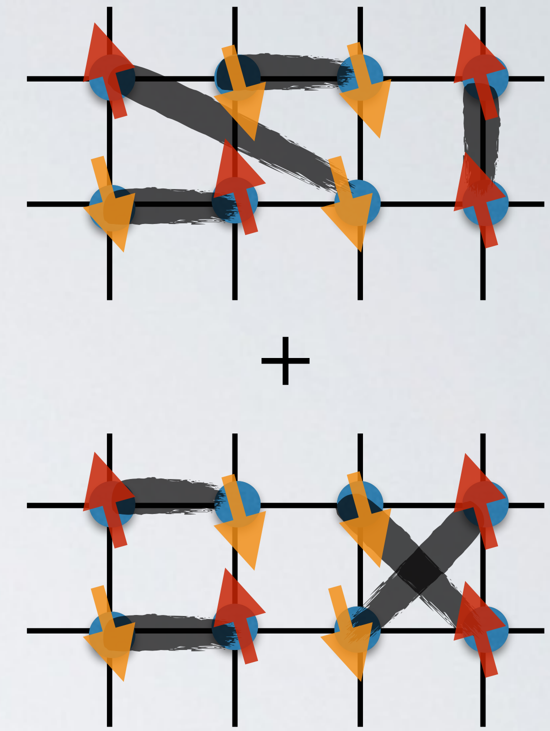
$$\sum_{[\text{dimer coverings } A]} \prod_{ij\in A} f_{ij}^{\text{boson}}(|\uparrow_i\downarrow_j\rangle - \downarrow_i\uparrow_j\rangle)$$

in the Sz basis

$$|\uparrow,\downarrow,\downarrow,\uparrow,\downarrow,\uparrow,\downarrow,\uparrow\rangle$$

Loop through all dimer configurations D consistent with pattern of valence bonds… (Q: how many?)

For each D, multiply the amplitudes and signs



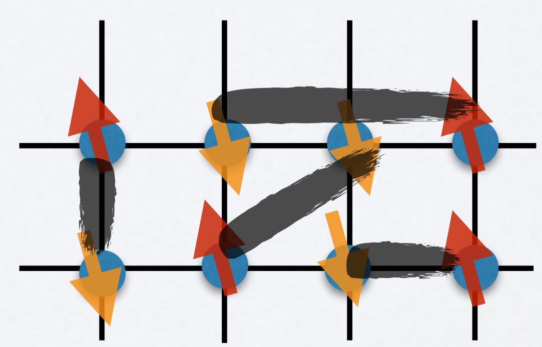
Q: Can we do better?

Permanent:

	1'st spin down (2)	2'nd spin down (3)	3'rd spin down (5)	4'th spin down (7)
1'st spin up (1)	f(1,2)	f(1,3)	f(1,5)	f(1,7)
2'nd spin up (4)	f(4,2)	f(4,3)	f(4,5)	f(4,7)
3'rd spin up (6)	f(6,2)	f(6,3)	f(6,5)	f(6,7)
4'th spin up (8)	f(8,2)	f(8,3)	f(8,5)	f(8,7)

Q: What are the "legal" valence bonds.
(1,2) (1,3) (1,5) (1,7) (4,2) (4,3) (4,5) …

Q: What is the amplitude of the one below?



Each dimer configuration corresponds to choosing one number from each row and column.

Then sum over them!

Suppose we want the amplitude of a bosonic RVB state…

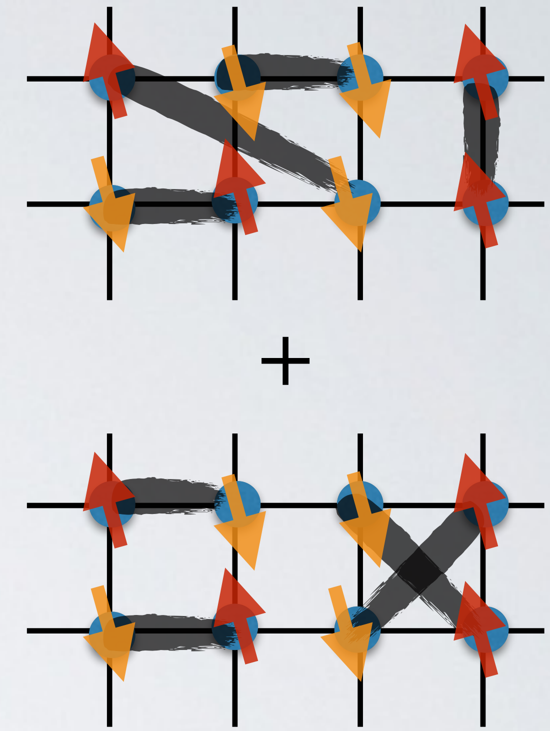
$$\sum_{[\text{dimer coverings } A]} \prod_{ij\in A} f_{ij}^{\text{boson}}(|\uparrow_i\downarrow_j\rangle - |\downarrow_i\uparrow_j\rangle)$$

in the Sz basis

$$|\uparrow,\downarrow,\downarrow,\uparrow,\downarrow,\uparrow,\downarrow,\uparrow\rangle$$

Loop through all dimer configurations D consistent with pattern of valence bonds… (Q: how many?)

For each D, multiply the amplitudes and signs



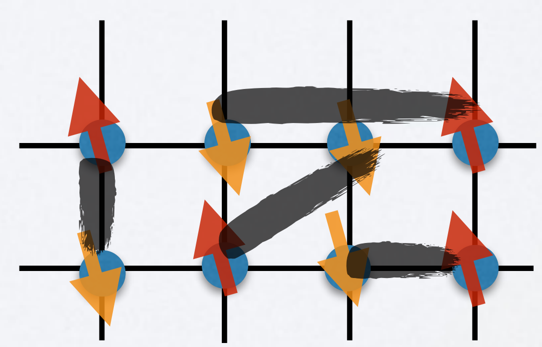
Q: Can we do better?

Permanent:

	1'st spin down (2)	2'nd spin down (3)	3'rd spin down (5)	4'th spin down (7)
1'st spin up (1)	f(1,2)	f(1,3)	f(1,5)	f(1,7)
2'nd spin up (4)	f(4,2)	f(4,3)	f(4,5)	f(4,7)
3'rd spin up (6)	f(6,2)	f(6,3)	f(6,5)	f(6,7)
4'th spin up (8)	f(8,2)	f(8,3)	f(8,5)	f(8,7)

Q: What are the "legal" valence bonds.
(1,2) (1,3) (1,5) (1,7) (4,2) (4,3) (4,5) …

Q: What is the amplitude of the one below?



Each dimer configuration corresponds to choosing one number from each row and column.

Then sum over them!

This is a permanent!

Suppose we want the amplitude of a bosonic RVB state… $\sum_{\text{[dimer coverings A]}} \prod_{ij \in A} f_{ij}^{\text{boson}} (|\uparrow_i \downarrow_j\rangle - \downarrow_i \uparrow_j \rangle)$

in the complete valence-bond basis

If you were working in a dimer basis, this would be good.

But still dealing with non-orthogonal states.

Suppose we want the amplitude of a fermionic RVB state…

$$\sum_{[\text{dimer coverings A}]} \prod_{ij\in A} f_{ij}\Theta_{ij}|0\rangle$$

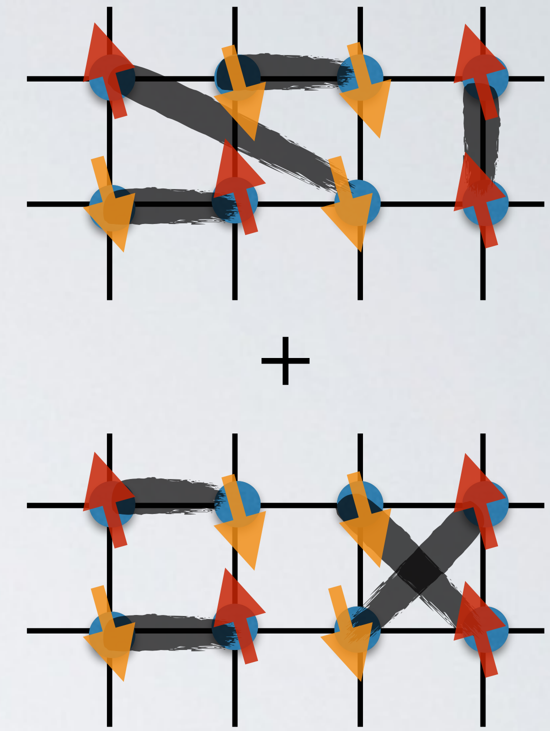
in the Sz basis

$$|\uparrow,\downarrow,\downarrow,\uparrow,\downarrow,\uparrow,\downarrow,\uparrow\rangle$$

Loop through all dimer configurations D consistent with pattern of valence bonds… (Q: how many?)

For each D, multiply the amplitudes and signs

As we loop through, signs are controlled by canonical ordering.

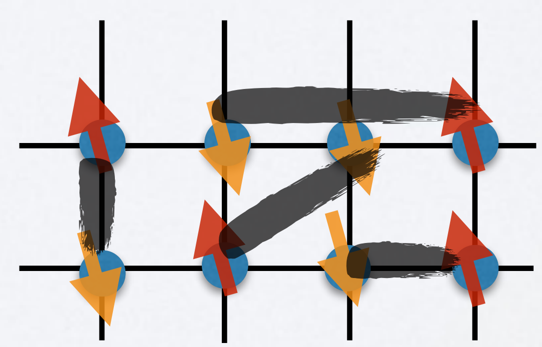


Q: Can we do better?

	1'st spin down (2)	2'nd spin down (3)	3'rd spin down (5)	4'th spin down (7)
1'st spin up (1)	f(1,2)	f(1,3)	f(1,5)	f(1,7)
2'nd spin up (4)	f(4,2)	f(4,3)	f(4,5)	f(4,7)
3'rd spin up (6)	f(6,2)	f(6,3)	f(6,5)	f(6,7)
4'th spin up (8)	f(8,2)	f(8,3)	f(8,5)	f(8,7)

Q: What are the "legal" valence bonds.
(1,2) (1,3) (1,5) (1,7) (4,2) (4,3) (4,5) …

Q: What is the amplitude of the one below?



Each dimer configuration corresponds to choosing one number from each row and column.

Canonical ordering is a fancy notation to ensure the constraint that if we flip electrons of the same spin, the sign is preserved.

Q: Are fermionic and bosons RVB states the same?

$$\sum_{[\text{dimer coverings A}]} \prod_{ij \in A} f_{ij}^{\text{boson}} (|\uparrow_i \downarrow_j\rangle - \downarrow_i \uparrow_j\rangle)$$

$$|\text{Bosonic RVB}\rangle = \sum_{[\text{dimer coverings A}]} \prod_{ij \in A} f_{ij}^{\text{boson}} (S_i^- - S_j^-)|F\rangle$$

$$S_i^- = c_{i,\downarrow}^\dagger c_{i,\uparrow}$$

$$|F\rangle = c_{1\uparrow}^\dagger c_{2\uparrow}^\dagger ... c_{n\uparrow}^\dagger |0\rangle$$

$$|\text{Bosonic RVB}\rangle = \sum_{[\text{dimer coverings A}]} \epsilon_{IJ} \prod_{ij \in A} \overset{\text{boson}}{f_{ij}} \Theta_{ij} |0\rangle$$

$$|\text{Fermionic RVB}\rangle = \sum_{[\text{dimer coverings A}]} \prod_{ij \in A} f_{ij} \Theta_{ij} |0\rangle$$

When is this possible?

$$\mathcal{H}_{BCS} = \sum_{R,R'\sigma} (t_{R-R'} - \mu\,\delta_{R-R'})\, c^{\dagger}_{R,\sigma} c_{R',\sigma} - \sum_{R,R'} \Delta_{R-R'}\, c^{\dagger}_{R,\uparrow} c^{\dagger}_{R',\downarrow} + H.c.$$
$$= \sum_{k\sigma} (\epsilon_k - \mu)\, c^{\dagger}_{k,\sigma} c_{k,\sigma} - \sum_{k} \Delta_k c^{\dagger}_{k,\uparrow} c^{\dagger}_{-k,\downarrow} + H.c.,$$

$$|pBCS\rangle = P_G\,|BCS\rangle = P_G \prod_k (u_k + v_k c^{\dagger}_{k,\uparrow} c^{\dagger}_{-k,\downarrow})|0\rangle$$

$$f_k = \frac{v_k}{u_k} = \frac{\Delta_k}{\epsilon_k + E_k}$$

$$f_{|r-r'|}$$

PEPS and RVB

$$|\omega\rangle = |01\rangle - |10\rangle + |22\rangle \in \mathbb{C}^3 \otimes \mathbb{C}^3$$

$$\mathcal{P}_4 = |0\rangle\big(\langle 0222| + \langle 2022| + \langle 2202| + \langle 2220|\big) + |1\rangle\big(\langle 1222| + \langle 2122| + \langle 2212| + \langle 2221|\big)$$